Lagrangian

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_{IZ_2} \tag{1}$$

$$\mathcal{L}_0 = \frac{1}{2}\partial_\mu\varphi_0\partial_\mu\varphi_0 + \frac{1}{2}m_0\varphi_0^2 + \lambda_0\varphi_0^4 \tag{2}$$

$$\mathcal{L}_1 = \frac{1}{2}\partial_\mu\varphi_1\partial_\mu\varphi_1 + \frac{1}{2}m_1\varphi_1^2 + \lambda_1\varphi_1^4 \tag{3}$$

$$\mathcal{L}_{IZ_2} = \mu\varphi_0^2\varphi_1^2 \tag{4}$$

$$\mathcal{L}_I = g\varphi_1\varphi_0^3 \tag{5}$$

On the lattice we can discretise the derivative $\partial_\mu\varphi(x) = \frac{1}{a}(\varphi(x+\mu)-\varphi(x))$. On the lattice the above lagrangian can be written in a more convenient way for simulations

$$\mathcal{L}_0 = \sum_x \left[-2\kappa_0 \sum_\mu \phi_0(x)\phi_0(x+\mu) + \lambda_L^0\left(\phi_0(x)^2 - 1\right)^2 + \phi_0(x)^2 \right], \tag{6}$$

$$\mathcal{L}_1 = \sum_x \left[-2\kappa_1 \sum_\mu \phi_1(x)\phi_1(x+\mu) + \lambda_L^1\left(\phi_1(x)^2 - 1\right)^2 + \phi_1(x)^2 \right], \tag{7}$$

$$\mathcal{L}_{IZ_2} = \mu_L \sum_x \phi_0(x)^2\phi_1(x)^2 \,, \tag{8}$$

$$\mathcal{L}_I = g_L \sum_x \phi_1(x)\phi_0(x)^3 \,. \tag{9}$$

With

$$m_0^2 = \frac{1-2\lambda_L^0}{\kappa_0} - 8\,, \quad \lambda_0 = \frac{\lambda_L^0}{4\kappa_0^2}\,, \quad \varphi_0 = \sqrt{2\kappa_0}\phi_0 \tag{10}$$

$$m_1^2 = \frac{1-2\lambda_L^1}{\kappa_1} - 8\,, \quad \lambda_1 = \frac{\lambda_L^1}{4\kappa_1^2}\,, \quad \varphi_1 = \sqrt{2\kappa_1}\phi_1\,, \tag{11}$$

and

$$\mu = \frac{\mu_L}{4\kappa_0\kappa_1}\,, \quad g = \frac{g_L}{4\sqrt{\kappa_0}\kappa_1^{3/2}} \tag{12}$$