University of Colorado - Boulder

ASEN 3200: Attitude Dynamics & Orbital Mechanics & Control

May 4th, 2023

Constellation Orbit Design

Author:
Hideyuki Nakanishi

Author:
Liam Mazzotta

Author:
Stefano Rivellini

Professor: Dr. Mark J. Moretto

Cheetah

Design, building, and placing spacecraft in orbit for a mission is a complex endeavor that only becomes more complex as the number of spacecraft increases. The objective of this lab is to minimize SV1, the number of satellites, and maximize SV2, the science value. The constellation developed consists of 3 satellites whose respective starting orbital elements are in the order of [a (km), e, i (deg), Ω (deg), ω (deg), θ (deg)] are [0.4643, 0, 60, 75, 90, −90], [0.4643, 0.18, 30, 45, 25.7, 180]**, and** [0.4643, 0.18, −45, 45, 25.7, 0]**. The Spacecraft respective cost values are** (J_{sv}) **are** 147536.04, 137368.19, 154420.06 **giving the full constellation a value of** 439324.29**.**

Ann and H.J. Smead
Aerospace Engineering Sciences
UNIVERSITY OF COLORADO **BOULDER**

Contents

I. Nomenclature

μ	=	Asteroid Gravitational Parameter $[km^3/s^2]$
m	=	Mass of each Spacecraft [kg]
A	=	Area of Solar Panels for Power $[m^2]$
d	=	Distance of a Target Point and Spacecraft at a time [m]
a	=	Semi-Major Axis of Orbit[km]
e	=	Eccentricity
r_p	=	Radius of Perigee [km]
r_a	=	Radius of Apogee [km]
i	=	Inclination Angle [rad/deg]
Ω	=	Right Ascension of the Ascending Node [rad/deg]
ω	=	Argument of Perigee [rad/deg]
θ	=	True Anomaly [rad/deg]
ϕ	=	Local Elevation Angle [rad/deg]
FOV	=	Field of View of Camera [rad/deg]
P_{SR}	=	Constant Solar Pressure $[N/m^2]$
C_R	=	Spacecraft Reflectivity
a_{SRP}	=	Acceleration due to Solar Radiation Pressure $[km/s^2]$
PQW	=	Perifocal Coordinate Frame
ACI	=	Asteroid Inertial Frame
$Body$	=	Asteroid Body Frame
$\hat{X}_I$	=	Inertial X-axis [1 0 0]
J_{sv}	=	Science Value
$SV1$	=	Number of Spacecraft
$SV2$	=	Science Value

II. Introduction

In the field of space science, the coupled relationship between time and money creates the bounds of projects. This being the case it is important for engineers to reduce the cost per amount of science. When determining the number of spacecraft and orbits to target scientific points of interest on the asteroid Bennu it is of great importance to create a satellite constellation solution that allows science to be conducted at all times for the duration of the missions lifespan while minimizing the resources used to send place the stats in the specified orbits. Satellites being expensive complex systems it is necessary that the number used in the proposed constellation is kept to a minimum. To determine the number of spacecraft in the constellation and their orbital elements the dynamics from the classical two-boy problem and added solar radiation pressure are used to propagate spacecraft orbits for varying orbital elements until an adequate J_{sv} is achieved. As the area and mass are functions of the number of spacecraft the J_{sv} and the orbit propagation are highly dependent on the number of satellites used.

III. Design Objectives & Constraints

The constellation design objectives are to collect useful scientific information on the surface of Bennu as much as possible for a week, minimizing the number of spacecraft. Along with determining the number of spacecraft, there are constraints that need to be considered. Increasing the number of spacecraft, each spacecraft gets smaller and less capable than their larger counterparts. For this reason, it requires a small amount of spacecraft with a high scientific value. About the characteristic of Bennu and spacecraft, the asteroid is spinning around the Z-axis in the ACI frame at a constant speed. The spacecraft can sense the surface of Bennu in the sunlight and in a local elevation angle of greater than or equal to 15 degrees. The Sun is assumted to be fixed and located in the negative X-axis in the ACI frame. In addition, the camera always points in the opposite radial axis and it needs to be less than or equal to the field of view of the camera (FOV) that is dependent on the number of satellites. Also, in space, there is constant solar radiation pressure in the negative X-axis in the ACI frame. It perturbates the motion of the spacecraft in orbit. Another orbit constraint for all spacecraft is the periapsis radius should be greater than or equal to 0.3 km and apoapsis radius should be less than or equal to 3km from the center of the asteroid.

