

RUHR-UNIVERSITÄT BOCHUM

# **Side-Channel Attacks On Implementations Of Lattice-Based Cryptosystems**

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## Abstract

The following text<sup>1</sup> describes what your abstract should be about.

The abstract should convey to the reader concisely and accurately within the space of a few sentences, the claim to knowledge that the authors are making. It should indicate the boundaries of space and time within which the enquiry has occurred. If there is a claim to generality beyond the boundaries of the enquiry the basis of that claim should be given, for example that a random sample is thought to be representative of a larger population. There should also be a hint of the method of enquiry.

The boundaries of an enquiry are important - and are unfortunately too often omitted from abstracts. This is due to the regrettable tendency for researchers to generalise their results from, for example, a few schools to all schools, and to imply that what was true at a particular time, is true for all time. Some reference to the geographical location of the children, or teachers, or schools on whom the claim to knowledge rests should be made. Because of the international nature of the research community it is worth making clear in what country the research took place. Also the period in which the data was collected should be stated.

The abstract should be a condensation of the substance of the paper, not a trailer, nor an introduction. Journals and thesis regulations usually put a limit of around 200 to 300 words to the length of an abstract. Trailer is a term borrowed from the cinema industry to describe a showing of a few highlights in order to win an audience. An Introduction tells that something is coming, but doesn't reveal its substance. These are not what is needed.

Abstracts are recycled in abstract journals and electronic networks and provide the main vehicle for other researchers to become aware of particular studies. Hence the more clearly they convey the claim to knowledge of the original paper the more useful they are in helping the reader to decide whether it is worth taking the trouble to obtain and read the original and possibly cite it in his/her own writing.

Both the abstract and the paper should make sense without the other.

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<sup>1</sup>Shamelessly ripped from <http://www.leeds.ac.uk/educol/abstract.htm>

# Contents

<b>1. Introduction</b>	<b>2</b>
1.1. Related Work . . . . .	2
1.2. Structure of this Paper . . . . .	3
<b>2. Theoretical Background</b>	<b>4</b>
2.1. Notation . . . . .	4
2.2. Ideal Lattices . . . . .	4
2.3. Learning with Errors Problem . . . . .	5
2.4. Discrete Gaussian Distribution . . . . .	5
2.5. Cryptographic Algorithms . . . . .	5
2.5.1. Ring-LWE Encryption Scheme . . . . .	5
2.5.2. BLISS Signature Scheme . . . . .	6
2.6. Side-Channel Attack Terminology . . . . .	7
<b>3. Masking the Ring-LWE Encryption Scheme I</b>	<b>8</b>
3.1. Implementation . . . . .	8
3.1.1. Overview . . . . .	8
3.1.2. Masked Decoder . . . . .	9
3.1.3. Masked Table Lookup . . . . .	9
3.2. Evaluation . . . . .	9
<b>4. Masking the Ring-LWE Encryption Scheme II</b>	<b>10</b>
4.1. Implementation . . . . .	10
4.2. Evaluation . . . . .	10
<b>5. Flush+Reload Cache Attack on Bliss</b>	<b>11</b>
5.1. Gaussian Sampling . . . . .	11
5.1.1. CDT Sampling . . . . .	11
5.1.2. Rejection Sampling . . . . .	11
5.2. Attacking the Sampling Algorithms . . . . .	11
5.3. Evaluation . . . . .	11
<b>6. Blinding Countermeasures</b>	<b>12</b>
6.1. Blinding Polynomial Multiplication . . . . .	12
6.2. Blinding Gaussian Sampling . . . . .	13

<b>7. Conclusion</b>	<b>15</b>
<b>A. Bibliography</b>	<b>18</b>

# Acronyms

# 1. Introduction

Despite the rapid progress in the development of quantum computers and the hereby increasingly urgent need for post-quantum cryptographic algorithms, no such algorithms has yet been standardized [CJL<sup>+</sup>16]. Current public-key cryptosystems like RSA, DHKE or even elliptic curve cryptography could easily be broken by a quantum computer, due to Shor’s algorithm for prime factorization and discrete logarithms [Sho97]. As most of today’s digital infrastructure depends (at least partially) on such public-key algorithms, the need for efficient and secure cryptography, that can withstand the power of quantum computation, is as high as never before.

