On an Algorithm for Evaluating the Validity of Singly-Quantified Monadic Predicate Logic Arguments

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An Example

Consider the following argument in the universal set of all beings.

"Every person is a rational being. There exists a being that is irrational and quantified. Therefore, there exists a quantified being that is not a person."

Px = x is a person, Qx = x is quantified , Rx = x is rational

$$(\forall x)\ (Px \supset Rx)$$

$$\underline{(\exists x)\ (\neg Rx \wedge Qx)}$$

$$\therefore (\exists x)\ (Qx \wedge \neg Px)$$

Example/Formal Proof

Steps	Reasons	$\therefore (\exists x)(Qx \wedge \neg Px)$
1. $(\forall x)(Px \supset Rx)$	1. premise	
2. $(\exists x)(\neg Rx \wedge Qx)$	2. premise	
3. $\neg Ry \wedge Qy$	3. 2 E.I.	
4. $Py \supset Ry$	4. 1 U.I.	
5. $\neg Ry$	5. 3 Simp.	
6. Qy	6. 3 Simp.	
7. $\neg Py$	7. 4,5 M.T.	
8. $Qy \wedge \neg Py$	8. 6,7 Conj.	
9. $\therefore (\exists x)(Qx \wedge \neg Px)$	9. 8 E.G.	

Q.E.D.

$$\boxed{\begin{array}{l} (\forall x)(Px \supset Rx) \\ \underline{(\exists x)(\neg Rx \wedge Qx)} \\ \therefore (\exists x)(Qx \wedge \neg Px) \end{array}}$$

Step 1

Write down the symbolic logic argument underlying each instantiation of the given predicate logic argument.

Consider the example given in Section 1 at the instance when $x = c$, we then have

$$
\begin{array}{l}
Pc \supset Rc \\
\underline{\neg Rc \wedge Qc} \\
\therefore Qc \wedge \neg Pc
\end{array}
$$

The symbolic logic argument underlying this one would be

$$
\begin{array}{l}
P \supset R \\
\underline{\neg R \wedge Q} \\
\therefore Q \wedge \neg P
\end{array}
$$

Step 2

Construct the truth table of the symbolic logic argument obtained in Step 1 and put at the topmost part of each column the quantifier of the proposition for that column. Each simple proposition that is not a premise of the argument is considered to be existentially quantified.

(Observe that the truth table has columns for the simple propositions, premises, and conclusion.)

Object Types	$\exists$	$\exists$	$\exists$	$\forall$	$\exists$	$\exists$
	P	Q	R	$P \supset R$	$\neg R \wedge Q$	$Q \wedge \neg P$
1	1	1	1	1	0	0
2	1	1	0	0	1	0
3	1	0	1	1	0	0
4	1	0	0	0	0	0
5	0	1	1	1	0	1
6	0	1	0	1	1	1
7	0	0	1	1	0	0
8	0	0	0	1	0	0

Step 3

Delete the rows of object types that have entries of 0 in a column of a universally quantified premise (i.e., rows with a 0 entry in a ∀ premise column).

For this particular example, Rows 2 and 4 are deleted (the 0 entries are marked with asterisks).

Object Types	∃	∃	∃	∀	∃	∃
	P	Q	R	$P \supset R$	$\neg R \wedge Q$	$Q \wedge \neg P$
1	1	1	1	1	0	0
2	1	1	0	0*	1	0
3	1	0	1	1	0	0
4	1	0	0	0*	0	0
5	0	1	1	1	0	1
6	0	1	0	1	1	1
7	0	0	1	1	0	0
8	0	0	0	1	0	0

Step 4

Delete the rows of object types that have entries of 1 in a column of an existentially quantified conclusion (i.e., rows with a 1 in a ∃ conclusion column).

For this particular example, Rows 5 and 6 are further deleted.

Object Types	∃	∃	∃	∀	∃	∃
	P	Q	R	P⊃R	¬R^Q	Q ^¬ P
1	1	1	1	1	0	0
3	1	0	1	1	0	0
5	0	1	1	1	0	1*
6	0	1	0	1	1	1*
7	0	0	1	1	0	0
8	0	0	0	1	0	0

Step 5

For each ∀ column, conjoin the entries of the remaining rows and indicate the combined truth value for the column at the bottom of the table.

Object Types	∃	∃	∃	∀	∃	∃
	P	Q	R	$P \supset R$	$\neg R \wedge Q$	$Q \wedge \neg P$
1	1	1	1	1	0	0
3	1	0	1	1	0	0
7	0	0	1	1	0	0
8	0	0	0	1	0	0
				1		

Step 6

For each ∃ column, disjoin the entries of the remaining rows and indicate the combined truth value for the column at the bottom of the table.

