

Tabla de integrales inmediatas	
Función	Primitiva
$[f(x)]^m f'(x), m \neq 1$	$\frac{[f(x)]^{m+1}}{m+1}$
$a^{f(x)} f'(x)$	$\frac{a^{f(x)}}{\ln(a)}$
$\cos[f(x)] f'(x)$	$\text{sen}[f(x)]$
$\cotg[f(x)] f'(x)$	$\ln  \text{sen}[f(x)] $
$\frac{f'(x)}{\text{sen}^2[f(x)]}$	$-\cotg[f(x)]$
$\frac{f'(x)}{1 + [f'(x)]^2}$	$-\arctg[f(x)]$
$\frac{f'(x)}{f(x)}$	$\ln  f(x) $
$\text{sen}[f(x)] f'(x)$	$-\cos[f(x)]$
$\text{tg}[f(x)] f'(x)$	$-\ln  \cos[f(x)] $
$\frac{f'(x)}{\cos^2[f(x)]}$	$\text{tg}[f(x)]$
$\frac{f'(x)}{\sqrt{1 - [f(x)]^2}}$	$\arcsen[f(x)]$

## Integrales trigonométricas

Cambio universal:

$$t = \text{tg}\left(\frac{x}{2}\right), dx = \frac{2}{1+t^2} dt, \text{sen}(x) = \frac{2t}{1+t^2}, \cos(x) = \frac{1-t^2}{1+t^2}$$

$$\int R[\text{sen}(x)] \cos(x) dx; t = \text{sen}(x), dt = \cos(x) dx$$

$$\int R[\cos(x)] \text{sen}(x) dx; t = \cos(x), dt = -\text{sen}(x) dx$$

$$\int R(\text{tg}(x)) dx; t = \text{tg}(x), x = \arctg(t), dx = \frac{dt}{1+t^2}$$

$$\int R(\text{sen}(x), \cos(x)) dx$$

$$\text{con } R(\cos(x), \text{sen}(x)) = \text{sen}^m(x) \cdot \cos^n(x)$$

(a) si algún exponente es impar, se realiza el cambio  $t =$  la otra función

(b) si  $m, n$  son pares no negativos:

$$\text{sen}^2(x) = \frac{1 - \cos(2x)}{2}, \cos^2(x) = \frac{1 + \cos(2x)}{2}$$

(c) si  $m, n$  son pares y alguno es negativo:

$$t = \text{tg}(x), dx = \frac{1}{1+t^2} dt, \text{sen}^2(x) = \frac{t^2}{1+t^2} dt,$$

$$\cos^2(x) = \frac{1}{1+t^2} dt$$

$$\int \cos(px) \cdot \cos(qx) dx, \int \text{sen}(px) \cdot \text{sen}(qx) dx,$$

$$\int \text{sen}(px) \cdot \cos(qx) dx$$

$$\cos(px) \cdot \cos(qx) = \frac{\cos[(p+q)x] + \cos[(p-q)x]}{2}$$

$$\text{sen}(px) \cdot \text{sen}(qx) = \frac{-\cos[(p+q)x] + \cos[(p-q)x]}{2}$$

$$\text{sen}(px) \cdot \text{sen}(qx) = \frac{\text{sen}[(p+q)x] + \text{sen}[(p-q)x]}{2}$$

**Integrales racionales:**  $\int \frac{P(x)}{Q(x)} dx$  se factoriza  $Q(x)$  y se escribe  $\frac{P(x)}{Q(x)}$  como suma de fracciones simples.

$$\int \frac{1}{x-a} dx = \ln|x-a|$$

$$\int \frac{1}{(x-a)^p} dx = \frac{(x-a)^{-p+1}}{-p+1}, p \neq 1$$

$$\int \frac{Mx+N}{(x-a)^2+b^2} dx = \frac{M}{2} |\ln|(x-a)^2+b^2| + \frac{N+Ma}{b} \arctg\left(\frac{x-a}{b}\right)$$

$$\text{Método de Hermite: } \int \frac{P(x)}{Q(x)} dx = \frac{Y(x)}{F(x)} + \int \frac{X(x)}{G(x)} dx$$

$$\text{Siendo } F(x) = M.C.D.(Q(x), Q'(x)), G(x) = \frac{Q(x)}{F(x)}$$

$Y(x)$  polinomio a determinar con  $\text{grado}(Y) < \text{grado}(F)$

$X(x)$  polinomio a determinar con  $\text{grado}(X) < \text{grado}(G)$

## Integrales irracionales

$$\int R\left(x, \left(\frac{ax+b}{cx+d}\right)^{m/n}, \dots, \left(\frac{ax+b}{cx+d}\right)^{r/s}\right) dx$$