Lattice-based cryptography is the most promising of all areas in post-quantum cryptography, as its underlying mathematics are already well understood and reasonably efficient implementations of some of the proposed cryptographic schemes are yet available. Our paper will give an overview over some selected lattice-based algorithms and their implementation in respect to their resistance to various side-channel attack techniques.

## 1.1. Related Work

This paper summarizes the content of several other papers, that have been published in recent years. Some of them are referred to below and we strongly recommend reading them.

Shortly after the ring-LWE problem was introduced in [LPR12] in 2012, the authors of [RRVV15] laid the groundwork for masked implementations of one ring-LWE encryption scheme and refined it in [RdCR<sup>+</sup>16] by getting rid of the need for a masked decoder. Just a year after the ring-LWE encryption scheme was introduced, the authors of [DDLL13] proposed the BLISS signature scheme, which is based on the ring-LWE problem as well. A possible side-channel attack on a slightly altered version of that signature scheme was then shown in [BHLY16] in 2016, which might be prevented by the blinding techniques used by the authors of [Saa16].

## 1.2. Structure of this Paper

In Section 2 we will start with an explanation of our notation and give an overview over the mathematic background needed to understand this paper. This includes introducing the reader to the concept of *(ideal) lattices*, the *learning with errors problem (LWE)*, *Discrete Gaussian Distributions*, the *ring-LWE Encryption Scheme* and the *BLISS Signature Scheme*. Additionally, we will give a short explanation of the side-channel attack terminology used throughout this paper. Section 3 will deal with the ring-LWE encryption scheme and will be split into two parts, starting with the description of a masked implementation of the decryption function, including a masked decoder build upon a masked table lookup. The second part of this section will be an evaluation of the proposed implementation in respect to its soundness to first- and second-order side-channel attacks.

A different approach to masking of the ring-LWE encryption scheme will be presented in Section 4 of our paper, which will as well be split into a description of the proposed scheme and an evaluation. Furthermore, the second masking scheme will be compared to the first one in respect to efficiency and complexity. Section 5 will discuss the **FLUSH+RELOAD** cache attacks on the Gaussian sampler used in the BLISS signature scheme. This part will start with a description of a perfect side channel attack on two Gaussian sampling algorithms, namely the cumulative distribution function (CDT sampling) and rejection sampling. This will be followed by an evaluation of the **FLUSH+RELOAD** attacks on an actual BLISS implementation, while running on modern CPUs.

Furthermore, in Section 6 we will be presenting two measures used for blinding polynomial multiplication and Gaussian sampling, which might help against the attacks described in Section 5.

Finally, Section 7 will summarize the content of our paper shortly and some conclusions will be drawn.

## 2. Theoretical Background

### 2.1. Notation

As we will only work with ideal lattices in our paper, all operations will be done within the ring  $R_q = \mathbb{Z}_q[x]/(f(x))$  with  $f(x)$  being an irreducible polynomial of degree  $n$  and all coefficients being reduced modulo  $q$ .

Polynomials will be written as bold lower case letters (**f**). As we will use the *NTT* for efficient polynomial multiplication within a ring  $R_q$ , polynomials in the NTT domain will be written as  $\tilde{\mathbf{f}}$ . Vectors will be denoted with an arrow on top of them ( $\vec{x}$ ), while matrices will be denoted by bold upper case letters (**A**). The entries of a vector  $\vec{x}$  will be called  $x_i$ , with  $i$  specifying the position within the vector starting at 0.

The notation for the  $l_p$  norm of a vector  $\vec{x}$  will be  $\|\vec{x}\|_p$ , only with the exception of the  $l_2$  norm, which will be referred to as  $\|\vec{x}\| = \sqrt{\sum_i x_i^2}$ .

