1

A

Because $C \sim N(5.7\%, (8\%)^2)$
$\frac{C-5.7\%}{8\%} \sim N(0,1)$
$P(C < 0) = \Phi(\frac{0-5.7\%}{8\%}) = 0.2380776$

B

Assume the value of my asset is α, then I will invest 0.5α in stock A, 0.25α in stock B and 0.25α in stock C.

My expected return is :

$$\begin{aligned} E(0.5\alpha \cdot A + 0.25\alpha \cdot B + 0.25\alpha \cdot C) &= 0.5\alpha E(A) + 0.25\alpha \cdot E(B) + 0.25\alpha \cdot E(C) \\ &= 0.5\alpha \times 1.2\% + 0.25\alpha \times 3.4\% + 0.25\alpha \times 5.7\% \\ &= 2.875\%\alpha \end{aligned}$$

My expected return rate:$\frac{E(0.5\alpha \cdot A + 0.25\alpha \cdot B + 0.25\alpha \cdot C)}{\alpha} = 2.85\%$

C

As the answer for A shows, the probability that the invest in stock C gets a negative return is $P(C < 0) = 0.2380776$.

Because $A \sim N(1.2\%, (0.8\%)^2)$:
$P(A < 0) = \Phi(\frac{0-1.2\%}{0.8\%}) = 0.0668072$
Similarly, because $B \sim N(3.4\%, (2.7\%)^2)$
$P(B < 0) = \Phi(\frac{0-3.4\%}{2.7\%}) = 0.1039684$

So the probability that all three stocks will earn a negative return is:
$P(A < 0 \& B < 0 \& C < 0) = P(A < 0) \times P(B < 0) \times P(C < 0) = 0.001653648$

2

A

Y \ X	1	0
1	$\frac{4\times 8}{40\times 39}$	$\frac{120}{40\times 39}$
0	$\frac{4\times 31}{40\times 39}$	$\frac{36\times 39}{40\times 39}-\frac{120}{40\times 39}$

B

Y	1	0
	$\frac{15}{100}$	$\frac{85}{100}$

C

X	1	0
	$\frac{10}{100}$	$\frac{90}{100}$

D

$P(X=1)P(Y=1)=\frac{15}{10000}$
$P(X=1\&Y=1)=\frac{3}{100}$
$P(X=1)P(Y=1)\neq P(X=1\&Y=1)$
So X and Y are not independent

E

$$E(Y)=P(Y=1)\times 1+P(Y=0)\times 0=0.15$$

$$D(Y)=(1-E(Y))^2\times P(X=1)+(0-E(Y))^2\times P(X=0)=0.1275$$

3

A

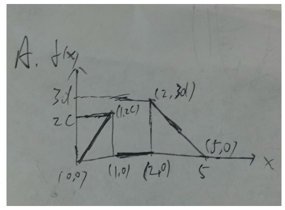

Figure 1:

B

I can't determine the specific values of c and d, but I can determine the relationship between c and d:

For the total area between the probability density curve is 1:

$$
\begin{aligned}
1 =& 2c \times 1 \times \frac{1}{2} + 3d \times (5-2) \times \frac{1}{2} \\
=& c + \frac{9}{2}d
\end{aligned}
$$

So we get $c + \frac{9}{2}d = 1$.

C

Given that that the probability that X is between 0 and 1 is four times as large as the probability that X falls between 2 and 5:

$$
\begin{aligned}
& 2c \times 1 \times \frac{1}{2} = 4 \times 3d \times (5-2) \times \frac{1}{2} \\
& c = 18d
\end{aligned}
$$

from the answer for 3.B we know that $c + \frac{9}{2}d = 1$.

$$
\begin{cases} c + \frac{9}{2}d = 1 \\ c = 18d \end{cases} \Rightarrow \begin{cases} d = \frac{2}{45} \\ c = \frac{4}{5} \end{cases}
$$

D

The area between 2 and 3 is ... $= \frac{1}{9}$

The area between 3 and 4 is ... $= \frac{1}{15}$

So:

$P(2 \leq x \leq 3) = \frac{1}{9}$
$P(3 \leq x \leq 4) = \frac{1}{15} < P(2 \leq x \leq 3)$
So I would bet that X would take on a value between 2 and 3.

