```
#include <bits/stdc++.h>
   #define FOR(i,a,b) for(int i=(a),_b=(b); i <=_b; ++i)
   #define FORD(i,a,b) for(int i=(a), b=(b); i>=b; --i)
3
   #define REP(i,a) for(int i=0,_a=(a); i < _a; ++i)
4
5
   #define DEBUG(X) { cout << #X << " = " << X << endl; }
6
   \# define \ PR(A,n) \ \{ cout << \# A << " = "; \ FOR(\_,1,n) \ cout << A[_] << ' '; \ cout <<
7
   endl; }
   #define PRO(A,n) { cout << #A << " = "; REP(\ ,n) cout << A[ ] << ' '; cout <<
8
   endl; }
9
   #define ll long long
10
   #define SZ(x) ((int) (x).size())
11
   #define sqr(x) ((x) * (x))
12
   #define double long double
13
   using namespace std;
14
15
   const double PI = acos((double) -1.0);
16
   const double EPS = 1e-10;
17
18
   struct Point {
19
20
        double x, y;
       Point() {}
21
       Point(double x, double y) : x(x), y(y) {}
22
       Point operator - (const Point& a) const {
23
            return Point(x-a.x, y-a.y);
24
25
       double len() {
26
27
            return hypot(x, y);
       }
28
   };
29
30
31
   struct Circle : Point {
       double r;
32
33
   };
34
   double cir area solve(double a, double b, double c) {
35
        return acos((a*a + b*b - c*c) / 2 / a / b);
36
37
   double cir area cut(double a, double r) {
38
       double s1 = a * r * r / 2;
39
       double s2 = sin(a) * r * r / 2;
40
        return s1 - s2;
41
   }
42
43
   double commonCircleArea(Circle c1, Circle c2) { //return the common area of two
44
        if (c1.r < c2.r) swap(c1, c2);
45
       double d = (c1 - c2).len();
46
            47
48
            if (d >= c1.r + c2.r - EPS) return 0.0;
       double a1 = cir area solve(d, c1.r, c2.r);
49
       double a2 = cir_area_solve(d, c2.r, c1.r);
50
        return cir area cut(a1*2, c1.r) + cir area cut(a2*2, c2.r);
51
   }
52
53
   int main() {
54
55
        cout << (fixed) << setprecision(12);</pre>
       ll x1, y1, r1, x2, y2, r2;
56
```

```
while (cin >> x1 >> y1 >> r1 >> x2 >> y2 >> r2) {
    Circle c1; c1.x = x1; c1.y = y1; c1.r = r1;
    Circle c2; c2.x = x2; c2.y = y2; c2.r = r2;
    cout << commonCircleArea(c1, c2) << endl;
}</pre>
```

```
#include <cstdio>
1
   #include <cstdlib>
   #include <cstring>
3
   #include <cassert>
   #include <ctime>
5
   #include <cmath>
   #include <algorithm>
7
   #include <string>
   #include <vector>
9
10
   #include <deque>
   #include <queue>
11
   #include <list>
12
   #include <set>
13
   #include <map>
14
15
   #define pb push_back
16
    #define mp make_pair
17
   #define TASKNAME ""
18
19
   #ifdef LOCAL
20
   #define eprintf(...) fprintf(stderr,__VA_ARGS__)
21
22
   #define eprintf(...)
