Unstable States: Community dynamics in a chaotic world

M.K. Lau

July 8, 2014

1 8 Jul 2014

Eigenvalues indicate system dynamics.

A system is stable if it eventually returns to a fixed point after a disturbance.

A system is unstable if it is not stable.

For imaginary (complex) eigenvalues:

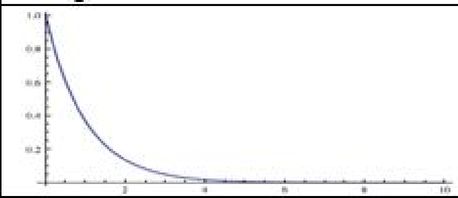
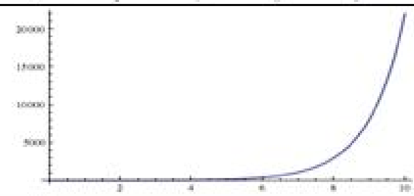
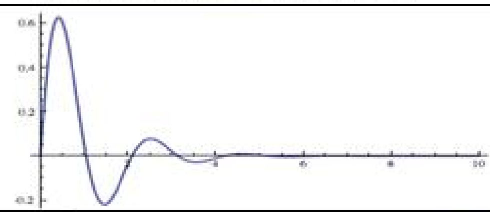
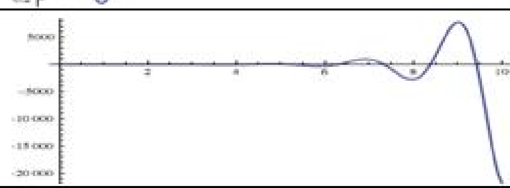
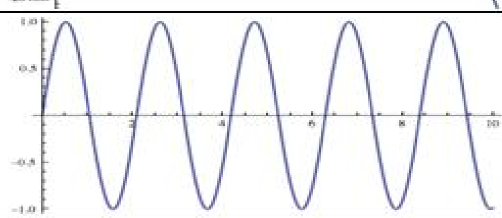
- Negative Real Part - Stable. Damping is a requirement of stability.
- Positive Real Part - Unstable. Increasing amplitude is unstable.
- Zero Real Part - Unstable. Oscillators are unstable, as they will not go back to a steady state post-disturbance
- The complex part will not affect the stability.

Real eigenvalues:

- Zero eigenvalues: unstable

- All Positive: unstable
- All Negative: unstable
- Both positive and negative: unstable, saddle.
- Repeated: needs further analysis. For the special case of two eigenvalues, both positive = unstable, both negative = stable.

Check out this figure:

Eigenvalues	Graph
All real and negative	
All real and one or more are positive	
All real eigenvalues are negative and there are imaginary parts	
One or more eigenvlaues have a positive real part and there are imaginary parts	
Real parts of the eigenvlaues are zero and there are imaginary parts	

```
>   source('../src/sens2_ews.R')
> library(ecodist)
> fps <- read.csv('~/Dropbox/Tipping Point MS/Model sensitivity analysis and second
> x <- read.csv('~/Dropbox/Tipping Point MS/Model sensitivity analysis and second pa
> x <- x[,-1]
> x <- x[1:12500,]
> x[x<0] <- 0
> x <- t(x)
>
> ##Obtain equations
>
>
```

2 3 Jul 2014

Foundations of Resilience Thinking

- Resilience = size and steepness of attractor basins
- Distinguishes between: near equilibrium behavior and long term persistence
- Panarchy = Processes operating at different scales permit organization

How does NMDS treat mutual information?

```
>   source('../src/sens2_ews.R')
> library(ecodist)
> fps <- read.csv('~/Dropbox/Tipping Point MS/Model sensitivity analysis and second
```

```
> x <- read.csv('~/Dropbox/Tipping Point MS/Model sensitivity analysis and second pa
> x <- x[,-1]
> x <- x[1:12500,]
> x[x<0] <- 0
> x <- t(x)
> ###kernel density information
> d1 <- density(x[1,])
> d2 <- density(x[2,])
> plot(density(x[1,])$x,density(x[3,])$x)
> points(-2:12,-2:12)
>
> ###
>
>
>
```