4

Assume that the standard deviation of scores distribution is σ,
X= the test scores
$X \sim N(70, \sigma) \Rightarrow \frac{X-70}{\sigma} \sim N(0,1)$
Since $P_{1.5} = 0.9332$ and $P(X < 85) = 0.9332$
$\frac{85-70}{\sigma} = 1.5 \Rightarrow \sigma = 10$

5

A

The probability that a current smoker is unemployed is:
$P(W=0|S=2) = \frac{P(W=0 \cap S=2)}{P(S=2)} = \frac{P(W=0 \cap S=2)}{P(S=2 \cap W=0)+P(S=2 \cap W=1)} = \frac{0.14}{0.14+0.22} = 0.3888889$

B

The probability that an individual is a former smoker is:
$P(S=1 \cap W=0) + P(S=1 \cap W=1) = 0.17 + 0.17 = 0.34$

C

$P(W=1) = P(W=1 \cap S=0) + P(W=1 \cap S=1) + P(W=1 \cap S=2) = 0.18 + 0.17 + 0.22 = 0.57$
$P(W=0) = 1 - P(W=1) = 0.43$
$E(W) = 1 \times P(W=1) + 0 \times P(W=0) = 0.57$

D

$P(S=0) = P(S=0 \cap W=0) + P(S=0 \cap W=1) = 0.12 + 0.18 = 0.3$
$P(S=0) \times P(W=1) = 0.171 \neq P(S=0 \cap W=1)$
So smoking and employment are not independent.

E

$P(S=0|W=1) = \frac{P(S=0 \cap W=1)}{P(W=1)} = \frac{0.18}{0.57} = 0.3157895$
$P(S=1|W=1) = \frac{P(S=1 \cap W=1)}{P(W=1)} = \frac{0.17}{0.57} = 0.2982456$
$P(S=2|W=1) = \frac{P(S=2 \cap W=1)}{P(W=1)} = \frac{0.22}{0.57} = 0.3859649$

6

A

If the size is 5, there are $\frac{20!}{(20-5)!} = 1860480$ ways

If the size is 7, there are $\frac{20!}{(20-7)!} = 390700800$ ways

If the size is 9, there are $\frac{20!}{(20-9)!} = 60949324800$ ways

If the size is 11, there are $\frac{20!}{(20-11)!} = 6704425728000$ ways

B

5

C

5

D

The probability of picking a committee of 9 representatives and all were Republicans is $\frac{C_{11}^{9}}{C_{20}^{9}} = \frac{55}{167960} = 0.0003274589$, so the governor probably didn't choose the committee randomly.

7

$0.05 \times 0.25 \times 0.4^2 \times 0.3^2 \times \frac{5!}{2!2!1!} = 0.00108$

8

A

$A_n = the\ National\ league\ team\ wins\ the\ n_{th}\ game$

$P(A_1) = 0.7$

$P(A_2|A_1) = 0.75, P(A_2|\overline{A_1}) = 0.4 \Rightarrow P(A_2 \cap A_1) = 0.525, P(A_2 \cap \overline{A_1}) = 0.21$

$P(A_2) = P(A_2 \cap A_1) + P(A_2 \cap \overline{A_1}) = 0.735$

$P(A_1) \times P(A_2) \neq P(A_1 \cap A_2)$

So the games are not independent.

