Théorème Central Limite

$X \sim \begin{matrix} \mu_0 \\ \sigma_0 \end{matrix}$ $\quad x_1, \ldots, x_N$

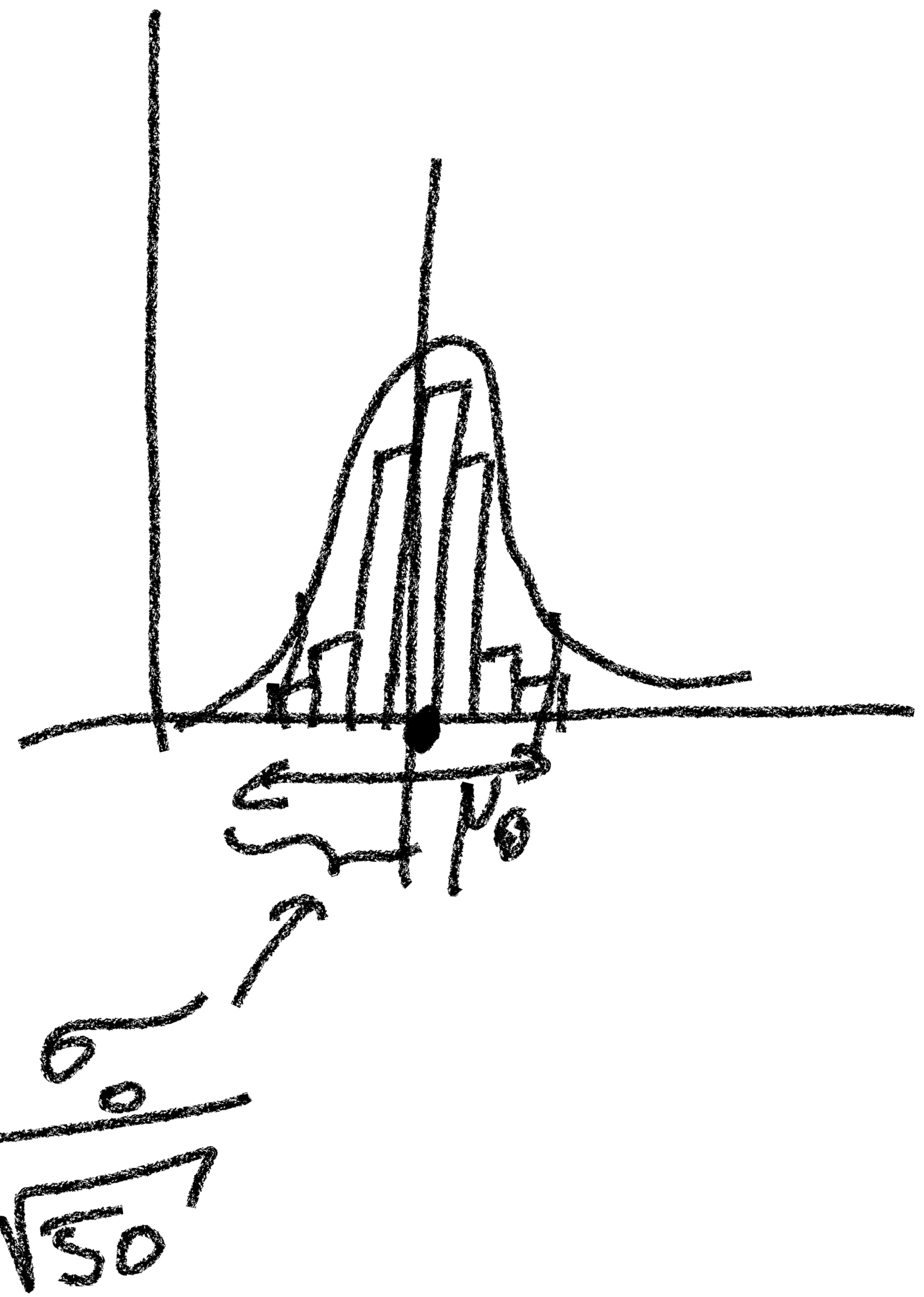
N expériences aléatoires

