1. **For the following discrete-time system, where y[n] and x[n] are, respectively, the output and the input sequences, determine whether the system is (1) linear, (2) shift-invariant, (3) stable, and (4) causal.** *Note that detailed steps should be given in the solutions.*

$$y[n] = 3(x[-n])^2 + 6$$

Solution:

(1) Let $y_1[n]$ and $y_2[n]$ be the output of the discrete-time system, whose input are denoted by $x_1[n]$ and $x_2[n]$, respectively. Then

$$y_1[n] = 3(x_1[-n])^2 + 6$$

And
$$y_2[n] = 3(x_2[-n])^2 + 6$$

When the input is a linear combination of $x_1[n]$ and $x_2[n]$, i.e., $\alpha x_1[n] + \beta x_2[n]$, where $\alpha$ and $\beta$ are both arbitrary real numbers, corresponding output $y_o[n]$ is given by

$$\begin{aligned} y_o[n] &= 3(\alpha x_1[-n] + \beta x_2[-n])^2 + 6 \\ &\neq \alpha y_1[n] + \beta y_2[n] \end{aligned}$$

The system is not linear.

(2) Since

$$\begin{aligned} 3(x[-n-n_0])^2 + 6 &= 3(x[-(n+n_0)])^2 + 6 \\ &\neq y[n-n_0] \end{aligned}$$

The system is not shift-invariant.

(3) If $|x[n]| \le B_x$ ($B_x$ is any positive bounded real number) for any integer value of $n$, then

$$|y[n]| = \left|3(x[-n])^2 + 6\right| \le 3|x[-n]|^2 + 6 \le 3B_x^2 + 6$$

Therefore, the discrete-time system is BIBO stable.

(4) When $n < 0$, The system output $y[n]$ depends on the input $x[-n]$ where $-n > 0$. Therefore, the discrete-time system is not causal.

---

**2. A LTI discrete-time system is cascaded by following two systems shown as Fig1:**

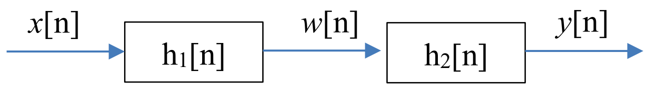



Fig 1

$h_1[n]$ is an ideal lowpass filter with $\omega_p = 0.5\pi$ ; The Linear Differential Equation of the second system is $y[n] = w[n] + w[n-1]$ with $y[n] = 0, n < 0$ ;

1) Determine the frequency response $H_2(e^{j\omega})$ of the second system.

2) Determine the frequency response $H(e^{j\omega})$ of the whole system. Sketch its magnitude and phase response.

3) If $x[n] = \cos(0.6\pi n) + 2\delta[n-5] + 2$, Determine the output $y[n]$.

Solution

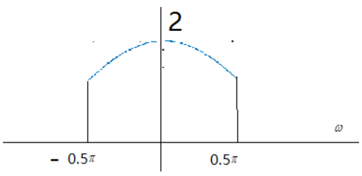
1) $Y(e^{j\omega}) = W(e^{j\omega}) + e^{-j\omega}W(e^{-\omega}), H_2(e^{j\omega}) = \frac{Y(e^{j\omega})}{W(e^{j\omega})} = 1 + e^{-j\omega}$

2) $H_1(e^{j\omega}) = \begin{cases} 1, & 0 \le |\omega| \le 0.5\pi \\ 0, & 0.5\pi \le |\omega| \le \pi \end{cases}, h_1[n] = \frac{\sin(0.5\pi n)}{\pi n}$

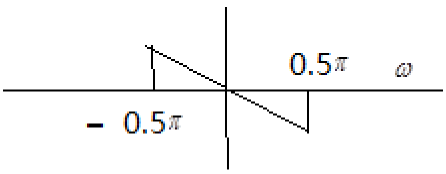
$$H(e^{-j\omega}) = H_1(e^{-j\omega})H_2(e^{-j\omega}) = \begin{cases} 1 + e^{-j\omega}, & 0 \le |\omega| \le 0.5\pi \\ 0, & 0.5\pi \le |\omega| \le \pi \end{cases}$$

Let $H(e^{-j\omega}) = |H(e^{j\omega})|e^{j\theta(\omega)}$, magnitude and phase response are:

$$\therefore |H(e^{j\omega})| = \begin{cases} \left|2\cos(\frac{\omega}{2})\right|, & 0 \le |\omega| \le 0.5\pi \\ 0, & 0.5\pi \le |\omega| \le \pi \end{cases}$$




$$\theta(\omega) = \begin{cases} -\frac{\omega}{2}, & 0 \le |\omega| \le 0.5\pi \\ 0, & 0.5\pi \le |\omega| \le \pi \end{cases}$$