B

When the series lasts 4 games:

The probability that the National league team wins:

$P = 0.7 \times (0.75)^3 = 0.2953125$

The probability that the American League wins:

$0.3 \times 0.6^3 = 0.0648$

The probability that the series lasts 4 games:$0.2953125 + 0.0648 = 0.3601125$

C

When the series lasts 5 games:

The probability that the National league team wins:

$P1_5 = P(\overline{A_1} \cap A_2 \cap A_3 \cap A_4 \cap A_5) = 0.3 \times 0.4 \times (0.75)^3 = 0.050625$

$P2_5 = P(A_1 \cap \overline{A_2} \cap A_3 \cap A_4 \cap A_5) = 0.7 \times 0.25 \times 0.4 \times (0.75)^2 = 0.039375$

$P3_5 = P(A_1 \cap A_2 \cap \overline{A_3} \cap A_4 \cap A_5) = 0.7 \times 0.75 \times 0.25 \times 0.4 \times 0.75 = 0.039375$

$P4_5 = P(A_1 \cap A_2 \cap A_3 \cap \overline{A_4} \cap A_5) = 0.7 \times 0.75 \times 0.75 \times 0.25 \times 0.4 = 0.039375$

$P1_5 + P2_5 + P3_5 + P4_5 = 0.16875$

The probability that the American League wins:

$P1'_5 = P(\overline{A_1} \cap \overline{A_2} \cap \overline{A_3} \cap A_4 \cap \overline{A_5}) = 0.3 \times 0.6 \times 0.6 \times 0.4 \times 0.25 = 0.0108$

$P2'_5 = P(\overline{A_1} \cap \overline{A_2} \cap A_3 \cap \overline{A_4} \cap \overline{A_5}) = 0.3 \times 0.6 \times 0.4 \times 0.25 \times 0.6 = 0.0108$

$P3'_5 = P(\overline{A_1} \cap A_2 \cap \overline{A_3} \cap \overline{A_4} \cap \overline{A_5}) = 0.3 \times 0.4 \times 0.25 \times 0.6 \times 0.6 = 0.0108$

$P4'_5 = P(A_1 \cap \overline{A_2} \cap \overline{A_3} \cap \overline{A_4} \cap \overline{A_5}) = 0.7 \times 0.25 \times (0.6)^3 = 0.0378$

$P1'_5 + P2'_5 + P3'_5 + P4'_5 = 0.0702$

The probability that the series lasts 5 games:

$P1_5 + P2_5 + P3_5 + P4_5 + P1'_5 + P2'_5 + P3'_5 + P4'_5 = 0.23895$

When the series lasts 6 games:

The probability that the National league team wins:

$P1_6 = P(\overline{A_1} \cap \overline{A_2} \cap A_3 \cap A_4 \cap A_5 \cap A_6) = 0.3 \times 0.6 \times 0.4 \times 0.75^3 = 0.030375$

$P2_6 = P(\overline{A_1} \cap A_2 \cap \overline{A_3} \cap A_4 \cap A_5 \cap A_6) = 0.3 \times 0.4 \times 0.25 \times 0.4 \times 0.75^2 = 0.00675$

$P3_6 = P(\overline{A_1} \cap A_2 \cap A_3 \cap \overline{A_4} \cap A_5 \cap A_6) = 0.00675$

$P4_6 = P(\overline{A_1} \cap A_2 \cap A_3 \cap A_4 \cap \overline{A_5} \cap A_6) = 0.00675$

$P5_6 = P(A_1 \cap \overline{A_2} \cap \overline{A_3} \cap A_4 \cap A_5 \cap A_6) = 0.7 \times 0.25 \times 0.6 \times 0.4 \times 0.75^2 = 0.023625$

$P6_6 = P(A_1 \cap \overline{A_2} \cap A_3 \cap \overline{A_4} \cap A_5 \cap A_6) = 0.7 \times 0.25 \times 0.4 \times 0.25 \times 0.4 \times 0.75 = 0.00525$

$P7_6 = P(A_1 \cap \overline{A_2} \cap A_3 \cap A_4 \cap \overline{A_5} \cap A_6) = 0.00525$