### 2.2. Ideal Lattices

A lattice  $\Lambda$  is discrete subgroup of  $\mathbb{R}^n$  that is defined as a set of  $m \leq n$  linearly independent vectors  $\vec{b}_1, \dots, \vec{b}_m \in \mathbb{R}^n$  and is generated by all linear combinations of those  $\vec{b}_i$ 's with integer coefficients:

$$\Lambda(\vec{b}_1, \dots, \vec{b}_m) = \left\{ \sum_{i=1}^m x_i \vec{b}_i \mid x_i \in \mathbb{Z} \right\} \quad (2.1)$$

The set  $\{\vec{b}_1, \dots, \vec{b}_m\}$  of those vectors is called the basis of that lattice. Such a basis is commonly represented by a matrix  $\mathbf{B} = (\vec{b}_1, \dots, \vec{b}_m)$ .

Furthermore, an ideal lattice is a lattice that corresponds to ideals in a ring  $R_q$ . This basically means, that we can deal with polynomials instead of matrices, which makes arithmetics used for cryptographic applications much more efficient. In our paper we will confine ourselves to those ideal lattices, as most of the current work in that area focuses around them. For more on the topic of ideal lattices, see [LPR12].



## 2.3. Learning with Errors Problem

## 2.4. Discrete Gaussian Distribution

The discrete Gaussian distribution with mean  $\mu$  and standard deviation  $\sigma$  is denoted as  $\mathcal{N}_{\mathbb{Z}}(\mu, \sigma^2)$ . In this paper we will focus on zero-centered distributions  $\mathcal{N}_{\mathbb{Z}}(0, \sigma^2)$  with a density function  $\rho_{\sigma}(x)$  given by:

$$\rho_{\sigma}(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}} \quad (2.2)$$

The probability function of discrete Gaussian distribution over  $\mathbb{Z}$  is then defined as  $D_{\sigma}(x) = \rho_{\sigma}(x)/\rho_{\sigma}(\mathbb{Z})$  with  $\rho_{\sigma}(\mathbb{Z}) = \sum_{y=-\infty}^{\infty} \rho_{\sigma}(y)$ . As we will sample whole vectors most of the time, we denote the discrete Gaussian distribution over  $\mathbb{Z}^m$  as  $\mathcal{N}_{\mathbb{Z}}^m(\mu, \sigma^2)$  and its probability function as  $D_{\sigma}^m(\vec{x}) = \rho_{\sigma}(\vec{x})/\rho_{\sigma}(\mathbb{Z})^m$  with  $\rho_{\sigma}(\vec{x})$  being defined as follows:

$$\rho_{\sigma}(\vec{x}) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{\|\mathbf{x}\|^2}{2\sigma^2}} \quad (2.3)$$

## 2.5. Cryptographic Algorithms

This Section will give a short overview of two cryptographic algorithms, that are based on ideal lattices and the ring-LWE problem. While the first algorithm can exclusively be used for encryption, the second one is a signature algorithm. For more information on the mathematical background of those algorithms, we would like to refer you to the cited papers.

### 2.5.1. Ring-LWE Encryption Scheme

In our paper we focus on the encryption scheme published in [LPR12], which will be referred to as *LPR*. This scheme consists of three main operations: *Key Generation*, *Encryption* and *Decryption*. The globally known parameters of this scheme are  $(n, q, \sigma)$  and a polynomial  $\mathbf{g}$ . The dimension of the polynomial ring  $R_q$  is defined by  $n$ , while  $q$  is the modulus. The standard deviation of the discrete Gaussian distribution is given by  $\sigma$ .

**Key Generation:** In this step, the coefficients of the two polynomials  $\mathbf{r}$  and  $\mathbf{s}$  are sampled according to the discrete Gaussian distribution  $\mathcal{N}_{\mathbb{Z}}^n(0, \sigma^2)$ . Then the public key  $\mathbf{p}$  is computed by  $\mathbf{p} = \mathbf{r} - \mathbf{g} \cdot \mathbf{s}$ . The resulting output is a key pair  $(\mathbf{p}, \mathbf{s})$  with  $\mathbf{p}$  being the public key and  $\mathbf{s}$  being the secret key.

