

# Effective mass

MG

The plots for each ensemble are

- effective mass meff0 from  $\langle \phi_0(t)\phi_0(0) \rangle$
- effective mass meff1 from  $\langle \phi_1(t)\phi_1(0) \rangle$ 
  - meff is extracted fitting the correlator as

$$c_1(t) = |A_{1-0}|^2 \exp(-M\frac{T}{2}) \cosh\left(M(t - \frac{T}{2})\right)$$

- Two particle energy for  $\langle \phi_0^2(t)\phi_0^2(0) \rangle$
- Two particle energy for  $\langle \phi_1^2(t)\phi_1^2(0) \rangle$
- Two particle energy for  $\langle \phi^2(t)\phi^{*2}(0) \rangle$ 
  - The two particle energy  $E_2$  is extracted from the correlator

$$c_2(t) = |A_{2-0}|^2 \exp(-E_2\frac{T}{2}) \cosh\left(E_2(t - \frac{T}{2})\right) + |A_{1-1}|^2 \exp(-MT).$$

to remove the extra term  $|A_{1-1}|^2 \exp(-MT)$  we take the difference

$$\tilde{c}_2(t) = c(t) - c(t+1) = |A_{2-0}|^2 \exp(-E_2\frac{T}{2}) \sinh\left(E_2(t - \frac{T-1}{2})\right).$$

The third correlator is constructed with the complex field

$$\phi = \phi_0 + i\phi_1$$

- Two particle energy for  $\langle \phi_0^3(t)\phi_0^3(0) \rangle$
- Two particle energy for  $\langle \phi_1^3(t)\phi_1^3(0) \rangle$
- Two particle energy for  $\langle \phi^3(t)\phi^{*3}(0) \rangle$ 
  - The three particle energy  $E_3$  is extracted from the correlator

$$c_3(t) = |A_{3-0}|^2 \exp(-E_3\frac{T}{2}) \cosh\left(E_3(t - \frac{T}{2})\right) + |A_{2-1}|^2 \exp(-(E_2+M)\frac{T}{2}) \cosh\left((E_2 - M)(t - \frac{T}{2})\right).$$

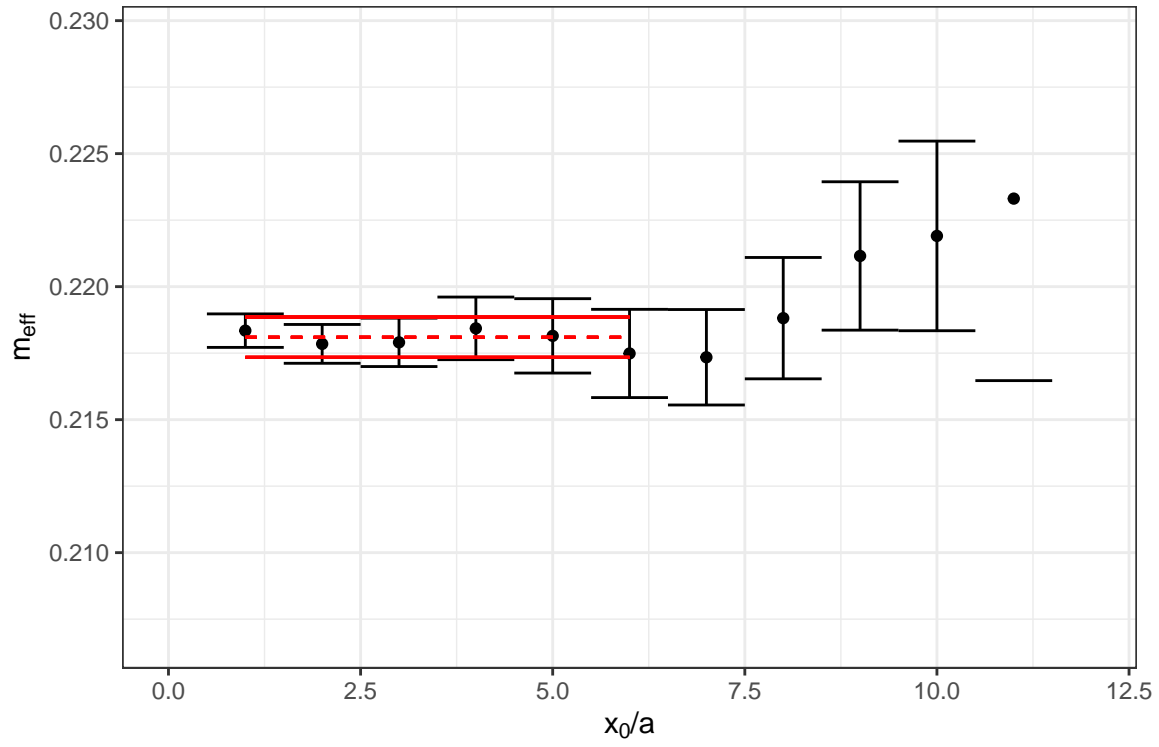
- BH four point function  $\langle \phi_0(\frac{T}{2})\phi_0(t)\phi_0(\frac{T}{8})\phi_0(0) \rangle$
- BH four point function  $\langle \phi_1(\frac{T}{2})\phi_1(t)\phi_1(\frac{T}{8})\phi_1(0) \rangle$
- BH four point function  $\langle \phi_0(\frac{T}{2})\phi_1(t)\phi_1(\frac{T}{8})\phi_0(0) \rangle$