(3)

$$X(e^{j\omega}) = \pi\{\delta(\omega+0.6\pi)+\delta(\omega-0.6\pi)\}+2e^{-j5\omega}+4\pi\delta(\omega)\quad |\omega|\le\pi$$

$$W(e^{j\omega}) = X(e^{j\omega})H_1(e^{j\omega}) = 2e^{-j5\omega}+4\pi\delta(\omega), |\omega|\le 0.5\pi$$

$$W[n] = 2\frac{\sin(0.5\pi(n-5))}{\pi(n-5)}+2$$

$$y[n] = W[n]+W[n-1] = 2\frac{\sin(0.5\pi(n-5))}{\pi(n-5)}+2\frac{\sin(0.5\pi(n-6))}{\pi(n-6)}+4$$

---------------------------------

3. Consider a discrete-time LTI system where the frequency response $H(e^{j\omega})$ is described by

$$H(e^{j\omega}) = \begin{cases} 1-2j, & 0<\omega<\pi \\ 1+2j, & -\pi<\omega<0 \end{cases}$$

(a) Is the impulse response of the system $h[n]$ real-valued?

(b) Calculate the following: $\sum_{n=-\infty}^{\infty}\left|\frac{1}{\sqrt{3}}h[n]\right|^2$

(c) Determine the response of the system to the input $x[n]=\cos(\omega_c n)$, where $0<\omega_c<\frac{\pi}{2}$.

***Solution*:**

(a) The DTFT of $h[n]$ is conjugate symmetric. Therefore, $h[n]$ is real-valued.

(b) The calculation can be completed using Parseval's theorem. That is,

$$\sum_{n=-\infty}^{\infty}\left|\frac{1}{\sqrt{3}}h[n]\right|^2 = \frac{1}{3}\sum_{n=-\infty}^{\infty}|h[n]|^2 = \frac{1}{6\pi}\int_{-\pi}^{\pi}\left|H(e^{j\omega})\right|^2 d\omega = \frac{5}{3}$$

(c) $x[n]=\cos(\omega_c n)=\frac{1}{2}\left(e^{j\omega_c n}+e^{-j\omega_c n}\right)$

Considering that the output to $e^{j\omega_c n}$ is $e^{j\omega_c n}H(e^{j\omega_c})$ and the output to $e^{-j\omega_c n}$ is $e^{-j\omega_c n}H(e^{-j\omega_c})$, the response of the system to the input $x[n]=\cos(\omega_c n)$ is given by

$$y[n]=\cos(\omega_c n)+2\sin(\omega_c n)$$

4、Let $x[n]$ 和 $X(e^{j\omega})$ represent a sequence and its Fourier transform, respectively. Determine, in terms of $X(e^{j\omega})$, the transform of $y_a[n]$, and $y_b[n]$. In each case, sketch $Y(e^{j\omega})$ for $X(e^{j\omega})$ as shown in Figure 9.

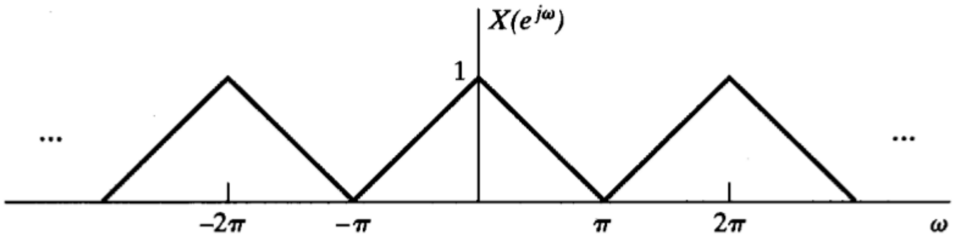



Figure 9

(a)

$$y_a[n] = \begin{cases} x[n], & n \text{ even}, \\ 0, & n \text{ odd}. \end{cases}$$

(b)

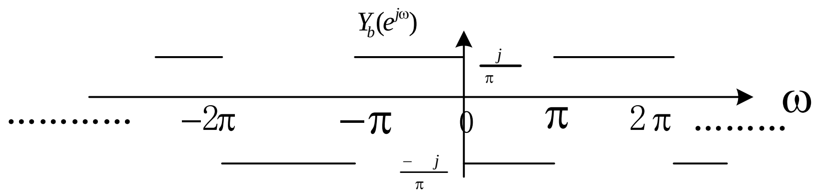
$$y_b[n] = nx[n]$$

Solution

(a) Note that $y_a[n] = \frac{1}{2}\left\{x[n] + (-1)^n x[n]\right\}$ and $-1 = e^{j\pi}$.

$$Y_a(e^{j\omega}) = \frac{1}{2}\{X(e^{j\omega}) + X(e^{j(\omega-\pi)})\} = 1$$

**(b)** $Y_b(e^{j\omega}) = j\dfrac{d}{d\omega}X(e^{j\omega})$




---------------------------------------------------------

5、**About sampling:**

(1) Please determine the smallest sampling rate for the following amplitude modulation (AM) signal $x_a(t)$, which satisfies the Nyquist rule.

$$x_a(t) = [1 + \cos(2\pi \times 100t)]\cos(2\pi \times 600t)$$

(2) Given $x_b(t)=\cos(2\pi ft),\ f<1KHz$

Sampling $x_b(t)$ with $f_s=600Hz$, after recovery we obtain a frequency of 150Hz.

