1

A

To get our first success on the xth toss means we have failed in all x-1 times before.

Since the results of every toss are independent, the probability that we have failed in all x-1 times before is :

$(1-P)^{x-1}$

And given that we have failed in all x-1 times before, the probability that we success in the xth toss is P.

So the probability of getting our first success on the xth toss of is given by:

$(1-P)^{x-1} \times P = (1-P)^{x-1}P$

B

If I toss a fair die, the probability that I get a 6 in a single toss is $\frac{1}{6}$.

If X equals the toss on which you first throw a 6, the probability distribution of X is:

$P(X = n) = (1 - \frac{1}{6})^{n-1} \times \frac{1}{6} = (\frac{5}{6})^{n-1} \times \frac{1}{6}$

C

If the die was unfair and had a 25% chance of landing on 6 but only a 15% chance of landing onall the other values, the probability that I get a 6 in a single toss is 0.25.

The probability distribution of X is:

$P(X = n) = (1 - 0.25)^{n-1} \times 0.25 = 0.75^{n-1} \times 0.25$

D

If the die is fair as in (B), the probability of getting the first success on the second trial is :

$P(X = 2) = (1 - \frac{1}{6})^{2-1} \times \frac{1}{6} = \frac{5}{36}$

If the die is unfair as in (C), the probability of getting the first success on the second trial is :

$P(X = 2) = (1 - 0.25)^{2-1} \times 0.25 = \frac{3}{16}$

D

Because the probability of getting a 6 in a single toss is different in my answer to (D), so I got different probabilities.

2

A

The joint probability distribution of X and Y:

Y \ X	0	1
0	$\frac{3\times2\times1}{10\times9\times8}$	0
1	$2\times\frac{3\times2\times7}{10\times9\times8}$	$\frac{7\times3\times2}{10\times9\times8}$
2	$\frac{3\times7\times6}{10\times9\times8}$	$2\times\frac{7\times6\times3}{10\times9\times8}$
3	0	$\frac{7\times6\times5}{10\times9\times8}$

Y \ X	-2	9	13
0	$\frac{6}{720}$	0	
1	$\frac{84}{720}$	$\frac{42}{720}$	
2	$\frac{126}{720}$	$\frac{252}{720}$	
3	0	$\frac{210}{720}$	

B

$P(X=0)=\frac{6}{720}+\frac{84}{720}+\frac{126}{720}+0=0.3$

$P(Y=3)=\frac{7}{24}$

$P(X=0\cap Y=3)=0\neq P(X=0)\times P(Y=3)$

So X and Y are not independent.

3

A

The joint probably distribution of X and Y:

Y \ X	1	0
1	0.18×0.2	$0.08\times(1-0.2)$
0	$(1-0.18)\times0.2$	$(1-0.08)\times(1-0.2)$

Y \ X	1	0
1	0.036	0.064
0	0.164	0.736

$0.036+0.064+0.164+0.736=1$

B

Given Y=1, the conditional distribution of X is:

X	1	0
P	$\frac{0.036}{0.036+0.064}$	$\frac{0.064}{0.036+0.064}$

X	1	0
P	0.36	0.64

C

The marginal distribution of X is:

X	1	0
P	0.2	0.8

The marginal distribution of Y is:

Y	1	0
P	0.1	0.9

D

The variance of X is $0.2 \times 0.8 = 0.16$
The variance of Y is $0.1 \times 0.9 = 0.09$
The covariance of X,Y is $Cov(X, Y) = E(XY) - E(X)E(Y)$
$P(XY = 1) = P(X = 1 \cap Y = 1) = 0.036 \Rightarrow E(XY) = 0.036$
$E(X) = 0.2, E(Y) = 0.1$
$Var(X) = 0.2 \times 0.8 = 0.16, Var(Y) = 0.1 \times 0.9 = 0.09$
$Cov(X, Y) = 0.036 - 0.02 = 0.016$
The correlation coefficient is:
$\frac{0.016}{\sqrt{0.16 \times 0.09}} = \frac{2}{15}$

