



Asynchronous Decentralized Online Convex Optimization

—A Novel Framework for Asynchronous Decentralized Online Learning

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Conference/Journal Submission

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2025 年 11 月 6 日



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Motivation and Background

- **Online learning:** Process large-scale streaming data efficiently
- **Existing decentralized algorithms:** Operate under synchronous setting
 - Leads to **straggler problem**: Fast learners wait for slow learners
- **This paper's purpose:** Design asynchronous counterpart of synchronous algorithms

Key challenges:

- Each learner has five basic types of actions: prediction, weight update, message sending, message receiving, and model averaging.
- Asynchronization: Each learner executes its own actions independently \Rightarrow the orders of the actions of different learners are intermixed together.



Contributions

Novel Framework: Asynchronous Decentralized Online Convex Optimization (AD-OCO)

- First formal framework that gives a complete characterization of the whole interaction dynamics of asynchronous decentralized online learning.
- Individual view: Specify the action sequence of each learner using its own indices.
- System view: Propose a novel event indexing system \Rightarrow mapping all learners into a single time axis.

AD-OGP Algorithm:

- A gossip-based communication scheme
 - Each learner is only allowed to communicate with its immediate neighbors.
 - Adopts "push-sum" strategy (a common way to realize asymmetric gossiping), which does not require any local synchronization among learners.
- A local update scheme build upon online gradient descent (OGD).
 - Design a novel **weighted projection** operation to avoid predictions outside the feasible domain.
- **Instantaneous model averaging**: Each learner performs model averaging as long as it's unoccupied and its receiving buffer is not empty.



Theoretical Contributions:

- Conduct the **first** regret analysis of asynchronous decentralized online learning.
- Extend the graph augmentation technique to online setting to handle message delays.
- Disentangle the complex effects of predictions, weight update and model averaging via our proposed event indexing system.
- Derive a non-trivial regret bound of AD-OGP: it is the same order $O(\sqrt{T})$ as that of its synchronous counterpart.
- Make an extra contribution to the convergence analysis of push-sum: reduce a factor of \sqrt{n} (n is the dimension of model parameters) in the convergence rate of push-sum compared to the previous results, and the final bound is now independent of n .



Problem Formulation

Network Structure:

- Undirected graph $\mathcal{G} = (V, E)$ with vertex set $V = \{1, \dots, m\}$
- Each learner $i \in V$ maintains a local model $w_i \in \mathcal{K}$ (convex compact decision set)
- Each learner i can only communicate with its immediate neighbors $\mathcal{N}(i) = \{j \in V \mid (i, j) \in E\}$

Individual View: Specifies the action sequence of each individual learner.

- Learner $i \in V$ is selected adversarially by the environment to make predictions at certain time point.
- Index the prediction rounds of learner i as $\tau = 1, \dots, N_i$, learner i predicts with its most recent model w_i^τ .
- Environment reveals a convex loss function $f_i^\tau : \mathcal{K} \mapsto \mathbb{R}$, and learner suffers from the loss $f_i^\tau(w_i^\tau)$.
- After finishing the calculation, it uses the gradient to update its model $w_i \leftarrow \mathcal{U}(w_i; \nabla f_i^\tau(w_i^\tau))$, where \mathcal{U} is the local update scheme.



