POSITIVE BOREL MEASURES

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Vector Spaces

2.1 Definition

A complex vector space is a set V, whose elements are called vectors and in which two operations, called addition and scalar multiplication, are defined, with the following familiar algebraic properties:

To every pair of vectors x and y there corresponds a vector $x + y$, in such a way that $x + y = y + x$ and $x + (y + z) = (x + y) + z$; V contains a unique vector *0* (*the zero vector or origin of* V) such that $x + 0 = x$ for every $x \in V$; and to each $x \in V$ there corresponds a unique vector $-x$ such that $x + (-x) = 0$.

To each pair $(\alpha x, x)$, where $x \in V$ and ex is a scalar, there is associated a vector exx E V, in such a way that 1x = x, ex(px) = (exP)x, and such that the two distributive laws

$$\alpha(x + y) = \alpha x + \alpha y, (\alpha + \beta)x = \alpha x + \beta x$$

hold.

A linear transformation of a vector space V into a vector space Vi is a mapping A of V into Vi such that