Sampling $x_b(t)$ with $f_s=550Hz$, after recovery we obtain a frequency of 200Hz.

Please determine the value of $f$?

Solution

(1) $x_a(t)=\cos(1200\pi t)+\cos(200\pi t)\cos(1200\pi t)$

$$=\cos(1200\pi t)+\frac{1}{2}\cos(1000\pi t)+\frac{1}{2}\cos(1400\pi t)$$

频率$f$：600HZ，500HZ，700HZ。

采样频率 $f_{sampling}=2\times f_{\max}=1400HZ$

so the smallest sampling rate is 1400HZ.

(2) we can know that $f>300HZ$ from the question

so $300HZ<f<1000HZ$

then $f$=150+600 =750HZ or $f$=600−150 =450HZ

and $f$=200+550 =750HZ or $f$=550−200 =350HZ

In summary: $f=750\text{HZ}$ The value of $f$ is 750HZ.

------------------------------------------------------------

6、A digital signal processing system is represented as figure 11.1. The frequency spectrum $X_a(j\Omega)$ of an analog input signal $x_a(t)$ is given in figure 11.2. The analog signal is being sampled at sampling interval $T=\dfrac{4\pi}{3\Omega_m}$. And pass the sampled sequence g[n] through an ideal lowpass filter $H(e^{j\omega})$ with cutoff frequency $\omega_c=\dfrac{\Omega_m T}{2}$, as shown in figure 11.3. Then put the filter's output y[n] into the ideal interpolator, an analog output $y_a(t)$ can be got. Please sketch:

(1) The frequency spectrum $G(e^{j\omega})$ of the sampled sequence g[n].

(2) The frequency spectrum $Y(e^{j\omega})$ of the filter output y[n].

(3) The frequency spectrum $Y_a(j\Omega)$ of the system output $y_a(t)$.

$$x_a(t)\longrightarrow\boxed{\text{Ideal Sampler}}\xrightarrow{g[n]}\boxed{H(e^{j\omega})}\xrightarrow{y[n]}\boxed{\text{Ideal Interpolator}}\longrightarrow y_a(t)$$

Figure 11.1 A digital signal processing system

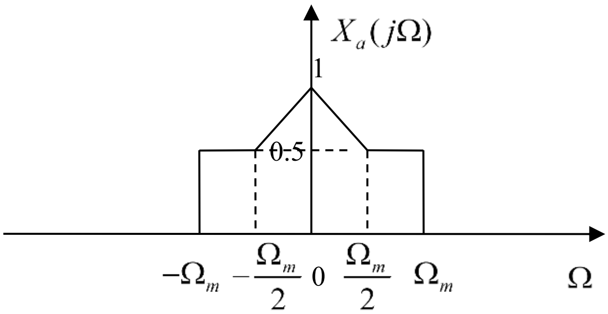



Figure 11.2 Spectrum of $x_a(t)$

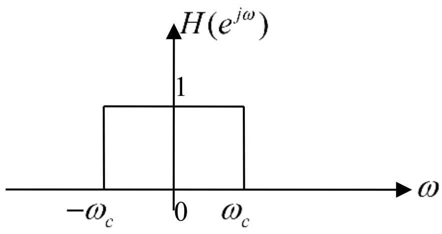
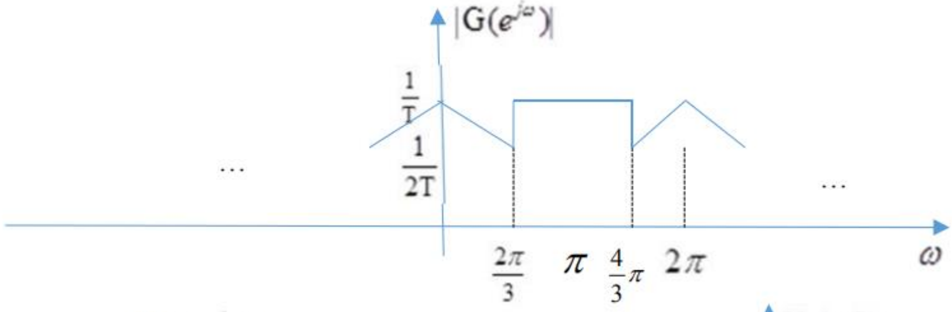
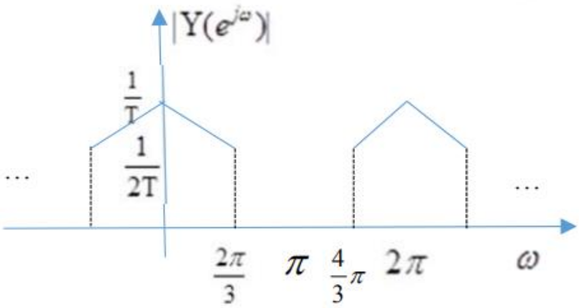