4

A

$$\begin{aligned} Var(2X - 3Y) =& E((2X - 3Y)^2)) - E^2(2X - 3Y) \\ =& E(4X^2 - 12XY + 9Y^2) - (4E^2(X) - 12E(X)E(Y) + 9E^2(Y)) \\ =& 4(E(X^2) - E^2(X))) + 9(E(Y^2) - E^2(Y))) - 12(E(XY) - E(X)E(Y)) \\ =& 4 * Var(X) + 9 * Var(Y) - 12 * Cov(X, Y) \end{aligned}$$

B

$$\begin{aligned} Cov(X + 3Y, Z) &= E((X + 3Y) * Z) - E(X + 3Y)E(Z) \\ &= E(XZ + 3YZ) - (E(X)E(Z) + 3E(Y)E(Z)) \\ &= E(XZ) + 3E(YZ) - E(X)E(Z) - 3E(Y)E(Z) \\ &= Cov(X, Z) + 3Cov(Y, Z) \end{aligned}$$

C

W=a+bX+cY

$$\begin{aligned} E(W) &= E(a+bX+cY) \\ &= E(a)+E(bX)+E(cY) \\ &= a+bE(X)+cE(Y) \\ &= a+b\mu_X+c\mu_Y \end{aligned}$$

$$\begin{aligned} Var(W) &= Var(a+bX+cY) \\ &= Var(a)+Var(bX)+Var(cY)+2Cov(a,bX)+2Cov(a,cY)+2Cov(bX,cY) \\ &= 0+b^2Var(X)+c^2Var(Y)+0+0+2bcCov(X,Y) \\ &= b^2\sigma_X^2+c^2\sigma_Y^2+2bcCov(X,Y) \end{aligned}$$

5

A

Joint distribution:

Y \ X	-2	9	13
3	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{3}$
5	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{6}$

Marginal distribution:

X	-2	9	13
P	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$

Y	3	5
P	$\frac{2}{3}$	$\frac{1}{3}$

B

We can exam that for all given value a and b, $P(X=a\cap Y=b)=P(X=a)\times P(Y=b)$.

So X and Y are independent.

C

To get the conditional probability of X given Y, we can find in the answer to (A) that:

$P(X=-2|Y=3)=\frac{\frac{1}{6}}{\frac{1}{6}+\frac{1}{6}+\frac{1}{3}}=\frac{1}{4}$

$P(X=9|Y=3)=\frac{\frac{1}{6}}{\frac{1}{6}+\frac{1}{6}+\frac{1}{3}}=\frac{1}{4}$

$P(X=13|Y=3)=\frac{\frac{1}{3}}{\frac{1}{6}+\frac{1}{6}+\frac{1}{3}}=\frac{1}{2}$

D

$E(X) = P(X = -2) \times (-2) + P(X = 9) \times 9 + P(X = 13) \times 13 = 8.25$

$P(X = -2|Y = 5) = \frac{\frac{1}{12}}{\frac{1}{12}+\frac{1}{12}+\frac{1}{6}} = \frac{1}{4}$
$P(X = 9|Y = 5) = \frac{\frac{1}{12}}{\frac{1}{12}+\frac{1}{12}+\frac{1}{6}} = \frac{1}{4}$
$P(X = 13|Y = 5) = \frac{\frac{1}{6}}{\frac{1}{12}+\frac{1}{12}+\frac{1}{6}} = \frac{1}{2}$
$E(X|Y = 5) = P(X = -2|Y = 5) \times (-2) + P(X = 9|Y = 5) \times 9 + P(X = 13|Y = 5) \times 13 = 8.25$

E

$E(Y) = 3 \times \frac{2}{3} + 5 \times \frac{1}{3} = \frac{11}{3}$
$P(XY = -6) = \frac{1}{6}$
$P(XY = -10) = \frac{1}{12}$
$P(XY = 27) = \frac{1}{6}$
$P(XY = 39) = \frac{1}{3}$
$P(XY = 45) = \frac{1}{12}$
$P(XY = 65) = \frac{1}{6}$
$E(XY) = \frac{1}{6} \times (-6) + \frac{1}{12} \times (-10) + \frac{1}{6} \times 27 + \frac{1}{3} \times 39 + \frac{1}{12} \times 45 + \frac{1}{6} \times 65 = \frac{121}{4}$
$E(X) \times E(Y) = \frac{33}{4} \times \frac{11}{3} = \frac{121}{4} = E(XY)$
$Cov(X, Y) = 0$

