

From the classical EM derivation at iteration t , we have, with \mathbf{X} the observed random variables and \mathbf{Z} the hidden random variables, for all sets of parameters $\boldsymbol{\theta}$ and for a fixed set of parameters $\boldsymbol{\theta}^{(t)}$:

$$\underbrace{\log p_{\boldsymbol{\theta}}(\mathbf{x})}_{l(\boldsymbol{\theta})} = \underbrace{\mathbb{E}_{p_{\boldsymbol{\theta}^{(t)}}(\mathbf{z}|\mathbf{x})}[\log p_{\boldsymbol{\theta}}(\mathbf{z}, \mathbf{x})]}_{Q(\boldsymbol{\theta}|\boldsymbol{\theta}^{(t)})} - \underbrace{\mathbb{E}_{p_{\boldsymbol{\theta}^{(t)}}(\mathbf{z}|\mathbf{x})}[\log p_{\boldsymbol{\theta}}(\mathbf{z}|\mathbf{x})]}_{H(\boldsymbol{\theta}|\boldsymbol{\theta}^{(t)})},$$

where l is the model log-likelihood, Q is the classical EM quantity and H is the entropy. Hence, $\forall \boldsymbol{\theta}$:

$$\begin{aligned} l(\boldsymbol{\theta}) &= Q(\boldsymbol{\theta}|\boldsymbol{\theta}^{(t)}) + H(\boldsymbol{\theta}|\boldsymbol{\theta}^{(t)}) \\ \implies l(\boldsymbol{\theta}) &\geq Q(\boldsymbol{\theta}|\boldsymbol{\theta}^{(t)}) + H(\boldsymbol{\theta}^{(t)}|\boldsymbol{\theta}^{(t)}) \\ &\quad \text{because } \forall \boldsymbol{\theta}, H(\boldsymbol{\theta}|\boldsymbol{\theta}^{(t)}) \geq H(\boldsymbol{\theta}^{(t)}|\boldsymbol{\theta}^{(t)}) \\ \implies l(\boldsymbol{\theta}) - Q(\boldsymbol{\theta}|\boldsymbol{\theta}^{(t)}) &\geq H(\boldsymbol{\theta}^{(t)}|\boldsymbol{\theta}^{(t)}). \end{aligned}$$

Then we have that the minimum of $l(\boldsymbol{\theta}) - Q(\boldsymbol{\theta}|\boldsymbol{\theta}^{(t)})$ is $H(\boldsymbol{\theta}^{(t)}|\boldsymbol{\theta}^{(t)})$ but we know that $l(\boldsymbol{\theta}^{(t)}) - Q(\boldsymbol{\theta}^{(t)}|\boldsymbol{\theta}^{(t)}) = H(\boldsymbol{\theta}^{(t)}|\boldsymbol{\theta}^{(t)})$. Thus $\boldsymbol{\theta} = \boldsymbol{\theta}^{(t)}$ is the place of the minimum, and by differentiating with respect to $\boldsymbol{\theta}$:

$$\begin{aligned} \nabla_{\boldsymbol{\theta}} \{l(\boldsymbol{\theta}) - Q(\boldsymbol{\theta}|\boldsymbol{\theta}^{(t)})\} |_{\boldsymbol{\theta}=\boldsymbol{\theta}^{(t)}} &= 0 \\ \implies \nabla_{\boldsymbol{\theta}} \{l(\boldsymbol{\theta})\} |_{\boldsymbol{\theta}=\boldsymbol{\theta}^{(t)}} &= \nabla_{\boldsymbol{\theta}} \{Q(\boldsymbol{\theta}|\boldsymbol{\theta}^{(t)})\} |_{\boldsymbol{\theta}=\boldsymbol{\theta}^{(t)}}. \end{aligned}$$

Thus a gradient ascent over the log-likelihood is equivalent to a gradient ascent over Q .