23
   #endif
24
25
   #define TIMESTAMP(x) eprintf("[" \#x "] Time = %.3lfs\n",clock()*1.0/
26
    CLOCKS_PER_SEC)
27
   #ifdef linux
28
   #define LLD "%lld"
29
   #else
30
   #define LLD "%I64d"
31
32
   #endif
33
   #define sz(x) ((int)(x).size())
34
35
   using namespace std;
36
37
38
    typedef double ld;
    typedef long long ll;
39
    typedef vector<ll> vll;
40
    typedef vector<int> vi;
41
    typedef vector<vi> vvi;
42
    typedef vector<bool> vb;
43
    typedef vector<vb> vvb;
44
    typedef pair<int, int> pii;
45
46
    const int inf = 1e9;
47
    const double eps = 1e-9;
48
    const double INF = inf;
49
50
    const double EPS = eps;
51
    struct point{
52
53
      ld x, y, z;
      point(ld x, ld y, ld z):x(x), y(y), z(z){}
54
      point(){ x = y = z = 0;}
55
      bool operator==(const point& a) const{
56
        return fabs(x - a.x) < eps && fabs(y - a.y) < eps && fabs(z - a.z) < eps;
57
58
```

```
bool load(){
59
         double _x,_y,_z;
60
         if (scanf("%lf%lf%lf",&_x,&_y,&_z) != 3) return 0;
61
62
         x = _x, y = _y, z = _z;
63
         return 1;
64
      void print(){
65
         printf("%lf %lf %lf\n",(double)x, (double)y, (double)z);
66
67
68
      void eprint(){
         eprintf("%lf %lf %lf\n",(double)x, (double)y, (double)z);
69
70
      ld dist2() const {
71
         return x*x+y*y+z*z;
72
73
      ld dist() const{
74
75
         return sqrt(dist2());
      }
76
    };
77
78
    inline point operator+(const point& a,const point& b){
79
      return point(a.x+b.x,a.y+b.y,a.z+b.z);
80
    }
81
82
    inline point operator-(const point& a,const point& b){
83
      return point(a.x-b.x,a.y-b.y,a.z-b.z);
84
85
86
    inline point operator*(const point& a,ld t){
87
      return point(a.x*t, a.y*t, a.z*t);
88
89
90
91
    inline ld det(ld a,ld b,ld c,ld d){
      return a*d - b*c;
92
93
94
    inline ld det(ld a11,ld a12,ld a13,ld a21,ld a22, ld a23, ld a31, ld a32,ld
95
    a33){
      return a11*det(a22,a23,a32,a33) - a12*det(a21,a23,a31,a33) + a13*det
96
    (a21,a22,a31,a32);
97
    }
98
    int sgn(ld x){
99
      return (x > eps) - (x < -eps);
100
101
102
    inline ld det(const point& a,const point& b,const point& c){
103
      return det(a.x,a.y,a.z,b.x,b.y,b.z,c.x,c.y,c.z);
104
105
106
107
    point vp(const point& a,const point& b){
      return point(det(a.y,a.z,b.y,b.z),-det(a.x,a.z,b.x,b.z),det
108
     (a.x,a.y,b.x,b.y));
109
110
    ld sp(const point& a,const point& b){
111
      return a.x * b.x + a.y * b.y + a.z * b.z;
112
113
    }
114
```

```
// BEGIN ALGO
115
116
     // vp is vector product (point)
117
     // sp is scalar product (ld)
118
119
     struct line{
120
       point p,v;
121
       line(){}; /*BOXNEXT*/
122
       line(const point& p,const point& v):p(p),v(v){
123
124
         assert(!(v == point()));
125
       bool on(const point& pt) const{
126
         return vp(pt - p, v) == point();
127
128
129
     };
     struct plane {
130
131
       point n;
       ld d;
132
       plane() : d(0) {}
133
       plane(const point &p1, const point &p2,
134
             const point &p3) {
135
136
         n = vp(p2 - p1, p3 - p1);
         d = -sp(n, p1);
137
         assert(side(p1) == 0);
138
         assert(side(p2) == 0);
139
         assert(side(p3) == 0);
140
       }
141
       int side(const point &p) const {
142
         return sgn(sp(n, p) + d);
143
       }
144
     };
145
146
     int intersec(const line& l1, const line& l2,
147
                   point& res){
148
       assert(!(l1.v == point()));
149
       assert(!(l2.v == point()));
150
       if (vp(l1.v,l2.v) == point()){
151
         if (vp(l1.v, l1.p - l2.p) == point())
152
153
           return 2; // same
         return 0; // parallel
154
155
       point n = vp(l1.v, l2.v);
156
       point p = l2.p - l1.p;
157
       if (sgn(sp(n,p)))
158
         return 0; // skew
159
160
       ld t;
       if (sqn(n.x))
161
         t = (p.y * l2.v.z - p.z * l2.v.y) / n.x;
162
       else if (sgn(n.y))
163
         t = (p.z * l2.v.x - p.x * l2.v.z) / n.y;
164
165
       else if (sgn(n.z))
         t = (p.x * l2.v.y - p.y * l2.v.x) / n.z;
166
167
       else
         assert(false);
168
       res = l1.p + l1.v * t;
169
       assert(l1.on(res)); assert(l2.on(res));
170