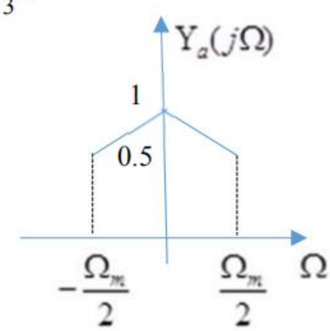

Figure 11.3 The spectrum of ideal lowpass filter

Solution:

$$\because T=\frac{4\pi}{3\Omega_m},\therefore \Omega_s=\frac{2\pi}{T}=\frac{3\Omega_m}{2},\therefore \omega_m=\Omega_m T=\frac{4}{3}\pi$$




------------------------------------------------------------------------

7、The transfer function of a casual LTI DT system is

$$H(z)=\frac{2+0.25z^{-1}}{1+0.25z^{-1}-0.06z^{-2}}$$

(1) Give the system's impulse response h[n].

(2) When the input is $x[n]=2(0.4)^n\mu[n]$

compute the system output y[n], for all n.

Solution: (1)因果性，且有 $H(z)=\frac{1}{1+0.4z^{-1}}+\frac{1}{1-0.15z^{-1}}$

$$h[n]=(-0.4)^n\mu[n]+(0.15)^n\mu[n]$$

(2) $X(z)=\dfrac{2}{1-0.4z^{-1}}, |z|>0.4$

$$Y(z)=H(Z)X(z)=\frac{1}{1+0.4z^{-1}}+\frac{4.2}{1-0.4z^{-1}}-\frac{1.2}{1-0.15z^{-1}}$$

$$y[n]=\mathrm{IZT}(Y(z))=(-0.4)^n\mu[n]-4.2(0.4)^n\mu[n]-1.2(0.15)^n\mu[n]$$

---

8、A causal LTI discrete time system is described by the difference equation:

$y[n]=0.5y[n-1]+0.25x[n]-x[n-1]$

x[n] and y[n] are the input and output sequences of the system

(1) Calculate the transfer function H(z) of this system.

(2) Give the zeros and poles of the system.

(3) Find out a minimum phase system $H_{min}(z)$ which has the same magnitude response as H(z) (give your reason).

Solution

(1) $y[n]=0.5y[n-1]+0.25x[n]-x[n-1]$

$Y(z)=0.5z^{-1}Y(z)+0.25X(z)-z^{-1}X(z)$ $\quad\quad$ $H(z)=\frac{Y(z)}{X(z)}=\frac{0.25-z^{-1}}{1-0.5z^{-1}}$

(2) Poles: $1-0.5z^{-1}=0,\ p_1=0.5$ $\quad$ Zeros: $0.25-z^{-1}=0,\ z_1=4$

(3) $\quad H(z)=\frac{0.25-z^{-1}}{1-0.5z^{-1}}=\frac{0.25-z^{-1}}{1-0.25z^{-1}}\times\frac{1-0.25z^{-1}}{1-0.5z^{-1}}=A(z)H_{min}(z)$

$A(z)==\frac{0.25-z^{-1}}{1-0.25z^{-1}}$ is an All-pass system.

$H_{min}(z)=\frac{1-0.25z^{-1}}{1-0.5z^{-1}}$ is a minimum phase system.