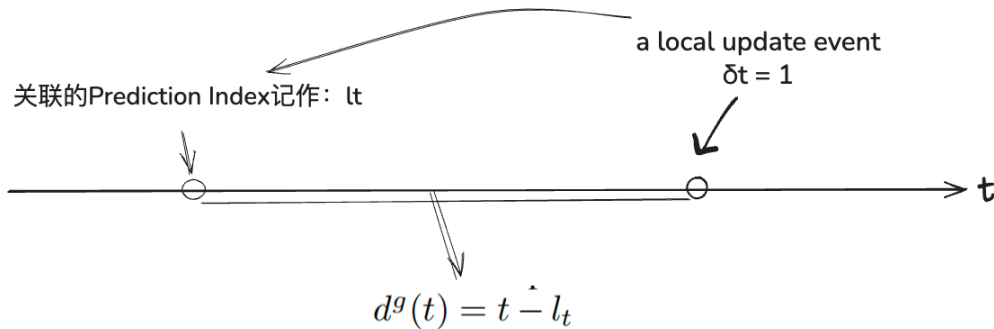
- Then it selects a subset of neighbors, and *sends* a copy of its updated model to each of them.
- Besides sending out its own model copies, learner i will also *receive* its neighbors' model copies at certain time points. These copies will be stored in receiving buffer \mathcal{B}_i .
- Whenever learner i is not occupied with gradient calculation or local update, and its buffer is not empty, it can *average* its model with the model copies stored in the buffer, i.e., $w_i \leftarrow \mathcal{A}(w_i; \mathcal{B}_i)$, where \mathcal{A} is the model averaging scheme.



System View and Event Indexing

Global Event Sequence: Due to asynchronization, each learner executes its actions independently. \Rightarrow The order of actions of all learners are intermixed.

- We need to specify the complete orders of the actions of all learners by mapping them into a single time axis.
- We view either a **predictions** or a **local update** of any learner as an event, and use a virtual counter t to index these events.
- Let $T = \sum_{i \in V} N_i$ be the total number of **predictions** of all learners, then the total number of **local updates** is also T .
- For each event $t \in \{1, \dots, 2T\}$, we use $\delta_t \in \{0, 1\}$ to distinguish the event type:
 - $\delta_t = 0$ if it is a prediction event.
 - $\delta_t = 1$ if it is a local update event.
- We further use i_t to denote the learner that executes the event t , and f_t to denote the associated feedback.
- Then there exists some $\tau \in \{1, \dots, N_{i_t}\}$ such that $f_t = f_{i_t}^\tau$.



The gap between these two indices

图: An example of Prediction and local update.



Delays and Model Evolution

Processing delay: Since any prediction must occur before its corresponding local update, here we have $t \geq 2$, $l_t < t$ and $d^g(t) > 0$.

$$d^p(t) = \text{card}\{l_t < s < t \mid \delta_s = 1\}$$

i.e., the number of local updates between prediction and its associated local update.

Message delay: After any local update event t , the learner i_t that executes this event will send out its model copy to a subset S_t of its neighbors $\mathcal{N}(i_t)$. We focus on the copy sent to any neighbor $j \in S_t$. We assume such copy is used to average learner j 's model between events r_{tj} and $(r_{tj} + 1)$ for some $r_{tj} \geq t$. For the message sent from learner i_t to learner j after event t , we introduce a quantity termed the message delay $d_{i_t}^m(t) = r_{tj} - t$.



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图: The evolution of model w_i .



Model Evolution and Regret Definition

Model Evolution: We denote $w_i(t)$ as the most recent model before event t occurs, and $u_i(t)$ as the model immediately after event t .

- For an update event executed by learner i_t , its associated gradient is calculated based on learner i_t 's prediction using model $w_{i_t}(l_t)$ at event l_t :

$$u_{i_t} = \mathcal{U}(w_{i_t}(t); \nabla f_{l_t}(w_{i_t}(l_t)))$$

The models of other learners do not change at i_t 's local update event, so $u_j(t) = w_j(t)$ for $j \neq i_t$.

- Model averaging operation is not regarded as an event. It happens between t and $t+1$ if learner i is not occupied.
 - Model at the beginning: $u_i(t)$
 - Model at the end: $w_i(t+1)$

So, the overall effect of model averaging on learner i between event t and $t+1$ can be expressed as:

$$w_i(t+1) = \begin{cases} u_i(t), & s = 0 \\ \mathcal{A}(\cdots (\mathcal{A}(\mathcal{A}(u_i(t), \mathcal{B}_i^1(t)), \mathcal{B}_i^2(t)) \cdots), \mathcal{B}_i^s(t)), & s \geq 1 \end{cases}$$



Algorithm Overview

Initial State: Each node i maintains a pair of variables (x_i, y_i) , and predicts with

$$w_{i_t}(t) = \frac{x_{i_t}(t)}{y_{i_t}(t)}.$$

- x_i is a **weighted model**, y_i is a **weight scalar** (always > 0). Both are used for push-sum.
- Initialization: $x_i = w_0 \in \mathcal{K}$ (\mathcal{K} is a convex compact decision set), $y_i = 1$.