3 2 Jul 2014

Mutual Information for Time Series

This is an attempt to implement methods from Moniz et al. (2007) Application of Information Theory Methods to Food Web Reconstruction

```
>   source('../src/sens2_ews.R')
> library(entropy)
> library(vegan)
> library(ecodist)
```

```
> fps <- read.csv('~/Dropbox/Tipping Point MS/Model sensitivity analysis and second
> x <- read.csv('~/Dropbox/Tipping Point MS/Model sensitivity analysis and second pa
> x <- x[,-1]
> x <- x[1:12500,]
> x[x<0] <- 0
> x <- t(x)
> ###
> ## mid.m <- array(0,dim=c(nrow(x),nrow(x)))
> ## for (i in 1:nrow(x)){
> ##   for (j in 1:nrow(x)){
> ##     mid.m[i,j] <- mi.Dirichlet(x[c(i,j),],0.5)
> ##   }
> ## }
> mid.m <- dget(file='../data/midm.rda')
> mid.m <- round(mid.m,7)
> pca <- princomp(as.dist(mid.m))
> nms <- nmds.min(nmds(as.dist(mid.m),2,2))
> plot(nms)
> plot(envfit(env=fps,ord=nms))
> envfit(env=fps,ord=nms)
>

>   library(entropy)
> y2d <- rbind( sample(1:100,100), sample(1:100,100) )
> y2d. = rbind( sample(1:100,100), sample(1:100,100) )
> test <- test. <- 0
```

```
> a <- seq(0,1,by=0.01)
> for (i in 1:length(a)){test[i] <- mi.Dirichlet(y2d,a=a[i])}
> for (i in 1:length(a)){test.[i] <- mi.Dirichlet(y2d.,a=a[i])}
> plot(test.~test);abline(lm(range(test.)~range(test)))
>
```

4 1 Jul 2014

Reading Beltrami:

```
> ##For N. = rN
> ##N(t) = N(0)e^rt
>
> N0 <- seq(0,100,by=0.1)
> t <- 0:100
> Nt <- sapply(N0, function(N0,t,r) N0*exp(r*t),t=t,r=2)
> plot(Nt[,1]~t,xlim=range(t),ylim=range(Nt),type='l',)
> for (i in 2:ncol(Nt)){lines(Nt[,i]~t)}
> plot(apply(Nt,2,max)~N0,type='l')
> plot(Nt[,ncol(Nt)]~t,xlim=range(t),ylim=range(Nt),type='l',)
>
```

5 30 Jun 2014

```
>   library('nimble')
>
```

6 27 Jun 2014

Alternative state detection? Should we be looking for break points?

```
> source('../src/sens2_ews.R')
>   fps <- read.csv('~/Dropbox/Tipping Point MS/Model sensitivity analysis and secon
> x <- read.csv('~/Dropbox/Tipping Point MS/Model sensitivity analysis and second pa
> x <- x[,-1]
> x <- x[1:12500,]
> x[x<0] <- 0
> ###
> my.col <- rainbow(nlevels(factor(fps$Prey)))[as.numeric(factor(fps$Prey))]
> ###
> cx <- x[,5]
> cx[cx<=0.03] <- 0
> plot(cx)
> abline(v=find.days(cx)$start)
> abline(v=find.days(cx)$end)
> days <- find.days(cx)
> x.days <- apply(x,2,function(x,d) x[d],d=sort(unlist(days)))
> x.cor <- list()
> for (k in 1:ncol(x)){
+   out <- 0
+   for (i in 1:(length(days)-1)){
+     d1 <- days[[i]]
+     d2 <- days[[(i+1)]]
+     xt <- x[d1[1:min(c(length(d1),length(d2)))],k]
```

```
+     xt1 <- x[d2[1:min(c(length(d1),length(d2)))],k]
+     out[i] <- cor(xt,xt1,method='k')
+   }
+   x.cor[[k]] <- out
+ }
> plot(x.cor[[1]],type='l',ylim=c(-0.5,1),xlab='Day',col=my.col[[1]])
> for (i in 1:length(x.cor)){lines(x.cor[[i]],col=my.col[[i]])}
> ###
> par(mfcol=c(2,1))
> i <- 1
> plot(density(x[,i]),main='',xlab=expression(O[2]),
+      xlim=range(x)+c(-1,1),ylim=c(0,1),col=my.col[i])
> for (i in 2:ncol(x)){
+   x.dens <- density(x[,i])
+   x.dens$y <- x.dens$y/max(x.dens$y)
+   x.dens$y <- x.dens$y+runif(1,-0.025,0.025)
+   lines(x.dens,col=my.col[i])
+ }
> legend('topright',legend=levels(factor(fps$Prey)),lty=1,col=rainbow(nlevels(factor
> ##
> my.col <- rainbow(nlevels(factor(fps$Prey)))[as.numeric(factor(fps$Prey))]
> i <- 1
> plot(1:nrow(x)~x[,i],col=my.col[i],xlab=expression(O[2]),ylab='time',type='l')
> for (i in 2:ncol(x)){
+   lines(1:nrow(x)~x[,i],col=my.col[i],xlab=expression(O[2]),ylab='time',type='l')
+ }
```

```
>
> ###Calculate the correlation between days
>
>
```