$$\text{con } t^k = \frac{ax+b}{cx+d}, k = m.c.m.\{n, \dots, s\}$$

$\int R\left(x, \sqrt{x^2+a^2}\right) dx$	$x = a \text{ tg}(t), \sqrt{x^2+a^2} = \frac{a}{\cos(t)},$ $dx = \frac{a}{\cos^2(t)} dt$
	$x = a \cotg(t), \sqrt{x^2+a^2} = \frac{a}{\text{sen}(t)}$ $dx = \frac{-a}{\text{sen}^2(t)} dt$
$\int R\left(x, \sqrt{x^2-a^2}\right) dx$	$x = a \sec(t), \sqrt{x^2-a^2} = a \text{ tg}(t),$ $dx = a \text{ tg}(t) \sec(t) dt$
	$x = a \text{ cosec}(t), \sqrt{x^2-a^2} = a \cotg(t),$ $dx = -a \cotg(t) \text{ cosec}(t) dt$
$\int R\left(x, \sqrt{a^2-x^2}\right) dx$	$x = a \text{ sen}(t), \sqrt{a^2-x^2} = a \cos(t),$ $dx = a \cos(t) dt$
	$x = a \cos(t), \sqrt{a^2-x^2} = a \text{ sen}(t),$ $dx = -a \text{ sen}(t) dt$

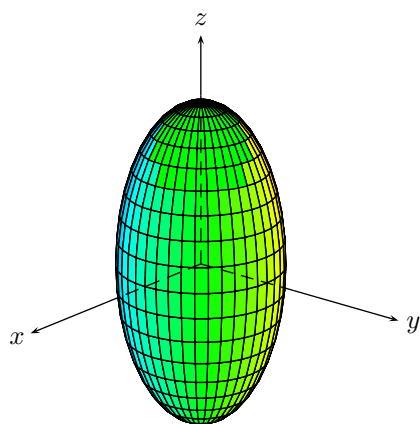
$$\int R\left(x, \sqrt{x^2+bx+c}\right) dx$$

$$a > 0; \sqrt{x^2+bx+c} = \sqrt{a}x + t, x = \frac{t^2-c}{b-2\sqrt{at}}$$

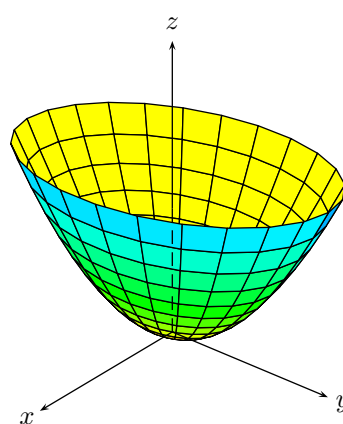
$$c > 0; \sqrt{x^2+bx+c} = xt + \sqrt{c}, x = \frac{2t\sqrt{c}-b}{a-t^2}$$

si  $a < 0$  y  $\alpha$  es una raíz de  $ax^2+bx+c$  se realiza el cambio:

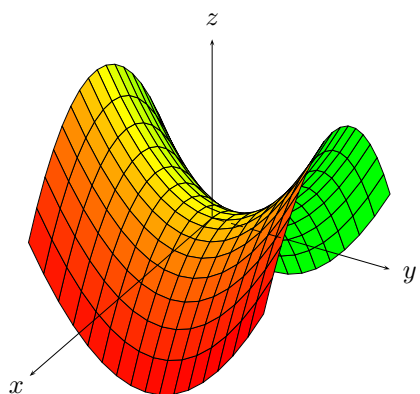
$$\sqrt{x^2+bx+c} = (x-\alpha)t$$



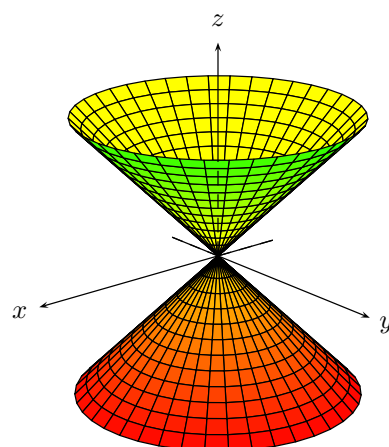
**Figura 1:** Elipsoide  $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$



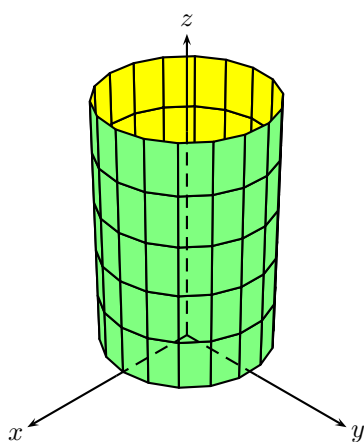
**Figura 2:** Paraboloide elíptico  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = z$



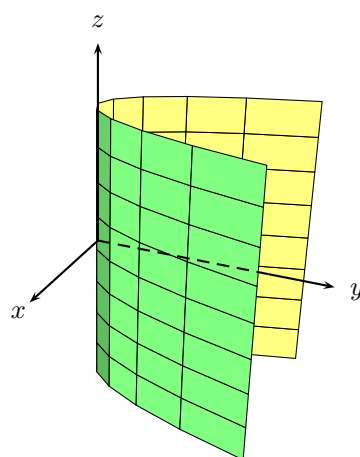
**Figura 3:** Paraboloide hiperbólico  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = z$



**Figura 4:** Cono elíptico  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = z^2$



**Figura 5:** Cilindro elíptico  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



**Figura 6:** Cilindro parabólico  $y = ax^2$ ,  $a > 0$