construct baby rudin Ch.1

Dawei Wang

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INTRUODCTION

Example

the equation

$$p^2 = 2$$

is not satisfied by any rational p.

Examine this situation more closely. Let A be the set of all positive rationals p such that $p^2 < 2$ and let B be the set of all positive rationals p such that $p^2 > 2$. We shall now show that A contains no largest number and B contains no smallest.

Definition

A is a set,
$x \in A$, $x \notin A$, *empty set*, *nonempty*, $A \in B$, $B \in A$, $A = B$, $A \neq B$

Q

ORDER SETS

Definition

order,
ordered set,
upper bound(lower bound),
least upper bound(supremum),
greatest lower bound(infimum)
least upper bound property

Theorem

Suppose S is an ordered set with the least-upper-bound property, $B \in S$, B is not empty, and B is bounded below. Let L be the set of all lower bound of B. Then

$$\alpha = supL$$

exists in S and $\alpha = infB$.
In particular, inf B exists in S.

FILEDS

Definition

field, ordered field

THE REAL FIELD

Theorem

There exists an ordered field R which has the least-upper-bound property.
Moreover, R contains Q as a subfield.

Theorem

(a) If $x \in R$, $y \in R$, and $x > 0$, then there is a positive integer n such that

$$nx > y$$

(b) If $x \in R$, $y \in R$, and $x < y$, then there exists a $p \in Q$ such that x¡p¡y.