7 26 Jun 2014

- Sub-sample - Standardize, detrend and decycle - Euclidean distance - Plot of eigen values - PC plot - overlay vectors

```
> library(vegan)
> library(ecodist)
> source('../src/sens2_ews.R')
>   fps <- read.csv('~/Dropbox/Tipping Point MS/Model sensitivity analysis and secon
> x <- read.csv('~/Dropbox/Tipping Point MS/Model sensitivity analysis and second pa
> x <- x[,-1]
> x <- x[1:12500,]
> x[x<0] <- 0
> ###standardize, detrend and decycle
> sdd.x <- list()
> index <- apply(fps,1,function(x) paste(x[3:6],collapse=' '))
> cn <- (1:nrow(fps))[fps$Prey==0&index==paste(fps[1,3:6],collapse=' ')]
> for (i in 1:ncol(x)){
+   if (all(x[,i]==0)){sdd.x[[i]] <- NA}else{
+     cn <- (1:nrow(fps))[fps$Prey==0&index==paste(fps[i,3:6],collapse=' ')]
+     sdd.x[[i]] <- sddSens(x[,i],x[,cn],eval=6250)
```

```
+   }
+ }
> sdd.x <- do.call(cbind,sdd.x)
> sdd.x[is.na(sdd.x)] <- 0
> ###ordinate
> sdd.d <- dist(t(sdd.x))
> hist(sdd.d)
> plot(hclust(sdd.d))
> ##
> par(mfrow=c(1,3))
> plot(sdd.x[,1],xlab='time',ylab=expression(O[2]),ylim=c(-1,1),pch=19,cex=0.05)
> for (i in 1:ncol(sdd.x)){points(sdd.x[,i],pch=19,cex=0.05)}
> sdd.ord <- princomp(sdd.d)
> names(sdd.ord)
> plot(sdd.ord$sdev/sum(sdd.ord$sdev))
> abline(h=0.05,lty=2)
> plot(sdd.ord$scores[,1:2])
> plot(envfit(ord=sdd.ord$scores[,1:2],env=fps))
> ###
> envfit(ord=sdd.ord$scores[,1:2],env=fps)
> envfit(ord=sdd.ord$scores[,2:3],env=fps)
> envfit(ord=sdd.ord$scores[,1:3],env=fps)
> pairs(fps)
> pairs(sdd.ord$scores[,1:3],pch=19,cex=1)
> par(mfrow=c(3,3))
> for (i in 1:3){
```

```
+   for (j in 1:3){
+     if (i==j){
+       plot(c(-1,1),c(-1,1),pch='',xlab='',ylab='',xaxt='n',yaxt='n')
+       text(c(0,0),c(0,0),labels=i,cex=5)
+     }else{
+       plot(sdd.ord$scores[,j:i],pch=19,cex=0.75)
+       plot(envfit(ord=sdd.ord$scores[,j:i],env=fps))
+   }
+   }
+ }
> library(rgl)
> plot3d(sdd.ord$scores[,1:3],type='s',col=rainbow(nlevels(factor((fps$Prey))))[as.n
> plot3d(sdd.ord$scores[,1:3],type='s',col=rainbow(nlevels(factor((fps$b))))[as.nume
> plot3d(sdd.ord$scores[,1:3],type='s',col=rainbow(nlevels(factor((fps$a))))[as.nume
> plot3d(sdd.ord$scores[,1:3],type='s',col=rainbow(nlevels(factor((fps$K))))[as.nume
> plot3d(sdd.ord$scores[,1:3],type='s',col=rainbow(nlevels(factor((fps$d))))[as.nume
>
```