IV. Design Approach

The design approach to this complex problem began with the fundamental dynamics of a classical two-body problem and then add complexity step by step. The foundation of the project relies on being able to propagate orbits through time using MATLAB's ODE45. So beginning with the two-body equation below.

Classical Two-Body Equation:

$$\ddot{\vec{r}} = \frac{-\mu}{||\vec{r}||^3}\vec{r} \tag{1}$$

Acceleration due to a constant solar radiation pressure was then added. At this stage in the design process, it is noted that the acceleration due to solar radiation pressure is dependent on both the area and mass of the satellites which is ultimately a function of the number of satellites used.

Satellite mass function:

$$m = \frac{1000}{1.05N_{sc}}[kg] \tag{2}$$

Satellite area function:

$$A = 5 - \frac{1}{5}(N_{sc} - 1)[m^2] \tag{3}$$

Function to calculate acceleration due to solar radiation pressure:

$$P_{sr} = 4.57e - 3; C_R = 1.2 \tag{4}$$

$$a_{SRP} = -\frac{P_{sr} * C_R * A}{m} * (-\hat{X}_I)[km/s^2] \tag{5}$$

Once the function to propagate the spacecraft orbits is developed the next step in the design approach is to look at the mass that the constellation is going to be orbiting. In this case, the constellation is being designed to orbit and collect data on the surface of the asteroid Bennu. To do this the gravitational parameter mu and the period of Bennu must be known. In addition to these key parameters, the targets of interest on the surface of Bennu must also be defined. The value of μ used in the design of this constellation is $4.892e-9[km^3/s^2]$ and the value of the period used is $1.5471e4[s]$. The surface of Bennu is defined by facet numbers and vertex vectors. Where each facet is composed of the space between three vertices each defined by a vector creating a polyhedron. Having a well-defined surface is particularly important for the case of the constellation design as the angle between the facets unit normal and the spacecraft directly relates to whether or not the spacecraft constellation can view the targeted facets for scientific observations.

Function to calculate Field of View:

$$FOV = \frac{\pi}{9N_{sc}} \tag{6}$$

Now that the orbits of the spacecraft and the Body has been defined the view of the spacecraft over time can be calculated and checked against the targeted list to determine whether or not a target is in view and further if the target is illuminated by the sun allowing the viewing of the target facet to be scientifically significant. This visibility check is needed to determine the value of the cost function J_{sv} that is trying to be maximized.

Phase Angle Mask function to determine whether or not the target is illuminated by the Sun:

$$H_i(t_k) = ceil(-\hat{X}_I \cdot \hat{r}_{targ,i}(t_k)) \tag{7}$$

All of the variables used to calculate the cost function such as the phase angle mask elevation angle and distance between the spacecraft and the viewable facet are calculated at each time step throughout the one-week simulation that the targets are viewable to each spacecraft.

Once the overall project puzzle was put together it is recognized that without being able to determine the initial orbit state vector to be propagated from the orbital elements optimizing the constellation design will be changing. To take the guesswork out of initializing the propagation of the spacecraft a function was developed to convert the initial orbital elements into an initial state vector to be propagated. This allowed visual judgment to be used to optimize the constellation's orbits.

Initializing the orbit orientation is important to maximize the scientific value (i.e. The cost function J_{sv}). To begin with, to acquire a large amount of scientific information, it is necessary for satellites to be situated at a high angle on each target point as possible plus, at a close distance from the target points but without collision. For that reason, the semi-major axis of each orbit needs to be as small as possible.

