Asynchronous Decentralized Online Learning

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Outline

Motivation and Background

- **Online learning**: Process large-scale streaming data efficiently
- **Existing decentralized algorithms**: Operate under synchronous setting
  - Leads to straggler problem: Fast learners wait for slow learners
- **This paper's purpose**: Design asynchronous counterpart of synchronous algorithms

Key challenges:

- Each learner has five basic types of actions: prediction, weight update, message sending, message receiving, and model averaging.
- Asynchronization: Each learner executes its own actions independently $\Rightarrow$ the orders of the actions of different learners are intermixed together.

Contributions

Novel Framework: Asynchronous Decentralized Online Convex Optimization (AD-OCO)

- First formal framework that gives a complete characterization of the whole interaction dynamics of asynchronous decentralized online learning.
- Individual view: Specify the action sequence of each learner using its own indices.
- System view: Propose a novel event indexing system $\Rightarrow$ mapping all learners into a single time axis.

AD-OGP Algorithm:

- A gossip-based communication scheme
  - Each learner is only allowed to communicate with its immediate neighbors.
  - Adopts "push-sum" strategy (a common way to realize asymmetric gossiping), which does not require any local synchronization among learners.
- A local update scheme build upon online gradient descent (OGD).
  - Design a novel weighted projection operation to avoid predictions outside the feasible domain.
- Instantaneous model averaging: Each learner performs model averaging as long as it's unoccupied and its receiving buffer is not empty.

Theoretical Contributions:

- Conduct the **first** regret analysis of asynchronous decentralized online learning.
- Extend the graph augmentation technique to online setting to handle message delays.
- Disentangle the complex effects of predictions, weight update and model averaging via our proposed event indexing system.
- Derive a non-trivial regret bound of AD-OGP: it is the same order $O(\sqrt{T})$ as that of its synchronous counterpart.
- Make an extra contribution to the convergence analysis of push-sum: reduce a factor of $\sqrt{n}$ (n is the dimension of model parameters) in the convergence rate of push-sum compared to the previous results, and the final bound is now independent of n.

Problem Formulation

Network Structure:

- Undirected graph $\mathcal{G} = (V, E)$ with vertex set $V = \{1, \ldots, m\}$
- Each learner $i \in V$ maintains a local model $w_i \in \mathcal{K}$ (convex compact decision set)
- Each learner i can only communicate with its immediate neighbors $\mathcal{N}(i) = \{j \in V \mid (i, j) \in E\}$

Individual View: Specifies the action sequence of each individual learner.

- Learner $i \in V$ is selected adversarially by the environment to make predictions at certain time point.
- Index the prediction rounds of learner i as $\tau = 1, \ldots, N_i$, learner i predicts with its most recent model w_i^τ.
- Environment reveals a convex loss function $f_i^\tau : \mathcal{K} \mapsto \mathbb{R}$, and learner suffers from the loss $f_i^\tau(w_i^\tau)$.
- After finishing the calculation, it uses the gradient to update its model $w_i \leftarrow \mathcal{U}(w_i; \nabla f_i^\tau(w_i^\tau))$, where $\mathcal{U}$ is the local update scheme.

- Then it selects a subset of neighbors, and *sends* a copy of its updated model to each of them.
- Besides sending out its own model copies, learner i will also *receive* its neighbors' model copies at certain time points. These copies will be stored in receiving buffer $\mathcal{B}_i$.
- Whenever learner i is not occupied with gradient calculation or local update, and its buffer is not empty, it can *average* its model with the model copies stored in the buffer, i.e., $w_i \leftarrow \mathcal{A}(w_i; \mathcal{B}_i)$, where $\mathcal{A}$ is the model averaging scheme.

System View and Event Indexing

Global Event Sequence: Due to asynchronization, each learner executes its actions independently. $\Rightarrow$ The order of actions of all learners are intermixed.

- We need to specify the complete orders of the actions of all learners by mapping them into a single time axis.
- We view either a **predictions** or a **local update** of any learner as an event, and use a virtual counter t to index these events.
- Let $T = \sum_{i \in V} N_i$ be the total number of **predictions** of all learners, then the total number of **local updates** is also T.
- For each event $t \in \{1, \ldots, 2T\}$, we use $\delta_t \in \{0, 1\}$ to distinguish the event type:
  - $\delta_t = 0$ if it is a prediction event.
  - $\delta_t = 1$ if it is a local update event.
- We further use i_t to denote the learner that executes the event t, and f_t to denote the associated feedback.
- Then there exists some $\tau \in \{1, \ldots, N_{i_t}\}$ such that $f_t = f_{i_t}^{\tau}$.