**T 24 L 10**

$$m_0^2 = -4.9 \quad m_1^2 = -4.9 \quad \lambda_0^2 = 2.5 \quad \lambda_1^2 = 2.5 \quad \mu^2 = 5 \quad g^2 = 0 \quad \text{replica} = 0$$

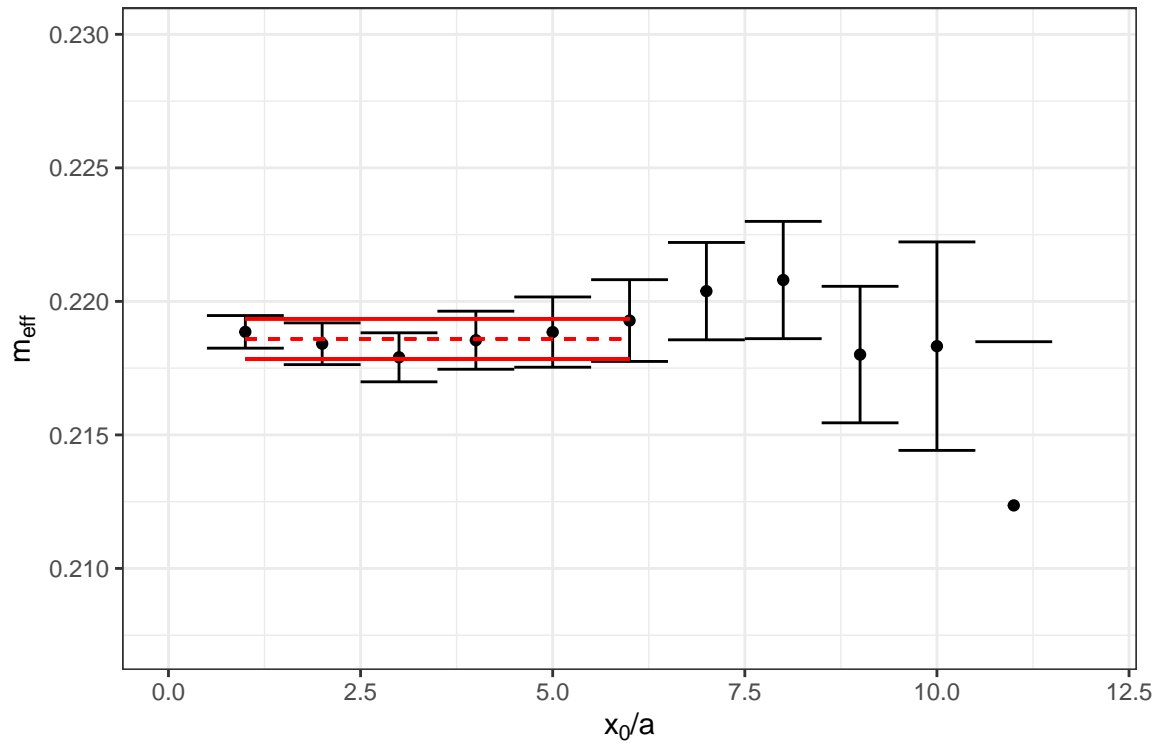