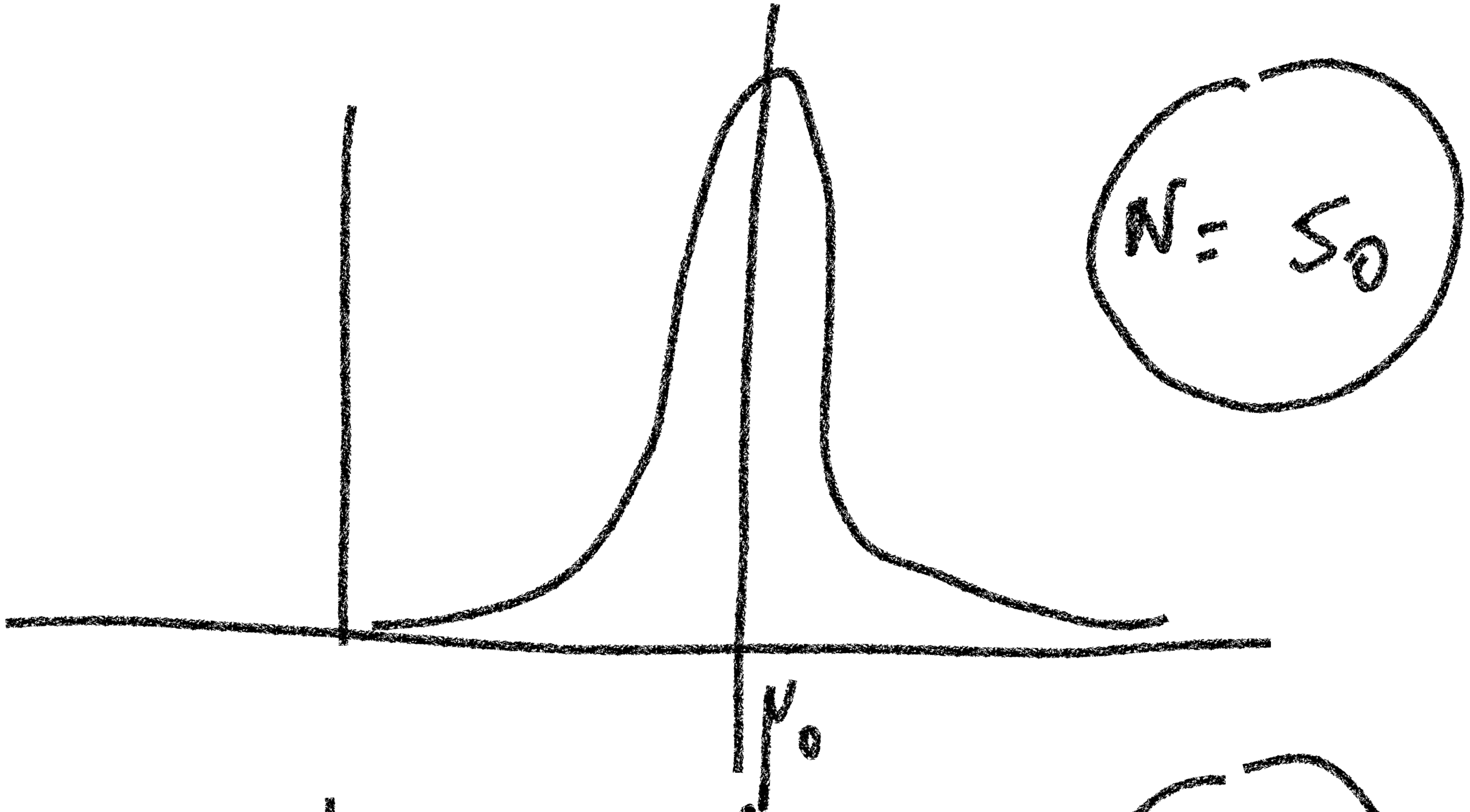
$$\underbrace{\frac{1}{N} \sum_{i=1}^{N} x_i}_{\text{moyenne empirique}} \sim \mathcal{N}\left(\mu_0, \frac{\sigma_0}{\sqrt{N}}\right)$$