AD-OGP Algorithm Steps:

Algorithm 1 Asynchronous Decentralized Online Gradient-Push (AD-OGP)

- 1: **Input:** Learning rate η , initial model $w_0 \in \mathcal{K}$
- 2: **Initialize:** For each learner $i \in V$, $x_i \leftarrow w_0$, $y_i \leftarrow 1$. Empty receiving buffer \mathcal{B}_i .
- 3: **Loop for each event** $t \in \{1, \dots, 2T\}$:
- 4: **If** $\delta_t = 0$ (prediction event) for learner i_t :
- 5: $w_{i_t}(t) \leftarrow x_{i_t}(t)/y_{i_t}(t)$
- 6: Make prediction with $w_{i_t}(t)$, receive loss function f_t .
- 7: **If** $\delta_t = 1$ (local update event) for learner i_t :
- 8: Let l_t be the prediction event corresponding to f_t .
- 9: Compute gradient $g_t = \nabla f_{l_t}(w_{i_t}(l_t))$.



Push-Sum Communication Strategy

Challenge: Decentralized model averaging typically involves linear averaging with neighbors. In an asynchronous and asymmetric setting, traditional doubly-stochastic averaging is not feasible.

- *Why?*

- Doubly-stochastic matrices require symmetric interactions ($p_{ij} = p_{ji}$) and global synchronization ($v(t+1) = Pv(t)$).
- In asynchronous asymmetric gossiping, learner i sends its model to j without waiting for acknowledgment or j 's model. This makes symmetric weights impossible to negotiate.
- Lack of global rounds means $\sum_j p_{ij} = 1$ (row stochastic) and $\sum_i p_{ij} = 1$ (column stochastic) cannot be maintained simultaneously in a global snapshot.
- Column stochasticity ($P_{ii} = \gamma_i, \forall j \in \mathcal{N}(i), P_{ji} = \gamma_i$) can be enforced locally for outgoing messages, but row stochasticity cannot.

- *How Push-sum addresses this?*

- Each learner maintains a tuple (x_i, y_i) . x_i carries model information, y_i is an auxiliary scalar.
- After local update, learner i calculates $\gamma_i = 1/(|\mathcal{N}(i)| + 1)$, updates $x'_i = \gamma_i x_i$, $y'_i = \gamma_i y_i$, and sends (x'_i, y'_i) to all neighbors.
- Neighbors accumulate these received quantities into their (x, y) pair (Instantaneous Model Averaging).



Regret Bound Analysis

Assumptions:

- **Assumption 1:** The message delays are bounded by some integer $D^{msg} \geq 0$.
- **Assumption 2:** Each learner performs local updates at least once every Γ_d steps for some integer $\Gamma_d > 0$.

Definitions:

- $\mathcal{Q}_{s,t} = \mathcal{Q}_T \cap (s, t)$ denotes local updates between events s and t .
- For any $t \in \mathcal{Q}_T$, $d^p(t) = |\mathcal{Q}_{l_t,t}|$.
- $g_t = \nabla f_t(x_{i_t}(l_t)/y_{i_t}(l_t))$, $\hat{g}_t = \nabla f_t(x_{j_t}(l_t)/y_{j_t}(l_t))$ for local update gradients.
- Total processing delay: $D^{proc} = \sum_{t \in \mathcal{Q}_T} d^p(t)$.

Key Techniques for Analysis:

- **Graph Augmentation Technique:** For each real node i , create D^{msg} virtual nodes. Message passing from i to j is viewed as a delay-free path on this augmented graph. This helps to handle message delays.
- **Mixing Analysis:** After a sufficient number of time steps (related to network diameter D and max delay D^{msg} , denoted as B), the multi-step transition matrix $\mathcal{Q}_{s,s+B}$ ensures that information from any node propagates to any other real node.