Mass index n= 0

$$\text{fit: } m_{eff} = 0.218098 \pm 0.000754$$



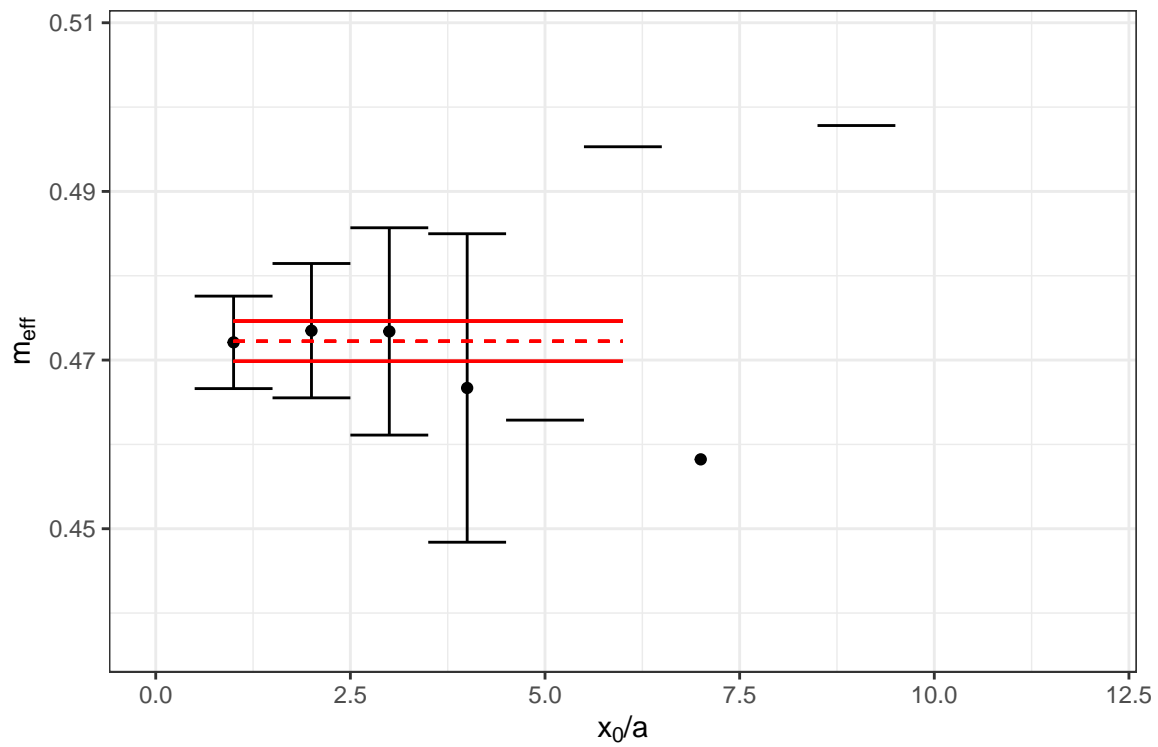
index  $n=1$

fit:  $m_{eff} = 0.218597 \pm 0.000751$



