Optimization of Football Player Rotations Using Gurobi

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1 Introduction

In football, managing player rotations is a critical task for ensuring optimal performance throughout a season. Coaches must balance player fatigue, physicality, and match difficulty to maximize team efficiency and performance. This problem becomes even more complex when considering multiple matches over a long season, as overusing key players or underestimating match difficulty can lead to suboptimal results.

In this paper, we propose a mathematical optimization model for selecting the best 3 attackers for each match during a football season. The objective is to maximize the total goals scored across all matches while adhering to constraints such as player physicality, match difficulty, and team composition. The model is implemented using Gurobi, a state-of-the-art optimization solver.

2 Mathematical Model

The model aims to maximize the total weighted goals scored across all matches, considering player performance (goals per game), match complexity (hard, medium, easy), and player physicality.

2.1 Decision Variables

- $x_{i,t}$: Binary variable, $x_{i,t} = 1$ if player i is selected for match t, 0 otherwise.

2.2 Parameters

- G_i: Goals per game for player i.
- P_i: Maximum consecutive matches player i can play, based on their physicality.
- C_t: Match complexity for match t, where $C_t \in \{0.5, 1.0, 1.5\}$ (easy, medium, hard).
- n_m: Total number of matches.
- n_p: Total number of players.

2.3 Objective Function

The objective is to maximize the total weighted goals scored across all matches:

$$\text{Maximize } Z = \sum_{t=1}^{n_m} \sum_{i=1}^{n_p} C_t \cdot G_i \cdot x_{i,t}$$

2.4 Constraints

The model includes the following constraints:

Team Size Constraint: Exactly 3 players must be selected for each match:

$$\sum_{i=1}^{n_p} x_{i,t} = 3, \quad \forall t \in \{1, \dots, n_m\}$$

Consecutive Match Constraint: A player cannot play more consecutive matches than their physicality limit:

$$\sum_{k=0}^{P_i} x_{i,t+k} \leq P_i, \quad \forall i \in \{1, \dots, n_p\}, \forall t \in \{1, \dots, n_m - P_i\}$$

Rest Key Players for Easier Matches: For matches with $C_t = 0.5$ (easy matches), at least 2 players with $G_i \leq 0.5$ must be selected:

$$\sum_{\{i:G_i \leq 0.5\}} x_{i,t} \geq 2, \quad \forall t \text{ where } C_t = 0.5$$

3 Explanation of the Model

3.1 Objective Function

The objective function maximizes the total weighted goals scored across the season. Higher weights (C_t) are assigned to harder matches, incentivizing the selection of stronger players for these matches.

3.2 Team Size Constraint

This constraint ensures that exactly 3 players are selected for each match, maintaining a balanced team composition.

3.3 Consecutive Match Constraint

This constraint ensures that a player does not exceed their physicality limit by capping the maximum consecutive matches they can play. The sliding window of size $P_i + 1$ ensures that the constraint is applied across all valid windows of matches.

3.4 Rest Key Players for Easier Matches

To preserve the physical condition of key players, this constraint enforces the selection of at least 2 weaker players (with $G_i \leq 0.5$) for easier matches.

4 Implementation and Results

The model is implemented using Python and Gurobi. The key components of the implementation include defining decision variables, parameters, constraints, and the objective function. The Gurobi optimizer solves the model, providing the optimal lineup for each match along with player statistics and total goals scored for the season.

5 Conclusion

This model demonstrates how mathematical optimization can be applied to solve real-world problems in sports scheduling. By incorporating player physicality, match complexity, and team composition, the model provides a robust framework for managing player rotations and maximizing team performance.