Function to calculate ground resolution per pixel:

$$GR_{i,j,k} = \frac{FOV}{2048 - \frac{1408}{9}(N_{sc}-1)} d_{i,j,k} \tag{8}$$

Minimizing the distance between the satellite minimizes the ground resolution term which has the effect of increasing the science value being maximized by the inverse of a square. To maintain a minimum distance the shape of the asteroid must be accounted for

The shape of the asteroid looks like a diamond shape. Thus, the orbits need to fit it by adding eccentricity. Next, determining the inclination and right ascension node is key to gaining high scientific information. Since we decided to operate three satellites, they need to cover all over the surface over time. therefore, one satellite orbits around close to the minor axis of the asteroid. The other two satellites orbit diagonally in opposite directions changing 45 degrees for both right ascension nodes. The argument of periapsis also needs to be taken into account to get closer to the surface. The initial true anomaly should be in the sunlight.

The Equation for calculating J_{sv}:

$$J_{sv} = \sum_{i=1}^{N_{targ}} \sum_{j=1}^{N_{sc}} \sum_{k=1}^{N_{V,i,j}} [\frac{0.007263}{GR_{i,j,k}^2} sin(\phi_{i,j,k}) H_i(t_k)] \tag{9}$$

Overall the design approach for minimizing the number of satellites used in the orbit while maximizing the cost function was to clear orbits that maximize their time in the sun with eccentricities that allow the orbits to conform to the shape of the body Beun. The conformity of the orbit throughout its propagation the distance between the spacecraft and the surface to remain at a minimum which in turn increases the value of the cost function.

V. Final Solution

The final solution for maximizing the value of SV2 and minimizing the value of SV1 consists of the three orbits with the following orbital parameters.

Spacecraft	a [km]	e	i [deg]	Ω [deg]	ω [deg]	θ [deg]
1	0.4643	0	60	75	90	-90
2	0.4643	0.18	30	45	25.7	180
3	0.4643	0.18	-45	45	25.7	0

Table 1 Maximized Orbital Elements at t = 0

Each of the set of initial orbital elements begins with the same semi-major axis value. Spacecraft 1 orbit around close to the minor axis of the asteroid. While Spacecraft 2 and Spacecraft 3 satellites orbit diagonally in opposite directions changing 45 degrees for both right ascension nodes.

Spacecraft Initial State (X0)						
X0	**x [km]**	**y [km]**	**z [km]**	v_x **[km/s]**	v_y **[km/s]**	v_z **[km/s]**
Spacecraft 1	0.12017	0.44849	-9.8488e-17	-4.9573e-05	1.3283e-05	8.8893e-05
Spacecraft 2	-0.20348	-0.49462	-0.11886	7.3462e-05	-2.0958e-05	-3.8546e-05
Spacecraft 3	0.15996	0.32516	-0.11681	-9.3247e-05	1.7692e-05	-7.8445e-05

Table 2 Initial State Vector for Each Satellite

Using the orbital elements the position and velocity vectors can be calculated in the Perifocal frame. Through a transformation using the angles i,Ω, and ω the initial state in the Perifocal fram can be converted into the ACI coordinate frame. The values of the initial state in the ACI frame are listed in the table above.

Constilation Cost Function Value (Jsv)	
Spacecraft 1	147536.04
Spacecraft 2	137368.19
Spacecraft 3	154420.06
Total	439324.29

Table 3 Science Value for Each Satellite

After calculating the cost function values for each orbit it can be seen that Spacecraft 3 has the greatest scientific value, followed by Spacecraft 1 and then Spacecraft 2. For a combined Science value of 439324.

VI. Discussion

Reflecting back on the objective to design a constellation that maximizes the amount of science for the fewest number of satellites. The final solution does just that producing a J_{sv} = 439324.29 with only three spacecraft while staying within the bounds of the design requirements. All three orbits propagated for the one-week time window avid collisions with the asteroid Bennu and each other while they collect scientific information.

