GENERAL ASSEMBLY

INTRO TO DATA SCIENCE

PROBABILITY AND NAIVE BAYESIAN CLASSIFICATION

LAST TIME:

- LINEAR REGRESSION
- BUILDING EFFECTIVE MODELS
- SCORING REGRESSION MODEL PERFORMANCE

QUESTIONS?

I. PROBABILITY

II. NAÏVE BAYESIAN CLASSIFICATION

EXERCISES:

III. IMPLEMENTING NB CLASSIFICATION

1% of women at age forty who participate in routine screening have breast cancer. 80% of women with breast cancer will get positive mammographies. 9.6% of women without breast cancer will also get positive mammographies. A woman in this age group had a positive mammography in a routine screening. What is the probability that she actually has breast cancer?

I. INTRO TO PROBABILITY

Q: What is a **probability***?*

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Examples

The probability of getting heads on a coin flip is .5

The probability of picking the 1 red ball in a bag of 8 balls is .125

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The probability of the sample space $P(\Omega)$ *is 1.*

Q: Consider two events A & B. How can we characterize the **intersection** *of these events?*

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A: With the **joint probability** *of A and B, written $P(AB)$.*

Examples

*The probability of rolling a die as an **odd**(A) **prime** (B) number is ...*

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Examples

The probability of rolling a die as an ***odd****(A)* ***prime*** *(B) number is ...*

	1	2	3	4	5	6
A:	O	E	O	E	O	E
B:	N	P	P	N	P	N

Q: Consider two events A & B. How can we characterize the **intersection** *of these events?*

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Examples

The probability of rolling a die as an ***odd*** *(A)* ***prime*** *(B) number is 2/6, or .333*

	1	2	3	4	5	6
A:	O	E	O	E	O	E
B:	N	P	P	N	P	N

Q: Consider two events A & B. How can we characterize the **intersection** *of these events?*

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Question:

What's the probability of rolling an ***even prime number?***

	1	2	3	4	5	6
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B:	N	P	P	N	P	N

Q: Consider two events A & B. How can we characterize the **intersection** *of these events?*

A: With the **joint probability** *of A and B, written $P(AB)$.*

Question:

What's the probability of rolling an ***even prime number? 1/6 (.1667)***

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This information about B transforms the sample space.

Take a moment to convince yourself of this!

Q: Suppose event B has occurred. What quantity represents the probability of A **given** *this information about B?*

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This is called the **conditional probability** *of A given B, written $P(A \mid B) = P(AB) \,/\, P(B)$.*

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Notice, with this we can also write $P(AB) = P(A \mid B) * P(B)$.

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Conditional probability: $P(A \mid B) = P(AB) \,/\, P(B)$

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$P(AB) = P(odd\ and\ prime) = .333 \quad (1/3, or\ 2/6)$
$P(B) = P(prime) = .5 \quad (1/5, or\ 3/6)$

Conditional probability: $P(A \mid B) = P(AB) \,/\, P(B)$

Question: *If someone announces they rolled an even number, what is the probability that it was prime?*

$P(AB) = P(\text{even and prime}) = .166 \quad (1/6)$
$P(B) = P(\text{even}) = .5 \quad (1/2)$

$P(A \mid B) = P(\text{prime given even}) = .166 \,/\, .5 = .333 \; (1/3)$

Review time. Determine conditional probability for each!

We have ten brown balls, 15 brown cubes, 18 green balls, and 25 green cubes in a bag. Michael takes an item out of the bag and announces...

1) It's green. What's the probability it's a cube?
2) It's brown. What's the probability it's a cube?
3) It's a ball. What's the probability it's green?
4) It's a cube. What's the probability it's a ball?

Conditional probability: $P(A \mid B) = P(AB) \,/\, P(B)$

Back to the roll problem: If someone rolls a prime number, what is the probability that they rolled an odd?

$P(AB) = P(\text{odd and prime}) = .333 \quad (1/3\text{, or }2/6)$
$P(B) = P(\text{prime}) = .5 \quad (1/5\text{, or }3/6)$

$P(A \mid B) = P(\text{odd given prime}) = .333 \,/\, .5 = .666\ (4/6)$

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Using the definition of the conditional probability, we can also write:

$$P(A \mid B) = P(AB) \,/\, P(B) = P(A) \;\rightarrow\; P(AB) = P(A) * P(B)$$

$$P(AB) = P(A \mid B) * P(B)$$

from last slide

$P(AB) = P(A \mid B) * P(B)$ *from last slide*

$P(BA) = P(B \mid A) * P(A)$ *by substitution*

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→ $P(A \mid B) * P(B) = P(B \mid A) * P(A)$ *by combining the above*

→ $P(A \mid B) = P(B \mid A) * P(A) \,/\, P(B)$ *by rearranging last step*

This result is called **Bayes' theorem**. *Here it is again:*

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Some facts:

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Some facts:

- This is a simple algebraic relationship using elementary definitions.