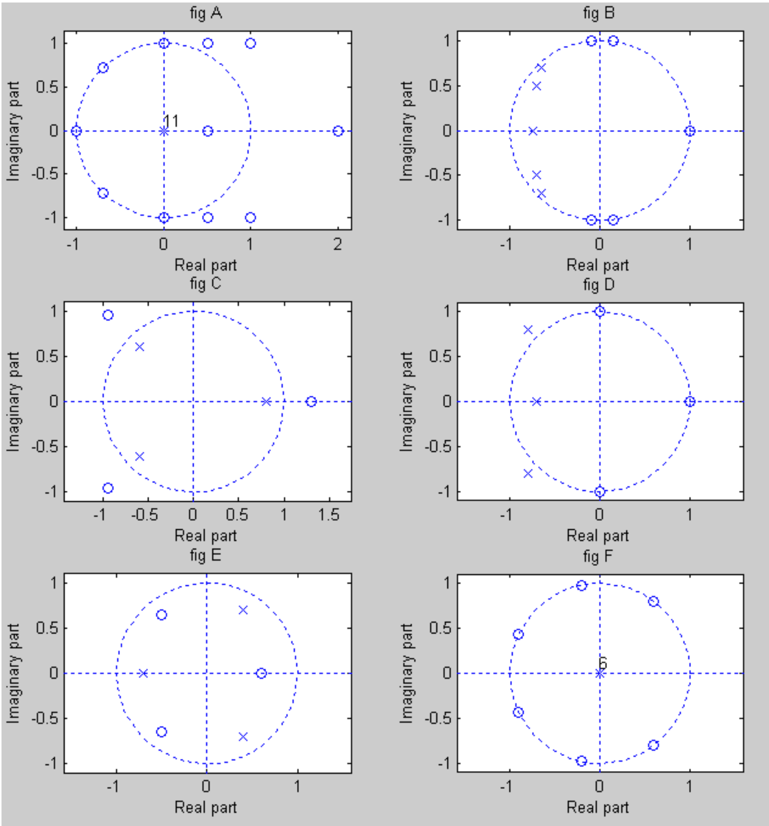
---

**9、The pole-zero plots in Figure 1 describe six different causal LTI systems.**

Answer the following questions about systems having the above pole-zero plots. In each case, an acceptable answer could be none or all.

1) Which systems are IIR system? Why?
2) Which systems are FIR system? Why?
3) Which systems are stable system? Why?
4) Which systems are minimum-phase system? Why?
5) Which systems are linear-phase system? Why?
6) Which systems have $|H(e^{j\omega})|$=constant for all $\omega$? Why?
7) Which systems has the shortest impulse response? Why?
8) Which systems have lowpass frequency responses? Why?

Figure1 Pole-zero plots of six different causal LTI system




**10、Let h[n] be a Type-4 real-coefficient linear-phase FIR filter .**

**1) If this filter has the following zeros:** $z_1 = 2, z_2 = 0.6j$ **, please determine the locations of the remaining zeros.**

**2) Please determine the FIR transfer function and realize it in cascade form.**

**Solution**:

$$(1).\ Type\ \ 4$$

$$h[n-N] = -h[n-N]$$

$$has\ \ a\ \ zero\ at\ z_3 = 1, (2\ \textbf{points})$$

$$and\ \ z_4 = \frac{1}{z_1} = \frac{1}{2}, (2\ \textbf{points})$$

$$real\ \ coefficient\ z_5 = z_2^* = -0.6j, (2\ \textbf{points})$$

$$z_6 = \frac{1}{z_2^*} = \frac{10}{6}j, (2\ \textbf{points})$$

$$z_7 = \frac{1}{z_2} = -j\frac{10}{6}. (2\ \textbf{points})$$

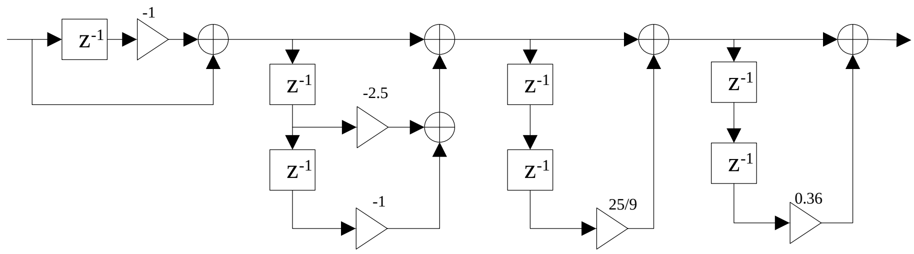
$$Length\quad N+1 = even, N\ is\ odd, let\ \ N = 7. (2\ \textbf{points})$$

$$(2)H(z)=(1-2z^{-1})(1-\frac{1}{2}z^{-1})(1-0.6jz^{-1})(1+0.6jz^{-1})(1-\frac{10}{6}jz^{-1})(1+\frac{10}{6}z^{-1})(1-z^{-1})\left(1\ \textbf{points}\right)$$

$$=(1-2.5z^{-1}+z^{-2})(1+0.36z^{-2})(1+\frac{25}{9}z^{-2})(1-z^{-1})\left(1\ \textbf{points}\right)$$

$$=1-3.5z^{-1}+6.637778z^{-2}-11.982z^{-3}+11.982z^{-4}-6.637778z^{-5}+3.5z^{-6}-z^{-7}\left(2\ \textbf{points}\right)$$

**cascade form:**




---

11、Known an 8-point real sequence $x[n]$(shown as following Figure 2 with 8-point DFT $X[k]$.

(1) Sketch $\left.\left(\frac{1}{8}\sum_{k=0}^{7}X[k]e^{j(2\pi/8)kn}\right)\right|_{n=9}$

(2) If $Y[k]=(-1)^k X[k]$, **sketch** $y[n]$ which is the 8-point IDFT of $Y[k]$ .