The Orbits chosen are either circular or near circular with a relatively small semi-major axis of 0.4643 km for the proposed design space that allows a semi-major axis to be as large as 3km for a circular orbit. As the cost function is inversely proportional to the square of the distance between the satellites and the target surface our design takes advantage of this with its short semi-major axis distance.

In addition, our design maximizes the time that the satellite is in the sun by inserting them into orbit with true anomalies where the surface of Benu is in the sun. This allows the spacecraft to begin collecting scientific data as soon as they are inserted into orbit. This proposed design utilizes the eccentricity of the orbit to better fit the diamond shape of the asteroid. With a complementary fit between the asteroid and the spacecraft, more data is able to be collected as compared to using all purely circular orbits.

When looking at the ground track the final constellation design stays between latitudes of 60 deg and -60 degrees as this is where the majority of the targets are concentrated. By having all of the orbits in the constellation track over the greatest population density of targets it is able to provide scientists with constant views of large amounts of scientifically interesting targets.

Average Elevation Angles [deg]	
Spacecraft 1	75.5
Spacecraft 2	70.9
Spacecraft 3	75.1
Total	73.8

Table 4 Average Elevation Angle for Each Satellite

The elevation angle of the spacecraft in the constellation is near normal to the visible target facets. With the elevation angle being high the scientific instruments have a near-normal view of the target points on the surface of Benu. The more normal the elevation angle the less chance it is for the scientific measurements to be distorted by the angle at which the spacecraft views the target. When varying the orbital elements our spacecraft were initially viewing the targets at shallow angles of elevation. The final solution constellation design improves upon this. The improved orbit designs average elevation angle of 74 degrees.

VII. Conclusion

It was determined that in order to maximize the amount of scientific value of the constellation three unique satellite orbits are needed. With the three chosen orbits the values of SV1 = 3 and the value of SV2 = 439324. The Final design is able to provide the scientist that utilizes the constellation of observations of the targeted points on the surface of Benu near normal vies with excellent ground resolution per pixel. The same concepts that are used to develop the final solution can also be applied to future missions to Benu and other bodies of interest. As the ground-up design approach allows for the progressive addition of complexity. Leaving room for the ability to increase the complexity of the orbits in steps allows this final design to be improved upon and iterated for higher fidelity orbit determination problems for both missions to Benu and Others in the future.

VIII. Appendix

A. Plots

Fig. 1 Time Series of the Number of Spacecraft in View of the Targets

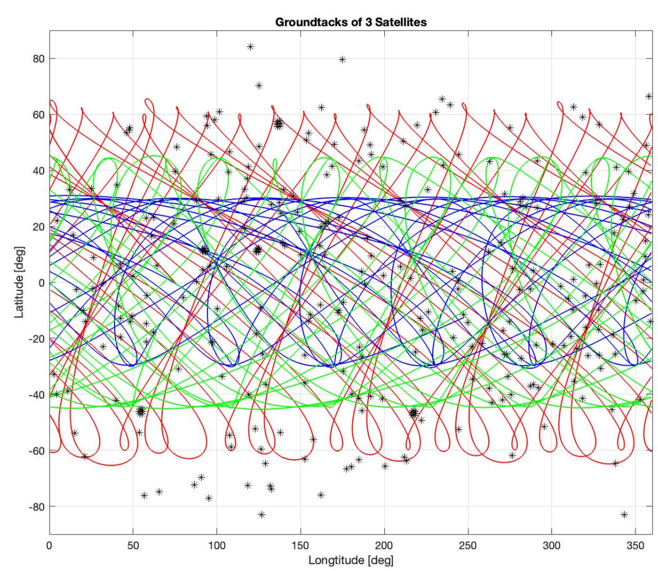

Fig. 2 Groundtracks of 3 Satellites (Red: Spacecraft 1 Blue: Spacecraft 2 Green: Spacecraft 3