8 20 Jun 2014

Plot tests (ordination and smoothed-detrended-decycled) are up on github.

EWS for simulations:

- Write a script that will record the early warning signals for all 180 simulations.

```
> source('../src/sens2_ews.R')
>   fps <- read.csv('~/Dropbox/Tipping Point MS/Model sensitivity analysis and secon
```

```
> x <- read.csv('~/Dropbox/Tipping Point MS/Model sensitivity analysis and second pa
> x <- x[,-1]
> #x <- x[1000:5400,]
> x[x<0] <- 0
> ###Isolate controls
> my.col <- rainbow(nlevels(factor(fps$Prey)))[as.numeric(factor(fps$Prey))]
> sdd.x <- list()
> index <- apply(fps,1,function(x) paste(x[3:6],collapse=' '))
> cn <- (1:nrow(fps))[fps$Prey==0&index==paste(fps[1,3:6],collapse=' ')]
> plot(sdd.x[[1]] <- sddSens(x[,1],x[,cn]),ylim=c(-1,1),pch=19,
+      cex=0.25,ylab=expression(O[2]),xlab='t',col=my.col[1])
> for (i in 1:ncol(x)){
+   if (all(x[,i]==0)){sdd.x[[i]] <- NA}else{
+     cn <- (1:nrow(fps))[fps$Prey==0&index==paste(fps[i,3:6],collapse=' ')]
+     points(sdd.x[[i]] <- sddSens(x[,i],x[,cn]),pch=19,cex=0.25,col=my.col[i])
+   }
+ }
> legend('topright',legend=unique(fps$Prey),col=rainbow(nlevels(factor(fps$Prey))),p
> ###
> par(mfrow=c(1,2))
> plot(x[,1])
> plot(sdd.x[[1]])
> ###
> my.col <- rainbow(nlevels(factor(fps$b)))[as.numeric(factor(fps$b))]
> sdd.x <- list()
> index <- apply(fps,1,function(x) paste(x[3:6],collapse=' '))
```

```
> cn <- (1:nrow(fps))[fps$Prey==0&index==paste(fps[1,3:6],collapse=' ')]
> plot(sdd.x[[1]] <- sddSens(x[,1],x[,cn]),ylim=c(-1,1),pch=19,
+      cex=0.25,ylab=expression(O[2]),xlab='t',col=my.col[1])
> for (i in 1:ncol(x)){
+   if (all(x[,i]==0)){sdd.x[[i]] <- NA}else{
+     cn <- (1:nrow(fps))[fps$Prey==0&index==paste(fps[i,3:6],collapse=' ')]
+     points(sdd.x[[i]] <- sddSens(x[,i],x[,cn]),pch=19,cex=0.25,col=my.col[i])
+   }
+ }
> legend('topright',legend=levels(factor(fps$b)),col=rainbow(nlevels(factor(fps$b)))
> library(strucchange)
>
>
```