**Encryption:** The encryption phase takes a  $n$ -bit message  $\mathbf{m}$  and the public key  $\mathbf{p}$  as input. Initially, the message  $\mathbf{m}$  is encoded as an element of the ring

$R_q$  by multiplying each of its bits by  $q/2$  and is then denoted by  $\mathbf{m}_{enc}$ . In the following step, three error polynomials  $\mathbf{e}_1$ ,  $\mathbf{e}_2$  and  $\mathbf{e}_3$  are sampled according to  $\mathcal{N}_{\mathbb{Z}}^n(0, \sigma^2)$  and will be used as noise. The ciphertext then consists of two parts  $(\mathbf{c}_1, \mathbf{c}_2)$ , with  $\mathbf{c}_1 = \mathbf{g} \cdot \mathbf{e}_1 + \mathbf{e}_2$  and  $\mathbf{c}_2 = \mathbf{p} \cdot \mathbf{e}_1 + \mathbf{e}_3 + \mathbf{m}_{enc}$ . The encryption algorithm returns the ciphertext  $(\mathbf{c}_1, \mathbf{c}_2)$ .

**Decryption:** For decryption, we start by computing  $\mathbf{m}_{enc} = \mathbf{c}_1 \cdot \mathbf{s} + \mathbf{c}_2$ . To decode  $\mathbf{m}_{enc}$ , we need a function  $\text{DECODE}(m_{enc,i})$  with  $m_{enc,i}$  being an element of  $\mathbf{m}_{enc}$ . One possible function is given below:

$$\text{DECODE}(x) = \begin{cases} 0, & \text{if } x \in (0, q/4) \cup (3q/4, q) \\ 1, & \text{if } x \in (q/4, 3q/4) \end{cases} \quad (2.4)$$

### 2.5.2. BLISS Signature Scheme

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#### Algorithm 1 BLISS KEY GENERATION

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**Output:** BLISS key pair  $(\mathbf{A}, \mathbf{S})$  with public key  $\mathbf{A} = (\mathbf{a}_1, \mathbf{a}_2) \in R_{2q}^2$  and secret key  $\mathbf{S} = (\mathbf{s}_1, \mathbf{s}_2) \in R_{2q}^2$ , such that  $\mathbf{AS} = \mathbf{a}_1 \cdot \mathbf{s}_1 + \mathbf{a}_2 \cdot \mathbf{s}_2 \equiv q \pmod{2q}$

- 1: Choose  $\mathbf{f}, \mathbf{g} \in R_{2q}$  uniformly at random with exactly  $d_1$  entries in  $\{\pm 1\}$  and  $d_1$  entries in  $\{\pm 2\}$
- 2:  $\mathbf{S} = (\mathbf{s}_1, \mathbf{s}_2) = (\mathbf{f}, 2\mathbf{g} + 1)$
- 3: **if**  $\mathbf{f}$  violates certain conditions (see [DDLL13]) **then**
- 4:     Restart
- 5: **end if**
- 6:  $\mathbf{a}_q = (2\mathbf{g} + 1)/\mathbf{f} \pmod{q}$  (restart if  $\mathbf{f}$  is not invertible)
- 7: **return**  $(\mathbf{A}, \mathbf{S})$  with  $\mathbf{A} = (2\mathbf{a}_q, q - 2) \pmod{2q}$

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#### Algorithm 2 BLISS SIGNATURE ALGORITHM

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**Input:** Message  $\mu$ , public key  $\mathbf{A} = (\mathbf{a}_1, q - 2)$ , secret key  $\mathbf{S} = (\mathbf{s}_1, \mathbf{s}_2)$

**Output:** Signature  $(\mathbf{z}_1, \mathbf{z}_2^\dagger, \mathbf{c}) \in \mathbb{Z}_{2q}^n \times \mathbb{Z}_p^n \times \{0, 1\}^n$