$P8_6 = P(A_1 \cap A_2 \cap \overline{A_3} \cap \overline{A_4} \cap A_5 \cap A_6) = 0.023625$

$P9_6 = P(A_1 \cap A_2 \cap \overline{A_3} \cap A_4 \cap \overline{A_5} \cap A_6) = 0.00525$

$P10_6 = P(A_1 \cap A_2 \cap A_3 \cap \overline{A_4} \cap \overline{A_5} \cap A_6) = 0.023625$

The probability that the American League wins:

$P1'_6 = P(A_1 \cap A_2 \cap \overline{A_3} \cap \overline{A_4} \cap \overline{A_5} \cap \overline{A_6}) = 0.7 * 0.75 * 0.25 * 0.6^3 = 0.02835$

$P2'_6 = P(A_1 \cap \overline{A_2} \cap A_3 \cap \overline{A_4} \cap \overline{A_5} \cap \overline{A_6}) = 0.7 \times 0.25 \times 0.4 \times 0.25 \times 0.6^2 = 0.0063$

$P3'_6 = P(A_1 \cap \overline{A_2} \cap \overline{A_3} \cap A_4 \cap \overline{A_5} \cap \overline{A_6}) = 0.0063$

$P4'_6 = P(A_1 \cap \overline{A_2} \cap \overline{A_3} \cap \overline{A_4} \cap A_5 \cap \overline{A_6}) = 0.0063$

$P5'_6 = P(\overline{A_1} \cap A_2 \cap A_3 \cap \overline{A_4} \cap \overline{A_5} \cap \overline{A_6}) = 0.0081$

$P6_6' = P(\overline{A_1} \cap A_2 \cap \overline{A_3} \cap A_4 \cap \overline{A_5} \cap \overline{A_6}) = 0.0018$
$P7_6' = P(\overline{A_1} \cap A_2 \cap \overline{A_3} \cap \overline{A_4} \cap A_5 \cap \overline{A_6}) = 0.0018$
$P8_6' = P(\overline{A_1} \cap \overline{A_2} \cap A_3 \cap A_4 \cap \overline{A_5} \cap \overline{A_6}) = 0.0081$
$P9_6' = P(\overline{A_1} \cap \overline{A_2} \cap A_3 \cap \overline{A_4} \cap A_5 \cap \overline{A_6}) = 0.0018$
$P10_6' = P(\overline{A_1} \cap \overline{A_2} \cap \overline{A_3} \cap A_4 \cap A_5 \cap \overline{A_6}) = 0.0081$
The probability that the series lasts 6 games:
$P1_6 + \cdots + P10_6 + P1_6' + \cdots + P10_6' = 0.2142$