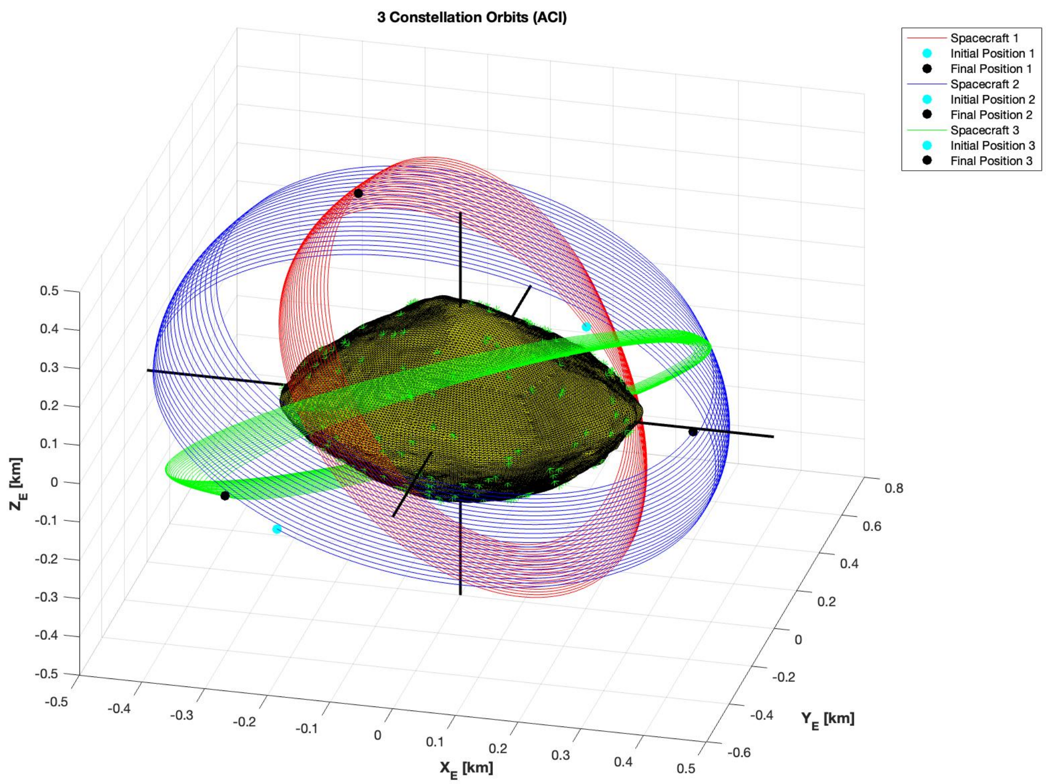

Fig. 3 3 Constellation Orbits in the ACI Frame

Spacecraft	a [km]	e	i [deg]	Ω [deg]	ω [deg]	θ [deg]
1	0.0.4622	0.35	66.1	78.6	-153.6	-125.3
2	0.4682	0.47	31.0	50.0	-126.5	98.5
3	0.4605	0.44	45.6	-129.7	61.8	-126.7