6

A

$P_1 = P(0\ \ ace \cap 0\ \ ten) = \frac{C_{44}^{13}}{C_{52}^{13}} = 0.08175499$
$P_2 = P(1\ \ ace \cap 1\ \ ten) = \frac{C_4^1 \times C_4^1 \times C_{44}^{11}}{C_{52}^{13}} = 0.1932391$
$P_3 = P(2\ \ ace \cap 2\ \ ten) = \frac{C_4^2 \times C_4^2 \times C_{44}^{9}}{C_{52}^{13}} = 0.04019048$
$P_4 = P(3\ \ ace \cap 3\ \ ten) = \frac{C_4^3 \times C_4^3 \times C_{44}^{7}}{C_{52}^{13}} = 0.000965537$
$P_5 = P(4\ \ ace \cap 4\ \ ten) = \frac{C_4^4 \times C_4^4 \times C_{44}^{5}}{C_{52}^{13}} = 1.710212 \times 10^{-6}$
The probability I you get exactly the same number of aces and tens in the cards that I pick:
$P_1 + P_2 + P_3 + P_4 + P_5 = 0.3161518$

B

Assume α = the amount of ace I will pick, α = the amount of ten I will pick.
$P(\alpha = \beta) = 0.3161518$
Since $P(\alpha > \beta) = P(\alpha < \beta), P(\alpha > \beta) + P(\alpha < \beta) + P(\alpha = \beta) = 1$ $P(\alpha > \beta) = \frac{1-P(\alpha=\beta)}{2} = 0.3419241$

7

A

$X \sim N(68, 2^2)$

$P(X > 71.5) \Rightarrow P(\frac{X-68}{2} > 1.75) = 1 - \Phi(1.75) = 0.04005916$

B

The probability that two randomly selected people will have a height that exceeds 71.5 inches:

$P^2(X > 71.5) = 0.001604736$

C

$\frac{X-68}{2} = 2.4 \Rightarrow X = 72.8$

D

$\Phi(0.8416212) = 0.8 \Rightarrow \frac{X-68}{2} = 0.8416212 \Rightarrow X = 69.68324$

E

10th percentile: $\Phi(-1.281552) = 0.1$

60th percentile: $\Phi(0.2533471) = 0.6$

$P(\frac{X-68}{2} > 0.2533471 | \frac{X-68}{2} > -1.281552) = \frac{\frac{X-68}{2} > 0.2533471}{\frac{X-68}{2} > -1.281552} = \frac{1-0.6}{1-0.1} = \frac{4}{9}$

8

X	1	0
P	0.5	0.5

Y	1	0
P	0.5	0.5

Z	1	0
P	0.5	0.5

Y+Z	-2	0	2
P	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

X \ Z	1	-1
1	$\frac{1}{4}$	$\frac{1}{4}$
-1	$\frac{1}{4}$	$\frac{1}{4}$

X \ Y	1	-1
1	$\frac{1}{4}$	$\frac{1}{4}$
-1	$\frac{1}{4}$	$\frac{1}{4}$

X \ Y+Z	-2	0	2
1	$\frac{1}{4}$	0	$\frac{1}{4}$
-1	0	$\frac{1}{2}$	0

For all $Y = \alpha, Z = \beta, X = \gamma$:
$P(X = \gamma \cap Y = \alpha) = P(X = \gamma) \times P(Y = \alpha)$
$P(X = \gamma \cap Z = \beta) = P(X = \gamma) \times P(Z = \beta)$
So X is independent of both Y and Z.
$P(X = 1 \cap Y + Z = 0) = 0 \neq P(X = 1) \times P(Y + Z = 0)$
So X is not independent of Y+Z.