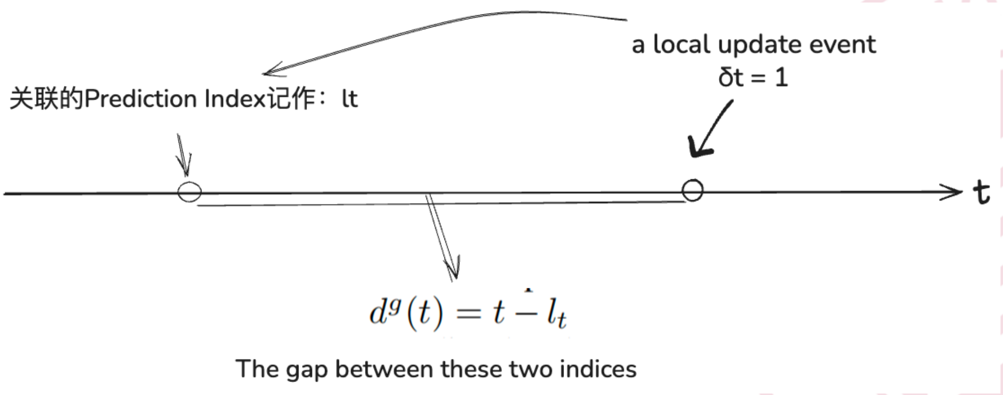

a local update event
δt = 1
关联的Prediction Index记作：lt
t
$d^g(t) = t - l_t$
The gap between these two indices

image: An example of Prediction and local update.

Delays and Model Evolution

Processing delay: Since any prediction must occur before its corresponding local update, here we have $t \geq 2$, $l_t < t$ and $d^g(t) > 0$.

$$d^p(t) = \text{card}\{l_t < s < t \mid \delta_s = 1\}$$

i.e., the number of local updates between prediction and its associated local update.

Message delay: After any local update event t, the learner i_t that executes this event will send out its model copy to a subset S_t of its neighbors $\mathcal{N}(i_t)$. We focus on the copy sent to any neighbor $j \in S_t$. We assume such copy is used to average learner j's model between events r_{tj} and $(r_{tj} + 1)$ for some $r_{tj} \geq t$. For the message sent from learner i_t to learner j after event t, we introduce a quantity termed the message delay $d^m_{i_t}(t) = r_{tj} - t$.

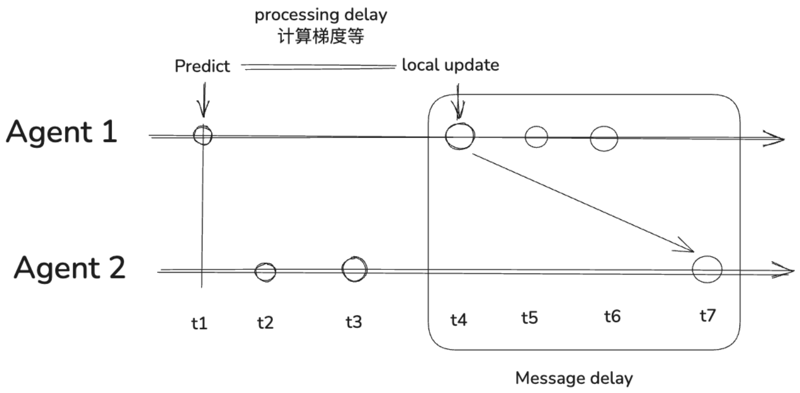


image: An example of Processing delay and Message delay.

Model Evolution and Regret Definition

Model Evolution: We denote $w_i(t)$ as the most recent model before event t occurs, and $u_i(t)$ as the model immediately after event t.

