$$W = \int_v P\,dv$$

$$= \int_v mRT \frac{dv}{v}$$

$$Pv = mRT$$

$$P = \frac{mRT}{v}$$

$$W_{23} = \int_2^3 mRT \frac{dv}{V}$$

$$= mRT_H \ln(V)\Big|_2^3$$

$$= mRT_H \ln\left(\frac{V_3}{V_2}\right)$$ where $V_2 =$ minimum V

$V_3 =$ maximum V

$$\boxed{W_{23} = mRT_H \ln\left(\frac{V_{max}}{V_{min}}\right)}$$

$$W_{41} = \int_4^1 mRT \frac{dv}{v}$$

$$= mRT_L \ln\left(\frac{V_1}{V_4}\right)$$ where

$V_1 =$ minimum V

$V_4 =$ maximum V

$$\boxed{W_{41} = mRT_L \ln\left(\frac{V_{min}}{V_{max}}\right)}$$

$$\boxed{Q_{23} = W_{23}}$$

* heat entering system must equal leaving system to remain at T_H

$$Q_{12} = m C_v \Delta T.$$

from the assumptions we made, we know

$$C_p - C_v = R \quad - ①$$

$$\frac{C_p}{C_v} = 1.4 \quad - ②.$$

rearrange equ. ②

$$C_p = 1.4 C_v$$

plug that into equ. ①

$$1.4 C_v - C_v = R.$$

$$0.4 C_v = R.$$

$$C_v = \frac{5}{2} R$$

thus,

$$\boxed{Q_{12} = \frac{5}{2} m R (T_H - T_L)}$$

$$W_{23} = mR\,T_H \ln\left(\frac{max}{V_{min}}\right)$$

$$W_{41} = mR\,T_L \ln\left(\frac{V_{min}}{V_{max}}\right).$$

$$Q_{23} = W_{23} = mR\,T_H \ln\left(\frac{V_{max}}{V_{min}}\right)$$

$$Q_{12} = \frac{5}{2} mR\,(T_H - T_L)$$

$$W_{net} = W_{23} + W_{41}$$

$$W_{net} = mR\left(T_H \ln\left(\frac{V_{max}}{V_{min}}\right) + T_L \ln\left(\frac{V_{min}}{V_{max}}\right)\right)$$

$$Q_{in} = Q_{12} + Q_{23}$$

$$Q_{in} = mR\left[T_H \ln\left(\frac{V_{max}}{V_{min}}\right) + \frac{5}{2}(T_H - T_L)\right]$$

$$\frac{1}{\eta} = \frac{Q_{in}}{W_{net}} = \frac{\cancel{mR}\left[T_H \ln\left(\frac{V_{max}}{V_{min}}\right) + \frac{5}{2}(T_H - T_L)\right]}{\cancel{mR}\left[T_H \ln\left(\frac{V_{max}}{V_{in}}\right) + T_L \ln\left(\frac{V_{min}}{V_{max}}\right)\right]}$$

To simplify the Denomerator Term.

$$T_H\left(\ln(V_{max})-\ln(V_{min})\right)+T_L\left(\ln(V_{min})-\ln(V_{max})\right)$$

$$= T_H\ln(V_{max})-T_L\ln(V_{max})-T_H\ln(V_{min})+T_L\ n(V_{min})$$

$$= \ln(V_{max})(T_H-T_L)-\ln(V_{min})(T_H-T_L)$$

$$= (T_H-T_L)\ln\left(\frac{V_{max}}{V_{min}}\right)$$

$$\frac{1}{\eta}=\frac{Q_{in}}{W_{net}}=\frac{T_H\ln\left(\frac{V_{max}}{V_{min}}\right)+\frac{5}{2}(T_H-T_L)}{\ln\left(\frac{V_{max}}{V_{min}}\right)(T_H-T_L)}$$

$$\frac{1}{\eta}=\frac{T_H}{T_H-T_L}+\frac{5}{2\ln\left(\frac{V_{max}}{V_{min}}\right)}$$

$$\eta=\left[\frac{1}{1-\frac{T_L}{T_H}}+\frac{5}{2\ln\left(\frac{V_{max}}{V_{min}}\right)}\right]^{-1}$$