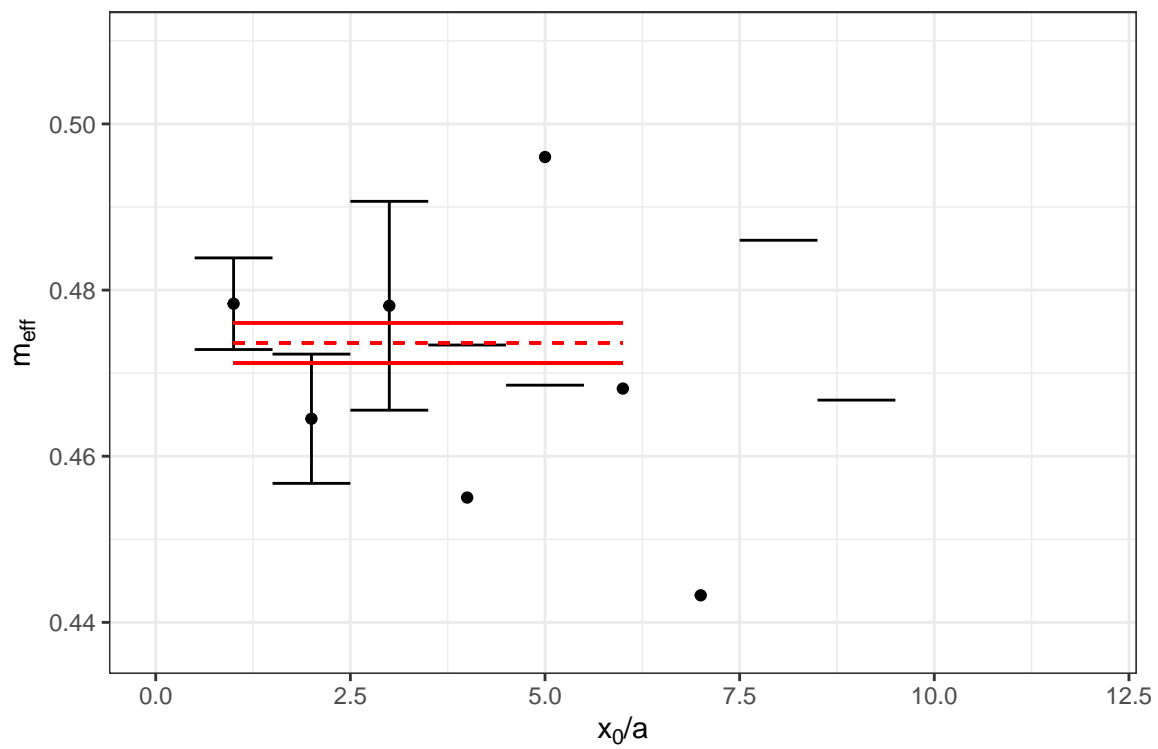
**Two particle energy** index  $n=0$

fit:  $m_{eff} = 0.472232 \pm 0.002379$



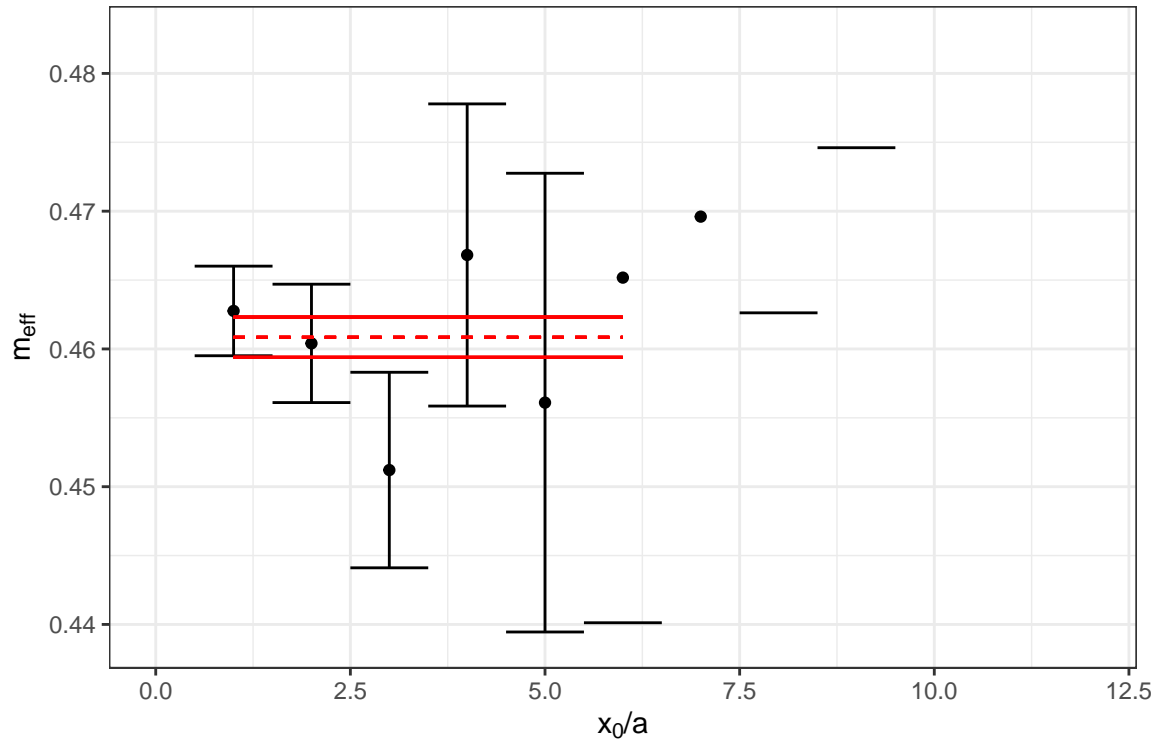
index n= 1