- 1:  $\mathbf{y}_1, \mathbf{y}_2 \leftarrow \mathcal{N}_{\mathbb{Z}}^n(0, \sigma^2)$
  - 2:  $\mathbf{u} = \zeta \cdot \mathbf{a}_1 \cdot \mathbf{y}_1 + \mathbf{y}_2 \pmod{2q}$
  - 3:  $\mathbf{c} = H(\lfloor \mathbf{u} \rfloor_d) \pmod{p}, \mu$
  - 4: Choose a random bit  $b$
  - 5:  $\mathbf{z}_1 = \mathbf{y}_1 + (-1)^b \mathbf{s}_1 \cdot \mathbf{c} \pmod{2q}$
  - 6:  $\mathbf{z}_2 = \mathbf{y}_2 + (-1)^b \mathbf{s}_2 \cdot \mathbf{c} \pmod{2q}$
  - 7: Continue with a probability based on  $\sigma, \|\mathbf{Sc}\|, \langle \mathbf{z}, \mathbf{Sc} \rangle$  (see [DDLL13]), else restart
  - 8:  $\mathbf{z}_2^\dagger = (\lfloor \mathbf{u} \rfloor_d - \lfloor \mathbf{u} - \mathbf{z}_2 \rfloor_d) \pmod{p}$
  - 9: **return**  $(\mathbf{z}_1, \mathbf{z}_2^\dagger, \mathbf{c})$
-

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**Algorithm 3** BLISS VERIFICATION ALGORITHM

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**Input:** Message  $\mu$ , public key  $\mathbf{A} = (\mathbf{a}_1, q - 2)$ , signature  $(\mathbf{z}_1, \mathbf{z}_2^\dagger, \mathbf{c})$ 

- 1: **if**  $\mathbf{z}_1, \mathbf{z}_2^\dagger$  violate certain conditions (see [DDLL13]) **then**
  - 2:     Reject
  - 3: **end if**
  - 4: **if**  $\mathbf{c} = H(\lfloor \zeta \cdot \mathbf{a}_1 \cdot \mathbf{z}_1 + \zeta \cdot q \cdot \mathbf{c} \rfloor_d + \mathbf{z}_2^\dagger \bmod p, \mu)$  **then**
  - 5:     Accept
  - 6: **end if**
- 

## 2.6. Side-Channel Attack Terminology

## 3. Masking the Ring-LWE Encryption Scheme I

Since most side-channel attacks focus on the decryption operation, this section will present an attempt to masking the decryption function of the *LPR* Ring-LWE encryption scheme. This masking approach was originally proposed in [RRVV15], for more details we would refer you to that paper.

### 3.1. Implementation

In this Section we will start by giving the reader an overview of the general setup, before going into more detail about the masked decoding algorithms. We will make strong use of the *Number Theoretic Transform* (NTT) in this chapter. We recall, that our notation for polynomials in the NTT domain is  $\tilde{\mathbf{f}}$ . The NTT operation itself will be denoted as  $\text{NTT}(\cdot)$ , while its inverse operation will be written as  $\text{INTT}(\cdot)$ . We want to stress, that  $\text{NTT}(\cdot)$  and  $\text{INTT}(\cdot)$  are linear operations, as we will use this characteristic for our blinding technique.

#### 3.1.1. Overview

This Subsection will give you a short overview of the blinding technique proposed in [RRVV15]. We start by splitting the secret key  $\mathbf{s}$  into two shares  $\mathbf{s}', \mathbf{s}'' \in R_q$  such that  $\mathbf{s} = \mathbf{s}' + \mathbf{s}''$ . Therefore we choose all coefficients of  $\mathbf{s}'$  uniformly at random and calculate  $\mathbf{s}'' = \mathbf{s} - \mathbf{s}'$ . In the NTT domain it follows, that  $\tilde{\mathbf{s}} = \tilde{\mathbf{s}}' + \tilde{\mathbf{s}}''$ . Due to the linearity of  $\text{INTT}(\cdot)$  and the multiplication, we can compute  $\mathbf{m}_{enc}$  as:

$$\mathbf{m}_{enc} = \text{INTT}(\tilde{\mathbf{s}} \cdot \tilde{\mathbf{c}}_1 + \tilde{\mathbf{c}}_2) = \text{INTT}(\tilde{\mathbf{s}}' \cdot \tilde{\mathbf{c}}_1 + \tilde{\mathbf{c}}_2) + \text{INTT}(\tilde{\mathbf{s}}'' \cdot \tilde{\mathbf{c}}_1) \quad (3.1)$$