       return 1; // intersects
171
172
     }
173
```

```
ld dist(const line& l1,const line& l2){
174
       point ret = l1.p - l2.p; /*BOXNEXT*/
175
       ret = ret - l1.v * (sp(l1.v,ret) / l1.v.dist2());
176
       point tmp = 12.v - 11.v *
177
                    (sp(l1.v,l2.v) / l1.v.dist2());
178
       if (sgn(tmp.dist2())) /*BOXNEXT*/
179
         ret = ret - tmp * (sp(tmp,ret) / tmp.dist2());
180
       assert(fabs(sp(ret,l1.v)) < eps);
181
       assert(fabs(sp(ret,tmp)) < eps);</pre>
182
183
       assert(fabs(sp(ret,l2.v)) < eps);</pre>
       return ret.dist();
184
185
    }
186
    void closest(const line& l1,const line& l2,
187
                   point& p1,point& p2){
188
       if (vp(l1.v,l2.v) == point()){
189
190
         p1 = 11.p;
         p2 = l2.p - l1.v * /*BOXNEXT*/
191
               (sp(l1.v,l2.p - l1.p) / l1.v.dist2());
192
         return;
193
194
195
       point p = l2.p - l1.p;
       ld t1 = (
196
                 sp(l1.v,p) * l2.v.dist2() -
197
                 sp(l1.v,l2.v) * sp(l2.v,p)
198
                ) / vp(l1.v,l2.v).dist2();
199
       ld t2 = (
200
                   sp(l2.v,l1.v) * sp(l1.v,p) -
201
                   sp(l2.v,p) * l1.v.dist2()
202
                ) / vp(l2.v,l1.v).dist2();
203
       p1 = l1.p + l1.v * t1;
204
       p2 = 12.p + 12.v * t2;
205
206
       assert(l1.on(p1));
       assert(l2.on(p2));
207
208
    }
209
    int cross(const line &l, const plane &pl,
210
               point &res) {
211
212
       ld d = sp(pl.n, l.v);
       if (sgn(d) == 0) {
213
         return (pl.side(l.p) == 0) ? 2 : 0;
214
215
       ld t = (-sp(pl.n, l.p) - pl.d) / d;
216
       res = l.p + l.v * t;
217
       #ifdef DEBUG
218
219
       assert(pl.side(res) == 0);
       #endif
220
       return 1;
221
222
223
224
    bool cross(const plane& p1,const plane& p2,
                 const plane& p3, point& res){
225
       ld d = det(p1.n,p2.n,p3.n);
226
       if (sgn(d) == 0) {
227
          return false;
228
229
       point px(p1.n.x, p2.n.x, p3.n.x);
230
231
       point py(p1.n.y, p2.n.y, p3.n.y);
       point pz(p1.n.z, p2.n.z, p3.n.z);
232
```

```
point p(-p1.d,-p2.d,-p3.d);
233
       res = point(
234
235
                    det(p,py,pz)/d,
                    det(px,p,pz)/d,
236
237
                    det(px,py,p)/d
238
      #ifdef DEBUG
239
       assert(p1.side(res) == 0);
240
       assert(p2.side(res) == 0);
241
242
       assert(p3.side(res) == 0);
       #endif
243
       return true;
244
    }
245
246
    int cross(const plane &p1, const plane &p2,
247
                line &res) {
248
249
       res.v = vp(p1.n, p2.n);
       if (res.v == point()) {
250
         if ( (p1.n * (p1.d / p1.n.dist2())) ==
251
               (p2.n * (p2.d / p2.n.dist2())))
252
           return 2;
253
254
         else
           return 0;
255
       }
256
       plane p3;
257
       p3.n = res.v;
258
       p3.d = 0;
259
       bool ret = cross(p1, p2, p3, res.p);
260
261
       assert(ret);
       assert(p1.side(res.p) == 0);
262
       assert(p2.side(res.p) == 0);
263
       return 1;
264
265
    }
    // END ALGO
266
267
268
269
    int main(){
270
271
       freopen(TASKNAME".in","r",stdin);
       #ifdef LOCAL
272
       freopen(TASKNAME".out", "w", stdout);
273
       #endif
274
275
276
         line l;
277
         l.p = point(1, 1, 1);
278
         l.v = point(1, 0, -1);
279
         plane p(point(10, 11, 12), point(9, 8, 7), point(1, 3, 2));
280
         point res;
281
         assert(cross(l, p, res) == 1);
282
283
       }
284
         plane p1(point(1, 2, 3), point(4, 5, 6), point(-1, 5, -4));
285
         plane p2(point(3, 2, 1), point(6, 5, 4), point(239, 17, -42));
286
         line l;
287
         assert(cross(p1, p2, l) == 1);
288
289
290
       {
         plane p1(point(1, 2, 3), point(4, 5, 6), point(-1, 5, -4));
291
```

```
plane p2(point(1, 2, 3), point(7, 8, 9), point(3, -1, 10));
292
293
294
         assert(cross(p1, p2, l) == 2);
       }
295
296
         plane p1(point(1, 2, 3), point(4, 5, 6), point(-1, 5, -4));
297
         plane p2(point(1, 2, 4), point(4, 5, 7), point(-1, 5, -3));
298
         line l;
299
         assert(cross(p1, p2, l) == 0);
300
301
302
      line l1.l2:
303
      while (l1.p.load()){
304
         l1.v.load(); l1.v = l1.v - l1.p;