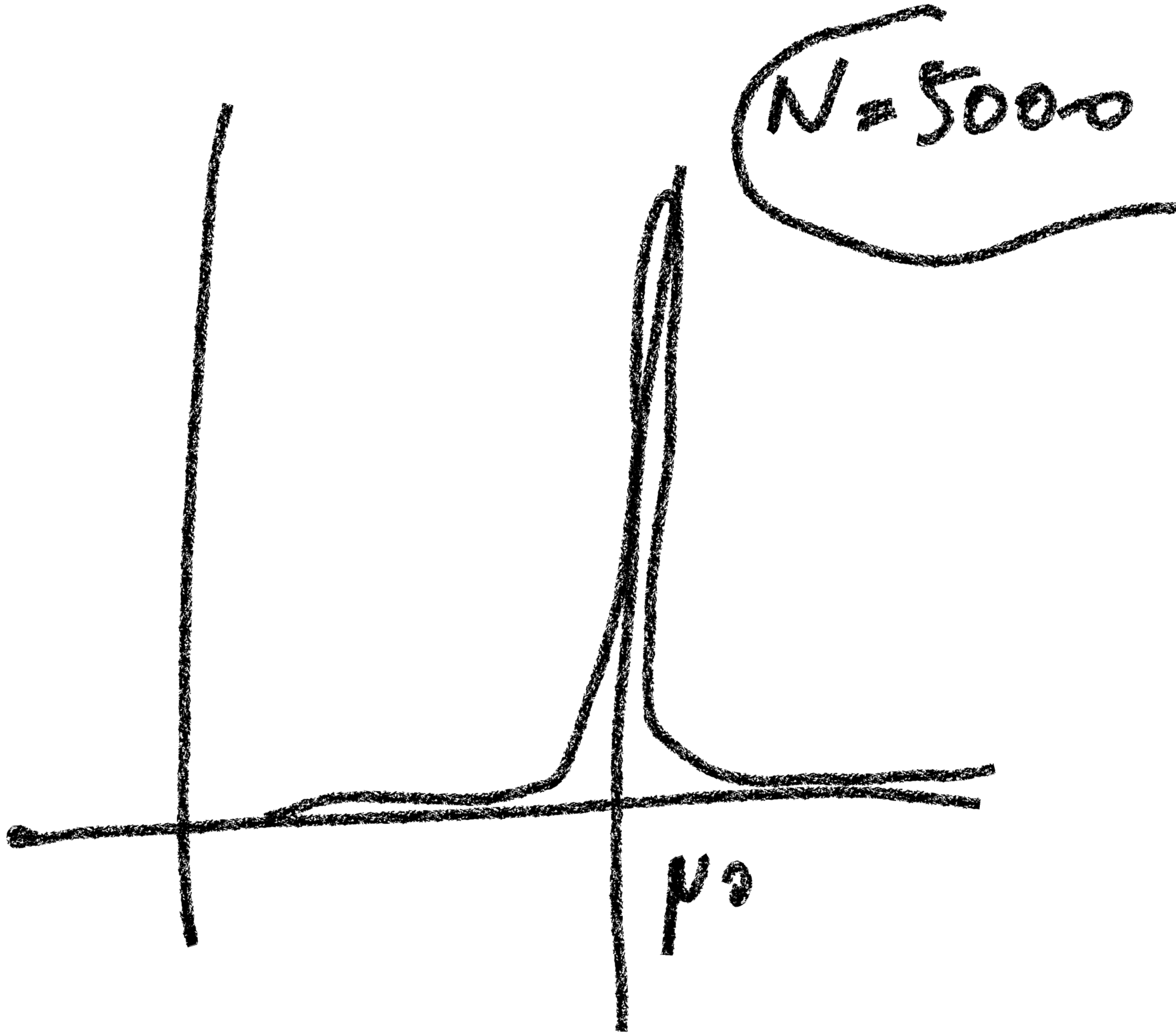
X = temp à Paris

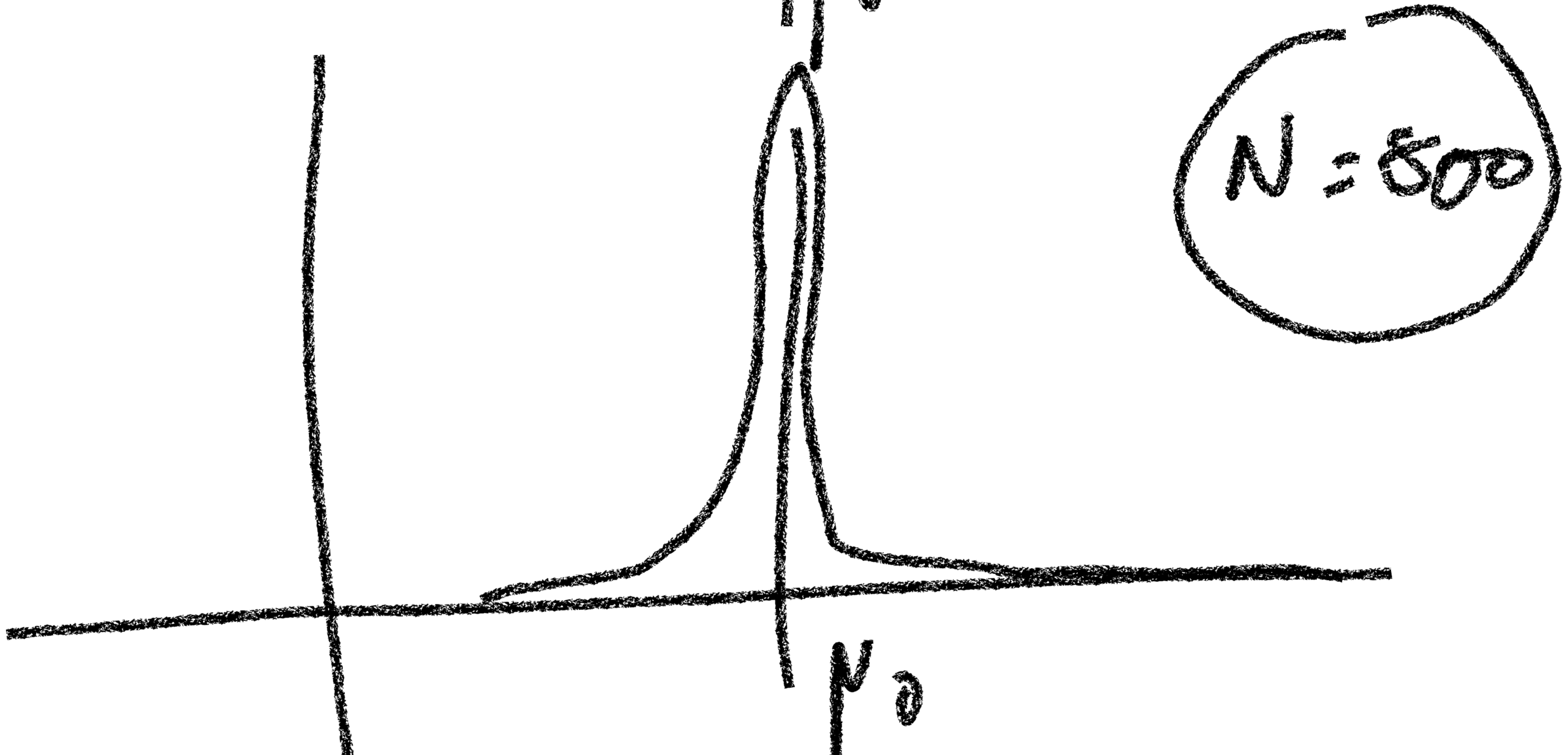
$$\begin{array}{cccc} x_1 & x'_1 & x''_1 & \\ \vdots & \vdots & \vdots & \cdots \\ x_{50} & x'_{50} & x''_{50} & \\ \downarrow & \downarrow & \downarrow & \\ \mu & \mu' & \mu'' & \end{array} \quad \Longrightarrow$$