When the series lasts 7 games(each team wins 3 games in the first 6 games):
$P1_{33} = P(A_1 \cap A_2 \cap A_3 \cap \overline{A_4} \cap \overline{A_5} \cap \overline{A_6}) = 0.7 \times 0.75^2 \times 0.25 \times 0.6^2 = 0.0354375$
$P2_{33} = P(A_1 \cap A_2 \cap \overline{A_3} \cap A_4 \cap \overline{A_5} \cap \overline{A_6}) = 0.7 \times 0.75 \times 0.25 \times 0.4 \times 0.25 \times 0.6 = 0.007875$
$P3_{33} = P(A_1 \cap A_2 \cap \overline{A_3} \cap \overline{A_4} \cap A_5 \cap \overline{A_6}) = 0.7 \times 0.75 \times 0.25 \times 0.6 \times 0.4 \times 0.25 = 0.007875$
$P4_{33} = P(A_1 \cap A_2 \cap \overline{A_3} \cap \overline{A_4} \cap \overline{A_5} \cap A_6) = 0.7 \times 0.75 \times 0.25 \times 0.6^2 \times 0.4 = 0.0189$
$P5_{33} = P(A_1 \cap \overline{A_2} \cap A_3 \cap A_4 \cap \overline{A_5} \cap \overline{A_6}) = 0.7 \times 0.25 \times 0.4 \times 0.75 \times 0.25 \times 0.6 = 0.007875$
$P6_{33} = P(A_1 \cap \overline{A_2} \cap A_3 \cap \overline{A_4} \cap A_5 \cap \overline{A_6}) = 0.7 \times 0.25 \times 0.4 \times 0.25 \times 0.4 \times 0.25 = 0.00175$
$P7_{33} = P(A_1 \cap \overline{A_2} \cap A_3 \cap \overline{A_4} \cap \overline{A_5} \cap A_6) = 0.7 \times 0.25 \times 0.4 \times 0.25 \times 0.6 \times 0.4 = 0.0042$
$P8_{33} = 100110 = 0.7 * 0.25 * 0.6 * 0.4 * 0.75 * 0.25 = 0.007875$
$P9_{33} = 100101 = 0.7 * 0.25 * 0.6 * 0.4 * 0.25 * 0.4 = 0.0042$
$P10_{33} = 100011 = 0.7 * 0.25 * 0.6 * 0.6 * 0.4 * 0.75 = 0.0189$
$P11_{33} = 011100 = 0.3 * 0.4 * 0.75 * 0.75 * 0.25 * 0.6 = 0.010125$
$P12_{33} = 011010 = 0.3 * 0.4 * 0.75 * 0.25 * 0.4 * 0.25 = 0.00225$
$P13_{33} = 011001 = 0.3 * 0.4 * 0.75 * 0.25 * 0.6 * 0.4 = 0.0054$
$P14_{33} = 010110 = 0.3 * 0.4 * 0.25 * 0.4 * 0.75 * 0.25 = 0.00225$
$P15_{33} = 010101 = 0.3 * 0.4 * 0.25 * 0.4 * 0.25 * 0.4 = 0.0012$
$P16_{33} = 010011 = 0.3 * 0.4 * 0.25 * 0.6 * 0.4 * 0.75 = 0.0054$
$P17_{33} = 001110 = 0.3 * 0.6 * 0.4 * 0.75 * 0.75 * 0.25 = 0.010125$
$P18_{33} = 001101 = 0.3 * 0.6 * 0.4 * 0.75 * 0.25 * 0.4 = 0.0054$
$P19_{33} = 001011 = 0.3 * 0.6 * 0.4 * 0.25 * 0.4 * 0.75 = 0.0054$
$P20_{33} = 000111 = 0.3 * 0.6 * 0.6 * 0.4 * 0.75 * 0.75 = 0.0243$
The probability that the series lasts 7 games:
$P1_{33} + \cdots + P20_{33} = 0.1867375$

D

0.3601125+0.23895+0.2142+0.1867375=1

E

9

A

X= scores

$X \sim N(440, 60) \Rightarrow \frac{X-440}{60} \sim N(0,1)$

$420 < X < 500 \Rightarrow -\frac{1}{3} < \frac{X-440}{60} < 1$

$P_1 - P_{-\frac{1}{3}} = 0.4719034$

B

$P_{1.644854} = 0.95 \Rightarrow \frac{X-440}{60} = 1.644854$

X=538.6912

C

$X = 500 \Rightarrow \frac{X-440}{60} = 1$

$P_1 = 0.8413447$

The probability that all there scores all below 500 is:

$P_1^2 = 0.7078609$

The probability that at least one of them scored more than 500 is:

$1 - P_1^2 = 0.2921391$

10

X= amount of dye discharged

$X \sim N(\Theta, 0.4) \Rightarrow \frac{X-\Theta}{0.4} \sim N(0,1)$

The probability that the shade is unacceptable is:

$1 - P(X \leq 6) = 1 - P(\frac{X-\Theta}{0.4} \leq \frac{6-\Theta}{0.4}) = 0.5\%$

because $P_{2.575829} = 0.995$

$\frac{6-\Theta}{0.4} = 2.575829$

$\Theta = 4.969668$