305
         12.p.load();
306
         12.v.load(); 12.v = 12.v - 12.p;
307
         if (l1.v == point() || l2.v == point()) continue;
308
         point res;
309
         int cnt = intersec(l1,l2,res);
310
         ld d = dist(l1,l2);
311
         if (fabs(d) < eps)</pre>
312
           assert(cnt >= 1);
313
         else
314
           assert(cnt == 0);
315
         point p1,p2;
316
         closest(l1,l2,p1,p2);
317
         assert(fabs((p1-p2).dist() - d) < eps);</pre>
318
319
       plane a(point(1,0,0),point(0,1,0),point(0,0,1));
320
       plane b(point(-1,0,0),point(0,-1,0),point(0,0,-1));
321
       line l;
322
       assert((cross(a,b,l))==0);
323
      TIMESTAMP(end);
324
       return 0;
325
    }
326
```

```
// Two-phase simplex algorithm for solving linear programs of the form
1
2
    //
3
    //
           maximize
                         c^T x
4
   //
           subject to
                         Ax \le b
   //
                         x >= 0
5
6
    // INPUT: A -- an m x n matrix
7
              b -- an m-dimensional vector
8
    //
              c -- an n-dimensional vector
9
    //
10
    //
              x -- a vector where the optimal solution will be stored
11
    //
    // OUTPUT: value of the optimal solution (infinity if unbounded
12
               above, nan if infeasible)
    //
13
14
    //
    // To use this code, create an LPSolver object with A, b, and c as
15
   // arguments. Then, call Solve(x).
16
17
   #include <iostream>
18
   #include <iomanip>
19
    #include <vector>
20
    #include <cmath>
21
   #include <limits>
22
23
   using namespace std;
24
25
    typedef long double DOUBLE;
26
    typedef vector<DOUBLE> VD;
27
    typedef vector<VD> VVD;
28
29
    typedef vector<int> VI;
30
    const DOUBLE EPS = 1e-9;
31
32
33
    struct LPSolver {
            int m, n;
34
            VI B, N;
35
            VVD D;
36
37
            LPSolver(const VVD &A, const VD &b, const VD &c): m(b.size()), n(c.size
38
    ()), N(n + 1), B(m), D(m + 2), VD(n + 2)
            {
39
                     for (int i = 0; i < m; i++)
40
                              for (int j = 0; j < n; j++)
41
                                      D[i][j] = A[i][j];
42
                     for (int i = 0; i < m; i++)
43
                     {
44
45
                              B[i] = n + i;
                              D[i][n] = -1;
46
                              D[i][n + 1] = b[i];
47
48
                     for (int j = 0; j < n; j++)
49
50
                     {
                              N[j] = j;
51
                              D[m][j] = -c[j];
52
53
                     N[n] = -1;
54
                     D[m + 1][n] = 1;
55
            }
56
57
            void Pivot(int r, int s)
58
```

```
{
59
                      for (int i = 0; i < m + 2; i++)
60
                               if (i != r)
61
                                        for (int j = 0; j < n + 2; j++)
62
                                                 if (j != s)
63
                                                          D[i][j] -= D[r][j] * D[i][s] /
64
     D[r][s];
                      for (int j = 0; j < n + 2; j++)
65
                               if (j != s)
66
67
                                        D[r][j] /= D[r][s];
                      for (int i = 0; i < m + 2; i++)
68
                               if (i != r)
69
                                        D[i][s] /= -D[r][s];
70
                      D[r][s] = 1.0 / D[r][s];
71
                      swap(B[r], N[s]);
72
             }
73
74
             bool Simplex(int phase)
75
76
                      int x = phase == 1 ? m + 1 : m;
77
                      while (true)
78
79
                      {
                               int s = -1;
80
                               for (int j = 0; j <= n; j++)
81
                               {
82
                                        if (phase == 2 \&\& N[i] == -1)
83
                                                 continue;
84
                                        if (s == -1 || D[x][j] < D[x][s] || D[x][j] ==
85
     D[x][s] && N[j] < N[s])
                                                 s = i;
86
87
                               if (D[x][s] > -EPS)
88
89
                                        return true;