Histogramme




N = 50
μ_0


N = 5000
μ_0


N = 500
μ_0

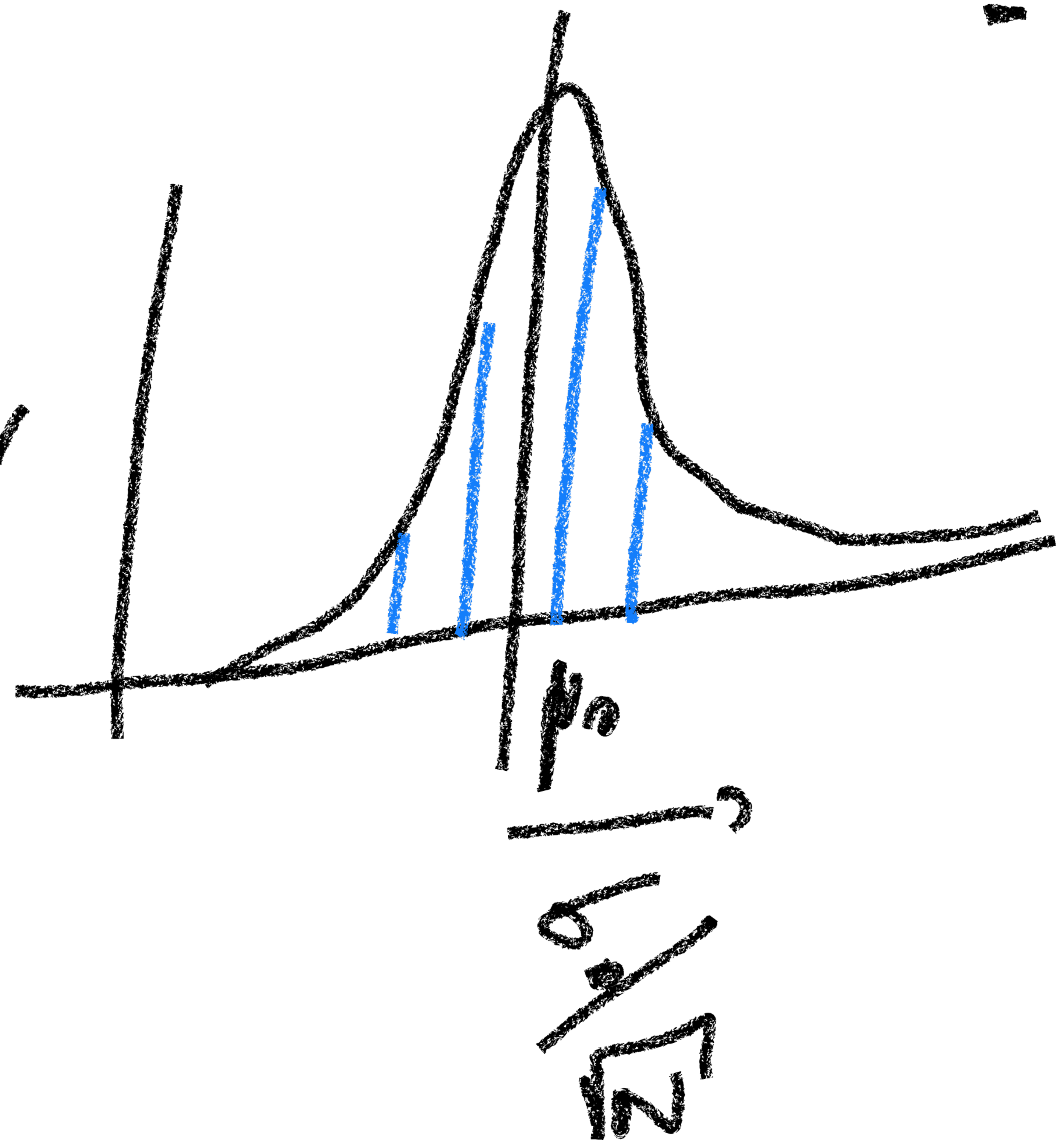
X
3.5
7.2
8.4
$\vdots$