- For an update event executed by learner i_t, its associated gradient is calculated based on learner i_t's prediction using model $w_{i_t}(l_t)$ at event l_t:

$$u_{i_t} = \mathcal{U}(w_{i_t}(t); \nabla f_{l_t}(w_{i_t}(l_t)))$$

  The models of other learners do not change at i_t's local update event, so $u_j(t) = w_j(t)$ for $j \neq i_t$.
- Model averaging operation is not regarded as an event. It happens between t and $t+1$ if learner i is not occupied.
  - Model at the beginning: $u_i(t)$
  - Model at the end: $w_i(t+1)$

  So, the overall effect of model averaging on learner i between event t and $t+1$ can be expressed as:

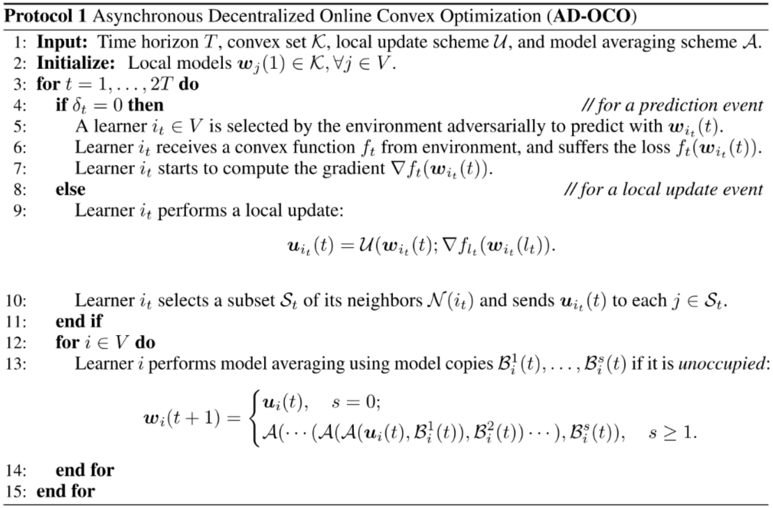
$$\boldsymbol{w}_i(t+1) = \begin{cases} \boldsymbol{u}_i(t), & s = 0 \\ \mathcal{A}\left(\cdots\left(\mathcal{A}\left(\mathcal{A}\left(\boldsymbol{u}_i(t), \mathcal{B}_i^1(t)\right), \mathcal{B}_i^2(t)\right)\cdots\right), \mathcal{B}_i^s(t)\right), & s \geq 1 \end{cases}$$

Regret Definition: In AD-OCO, the regret w.r.t an arbitrary reference learner $j \in V$ is defined as:

$$\text{Regret}_j = \sum_{t \in \mathcal{P}_T} f_t\left(\boldsymbol{w}_j(t)\right) - \sum_{t \in \mathcal{P}_T} f_t\left(\boldsymbol{w}^*\right)$$

where $\mathcal{P}_T = \{t \in \{1, \ldots, 2T\} \mid \delta_t = 0\}$ denotes the set of all prediction events, and $w^* \in \operatorname{argmin}_{\boldsymbol{w} \in \mathcal{K}} \sum_{t \in \mathcal{P}_T} f_t(\boldsymbol{w})$ refers to a **fixed model** chosen with hindsight.
Goal: Learners aim to make a prediction sequence $\{\boldsymbol{w}_i(t)\}_{i \in V, t \in \mathcal{P}_T}$, such that the regret of any reference learner j grows sublinearly with time, i.e.,

$$\lim_{T \to \infty} \text{Regret}_j / T = 0$$



Protocol 1 Asynchronous Decentralized Online Convex Optimization (**AD-OCO**)

```
1: Input: Time horizon $T$, convex set $\mathcal{K}$, local update scheme $\mathcal{U}$, and model averaging scheme $\mathcal{A}$.
2: Initialize: Local models $\boldsymbol{w}_j(1) \in \mathcal{K}, \forall j \in V$.
3: for $t = 1, \ldots, 2T$ do
4:     if $\delta_t = 0$ then                                   // for a prediction event
5:         A learner $i_t \in V$ is selected by the environment adversarially to predict with $\boldsymbol{w}_{i_t}(t)$.
6:         Learner $i_t$ receives a convex function $f_t$ from environment, and suffers the loss $f_t(\boldsymbol{w}_{i_t}(t))$.
7:         Learner $i_t$ starts to compute the gradient $\nabla f_t(\boldsymbol{w}_{i_t}(t))$.
8:     else                                                     // for a local update event
9:         Learner $i_t$ performs a local update:
               $\boldsymbol{u}_{i_t}(t) = \mathcal{U}(\boldsymbol{w}_{i_t}(t); \nabla f_{l_t}(\boldsymbol{w}_{i_t}(l_t)))$.
10:        Learner $i_t$ selects a subset $\mathcal{S}_t$ of its neighbors $\mathcal{N}(i_t)$ and sends $\boldsymbol{u}_{i_t}(t)$ to each $j \in \mathcal{S}_t$.
11:    end if
12:    for $i \in V$ do
13:        Learner $i$ performs model averaging using model copies $\mathcal{B}_i^1(t), \ldots, \mathcal{B}_i^s(t)$ if it is unoccupied:
               $\boldsymbol{w}_i(t+1) = \begin{cases} \boldsymbol{u}_i(t), & s = 0; \\ \mathcal{A}(\cdots(\mathcal{A}(\mathcal{A}(\boldsymbol{u}_i(t), \mathcal{B}_i^1(t)), \mathcal{B}_i^2(t))\cdots), \mathcal{B}_i^s(t)), & s \geq 1. \end{cases}$
14:    end for
15: end for
```