                               int r = -1;
90
                               for (int i = 0; i < m; i++)
91
92
                                        if (D[i][s] < EPS)
93
                                                 continue;
94
95
                                        if (r == -1 || D[i][n + 1] / D[i][s] < D[r][n</pre>
     + 1] / D[r][s] ||
                                                 (D[i][n + 1] / D[i][s]) == (D[r][n +
96
     1] / D[r][s]) && B[i] < B[r])
                                                 r = i;
97
98
                               if (r == -1)
99
                                        return false;
100
                               Pivot(r, s);
101
                      }
102
103
104
             DOUBLE Solve(VD &x)
105
106
                      int r = 0;
107
                      for (int i = 1; i < m; i++)
108
                               if (D[i][n + 1] < D[r][n + 1])
109
110
                      if (D[r][n + 1] < -EPS)
111
112
                      {
                               Pivot(r, n);
113
```

```
if (!Simplex(1) || D[m + 1][n + 1] < -EPS)
114
                                           return -numeric_limits<DOUBLE>::infinity();
115
                                 for (int i = 0; i < m; i++)
116
                                           if (B[i] == -1)
117
                                           {
118
                                                    int s = -1;
119
                                                    for (int j = 0; j \le n; j++)
120
                                                              if (s == -1 || D[i][j] < D[i]
121
     [s] \mid | D[i][j] == D[i][s] \&\& N[j] < N[s])
122
                                                                       s = j;
                                                    Pivot(i, s);
123
                                           }
124
125
                        if (!Simplex(2))
126
                                  return numeric_limits<DOUBLE>::infinity();
127
                        x = VD(n);
128
                        for (int i = 0; i < m; i++)
129
                                 if (B[i] < n)
130
                                           x[B[i]] = D[i][n + 1];
131
                        return D[m][n + 1];
132
              }
133
134
     };
135
     int main()
136
     {
137
              //
                       Example:
138
              //
                       maximize
                                     x1 - x2
139
              //
                       subject to
140
              //
                             6x1 - x2
                                              <= 10
141
              //
                             -x1 - 5x2
                                              <= -4
142
                              x1 + 5x2 + x3 \le 5
              //
143
              //
                             -x1 - 5x2 - x3 <= -5
144
              //
145
              //
                            x1 >= 0
146
                             x2 >= 0
              //
147
                            x3 >= 0
              //
148
149
              const int n = 4;
150
151
              const int m = 3;
              DOUBLE A[n][m] =
152
153
              {
                        \{6, -1, 0\},\
154
                        \{-1, -5, 0\},\
155
                        \{1, 5, 1\},\
156
                        \{-1, -5, -1\}
157
158
              };
              DOUBLE b[n] = \{10, -4, 5, -5\};
159
              DOUBLE c[m] = \{1, -1, 0\};
160
161
              VVD A(n);
162
              VD b(\underline{b}, \underline{b} + n);
163
              VD c(_c, _c + m);
for (int i = 0; i < n; i++)</pre>
164
165
                        A[i] = VD(_A[i], _A[i] + m);
166
167
              LPSolver solver(A, b, c);
168
              VD x;
169
              DOUBLE value = solver.Solve(x);
170
171
```

```
// C++ routines for computational geometry.
2
   #include <iostream>
3
   #include <vector>
4
   #include <cmath>
5
   #include <cassert>
7
   using namespace std;
8
9
10
   double INF = 1e100;
    double EPS = 1e-12;
11
12
   struct PT {
13
14
      double x, y;
      PT() {}
15
      PT(double x, double y) : x(x), y(y) {}
16
      PT(const PT \&p) : x(p.x), y(p.y)
17