fit:  $m_{\text{eff}} = 0.473648 \pm 0.002415$



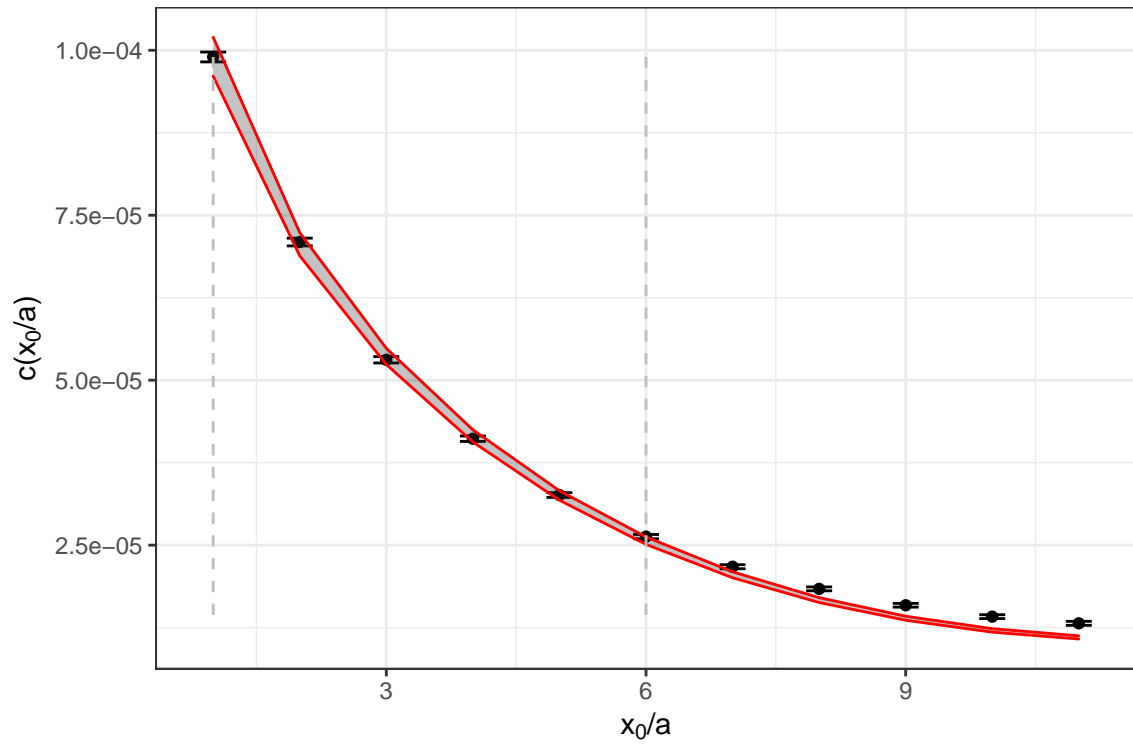
index n= 2

fit:  $m_{\text{eff}} = 0.460848 \pm 0.001456$



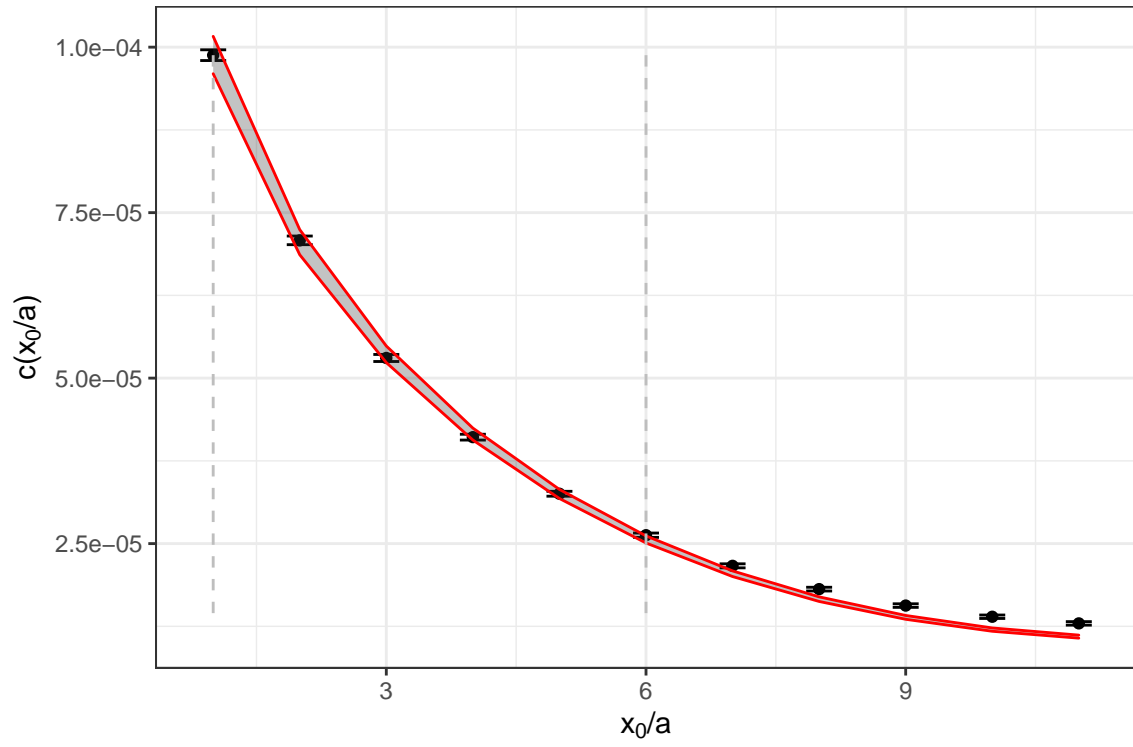