image: The AD-OGP Algorithm Framework (Algorithm 1).

Algorithm Overview

Initial State: Each node i maintains a pair of variables (x_i, y_i), and predicts with $w_{i_t}(t) = \frac{x_{i_t}(t)}{y_{i_t}(t)}$.

- x_i is a **weighted model**, y_i is a **weight scalar** (always > 0). Both are used for push-sum.
- Initialization: $x_i = w_0 \in \mathcal{K}$ ($\mathcal{K}$ is a convex compact decision set), $y_i = 1$.

AD-OGP Algorithm Steps:

Algorithm 1 Asynchronous Decentralized Online Gradient-Push (AD-OGP)

```
1: Input: Learning rate $\eta$, initial model $w_0 \in \mathcal{K}$
2: Initialize: For each learner $i \in V$, $x_i \leftarrow w_0$, $y_i \leftarrow 1$. Empty receiving buffer $\mathcal{B}_i$.
3: Loop for each event $t \in \{1, \dots, 2T\}$:
4:     If $\delta_t = 0$ (prediction event) for learner $i_t$:
5:         $w_{i_t}(t) \leftarrow x_{i_t}(t) / y_{i_t}(t)$
6:         Make prediction with $w_{i_t}(t)$, receive loss function $f_t$.
7:     If $\delta_t = 1$ (local update event) for learner $i_t$:
8:         Let $l_t$ be the prediction event corresponding to $f_t$.
9:         Compute gradient $g_t = \nabla f_{l_t}(w_{i_t}(l_t))$.
```

Push-Sum Communication Strategy

Challenge: Decentralized model averaging typically involves linear averaging with neighbors. In an asynchronous and asymmetric setting, traditional doubly-stochastic averaging is not feasible.

- *Why?*
  - Doubly-stochastic matrices require symmetric interactions ($p_{ij} = p_{ji}$) and global synchronization ($v(t+1) = Pv(t)$).
  - In asynchronous asymmetric gossiping, learner i sends its model to j without waiting for acknowledgment or j's model. This makes symmetric weights impossible to negotiate.
  - Lack of global rounds means $\sum_j p_{ij} = 1$ (row stochastic) and $\sum_i p_{ij} = 1$ (column stochastic) cannot be maintained simultaneously in a global snapshot.
  - Column stochasticity ($P_{ii} = \gamma_i$, $\forall j \in \mathcal{N}(i)$, $P_{ji} = \gamma_i$) can be enforced locally for outgoing messages, but row stochasticity cannot.
- *How Push-sum addresses this?*
  - Each learner maintains a tuple (x_i, y_i). x_i carries model information, y_i is an auxiliary scalar.
  - After local update, learner i calculates $\gamma_i = 1/(|\mathcal{N}(i)| + 1)$, updates $x_i' = \gamma_i x_i$, $y_i' = \gamma_i y_i$, and sends (x_i', y_i') to all neighbors.
  - Neighbors accumulate these received quantities into their (x, y) pair (Instantaneous Model Averaging).
  - The predicted model is $z_i = x_i/y_i$

Regret Bound Analysis

Assumptions:

- **Assumption 1**: The message delays are bounded by some integer $D^{msg} \geq 0$.
- **Assumption 2**: Each learner performs local updates at least once every Γ_d steps for some integer $\Gamma_d > 0$.