Look at the temporal correlation structure among simulations:

```
> ##Using the 100 min averages
> ##Runs are in columns
>   fps <- read.csv('~/Dropbox/Tipping Point MS/Model sensitivity analysis and secon
> x <- read.csv('~/Dropbox/Tipping Point MS/Model sensitivity analysis and second pa
> x <- x[,-1]
> plot(x[,1],xlab='t',ylab=expression(O[2]),ylim=c(0,max(x)),type='l',lwd=0.5,col=rai
> for (i in 2:ncol(x)){
+   lines(1:nrow(x),x[,i],lwd=0.5,col=rainbow(ncol(x))[i])
+ }
> abline(v=c(1000,5400))
> x <- x[1000:5400,]
```

```
> x[x<0] <- 0
> d.x <- dist(t(x))
> library(vegan)
> library(ecodist)
> eig.x <- eigen(d.x)
> biplot(princomp(d.x))
> vd.x <- vegdist(t(rbind(x,rep(1,ncol(x)))))
> nmds.x <- nmds(vd.x,2,2)
> envf.x <- envfit(env=fps,ord=nmds.min(nmds.x))
> plot(apply(nmds.min(nmds.x),2,jitter,factor=50),col=rainbow(ncol(x)))
> plot(envf.x)
> envf.x
>
```

9 19 Jun 2014

Generate plots for the tipping$_{p}oints2$

- Model description is in SarrModel-20140709-AME.lyx
- Model was coded in mathematica (Pitcher$_{P}lant_{T}hreshold_{M}odel.txt)Freeparameterspaceforsim$

  Note that the a and b terms for calculating w(t) are coupled in order to keep the integral (i.e. the area under the decomposition curve) equal across simulations

- Simulation output is in Raw$_{r}uns.csv, ten_{m}in_{a}verage.csv, hundred_{m}in_{a}verage.csvPARtoO2.docxdescri$

```
>   ##free parameter space
>   fps <- read.csv('~/Dropbox/Tipping Point MS/Model sensitivity analysis and secon
> ##Using the 100 min averages
> ##Runs are in columns
> x <- read.csv('/Users/Aeolus/Dropbox/Tipping Point MS/Model sensitivity analysis a
> x <- x[,-1]
> plot(x[,1],pch=19,cex=0.1)
> for (i in 2:ncol(x)){
+   points(1:length(x[,i]),x[,i],pch=19,cex=0.1)
+ }
> par(mfrow=c(1,2))
> i <- 2
> plot(x[,i],pch=19,cex=0.1,col=rainbow(length(x[,i])))
> plot(x[2:length(x[,i]),i]~x[1:(length(x[,i])-1),i],pch=19,cex=0.1,col=rainbow(len
> par(mfrow=c(1,2))
> for (i in 1:ncol(x)){
+   plot(x[,i],pch=19,cex=0.1,col=rainbow(length(x[,i])),
+        main=paste(paste(colnames(fps)[2:ncol(fps)],fps[i,2:ncol(fps)],sep='='),col
+   plot(x[2:length(x[,i]),i]~x[1:(length(x[,i])-1),i],pch=19,cex=0.1,col=rainbow(l
+        main=paste(paste(colnames(fps)[2:ncol(fps)],fps[i,2:ncol(fps)],sep='='),col
+   play(x[,i]^3/max(x^3))
+   Sys.sleep(0.15)
+ }
>
```

The pitcher plant respiration model:

Terms:

- t = time
- x = oxygen
- A = environmental factor promoting oxygenation (i.e. PAR = light)
- f(w,x) = loss/decay of oxygen
- g(x) = recovery of oxygen (augmented by mineralized nutrients)
- w = prey mass

$$\frac{dx}{dt} = A - f(w, x) + g(x) \tag{1}$$

$$x_{t+1} = a_t * sin(2\pi f t) - (m + a_t \frac{w_{t-1}}{K_w + w_{t-1}}) + D_t(x_t, a'_t) \tag{2}$$

```
>   a0 <- 10
> f <- 1/1440
> t <- rep((1:1440),10)
> d <- rep(1:10,each=1440)
> A <- a0*sin(2*pi*f*t)
> A[t>720] <- 0
> plot(A,type='l',ylab=expression(O[2]))
>
```

