

滤波过程一般分为两个步骤：**预测和更新**

预测：

$$\begin{aligned}
 & p(x_t | z_{1:t-1}) \\
 &= \int p(x_t, x_{t-1} | z_{1:t-1}) dx_{t-1} \\
 &= \int p(x_t | x_{t-1}) p(x_{t-1} | z_{1:t-1}) dx_{t-1}
 \end{aligned}$$

更新：

$$\begin{aligned}
 & p(x_t | z_{1:t}) \\
 &= \frac{p(x_t, z_{1:t})}{p(z_{1:t})} \\
 &= \frac{p(x_t, z_{1:t-1}, z_t)}{p(z_{1:t-1}, z_t)} \\
 &= \frac{p(z_t | x_t, z_{1:t-1}) p(x_t, z_{1:t-1})}{p(z_t, z_{1:t-1})} \\
 &= \frac{p(z_t | x_t, z_{1:t-1}) p(x_t | z_{1:t-1})}{p(z_t | z_{1:t-1})} \\
 &= \eta p(z_t | x_t) p(x_t | z_{1:t-1})
 \end{aligned}$$

t 时刻的观测值 z_t 只由状态值 x_t 决定，与 $z_{1:t-1}$ 无关

$$\begin{aligned}
 \frac{1}{\eta} &= \int p(z_t | z_{1:t-1}) dx_t \\
 &= \int p(z_t | x_t) p(x_t | z_{1:t-1}) dx_t
 \end{aligned}$$

$$\begin{aligned}
 & p(x_t | z_{1:t}) \\
 &= \eta p(z_t | x_t) p(x_t | z_{1:t-1}) \\
 &= \frac{p(z_t | x_t) p(x_t | z_{1:t-1})}{\int p(z_t | x_t) p(x_t | z_{1:t-1}) dx_t}
 \end{aligned}$$