Hyp: $X \sim \mu_0, \sigma_0$

$\mu_0 \approx ?$

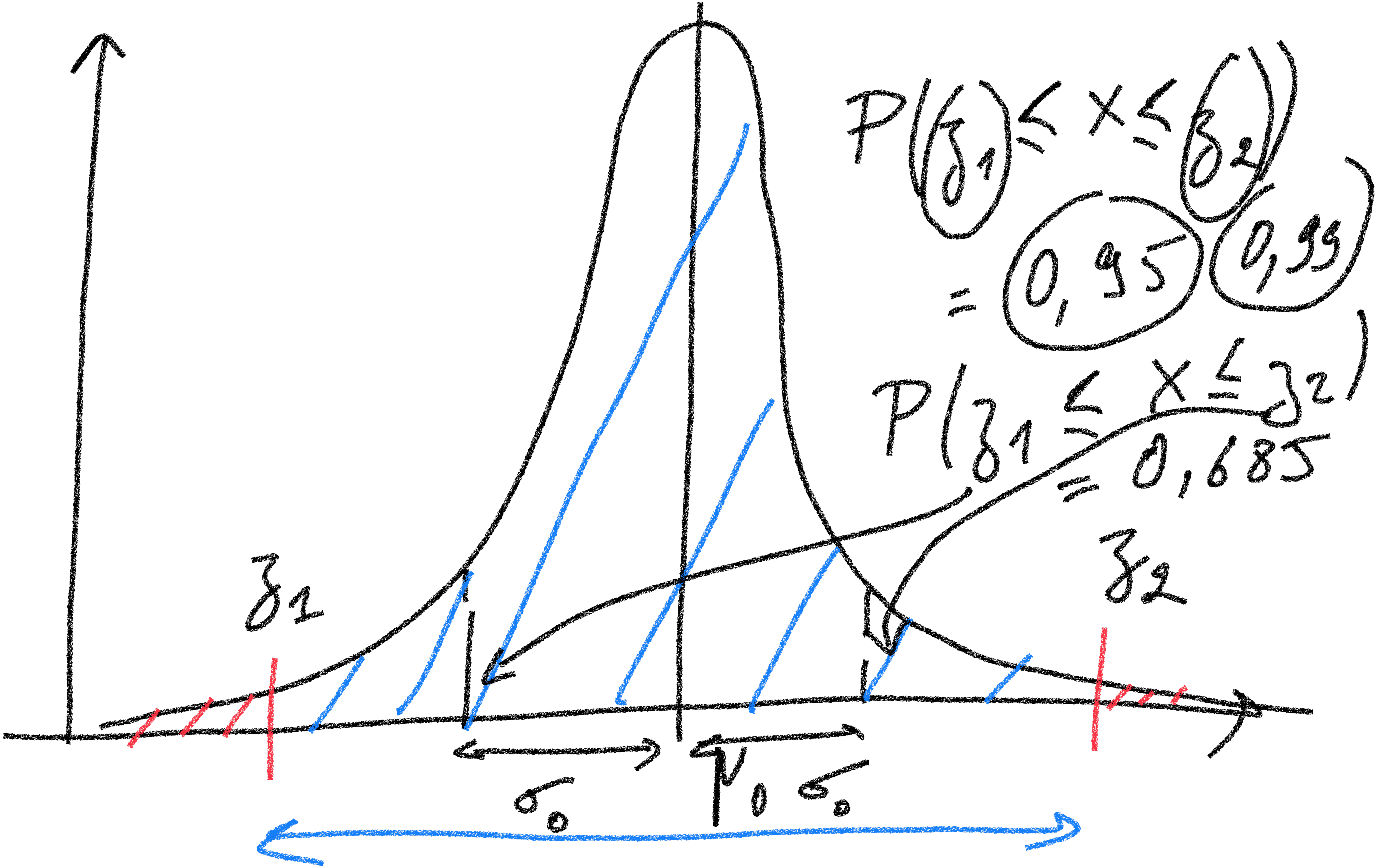
$\frac{1}{N}\sum_{i=1}^{N} x_i \approx \mu_0$