Table 5 Orbital Elements at the Final Time

B. Block Diagram

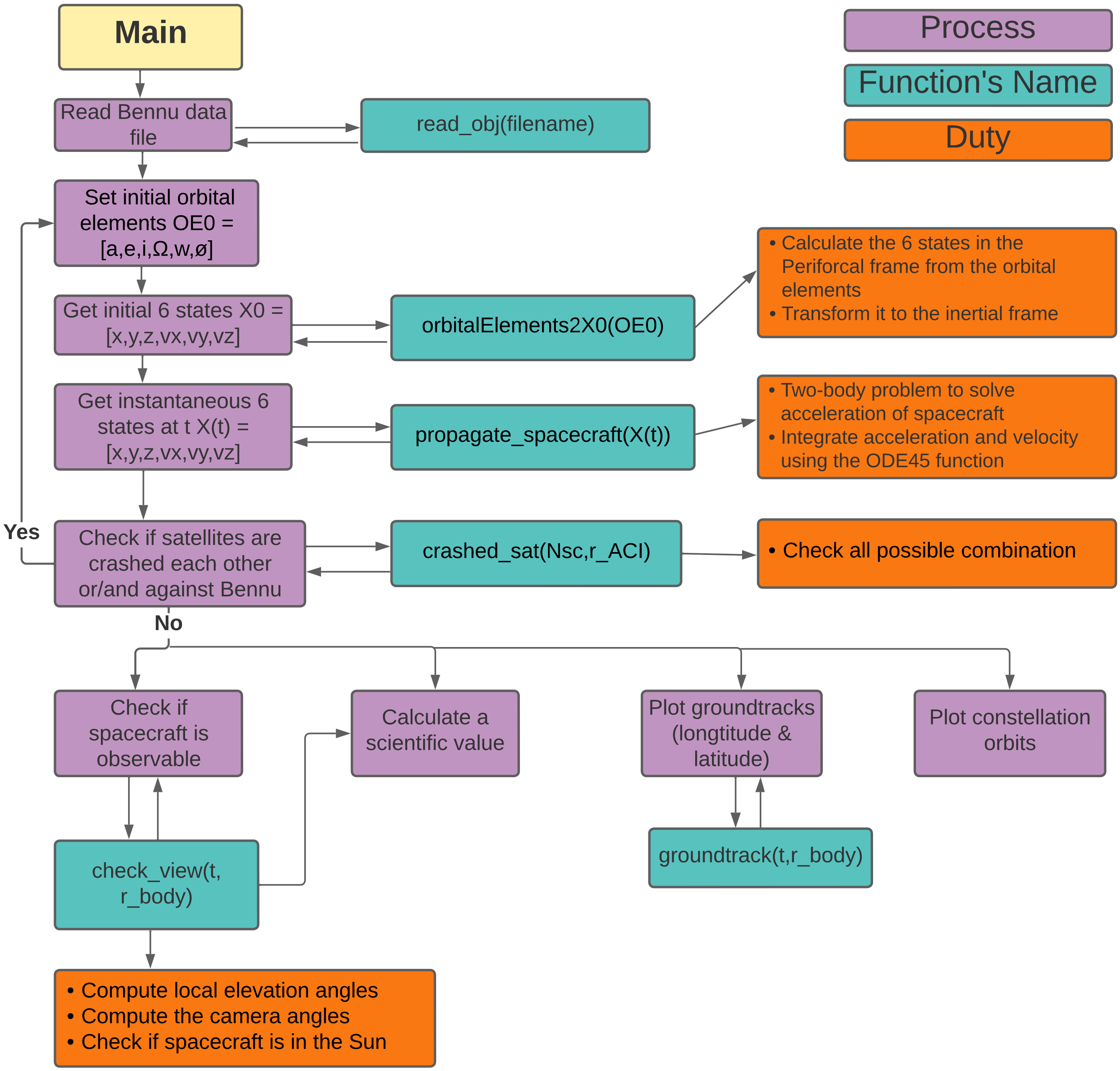

Fig. 4 Computational Block Diagram

C. Description of Block Diagram

1) The existing data file of Bennu needs to be imported to MATLAB and extracted main information such as facets and vertices by calling the 'read_obj()' function.
2) The initial orbital elements such as a, e, i, Ω, ω, θ for each satellite should be set.
3) Using these elements, the initial position and velocity vectors of each satellite can be computed by calling the 'orbitalElements2X0()' function. This function outputs the position and velocity vectors in ACI at a time based on the initial state.
4) Once getting the spacecraft state vectorat times, each satellite needs to be checked whether it collides with one another or/and with Bennu. The function 'crashed_sat()' tells if it is crashed or not.
5) If there are no collisions, the 'check_view()' function determines whether the satellites can observe the target points at times by checking if the targets are in sunlight, the elevation angles are greater than 15 degrees, and the field of the view of the camera is less than the camera angles.
6) Then a scientific value can be computed from the elvation angles and the distance between the target points and the spacecraft within the viewable range.
7) Also, groundtracks are required to calculate to see which satellite views which target point in the 'groundtrack()' function. It needs to transform the inertial position to the body position, using the function called 'ACI2body()'.
8) Finally, the constellation orbits, viewable facets for a week, and groundtacks are plotted.