(3) Dtermine the value of $X[4]$.

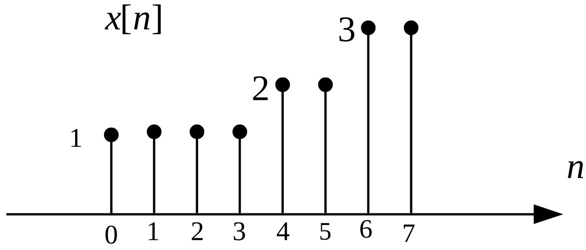



Fig 2

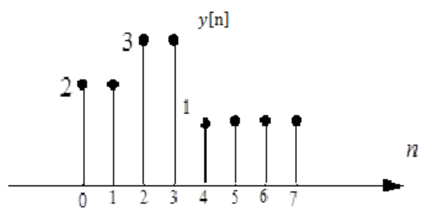
Solution (1) $X[k]=\sum_{n=0}^{7}x[n]W_8^{kn}$， $x[n]=\frac{1}{8}\sum_{n=0}^{7}X[k]W_8^{kn}=\frac{1}{8}\sum_{n=0}^{7}X[k]e^{j(2\pi/8)kn}$

$x[9]=\left(\frac{1}{8}\sum_{n=0}^{7}X[k]e^{j(2\pi/8)kn}\right)|_{n=9}$ $\quad x[<9>_8]=x[1]=1$

(2) $Y[k]=(-1)^k X[k]$

$y[n]=\frac{1}{8}\sum_{n=0}^{7}(-1)^k X[k]W_8^{kn}=\frac{1}{8}\sum_{n=0}^{7}X[k]W_8^{kn}W_8^{-k4}$

$=x[<n-4>_8]$



12、

Consider a signal $x[t]=A\cos(2\pi f_1 t)+B\cos(2\pi f_2 t)$, $f_1=3k$ $f_2=4k$，Suppose

x(t) is sampled with the $f_T=20k$ to produce the discrete sequence x[n].

Give the x[n].

If *A=B=1*，**design a FIR filter** to block the $f_1$, which should be suppressed by 60dB at least. You should use the **fixed window** function method and compute the **minimum length of window**. Give the mathematical expression of your FIR filter.

Solution.

$$x[n]=A\cos(0.3\pi n)+B\cos(0.4\pi n)$$

$$\omega_p=0.4\pi \quad \omega_S=0.3\pi \qquad \alpha_s=60dB$$

**Choose Blackman window,** $5.56\pi/M=0.1\pi \Rightarrow M\approx 56 \quad \omega_c=0.35\pi$

$$w[n]=\left[0.42+0.5\cos\left(\frac{2\pi(n-M)}{2M+1}\right)+0.08\cos\left(\frac{4\pi(n-M)}{2M+1}\right)\right],0\le n\le 2M$$

$$\hat{h}_{HP}[n]=\begin{cases}-\dfrac{\sin(\omega_c(n-M))}{\pi(n-M)},0\le n\le 2M,except\ n=M\\ 1-\dfrac{\omega_c}{\pi},n=M\ only\end{cases}$$

**FIR filter is:** $h[n]=\hat{h}_{HP}[n]\cdot w[n],0\le n\le 2M$

13、

Use bilinear transformation method to design a low-pass Butterworth digital filter with the following specifications: the passband edge frequency $\omega_p=\dfrac{\pi}{3}$, the stopband edge frequency $\omega_s=\dfrac{2\pi}{3}$, the maximum passband attenuation $\alpha_p=0.6dB$ and the minimum stopband attenuation $\alpha_s=-10dB$.

1)Calculate the prewarp frequencies $\Omega_p$ and $\Omega_s$.

2)Determine minimum order $N$ of the Butterworth filter

3)Determine the 3-dB cutoff frequency $\Omega_c$ when the **stopband** specification is exactly matched.

4)Use the results calculated in steps 2) and 3) to determine the analog prototype filter $H(s)$.

5)Use bilinear transformation to convert $H(s)$ into the desired digital filter $H(z)$

6)Draw a canonic structure realization of $H(z)$.