9

A

$P(X = 1|pick\ \ fair) = 0.5$
$P(X = 1|pick\ \ unfair) = 0.6$
$P(X = 1 \cap pick\ \ fair) = 0.5 \times 0.5 = 0.25$
$P(X = 1 \cap pick\ \ unfair) = 0.6 \times 0.5 = 0.3$
$P(X = 1) = P(X = 1 \cap pick\ \ unfair) + P(X = 1 \cap pick\ \ fair) = 0.55$
$P(X = 0) = 1 - P(X = 1) = 0.45$

X	1	0
P	0.55	0.45

B

$P(X = 1 \cap Y = 2|pick\ \ fair\ \ 2\ \ times) = 0.5 \times 0.5 = 0.25$
$P(X = 1 \cap Y = 2|pick\ \ unfair\ \ 2\ \ times) = 0.6 \times 0.6 = 0.36$
$P(X = 1 \cap Y = 2|pick\ \ fair\ \ then\ \ pick\ \ unfair) = 0.5 \times 0.6 = 0.3$
$P(X = 1 \cap Y = 2|pick\ \ unfair\ \ then\ \ pick\ \ fair) = 0.6 \times 0.5 = 0.3$
$P(X = 1 \cap Y = 2) = (P(X = 1 \cap Y = 2 \cap pick\ \ fair\ \ 2\ \ times) + P(X = 1 \cap Y = 2 \cap pick\ \ unfair\ \ 2\ \ times) + P(X = 1 \cap Y = 2|pick\ \ fair\ \ then\ \ pick\ \ unfair) + P(X = 1 \cap Y = 2|pick\ \ unfair\ \ then\ \ pick\ \ fair)) \times (0.5 \times 0.5) = 0.3025$
$P(X = 1 \cap Y = 1) = P(X = 1) - P(X = 1 \cap Y = 2) - P(X = 1 \cap Y = 0) = 0.2475$
$P(X = 0 \cap Y = 0|pick\ \ fair\ \ 2\ \ times) = 0.5 \times 0.5 = 0.25$
$P(X = 0 \cap Y = 0|pick\ \ unfair\ \ 2\ \ times) = 0.4 \times 0.4 = 0.16$
$P(X = 0 \cap Y = 0|pick\ \ fair\ \ then\ \ pick\ \ unfair) = 0.5 \times 0.4 = 0.2$
$P(X = 0 \cap Y = 0|pick\ \ fair\ \ then\ \ pick\ \ unfair) = 0.4 \times 0.5 = 0.2$
$P(X = 0 \cap Y = 0) = 0.5 \times 0.5 \times (0.25 + 0.16 + 0.2 + 0.2) = 0.2025$
$P(X = 0 \cap Y = 1) = P(X = 0) - P(X = 0 \cap Y = 0) - P(X = 0 \cap Y = 2) = 0.2475$

X \ Y	0	1	2
0	0.2025	0.2475	0
1	0	0.2475	0.3025

C

$P(X=1)=0.55, P(Y=0)=0.2025$

$P(X=1\cap Y=0)=0\neq P(X=1)\times P(Y=0)$

So X and Y are not independent.

D

$P(7\ \ heads|pick\ \ fair)=C_8^7\times 0.5^7\times 0.5^1=0.03125$

$P(7\ \ heads|pick\ \ unfair)=C_8^7\times 0.6^7\times 0.4^1=0.08957952$

$P(7\ \ heads)=P(7\ \ heads\cap pick\ \ fair)+P(7\ \ heads\cap pick\ \ unfair)=\frac{0.03125+0.08957952}{2}=0.06041475$

10

A=a student enrolls in postsecondary education within three months of finishing high school

$\alpha=$the student did not have a parent who attended college

$\beta=$the student had a parent who attended some college

$\gamma=$the student have at least one parent who completed college

$P(A|\alpha)=0.58$

$P(A|\beta)=0.63$

$P(A|\gamma)=0.78$

$P(\alpha)=0.35$

$P(\beta)=0.3$

$P(\gamma)=0.35$

$P(A)=P(A|\alpha)\times P(\alpha)+P(A|\beta)\times P(\beta)+P(A|\gamma)\times P(\gamma)=0.35\times 0.58+0.3\times 0.63+0.35\times 0.78=0.665$