- It's interesting because it's kind of a "wormhole" between two different "interpretations" of probability.

- It's a very powerful computational tool.

Things to consider:

Flipping a coin to see if it's heads or tails is our ***test*** *to see which coin was chosen.*

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Flipping a coin to see if it's heads or tails is our ***test*** *to see which coin was chosen.*

Our probability can change ***dependent*** *on previous results. Two heads in a row did not confirm anything, but only changed our perception of probability for each coin.*

Briefly, the two interpretations can be described as follows:

II. NAÏVE BAYESIAN CLASSIFICATION

Suppose we have a dataset with features $x_1, \ldots, x_n$ and a class label C. What can we say about classification using Bayes' theorem?

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$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

Bayes' theorem can help us to determine the probability of a record belonging to a class, given the data we observe.

source: Data Analysis with Open Source Tools, by Philipp K. Janert. O'Reilly Media, 2011.

Each term in this relationship has a name, and each plays a distinct role in any Bayesian calculation (including ours).

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

This term is the **likelihood function**. *It represents the joint probability of observing features $\{x_i\}$ given that that record belongs to class C.*

$$P(\text{class } C \mid \{x_i\}) = \frac{\boxed{P(\{x_i\} \mid \text{class } C)} \cdot P(\text{class } C)}{P(\{x_i\})}$$

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We can observe the value of the likelihood function from the training data.

This term is the **prior probability** *of C. It represents the probability of a record belonging to class C before the data is taken into account.*

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot \boxed{P(\text{class } C)}}{P(\{x_i\})}$$

This term is the **prior probability** *of* C. *It represents the probability of a record belonging to class* C *before the data is taken into account.*

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

The value of the prior is also observed from the data.

This term is the **normalization constant.** *It doesn't depend on* C*, and is generally ignored until the end of the computation.*

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{\boxed{P(\{x_i\})}}$$

This term is the **normalization constant.** *It doesn't depend on C, and is generally ignored until the end of the computation.*

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

The normalization constant doesn't tell us much.

This term is the **posterior probability** *of* C. *It represents the probability of a record belonging to class* C *after the data is taken into account.*

$$\boxed{P(\text{class } C \mid \{x_i\})} = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

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The goal of any Bayesian computation is to find ("learn") the posterior distribution of a particular variable.

The idea of Bayesian inference, then, is to **update** *our beliefs about the distribution of C using the data ("evidence") at our disposal.*

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

Then we can use the posterior for prediction.

Q: What piece of the puzzle we've seen so far looks like it could intractably difficult in practice?

Remember the likelihood function?

$$P(\{x_i\} \mid C) = P(\{x_1, x_2, \ldots, x_n\}) \mid C)$$

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Observing this exactly would require us to have enough data for every possible combination of features to make a reasonable estimate.

Q: What piece of the puzzle we've seen so far looks like it could intractably difficult in practice?

A: Estimating the full likelihood function.

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$$P(\{x_i\} \mid C) = P(x_1, x_2, \ldots, x_n \mid C) \approx P(x_1 \mid C) * P(x_2 \mid C) * \ldots * P(x_n \mid C)$$

Q: What is this classification best suited for?

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A: More often than not, Naive Bayes makes a great **text classifier**.

(Classic) Example: Classifying email as either spam or ham

Q: What are our features?

A: The text available in emails

(Classic) Example: Classifying email as either spam or ham

Q: How do we turn the text into features?

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A: Word counts, frequency matrices, tf-idf

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Q: How do we turn the text into features?

A: Word counts, frequency matrices,

We can make alterations to this dictionary/features: dropping stop words, for example.

$$p(spam \mid word) = \frac{p(word \mid spam)p(spam)}{p(word)}$$

Expanding from words to a document, each email can be represented by a binary vector, whose ith entry is 1 or 0 depending on whether the ith word appears.

(Classic) Example: Classifying email as either spam or ham

html,	table,	Nigerian,	prince,	lunch,	break,	U.S.	spam
1	1	1	1	0	0	1	1
0	1	0	0	1	1	1	0

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html	table	Nigerian	prince	lunch	break	U.S.	spam
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0	1	0	0	1	1	1	0

Now we want to learn P(word|spam) ie, what’s the probability this word shows up given that it’s spam?

This is what makes it supervised learning!

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Now that we've trained our data, we want to compute probability for each class (spam = 1 and spam = 0)

Bayesian spam filtering adapts per user

The word ‘Nigeria’ is very indicative of advance fee fraud.

But a spouse’s name might be indicative of importance -- so there are not hard or fast rules

Supervised Classification Framework