Definitions:

- $\mathcal{Q}_{s,t} = \mathcal{Q}_T \cap (s, t)$ denotes local updates between events s and t.
- For any $t \in \mathcal{Q}_T$, $d^p(t) = |\mathcal{Q}_{l_t,t}|$.
- $g_t = \nabla f_{l_t}(x_{i_t}(l_t)/y_{i_t}(l_t))$, $\hat{g}_t = \nabla f_{l_t}(x_j(l_t)/y_j(l_t))$ for local update gradients.
- Total processing delay: $D^{proc} = \sum_{t\in\mathcal{Q}_T} d^p(t)$.

Key Techniques for Analysis:

- **Graph Augmentation Technique**: For each real node i, create D^{msg} virtual nodes. Message passing from i to j is viewed as a delay-free path on this augmented graph. This helps to handle message delays.
- **Mixing Analysis**: After a sufficient number of time steps (related to network diameter $\mathcal{D}$ and max delay D^{msg}, denoted as B), the multi-step transition matrix $\mathcal{Q}_{s,s+B}$ ensures that information from any node propagates to any other real node.

Regret Bound Derivation

General Regret Upper Bound:

$$\text{Regret}_j \leq \frac{m}{2\eta}\|\boldsymbol{w}_0 - \boldsymbol{w}^*\|^2 + \frac{2\eta}{m}\sum_{t\in\mathcal{Q}_T}\sum_{s\in\mathcal{Q}_{l_t,t+1}} g_t g_s + \eta\sum_{t\in\mathcal{Q}_T}\frac{g_t^2}{y_{i_t}(t)}$$
$$+2\eta\sum_{t\in\mathcal{Q}_T}\sum_{s\in\mathcal{Q}_{1,l_t}}\lambda^{\lfloor\frac{l_t-s}{2B}\rfloor}(g_t+2\hat{g}_t)\frac{g_s}{y_{i_s}(t)} + 2\eta\sum_{t\in\mathcal{Q}_T}\sum_{s\in\mathcal{Q}_{1,t+1}}\lambda^{\lfloor\frac{t-s}{2B}\rfloor} g_t\frac{g_s}{y_{i_s}(t)},$$

where $\lambda = 1 - m\alpha^4$, $\alpha = (1/m)^B$, $B = (\mathcal{D}+1)(D^{msg}+\Gamma_d)$.

Final Regret Bound (assuming $g_t \leq G, \hat{g}_t \leq G, \forall t \in \mathcal{Q}_T$ for $G < \infty$):

$$\textbf{Regret}_j \leq \frac{m}{2\eta}\|\boldsymbol{w}_0 - \boldsymbol{w}^*\|^2 + \frac{2\eta}{m}G^2(T + D^{proc}) + \frac{2\eta}{m}\left(\frac{8+\alpha^4}{\alpha^5}\right)BG^2T$$

By setting $\eta = (mF/2G)(D^{proc} + T + CBT)^{-1/2}$, where $C = (8+\alpha^4)/\alpha^5$, the final regret is $O((CBT + T + D^{proc})^{1/2})$.

Experimental Setup

Two Large-Scale Real-World Datasets:

- **Higgs dataset**: A benchmark dataset in high-energy physics for binary classification, consisting of 11 million instances with 28 features. Uses logical loss.
- **Poker-hand dataset**: A commonly used dataset in automatic rule generation for 10-class classification, with 1 million instances and 25 features. Uses multivariate logistic loss.

These datasets, with different tasks (binary vs. multi-class) and domains, test the algorithm's scalability and generalization.

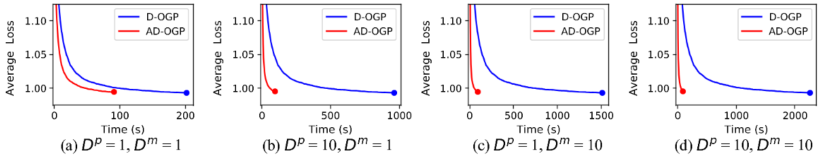
Comparison and Verification Goals:

- Compare AD-OGP with D-OGP (synchronous) to show the advantage of asynchronization.
- Verify the effectiveness of asymmetric gossiping and instantaneous model averaging.
- Verify our theoretical regret bound.