$$w(t + 1) = ae^{-bw_0 t} \tag{3}$$

Note that this is a correction from the printed version in PNAS

```
> y <- NULL
> a <- 20
> ##b
>
> for (i in 1:17){ y[i] <- 20*exp(-.1395*(i-1))}
> plot(c(0:16), y, xlim=c(0,16), ylim=c(0,20))
> par(list(pin=c(3,3),las=1))
> integrand <- function(x, a=4) {20*exp(-a*x)}
> plot(integrand, xlim=c(0,16), xlab="Days", ylab="Mass of prey remaining (mg)",
+                       lwd=2, col="red", font=2, font.lab=2)
> integrate(integrand, lower=0, upper=16)
> ###
> a <- 20
> b <- 4
> e <- exp(1)
> w <- 75/1000
> for (t in 2:20){
+   w[t] <- exp(-w[t-1])
+ }
> plot(w,type='l')
>
```

Here is a link to the pitcher-plant tipping point dropbox. To start, go to ../Model sensitivity analysis and second paper/data/ and read the metadata.csv file. The various raw and average files are the ones to think about plotting. We can talk about it later today.

10 18 Jun 2014

Question: Are transitions between chaotic states detectable?

Question: Are there classes of models that possess detectable chaotic transition warning signals?

Modeling Issues:

1. a in S2.2 can be confused with a in S2.1
2. What is the exponent in S2.2?
3. Why is b (S2.2) in days rather than minutes?
4. Why is w(t-1) used in S2.3?

```
>   ##Initial stab at the spo2 model
>   a0 <- 10
> f <- 1/(60*24) #total minutes in a day
> t <- rep((1:(60*24)),11)
> At <- a0 * sin(2*pi*f*t)
> At[t>720] <- 0
> plot(At~I(1:length(t)),type='l')
> a <- 20
> b <- 4
> wt <- 20
> wt <- a*exp(1)^(-b*wt)
> spo2 <- function(x,a,f,t,m,w,Kw,s,d){
+   At <- a0 * sin(2*pi*f*t)
+
```

```
+
+ }
>
```

11 17 Jun 2014

Chatted with Aaron

The data don't support a general pattern of alternative states.

If systems are inherently chaotic or stochastic, then what?

The data:

- Climate isn't stable
- Fossil records don't show stability
- Tropical systems don't show stability
- Even temperate systems break the rule of stability

Two goals:

1. Can we tell when we transition between states?
2. Is there an alternative philosphy? Mathematical framework?

Run simulation increasing r with variance over the chaos threshold

```
>  ##hold ni constant
>  ##hold sd constant at 0.1
>  ##record r
>  ##record n
```

```
>   ##record ews
>   source('../src/unstable_states.R')
> library(earlywarnings)
> cb8.16 <- 2.57 #choatic boundary, 8-16 cycles
> ri <- (cb8.16)-(0.02/2)
> rf <- (cb8.16)+(0.02/2)
> dmc <- list()
> ews <- list()
> mcn <- 100
> for (i in 1:mcn){
+   print(i)
+   dmc[[i]] <- disrupt.mc(N=10,sd=0.01,ri=ri,rf=rf,dump=TRUE)
+   ews[[i]] <- generic_ews(dmc[[i]]$N)
+   dev.off()
+ }
> ## dput(dmc,'../results/dmc.out')
> ## dput(ews,'../results/ews.out')
> ###
> ###
> ###
> dmc <- dget('../results/dmc.out')
> ews <- dget('../results/ews.out')
> ##
> yl <- apply(do.call(rbind,ews),2,min)
> yu <- apply(do.call(rbind,ews),2,max)
> ##
```

```
> par(mfrow=c(2,(ncol(ews[[1]])-1)/2))
> for (i in 2:ncol(ews[[1]])){
+   for (j in 1:length(ews)){
+     if (j==1){
+       plot(ews[[j]][,i]~ews[[j]][,1],ylim=c(yl[i],yu[i]),
+             ylab=colnames(ews[[1]])[i],xlab='t',
+             type='l',lwd=0.25)
+     }else{
+       lines(ews[[j]][,i]~ews[[j]][,1],lwd=0.25)
+     }
+   }
+ }
> ###Average plots
> ews. <- do.call(rbind,ews)
> t <- do.call(rbind,dmc)$t
> r <- do.call(rbind,dmc)$r
> r. <- r
> r[r.>=cb8.16] <- 2
> r[r.<cb8.16] <- 1
> r <- r[t>=ews[[1]][,1][1]]
> #pairs(ews.,cex=0.05,pch=19,col=r)
> par(mfrow=c(1,1))
> plot(ews.[,c(3,4)],pch=19,col=r,cex=(0.01+(0.5*(ews.[,1]/max(ews.[,1])))))
> unique(ews.[ews.[,3]>30.15&ews.[,4]>0.185,1])
> ews. <- apply(ews.,2,function(x,t) tapply(x,t,mean),t=ews.[,1])
> pairs(ews.,cex=0.10,pch=19)
```

```
>
```