                                            {}
      PT operator + (const PT &p)
                                    const { return PT(x+p.x, y+p.y); }
18
      PT operator - (const PT &p)
                                    const { return PT(x-p.x, y-p.y); }
19
      PT operator * (double c)
                                    const { return PT(x*c,
                                                               y*c );
20
      PT operator / (double c)
                                    const { return PT(x/c,
                                                               y/c
21
                                                                    ); }
22
    };
23
   double dot(PT p, PT q)
                                { return p.x*q.x+p.y*q.y; }
24
   double dist2(PT p, PT q)
                                { return dot(p-q,p-q); }
25
    double cross(PT p, PT q)
                                { return p.x*q.y-p.y*q.x; }
26
    ostream & operator << (ostream & os, const PT & p) {
27
      os << "(" << p.x << "," << p.y << ")";
28
29
    }
30
    // rotate a point CCW or CW around the origin
31
   PT RotateCCW90(PT p)
                          { return PT(-p.y,p.x); }
32
33
   PT RotateCW90(PT p)
                            { return PT(p.y,-p.x); }
    PT RotateCCW(PT p, double t) {
34
      return PT(p.x*cos(t)-p.y*sin(t), p.x*sin(t)+p.y*cos(t));
35
36
    }
37
   // project point c onto line through a and b
38
39
    // assuming a != b
    PT ProjectPointLine(PT a, PT b, PT c) {
40
      return a + (b-a)*dot(c-a, b-a)/dot(b-a, b-a);
41
42
43
    // project point c onto line segment through a and b
44
   PT ProjectPointSegment(PT a, PT b, PT c) {
45
      double r = dot(b-a,b-a);
46
      if (fabs(r) < EPS) return a;</pre>
47
      r = dot(c-a, b-a)/r;
48
      if (r < 0) return a;
49
      if (r > 1) return b;
50
51
      return a + (b-a)*r;
    }
52
53
    // compute distance from c to segment between a and b
54
    double DistancePointSegment(PT a, PT b, PT c) {
55
      return sqrt(dist2(c, ProjectPointSegment(a, b, c)));
56
57
58
   // compute distance between point (x,y,z) and plane ax+by+cz=d
59
```

```
double DistancePointPlane(double x, double y, double z,
60
                                double a, double b, double c, double d)
61
62
    {
      return fabs(a*x+b*y+c*z-d)/sqrt(a*a+b*b+c*c);
63
64
    }
65
    // determine if lines from a to b and c to d are parallel or collinear
66
    bool LinesParallel(PT a, PT b, PT c, PT d) {
67
      return fabs(cross(b-a, c-d)) < EPS;</pre>
68
69
70
    bool LinesCollinear(PT a, PT b, PT c, PT d) {
71
      return LinesParallel(a, b, c, d)
72
           && fabs(cross(a-b, a-c)) < EPS
73
           && fabs(cross(c-d, c-a)) < EPS;
74
    }
75
76
    // determine if line segment from a to b intersects with
77
    // line segment from c to d
78
    bool SegmentsIntersect(PT a, PT b, PT c, PT d) {
79
      if (LinesCollinear(a, b, c, d)) {
80
         if (dist2(a, c) < EPS || dist2(a, d) < EPS ||</pre>
81
           dist2(b, c) < EPS || dist2(b, d) < EPS) return true;</pre>
82
         if (dot(c-a, c-b) > 0 \&\& dot(d-a, d-b) > 0 \&\& dot(c-b, d-b) > 0)
83
           return false;
84
         return true;
85
86
      if (cross(d-a, b-a) * cross(c-a, b-a) > 0) return false;
87
      if (cross(a-c, d-c) * cross(b-c, d-c) > 0) return false;
88
      return true;
89
    }
90
91
    // compute intersection of line passing through a and b
92
    // with line passing through c and d, assuming that unique
93
    // intersection exists; for segment intersection, check if
94
    // segments intersect first
95
    PT ComputeLineIntersection(PT a, PT b, PT c, PT d) {
96
      b=b-a; d=c-d; c=c-a;
97
      assert(dot(b, b) > EPS \&\& dot(d, d) > EPS);
98
      return a + b*cross(c, d)/cross(b, d);
99
    }
100
101
    // compute center of circle given three points
102
    PT ComputeCircleCenter(PT a, PT b, PT c) {
103
      b=(a+b)/2;
104
105
      c=(a+c)/2;
      return ComputeLineIntersection(b, b+RotateCW90(a-b), c, c+RotateCW90(a-c));
106
    }
107
108
    // determine if point is in a possibly non-convex polygon (by William
109
110
    // Randolph Franklin); returns 1 for strictly interior points, 0 for
    // strictly exterior points, and 0 or 1 for the remaining points.