Network Topologies for Comparison:

- Complete graph (high connectivity)
- Watts-Strogatz graph (random graph with medium connectivity)
- Ring graph (low connectivity)

The Benefit of Asynchronization



(a) $D^p = 1, D^m = 1$ (b) $D^p = 10, D^m = 1$ (c) $D^p = 1, D^m = 10$ (d) $D^p = 10, D^m = 10$

image: Illustration of the benefit of asynchronization. The four plots compare AD-OGP and D-OGP on 64-node Watts-Strogatz graphs on poker-hand under varying delay levels.

- AD-OGP's running speed is significantly faster than D-OGP.
- The performance gap increases with higher processing and message delays.
- AD-OGP's average loss is comparable to D-OGP, indicating no sacrifice in learning performance.

The Effectiveness of Asymmetric Gossiping

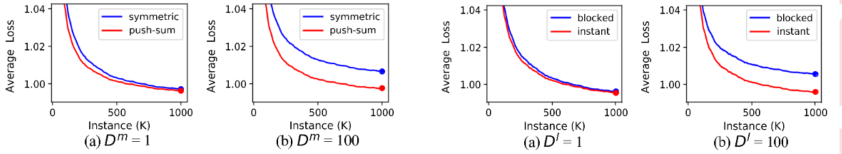


image: Illustration of the effectiveness of asymmetric gossiping.

- Increasing D^{msg} artificially creates communication bottlenecks.
- Experiments show that asymmetric gossip communication significantly improves performance, especially prominent with high communication delays.

The Effectiveness of Instantaneous Model Averaging

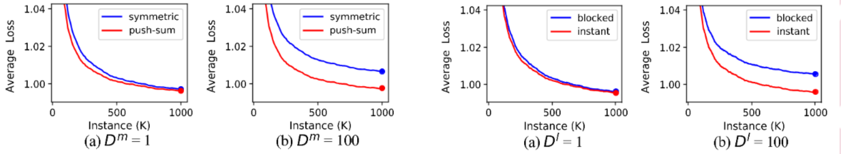
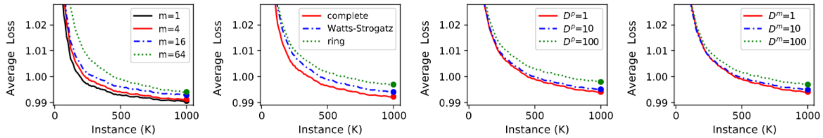


image: Illustration of the effectiveness of instantaneous model averaging.

- Adjusting D_l magnifies learner waiting time.
- Experiments demonstrate that instantaneous model averaging significantly boosts performance by utilizing idle time, especially when noticeable idle periods exist.

Verification of the Theoretical Bound



(a) network sizes (b) network topology (c) processing delays (d) message delays

image: Verification of our regret bound. The four plots show AD-OGP′ s performance on poker-hand with varying characteristics. (a) Varying network sizes. (b) Varying network topology. (c) Varying levels of processing delays. (d) Varying levels of message delays.

- Experiments adjust four factors in the regret bound formula across the four sub-figures.
- Results indicate that AD-OGP's loss is higher when:
  - The number of learners is larger.
  - Network connectivity is worse.
  - Processing delays are heavier.
  - Message delays are heavier.
- This shows a high consistency between experimental behavior and theoretical results.

Conclusions

Key Contributions:

- In this paper, we present the first systematic study of asynchronous decentralized online learning.
- We begin by formulating the framework of Asynchronous Decentralized Online Convex Optimization (AD-OCO), which gives a complete characterization of the whole interaction dynamics.
- Then we devise the Asynchronous Decentralized Online Gradient-Push (AD-OGP) algorithm, which is fully asynchronous and includes two novel innovations: weighted projection and instantaneous model averaging.
- Theoretically, we provide the first regret analysis paradigm for asynchronous decentralized online learning, deriving a non-trivial regret bound of $O(\sqrt{T})$.
- Finally, we conduct extensive experiments to demonstrate the benefit of asynchronization, verify the effectiveness of the two innovations, and corroborate the theoretical bound.
- Our work paves the way for future research investigating asynchronous decentralized online learning.

Limitations and Future Work

Limitations and Future Work:

- As the first step of studying asynchronous learning in the decentralized online setting, our framework formulation and theoretical analysis are conducted in the convex setting.
- Although such limitation does not affect its usage in the non-convex setting, we would like to establish a formal non-convex analysis in the future.
- Further explore more complex network scenarios and dynamic topologies.

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