Reading Hastings Reading Sheffer Reading Dakos

12 16 Jun 2014

- the distribution of the average of r is uniform
- ensemble distribution is normal
- EWS stats not correlated between 8to16 and 16to8
- Phase (Ni to Nf) spaces for EWS correlations
- EWS stats intercorrelations show correlations and break points

```
> ###Run repeated simulations for ews time series
> source('../src/unstable_states.R')
> library(earlywarnings)
> cb8.16 <- 2.57 #choatic boundary, 8-16 cycles
> ri <- (cb8.16)-(0.02/2)
> rf <- (cb8.16)+(0.02/2)
> ##Visualizing the error in r
> r8.16 <- list()
> r16.8 <- list()
> for (i in 1:138){
+   print(i)
+   r8.16[[i]] <- disrupt.mc(N=i,sd=0.01,ri=ri,rf=rf,dump=TRUE)$r
+   r16.8[[i]] <- disrupt.mc(N=i,sd=0.01,ri=rf,rf=ri,dump=TRUE)$r
```

```
+ }
> rmu8.16 <- apply(do.call(rbind,r8.16),2,mean)
> rmu16.8 <- apply(do.call(rbind,r16.8),2,mean)
> par(mfrow=c(1,2))
> plot(density(rmu8.16),main='',xlab='r')
> for (i in 1:length(r8.16)){
+   lines(density(r8.16[[i]]),col='grey',lwd=0.5)
+ }
> abline(v=cb8.16,lty=2);abline(v=mean(rmu8.16),lty=2,col='darkgrey')
> plot(density(rmu16.8),xlab='r',main='')
> for (i in 1:length(r16.8)){
+   lines(density(r16.8[[i]]),col='grey',lwd=0.5)
+ }
> abline(v=cb8.16,lty=2);abline(v=mean(rmu16.8),lty=2,col='darkgrey')
> ###Determine average threshold
> ##What is the point at which rbar has crossed the threshold?
> ##Directionality depends on direction of r
> rt8.16 <- (1:length(rmu8.16))[rmu8.16>=cb8.16][1]
> rt16.8 <- (1:length(rmu16.8))[rmu16.8<=cb8.16][1]
> rt8.16
> rt16.8
> ###EWS stats
> stats8.16 <- dget(file='../results/stats816.rdata')
> stats16.8 <- dget(file='../results/stats168.rdata')
> ###
> stats8.16 <- na.omit(do.call(rbind,stats8.16))
```

```
> stats16.8 <- na.omit(do.call(rbind,stats16.8))
> ###
> stats8.16 <- stats8.16[1:min(c(nrow(stats8.16),nrow(stats16.8))),]
> stats16.8 <- stats16.8[1:min(c(nrow(stats8.16),nrow(stats16.8))),]
> ###
> par(mfrow=c(2,ncol(stats8.16)/2),
+     mai=c(0.25,0.01,0.25,0.01))
> for (i in 1:ncol(stats8.16)){
+   plot(density(stats8.16[,i]),
+        main=colnames(stats8.16)[i],
+        xlim=c(min(c(stats8.16[,i],stats16.8[,i])),
+          max(c(stats8.16[,i],stats16.8[,i]))),
+        xaxt='n',yaxt='n',bty='n')
+   lines(density(stats16.8[,i]),lty=2)
+ }
> ###
> par(mfrow=c(2,ncol(stats16.8)/2))
> for (i in 1:ncol(stats8.16)){
+   plot(stats16.8[,i]~stats8.16[,i],
+        xlab=paste(colnames(stats8.16)[i],'8.16'),