111
    // Note that it is possible to convert this into an *exact* test using
112
    // integer arithmetic by taking care of the division appropriately
113
    // (making sure to deal with signs properly) and then by writing exact
114
    // tests for checking point on polygon boundary
115
    bool PointInPolygon(const vector<PT> &p, PT q) {
116
117
      bool c = 0;
      for (int i = 0; i < p.size(); i++){
118
```

```
int j = (i+1)%p.size();
119
         if ((p[i].y <= q.y && q.y < p[j].y ||</pre>
120
           p[j].y \le q.y \& q.y < p[i].y) \& 
121
           q.x < p[i].x + (p[j].x - p[i].x) * (q.y - p[i].y) / (p[j].y - p[i].y))
122
123
           c = !c;
124
      return c;
125
    }
126
127
128
    // determine if point is on the boundary of a polygon
    bool PointOnPolygon(const vector<PT> &p, PT q) {
129
      for (int i = 0; i < p.size(); i++)
130
         if (dist2(ProjectPointSegment(p[i], p[(i+1)%p.size()], q), q) < EPS)
131
           return true;
132
         return false;
133
    }
134
135
    // compute intersection of line through points a and b with
136
    // circle centered at c with radius r > 0
137
    vector<PT> CircleLineIntersection(PT a, PT b, PT c, double r) {
138
      vector<PT> ret:
139
      b = b-a;
140
      a = a-c;
141
      double A = dot(b, b);
142
      double B = dot(a, b);
143
      double C = dot(a, a) - r*r;
144
      double D = B*B - A*C;
145
      if (D < -EPS) return ret;</pre>
146
      ret.push back(c+a+b*(-B+sqrt(D+EPS))/A);
147
      if (D > EPS)
148
         ret.push back(c+a+b*(-B-sqrt(D))/A);
149
      return ret;
150
151
    }
152
    // compute intersection of circle centered at a with radius r
153
    // with circle centered at b with radius R
154
    vector<PT> CircleCircleIntersection(PT a, PT b, double r, double R) {
155
      vector<PT> ret;
156
157
      double d = sqrt(dist2(a, b));
      if (d > r+R || d+min(r, R) < max(r, R)) return ret;</pre>
158
      double x = (d*d-R*R+r*r)/(2*d);
159
      double y = sqrt(r*r-x*x);
160
      PT v = (b-a)/d;
161
       ret.push_back(a+v*x + RotateCCW90(v)*y);
162
      if (y > 0)
163
         ret.push back(a+v*x - RotateCCW90(v)*y);
164
      return ret;
165
    }
166
167
    // This code computes the area or centroid of a (possibly nonconvex)
168
169
    // polygon, assuming that the coordinates are listed in a clockwise or
    // counterclockwise fashion. Note that the centroid is often known as
170
    // the "center of gravity" or "center of mass".
171
    double ComputeSignedArea(const vector<PT> &p) {
172
      double area = 0;
173
      for(int i = 0; i < p.size(); i++) {</pre>
174
         int j = (i+1) % p.size();
175
176
        area += p[i].x*p[j].y - p[j].x*p[i].y;
      }
177
```

```
return area / 2.0;
178
    }
179
180
    double ComputeArea(const vector<PT> &p) {
181
       return fabs(ComputeSignedArea(p));
182
183
184
    PT ComputeCentroid(const vector<PT> &p) {
185
       PT c(0,0);
186
187
       double scale = 6.0 * ComputeSignedArea(p);
       for (int i = 0; i < p.size(); i++){
188
         int j = (i+1) % p.size();
189
         c = c + (p[i]+p[j])*(p[i].x*p[j].y - p[j].x*p[i].y);
190
191
       return c / scale;
192
    }
193
194
    // tests whether or not a given polygon (in CW or CCW order) is simple
195
    bool IsSimple(const vector<PT> &p) {
196
       for (int i = 0; i < p.size(); i++) {</pre>
197
         for (int k = i+1; k < p.size(); k++) {</pre>
198
           int j = (i+1) % p.size();
199
           int l = (k+1) % p.size();
200
           if (i == l || j == k) continue;
201
           if (SegmentsIntersect(p[i], p[j], p[k], p[l]))
202
             return false;
203
204
         }
205
206
       return true;
    }
207
208
    int main() {
209
210
       // expected: (-5,2)
211
       cerr << RotateCCW90(PT(2,5)) << endl;</pre>
212
213
       // expected: (5,-2)
214
       cerr << RotateCW90(PT(2,5)) << endl;</pre>
215
216
       // expected: (-5,2)
217
       cerr << RotateCCW(PT(2,5),M PI/2) << endl;</pre>
218
219
       // expected: (5,2)
220
       cerr << ProjectPointLine(PT(-5, -2), PT(10, 4), PT(3, 7)) << endl;
221
222
       // expected: (5,2) (7.5,3) (2.5,1)
223
       cerr << ProjectPointSegment(PT(-5,-2), PT(10,4), PT(3,7)) << " "