$\Rightarrow \mathcal{N}$



Loi des Grands Nombres

Intervalles de Confiance


P(z1 ≤ x ≤ z2)
= 0,95 0,99
P(z1 ≤ x ≤ z2)
= 0,685
z1
z2
σ0
μ0
σ0

IC : Seuil de confiance $1-\alpha = 95\% \ / \ \alpha = 5\%$

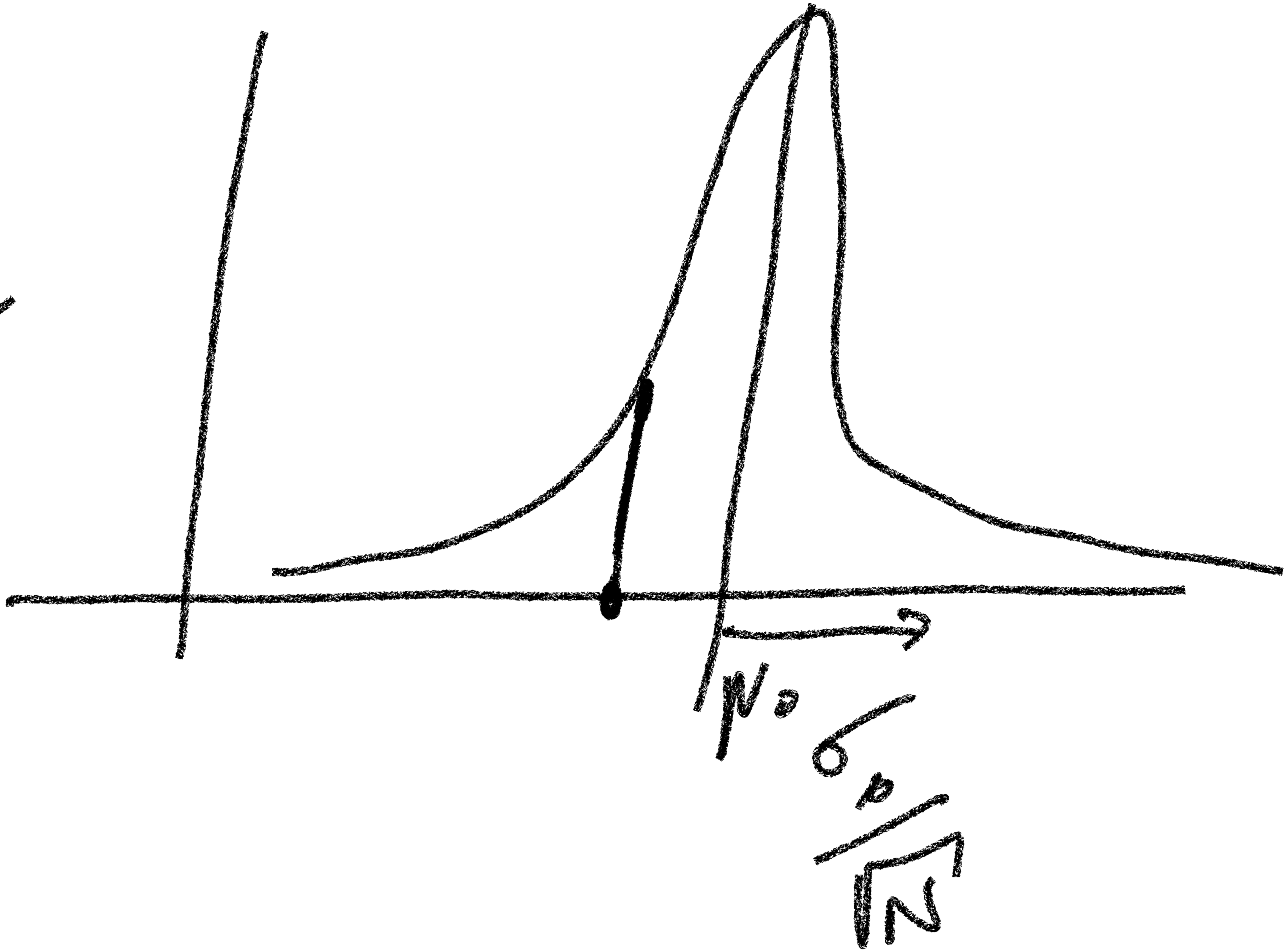
$$[z_1, z_2] \ / \ P(z_1 \leq \mu_0 \leq z_2) = 1-\alpha$$