This enables us to split the whole equation into two branches, calculating  $\mathbf{m}'_{enc}$  and  $\mathbf{m}''_{enc}$  in the following way:

$$\mathbf{m}'_{enc} = \text{INTT}(\tilde{\mathbf{s}}' \cdot \tilde{\mathbf{c}}_1 + \tilde{\mathbf{c}}_2) \quad (3.2)$$

$$\mathbf{m}''_{enc} = \text{INTT}(\tilde{\mathbf{s}}'' \cdot \tilde{\mathbf{c}}_1) \quad (3.3)$$

Those computations can be done on a arithmetic processor without any protection against side-channel attacks like *DPA*, as both branches are totally independent of our secret key  $\mathbf{s}$ .

However, the  $\text{DECODE}(m_{enc,i})$  function in the decryption stage of the *LPR* scheme is non-linear and cannot easily be split into two parts. For this reason, we will present a masked decoder in the next Subsection, that takes  $\mathbf{m}'_{enc}$  and  $\mathbf{m}''_{enc}$  as inputs to compute two shares  $\mathbf{m}'$ ,  $\mathbf{m}''$  of the decoded message  $\mathbf{m}$  in a fairly efficient way.

### 3.1.2. Masked Decoder

### 3.1.3. Masked Table Lookup

## 3.2. Evaluation

## **4. Masking the Ring-LWE Encryption Scheme II**

### **4.1. Implementation**

### **4.2. Evaluation**

## **5. Flush+Reload Cache Attack on Bliss**

### **5.1. Gaussian Sampling**

#### **5.1.1. CDT Sampling**

#### **5.1.2. Rejection Sampling**

### **5.2. Attacking the Sampling Algorithms**

### **5.3. Evaluation**

## 6. Blinding Countermeasures

Blinding is a countermeasure commonly used to prevent side-channel attacks like *Differential Power Analysis* (DPA) [KJJ99]. It is used to add additional randomness to mathematical operations in a way, that the attacker can not easily draw conclusions from his observations. This Section summarizes two blinding countermeasures presented in [Saa16], which appear to be of special interest for Ring-LWE cryptosystems. While the first countermeasure will be an approach to blinding of polynomial multiplication within a ring  $R_q$ , the second one will be a blinding countermeasure for Gaussian sampling. All those techniques are believed to help against the attacks we described in 5, yet this has not been verified.

### 6.1. Blinding Polynomial Multiplication

There are two pretty types of blinding for polynomial multiplications, the first of whom is the multiplication of each polynomial with a constant. For two polynomials  $\mathbf{f}, \mathbf{g} \in R_q$  and constants  $a, b \in \mathbb{Z}_q$  the blinding operation and the inverse operation look as follows:

$$\mathbf{h} = a\mathbf{f} \cdot b\mathbf{g} \tag{6.1}$$

$$\mathbf{f} \cdot \mathbf{g} = (ab)^{-1}\mathbf{h} \tag{6.2}$$

The second type would be circularly shifting the coefficients in each of the polynomials. As a polynomial can be written as  $\mathbf{f} = \sum_{i=0}^{n-1} f_i x^i$  with  $f_i$  being the  $i$ -th coefficient of the polynomial  $\mathbf{f}$ , a shift by  $j$  positions would be equal to the following computation:

$$x^j \mathbf{f} = \sum_{i=0}^{n-1} f_i x^{i+j} = \sum_{i=0}^{n-1} f_{i-j} x^i \tag{6.3}$$

Both of those blinding operation can be combined within one function, which will be called  $\text{POLYBLIND}(\mathbf{v}, s, c)$  from now on. This function works on the coefficient vectors of polynomials of degree  $n$  and is given in Algorithm 4.