(1) $\Omega_p = \tan(\frac{w_p}{2}) = \tan(\frac{\pi}{6}) = \frac{\sqrt{3}}{3}$ (3 分)

$\Omega_s = \tan(\frac{w_s}{2}) = \tan(\frac{\pi}{3}) = \sqrt{3}$ (3 分)

(2) $\alpha_p = 20\log_{10}\sqrt{1+\varepsilon^2} \Rightarrow \varepsilon^2 = 10^{\frac{0.6}{10}} - 1 = 0.148$

$$\alpha_s = 20\log_{10}\frac{1}{A} \Rightarrow A = 10^{\frac{1}{2}} \Rightarrow A^2 = 10$$

所以 $k = \frac{\Omega_p}{\Omega_s} = \frac{\sqrt{3}/3}{\sqrt{3}} = \frac{1}{3}$ $\qquad k_1 = \frac{\varepsilon}{\sqrt{A^2-1}} = \frac{\sqrt{0.148}}{\sqrt{9}} = \frac{\sqrt{0.148}}{3}$

$N = \frac{\log_{10}(\frac{1}{k_1})}{\log_{10}(\frac{1}{k})} = \frac{\log_{10}(\frac{3}{\sqrt{0.148}})}{\log_{10}3} = 1.8695$ 所以 N = 2 (共 6 分)

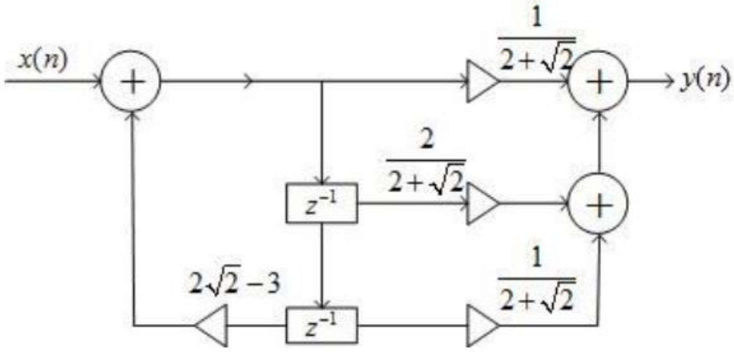
(3) $\frac{1}{(1+\frac{\Omega_s}{\Omega_c})^{2N}} = \frac{1}{A^2} \Rightarrow \Omega_c = \Omega_s(A^2-1)^{-\frac{1}{2N}} = \sqrt{3}\times(10-1)^{-\frac{1}{4}} = 1$ (3 分)

(4) $H(s) = \frac{1}{s^2+\sqrt{2}s+1}$ (3 分)

(5) $H(z) = H(s)\Big|_{s=\frac{1-z^{-1}}{1+z^{-1}}} \Rightarrow H(z) = \frac{1}{\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^2 + \sqrt{2}\frac{1-z^{-1}}{1+z^{-1}} + 1}$

$= \frac{1+2z^{-1}+z^{-2}}{(2+\sqrt{2})+(2-\sqrt{2})z^{-2}}$ (3 分)

(6) (3 分)



14、

Design a digital low pass filter to reduce the noise of an audio signal. The bandwidth of the signal is 3400Hz and the sampling frequency is 13600Hz. Based on a 3-order butterworth prototype filter using the bilinear transformation method to design an IIR filter and **give it's transfer function** ,then **sketch its canonic direct form structure**.

Solution.

$\omega_C = 2\pi \times \frac{3400(Hz)}{13600(Hz)} = 0.5\pi$, (2points )

The 3-order normalizedButterworth filter $H_{an}(s) = \frac{1}{(s+1)(s^2+s+1)}$ (3points )

prewarp: $\Omega_C = \mathrm{tg}(\omega_C/2) = 1$, (3points )

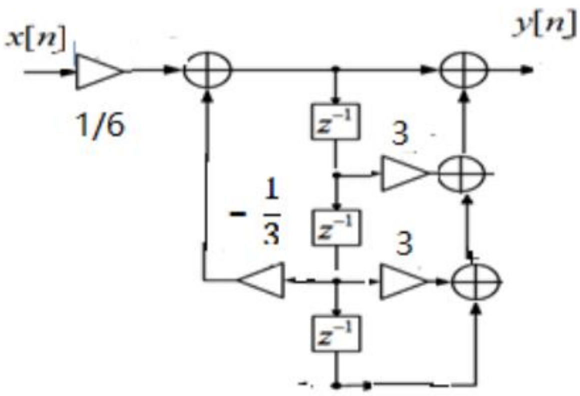
prototype filter is $H(s) = H_{an}(\frac{s}{\Omega_C}) = \frac{1}{(s+1)(s^2+s+1)}$ (2points )

in bilinear transform method,

$$G(z) = H(s)|_{s=\frac{1-z^{-1}}{1+z^{-1}}} = \frac{1}{(s+1)(s^2+s+1)}\Big|_{s=\frac{1-z^{-1}}{1+z^{-1}}}$$

$$G(z) = \frac{1+3z^{-1}+3z^{-2}+z^{-3}}{6\times(1+\frac{1}{3}z^{-2})}$$ (5points )

**its canonic direct form structure**




(5points )

1．Use **fixed** windows to design a lowpass FIR filter with a linear phase. When the following signal x[n] pass this filter, the higher frequency should be suppressed by at least 50dB. Give the specifications of this filter, choose a window and explain your reasons, and give the impulse response of this filter.

$$x[n] = \cos(\frac{\pi n}{6}) + \frac{3}{4}\cos(\frac{\pi n}{4})$$

2 .If use a rectangular window with length-N to multiply x[n], and then do N-point DFT analysis.