+        ylab=paste(colnames(stats16.8)[i],'16.8'))
+   abline(lm(stats16.8[,i]~stats8.16[,i]))
+ }
> ###
> par(mfrow=c(4,ncol(stats8.16)/2),
+     mai=c(0,0,0,0))
```

```
> for (i in 1:ncol(stats8.16)){
+   plot(stats8.16[1:(nrow(stats8.16)-1),i]~stats8.16[2:(nrow(stats8.16)),i],
+        type='l',xaxt='n',yaxt='n',bty='n')
+ }
> for (i in 1:ncol(stats16.8)){
+   plot(stats16.8[1:(nrow(stats16.8)-1),i]~stats16.8[2:(nrow(stats16.8)),i],
+        type='l',col='darkgrey',xaxt='n',yaxt='n',bty='n')
+ }
> ###Ensemble N~stats
> pairs(data.frame(Ni=(1:nrow(stats8.16)),stats8.16),pch=19,cex=0.10,col='black')
> pairs(data.frame(Ni=(1:nrow(stats16.8)),stats16.8),cex=0.10,col='black')
>

> library(earlywarnings)
> set.seed(1)
> drmc8.16 <- disrupt.mc(sd=0.01,ri=ri,rf=rf,dump=TRUE)
> set.seed(1)
> drmc16.8 <- disrupt.mc(sd=0.01,ri=rf,rf=ri,dump=TRUE)
> ews8.16 <- generic_ews(drmc8.16$N)
> ews16.8 <- generic_ews(drmc16.8$N)
> stats8.16 <- cor(ews8.16,method='ken')
> stats16.8 <- cor(ews8.16,method='ken')
>
>

>   ##Re-doing noise in r shifting up and down across 8-16
> source('../src/unstable_states.R')
```

```
> cb8.16 <- 2.57 #choatic boundary, 8-16 cycles
> ri <- (cb8.16)-(0.02/2)
> rf <- (cb8.16)+(0.02/2)
> set.seed(1)
> drmc8.16 <- disrupt.mc(sd=0.01,ri=ri,rf=rf,dump=TRUE)
> set.seed(1)
> drmc16.8 <- disrupt.mc(sd=0.01,ri=rf,rf=ri,dump=TRUE)
> par(mfrow=c(3,2))
> plot(drmc8.16$r,xlab='time',ylab='r')
> abline(h=cb8.16,col=2)
> plot(drmc16.8$r,xlab='time',ylab='r')
> abline(h=cb8.16,col=2)
> plot(drmc16.8$N,xlab='time',ylab='N')
> plot(drmc8.16$N,xlab='time',ylab='N')
> plot(drmc16.8$N~drmc16.8$r,xlab='r',ylab='N',type='l')
> plot(drmc8.16$N~drmc8.16$r,xlab='r',ylab='N',type='l')
> ###Phase space
> par(mfrow=c(1,2))
>                                          #plus n time steps
> for (n in 25:50){
+   plot(drmc16.8$N[(1:(length(drmc16.8$N)-n))],drmc16.8$N[((n+1):(length(drmc16.8$
+        xlab='N',ylab='N+1',type='l')
+   plot(drmc8.16$N[(1:(length(drmc8.16$N)-n))],drmc8.16$N[((n+1):(length(drmc8.16$
+        xlab='N',ylab='N+1',type='l')
+ }
> #EWS
```

```
> library(earlywarnings)
> ews8.16 <- generic_ews(drmc8.16$N)
> ews16.8 <- generic_ews(drmc16.8$N)
>
```

Summary to date (going back in time):

- At the 8-16 cycle threshold, error in r leads to early, sudden shifts
- Slow ramping can also be seen visually
- Sudden jumps across cycle boundaries can be seen visually