Object Types	∃	∃	∃	∀	∃	∃
	P	Q	R	$P \supset R$	$\neg R \wedge Q$	$Q \wedge \neg P$
1	1	1	1	1	0	0
3	1	0	1	1	0	0
7	0	0	1	1	0	0
8	0	0	0	1	0	0
	1	1	1	1	0	0

Step 7

Determine the validity of the argument according to the following rules:

I. *If all object types are deleted then **the argument is valid.***

II. *If in the universe that contains these, and only these, remaining object types, it is not the case that all the premises are true and the conclusion false, then **the argument is valid.***

III. *A) If in the universe that contains these, and only these, remaining object types, all the existentially quantified simple propositions are true, the premises are true, and the conclusion is false, then **the argument is invalid.***

*B) However, if in the universe just described not all of the existentially quantified simple propositions are true but the premises are all true and the conclusion false, then **the argument is valid** in universes in which it has existential import and **invalid otherwise.***

Object Types	∃	∃	∃	∀	∃	∃
	P	Q	R	P⊃R	¬R^Q	Q ^¬ P
1	1	1	1	1	0	0
3	1	0	1	1	0	0
7	0	0	1	1	0	0
8	0	0	0	1	0	0
	1	**1**	**1**	**1**	**0**	**0**

For this particular example, Rule II is applicable and we conclude that the argument is valid (of course, the formal proof presented earlier already shows this).

$(\forall x)(Px \supset Qx)$

$\underline{(\forall x)(\neg Qx \qquad)}$

$\therefore (\exists x)(\neg Px \qquad)$

An example in which all the object types are deleted. Rule I applies and the argument is valid.

Object Types	$\exists$	$\exists$	$\forall$	$\forall$	$\exists$
	P	Q	$P \supset Q$	$\neg Q$	$\neg P$
1	1	1	1	0*	0
2	1	0	0*	1	0
3	0	1	1	0*	1*
4	0	0	1	1	1*

$(\forall x)(Px \supset Qx)$
$(\exists x)(Qx \wedge Rx)$
$\underline{(\exists x)(\neg Qx \vee \neg Rx)}$
$\therefore (\forall x)(Px \supset \neg Qx)$

An example of an argument that is invalid by Rule III.A).

Object Types	∃	∃	∃	∀	∃	∃	∀
	P	Q	R	$P \supset Q$	$Q \wedge R$	$\neg Q \vee \neg R$	$P \supset \neg Q$
1	1	1	1	1	1	0	0
2	1	1	0	1	0	1	0
3	1	0	1	0*	0	1	1
4	1	0	0	0*	0	1	1
5	0	1	1	1	1	0	1
6	0	1	0	1	0	1	1
7	0	0	1	1	0	1	1
8	0	0	0	1	0	1	1

Object Types	∃	∃	∃	∀	∃	∃	∀
	P	Q	R	$P \supset Q$	$Q \wedge R$	$\neg Q \vee \neg R$	$P \supset \neg Q$
1	1	1	1	1	1	0	0
2	1	1	0	1	0	1	0
5	0	1	1	1	1	0	1
6	0	1	0	1	0	1	1
7	0	0	1	1	0	1	1
8	0	0	0	1	0	1	1
	1	**1**	**1**	**1**	**1**	**1**	**0**

$$(\forall x)(Px \supset Qx)$$
$$\underline{(\forall x)(Qx \supset Rx)}$$
$$\therefore (\exists x)(Px \wedge Rx)$$

An example of an argument that is invalid except in universes in which it has existential import. Rule III.B) is applicable.

Object Types	∃	∃	∃	∀	∀	∃
	P	Q	R	$P \supset Q$	$Q \supset R$	$P \wedge R$
1	1	1	1	1	1	1*
2	1	1	0	1	0*	0
3	1	0	1	0*	1	1*
4	1	0	0	0*	1	0
5	0	1	1	1	1	0
6	0	1	0	1	0*	0
7	0	0	1	1	1	0
8	0	0	0	1	1	0

Object Types	∃	∃	∃	∀	∀	∃
	P	Q	R	$P \supset Q$	$Q \supset R$	$P \wedge R$
5	0	1	1	1	1	0
7	0	0	1	1	1	0
8	0	0	0	1	1	0
	0	**1**	**1**	**1**	**1**	**0**

End of Presentation

THANKS FOR YOUR KIND ATTENTION!

Have a good day!