Regret Bound Derivation

General Regret Upper Bound:

$$\begin{aligned} \text{Regret}_j \leq & \frac{m}{2\eta} \|\mathbf{w}_0 - \mathbf{w}^*\|^2 + \frac{2\eta}{m} \sum_{t \in \mathcal{Q}_T} \sum_{s \in \mathcal{Q}_{l_t, t+1}} g_t g_s + \eta \sum_{t \in \mathcal{Q}_T} \frac{g_t^2}{y_{i_t}(t)} \\ & + 2\eta \sum_{t \in \mathcal{Q}_T} \sum_{s \in \mathcal{Q}_{1, l_t}} \lambda^{\lfloor \frac{l_t - s}{2B} \rfloor} (g_t + 2\hat{g}_t) \frac{g_s}{y_{i_s}(t)} + 2\eta \sum_{t \in \mathcal{Q}_T} \sum_{s \in \mathcal{Q}_{1, t+1}} \lambda^{\lfloor \frac{t-s}{2B} \rfloor} g_t \frac{g_s}{y_{i_s}(t)}, \end{aligned}$$

where $\lambda = 1 - m\alpha^4$, $\alpha = (1/m)^B$, $B = (\mathcal{D} + 1)(D^{msg} + \Gamma_d)$.

Final Regret Bound (assuming $g_t \leq G$, $\hat{g}_t \leq G$, $\forall t \in \mathcal{Q}_T$ for $G < \infty$):

$$\text{Regret}_j \leq \frac{m}{2\eta} \|\mathbf{w}_0 - \mathbf{w}^*\|^2 + \frac{2\eta}{m} G^2 (T + D^{proc}) + \frac{2\eta}{m} \left(\frac{8 + \alpha^4}{\alpha^5} \right) B G^2 T$$

By setting $\eta = (mF/2G)(D^{proc} + T + CBT)^{-1/2}$, where $C = (8 + \alpha^4)/\alpha^5$, the final regret is $O((CBT + T + D^{proc})^{1/2})$.



Experimental Setup

Two Large-Scale Real-World Datasets:

- **Higgs dataset:** A benchmark dataset in high-energy physics for binary classification, consisting of 11 million instances with 28 features. Uses logical loss.
- **Poker-hand dataset:** A commonly used dataset in automatic rule generation for 10-class classification, with 1 million instances and 25 features. Uses multivariate logistic loss.

These datasets, with different tasks (binary vs. multi-class) and domains, test the algorithm's scalability and generalization.

Comparison and Verification Goals:

- Compare AD-OGP with its synchronous counterpart (D-OGP) to show the advantage of asynchronization.
- Verify the effectiveness of asymmetric gossiping and instantaneous model averaging.
- Verify our theoretical regret bound.

Network Topologies for Comparison:

- Complete graph (high connectivity)
- Watts-Strogatz graph (random graph with medium connectivity)
- Ring graph (low connectivity)

The Benefit of Asynchronization

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Effectiveness of Asymmetric Gossiping and Instantaneous Model Averaging

4.3.1 The Effectiveness of Asymmetric Gossiping

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Verification of the Theoretical Bound

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Conclusions and Future Work




Key Contributions:

- In this paper, we present the first systematic study of asynchronous decentralized online learning.
- We begin by formulating the framework of Asynchronous Decentralized Online Convex Optimization (AD-OCO), which gives a complete characterization of the whole interaction dynamics.
- Then we devise the Asynchronous Decentralized Online Gradient-Push (AD-OGP) algorithm, which is fully asynchronous and includes two novel innovations: weighted projection and instantaneous model averaging.
- Theoretically, we provide the first regret analysis paradigm for asynchronous decentralized online learning, deriving a non-trivial regret bound of $O(\sqrt{T})$.
- Finally, we conduct extensive experiments to demonstrate the benefit of asynchronization, verify the effectiveness of the two innovations, and corroborate the theoretical bound.
- Our work paves the way for future research investigating asynchronous decentralized online learning.

Limitations and Future Work:



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