The inverse operation can be denoted by  $\text{POLYBLIND}(\vec{v}', -s, c^{-1})$ . Due to the isometries of the ring  $R_q$ , the multiplication of two polynomials (here: their coefficient vectors) can be blinded using the  $\text{POLYBLIND}$  function in the following way:



**Algorithm 4** POLYBLIND**Input:** coefficient vector  $\vec{v}$ , number of shifts  $s$ , constant  $c$ **Output:** blinded coefficient vector  $\vec{v}'$ 


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```

1: for  $i = 0, \dots, n - s - 1$  do
2:    $v'_i = cv_{i+s} \bmod q$ 
3: end for
4: for  $i = n - s, \dots, n - 1$  do
5:    $v'_i = q - cv_{i+s-n} \bmod q$ 
6: end for
7: return  $\vec{v}'$ 

```

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$$\begin{aligned}
\vec{f}' &= \text{POLYBLIND}(\vec{f}, r, a) \text{ with } r \in_R 0, \dots, n - 1 \text{ and } a \in_R \mathbb{Z}_q \\
\vec{g}' &= \text{POLYBLIND}(\vec{g}, s, b) \text{ with } s \in_R 0, \dots, n - 1 \text{ and } b \in_R \mathbb{Z}_q \\
\vec{h}' &= \vec{f}' \cdot \vec{g}' \\
\vec{h} &= \text{POLYBLIND}(\vec{h}', -(r + s), (ab)^{-1})
\end{aligned}$$

## 6.2. Blinding Gaussian Sampling

As with the blinding of polynomial multiplication in the last subsection, there are two pretty easy ways to blind the coefficient vectors during the process of Gaussian sampling. We will again give a short description of both of them and present a function, that combines both methods.

We define a function  $\text{VECTORSAMPLE}(n, \sigma)$ , that samples and returns a vector according to the discrete Gaussian distribution  $\mathcal{N}_{\mathbb{Z}}^n(0, \sigma^2)$ . A naive implementation of this function could lead to leakage of information to an attacker using e.g. DPA. This has been done in the cache attack from [BHL16] we described in Section 5.

The first approach to blinding would be to randomly shuffle the elements in the coefficient vector. The function  $\text{VECTORSHUFFLE}(\vec{x})$  is doing exactly that, so that  $\text{VECTORSHUFFLE}(\text{VECTORSAMPLE}(n, \sigma))$  would increase security to a certain extend.

For the second approach we need to take a short detour through probability theory. For two Gaussian distributions  $X = \mathcal{N}_{\mathbb{Z}}^n(\mu_X, \sigma_X^2)$  and  $Y = \mathcal{N}_{\mathbb{Z}}^n(\mu_Y, \sigma_Y^2)$  it holds that their sum is equal to  $X + Y = \mathcal{N}_{\mathbb{Z}}^n(\mu_X + \mu_Y, \sigma_X^2 + \sigma_Y^2)$ . As we focus on zero-centered distributions, the center does not change for us. For the standard deviation it follows, that  $\sigma_{X+Y} = \sqrt{\sigma_X^2 + \sigma_Y^2}$ . Algorithm 5 is one possible way to make use of those characteristics of Gaussian distributions. It also makes use of the  $\text{VECTORSHUFFLE}(\vec{x})$  function to increase overall security. Another, very similar approach, has been described in [PDG14].

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**Algorithm 5** VECTORBLINDSAMPLE

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**Input:** length of the vector  $n$ , number of iterations  $m$ , standard deviation  $\sigma$ **Output:** sampled vector  $\vec{x}$ 

- 1:  $\mathbf{x} = \mathbf{0}$
  - 2: **for**  $i = 1, \dots, m$  **do**
  - 3:    $\vec{x} = \vec{x} + \mathcal{N}_{\mathbb{Z}}^n(0, (\frac{1}{\sqrt{m}}\sigma)^2)$
  - 4:    $\vec{x} = \text{VECTORSHUFFLE}(\vec{x})$
  - 5: **end for**
  - 6: **return**  $\vec{x}$
-

## 7. Conclusion

Conclude your thesis and discuss your results...

## List of Figures

## List of Tables

## A. Bibliography

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