What is the smallest N, which can clearly resolve(分辨) these two sinusoids of x[n] in magnitude of DFT? Sketch the magnitude of DFT.

1）choosing Hamming window for 50dB suppression.

$$\omega_p = \frac{\pi}{6}, \quad \omega_s = \frac{\pi}{4}, \quad \Delta\omega = \omega_s - \omega_p = \frac{\pi}{12}, \quad \omega_c = \frac{\omega_p + \omega_s}{2} = \frac{5\pi}{24}$$

From table 10.2, $\frac{3.32\pi}{M} \le \frac{\pi}{12} \quad \Rightarrow \quad M \ge 39.84, let\ M = 40$

Ideal LPF $h_d[n] = \frac{\sin(\omega_c n)}{\pi n} = \frac{\sin\left(\frac{5\pi n}{24}\right)}{\pi n}$

$$w[n] = 0.54 + 0.46\cos\left(\frac{\pi n}{40}\right), \quad -40 \le n \le 40$$

So, the causal LPF is

$$h_t[n] = h_d[n-M]w[n-M] = \frac{\sin\left(\frac{5\pi(n-40)}{24}\right)}{\pi(n-40)}[0.54 + 0.46\cos\left(\frac{\pi(n-40)}{40}\right)], 0 \le n \le 80$$

N-point DFT has resolution $\frac{2\pi}{N}$. When x[n] is analyzed with N-point DFT, it is required $\frac{2\pi}{N} \le \Delta\omega = \frac{\pi}{12} \Rightarrow N \ge 24$

We can roughly sketch the magnitude of 24-point DFT as following


----------------------------------------------------------------

1. Design a linear-phase FIR system $H(e^{j\omega})$ by windowing. The specifications are given as follow

$$\begin{cases} 0.9 \le \left|H\left(e^{j\omega}\right)\right| \le 1, & |\omega| \le 0.1\pi \\ \left|H\left(e^{j\omega}\right)\right| \le 0.01, & 0.5\pi \le |\omega| < \pi \end{cases}$$

(1) Find the **possible choices** for the type of fixed window

(2) If the **Hamming** window is employed, determine the **minimum N** and **group delay** of the design for **type II** linear-phase FIR, where M is defined as follow

$$\text{Window}[n] = \begin{cases} \text{Non}-\text{zero}, & 0 \le n \le N \\ 0, & others \end{cases}$$

(3) Based on question2, if we want to construct the filter by **Linear-phase direct form**, how many **multipliers, delayers and adders** shall we need?

(4) If we want to design an IIR filter by **bilinear transform** to satisfy $\left|H(e^{j\omega})\right|$, indicate the specifications of analog prototype filter(模拟原型滤波器). (Note: sampling frequency T=2)

(5) Compare the **advantages** of IIR and FIR filter.

A: (1)窗类型决定了滤波器设计的逼近误差

通带逼近误差$\delta_1 < 0.1$，阻带逼近误差$\delta_2 < 0.01$

$$\to \delta < min(\delta_1, \delta_2) = 0.01 = -40dB$$

$\to$Hanning , Hamming 和 Blackman 均可使用.

(2) 采用 Hamming 窗，其过渡带$\Delta\omega = 3.32\pi/M$

滤波器设计要求过渡带$\Delta\omega = 0.5\pi - 0.1\pi = 0.4\pi$

$M > 8.3$, 由于采用 TypeII 类型，要求 N 取奇数，N=17, 其群延迟=8.5

(3) TypeII 线性相位结构需要运算资源：

加法器数量：17 个；延迟单元数量：17 个；乘法器数量：9 个

(4) 双线性变换，模拟原型滤波器设计指标为

$$\begin{cases} 0.9 \le |H(j\Omega)| \le 1, & |\Omega| \le \tan(0.05\pi) \\ |H(j\Omega)| \le 0.01, & |\Omega| \ge \tan(0.25\pi) \end{cases}$$

(5) IIR 滤波器与 FIR 滤波器比较

① 相同的技术指标下，IIR 滤波器的阶数比 FIR 滤波器的少。
② FIR 滤波器可得到严格线性相位，而 IIR 滤波器则难以做到。
③ IIR 滤波器只有当所有极点都在单位圆内时滤波器才是稳定的，FIR 滤波器均稳定。
④ 相同阶数下，FIR 滤波器由于可采用 FFT 来实现，因此运算速度更快
⑤ IIR 滤波器可利用 C-T 滤波器现成的闭合公式、数据和表格来设计，计算工作量较小。