224
            << ProjectPointSegment(PT(7.5,3), PT(10,4), PT(3,7)) << " "</pre>
225
            << ProjectPointSegment(PT(-5,-2), PT(2.5,1), PT(3,7)) << endl;
226
227
228
       // expected: 6.78903
       cerr << DistancePointPlane(4, -4, 3, 2, -2, 5, -8) << endl;</pre>
229
230
       // expected: 1 0 1
231
       cerr << LinesParallel(PT(1,1), PT(3,5), PT(2,1), PT(4,5)) << " "
232
            << LinesParallel(PT(1,1), PT(3,5), PT(2,0), PT(4,5)) << " "
233
            << LinesParallel(PT(1,1), PT(3,5), PT(5,9), PT(7,13)) << endl;
234
235
       // expected: 0 0 1
236
```

```
cerr << LinesCollinear(PT(1,1), PT(3,5), PT(2,1), PT(4,5)) << " "
237
            << LinesCollinear(PT(1,1), PT(3,5), PT(2,0), PT(4,5)) << " "
238
            << LinesCollinear(PT(1,1), PT(3,5), PT(5,9), PT(7,13)) << endl;
239
240
       // expected: 1 1 1 0
241
       cerr << SegmentsIntersect(PT(0,0), PT(2,4), PT(3,1), PT(-1,3)) << " "
242
            << SegmentsIntersect(PT(0,0), PT(2,4), PT(4,3), PT(0,5)) << " "
243
            << SegmentsIntersect(PT(0,0), PT(2,4), PT(2,-1), PT(-2,1)) << " "
244
            << SegmentsIntersect(PT(0,0), PT(2,4), PT(5,5), PT(1,7)) << endl;
245
246
       // expected: (1,2)
247
       cerr << ComputeLineIntersection(PT(0,0), PT(2,4), PT(3,1), PT(-1,3)) << endl;
248
249
       // expected: (1,1)
250
       cerr << ComputeCircleCenter(PT(-3,4), PT(6,1), PT(4,5)) << endl;
251
252
       vector<PT> v;
253
       v.push back(PT(0,0));
254
       v.push back(PT(5,0));
255
       v.push_back(PT(5,5));
256
       v.push back(PT(0,5));
257
258
       // expected: 1 1 1 0 0
259
       cerr << PointInPolygon(v, PT(2,2)) << " "</pre>
260
            << PointInPolygon(v, PT(2,0)) << " "</pre>
261
            << PointInPolygon(v, PT(0,2)) << " "</pre>
262
            << PointInPolygon(v, PT(5,2)) << " "</pre>
263
            << PointInPolygon(v, PT(2,5)) << endl;</pre>
264
265
       // expected: 0 1 1 1 1
266
       cerr << PointOnPolygon(v, PT(2,2)) << " "</pre>
267
            << PointOnPolygon(v, PT(2,0)) << " "</pre>
268
            << PointOnPolygon(v, PT(0,2)) << " "
269
            << PointOnPolygon(v, PT(5,2)) << " "</pre>
270
            << PointOnPolygon(v, PT(2,5)) << endl;</pre>
271
272
       // expected: (1,6)
273
                     (5,4)(4,5)
       //
274
                     blank line
275
       //
       //
                     (4,5) (5,4)
276
                     blank line
       //
277
                     (4,5) (5,4)
       //
278
       vector<PT> u = CircleLineIntersection(PT(0,6), PT(2,6), PT(1,1), 5);
279
       for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;</pre>
280
       u = CircleLineIntersection(PT(0,9), PT(9,0), PT(1,1), 5);
281
       for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;</pre>
282
       u = CircleCircleIntersection(PT(1,1), PT(10,10), 5, 5);
283
       for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;</pre>
284
      u = CircleCircleIntersection(PT(1,1), PT(8,8), 5, 5);
for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;</pre>
285
286
287
       u = CircleCircleIntersection(PT(1,1), PT(4.5,4.5), 10, sqrt(2.0)/2.0);
       for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;</pre>
288
       u = CircleCircleIntersection(PT(1,1), PT(4.5,4.5), 5, sqrt(2.0)/2.0);
289
       for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;</pre>
290
291
       // area should be 5.0
292
       // centroid should be (1.1666666, 1.166666)
293
       PT pa[] = \{ PT(0,0), PT(5,0), PT(1,1), PT(0,5) \};
294
       vector<PT> p(pa, pa+4);
295
```

```
PT c = ComputeCentroid(p);
cerr << "Area: " << ComputeArea(p) << endl;
cerr << "Centroid: " << c << endl;
return 0;
301 }</pre>
```

```
@ECHO OFF
1
2
   SET generator=%1
3
   SET checkProgram=%2
4
   SET correctProgram=%3
5
   for /l %x in (1, 0, 1) do (
7
   %generator% > test.in
8
   if %errorlevel% neq 0 exit
9
   %checkProgram% < test.in > test1.out
10
   if %errorlevel% neq 0 exit
11
   %correctProgram% < test.in > test2.out
12
   if %errorlevel% neq 0 exit
13
   FC test1.out test2.out
14
   if %errorlevel% neq 0 exit
15
   del test.in test1.out test2.out
16
   timeout 1 /NOBREAK
17
18
   )
```