$1-\alpha = 95\%$

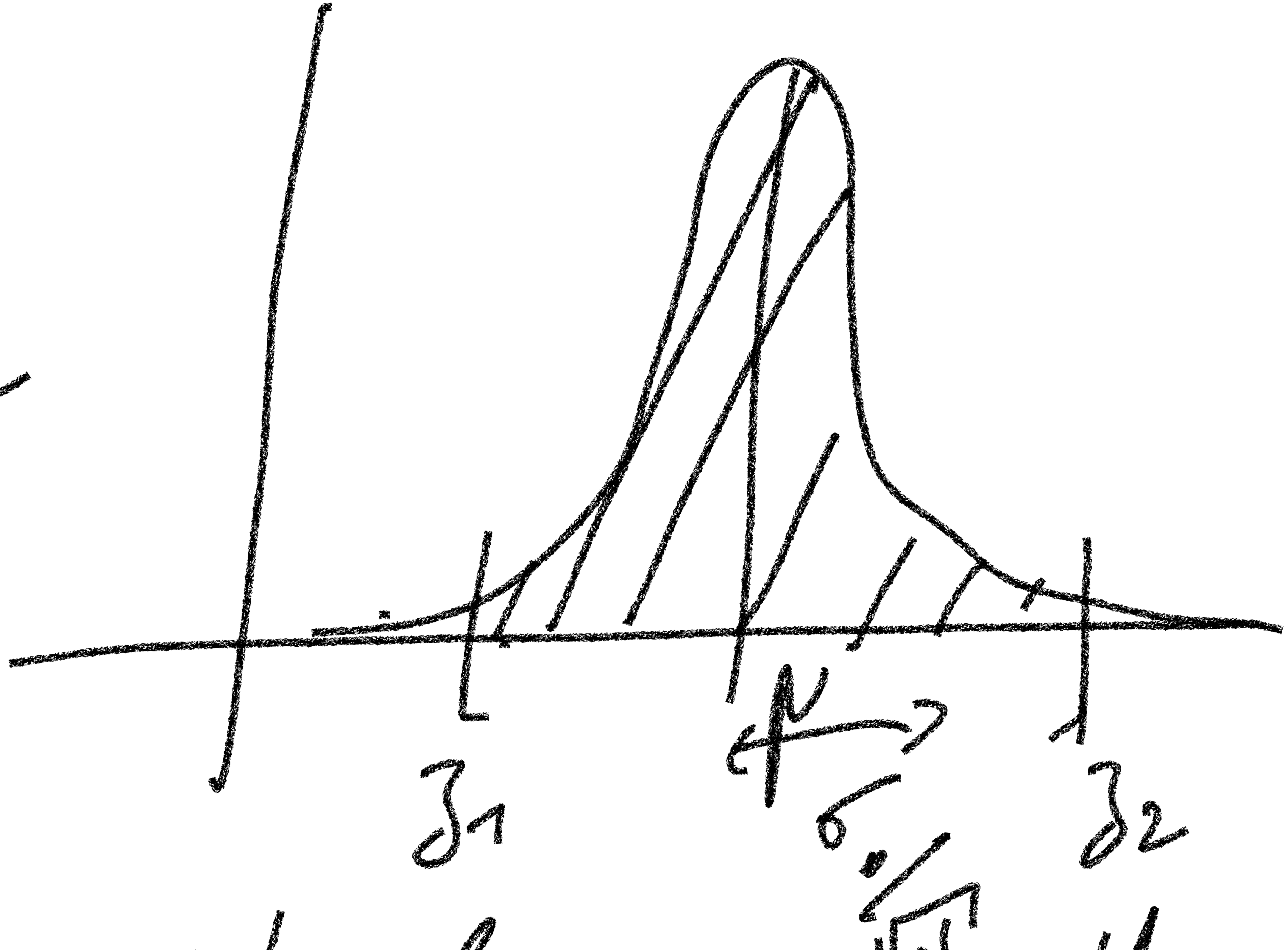
ex) : $[19{,}5\,°C , 23{,}7\,°C]$

μ_0 a 95% de chances de se trouver dans $[19{,}5 , 23{,}7]$

$\mu \sim$



$\mu_0 \sim$



scipy.stats.norm.interval(mu = emp, std = $\frac{\text{emp\_std}}{\sqrt{N}}$, alpha = 0.05)

$$\text{interval}(\ldots) = \left[19{,}5 \quad , \quad 23{,}7 \right]$$

$N = 10\,000$ échantillons

$\sigma_0 = 5$ ans

$\mu_0 = ?$

$\mu = \frac{1}{N} \sum$ âges

$\alpha = 0.05$

scipy.stats.norm.interval(mu = μ, std = $\frac{\sigma_0}{\sqrt{10000}}$, alpha = $\cancel{0.05}$ 0.01)

$= [37.9, 42.1]$

Hyp	Loi de Prob	Ecart type
σ_0 conn μ_0 inc N samples	Loi Normale	$\frac{\sigma_0}{\sqrt{N}}$
μ_0, σ_0 inc N samples p : tx conv emp	Loi Normale	$\sqrt{\frac{p(1-p)}{N}}$
N samples σ = var emp	Loi du χ^2 (Chi 2)	stats.chi2.interval($1-\alpha$, $N-1$) "$[z_1, z_2]$" $\left[\frac{\sigma^2 N}{z_2}, \frac{\sigma^2 N}{z_1}\right]$

Hyp	Loi	Écart type
μ_0 inc $\to \mu_0$? σ_0 inc N samples σ écart type emp	Loi de Student	$\frac{\sigma}{\sqrt{N-1}}$