Three particle energy index  $n=0$

fit:  $m_{\text{eff}} = 1.535108 \pm 0.033457$



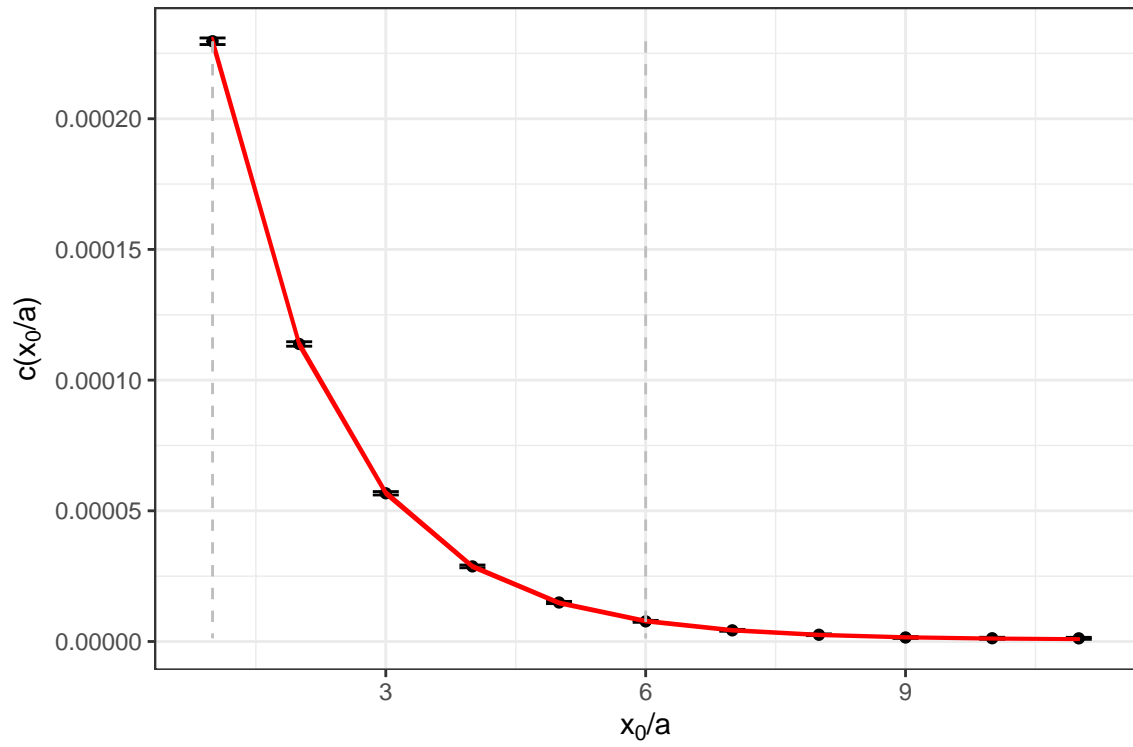
index  $n=1$

fit:  $m_{\text{eff}} = 1.584059 \pm 0.153770$



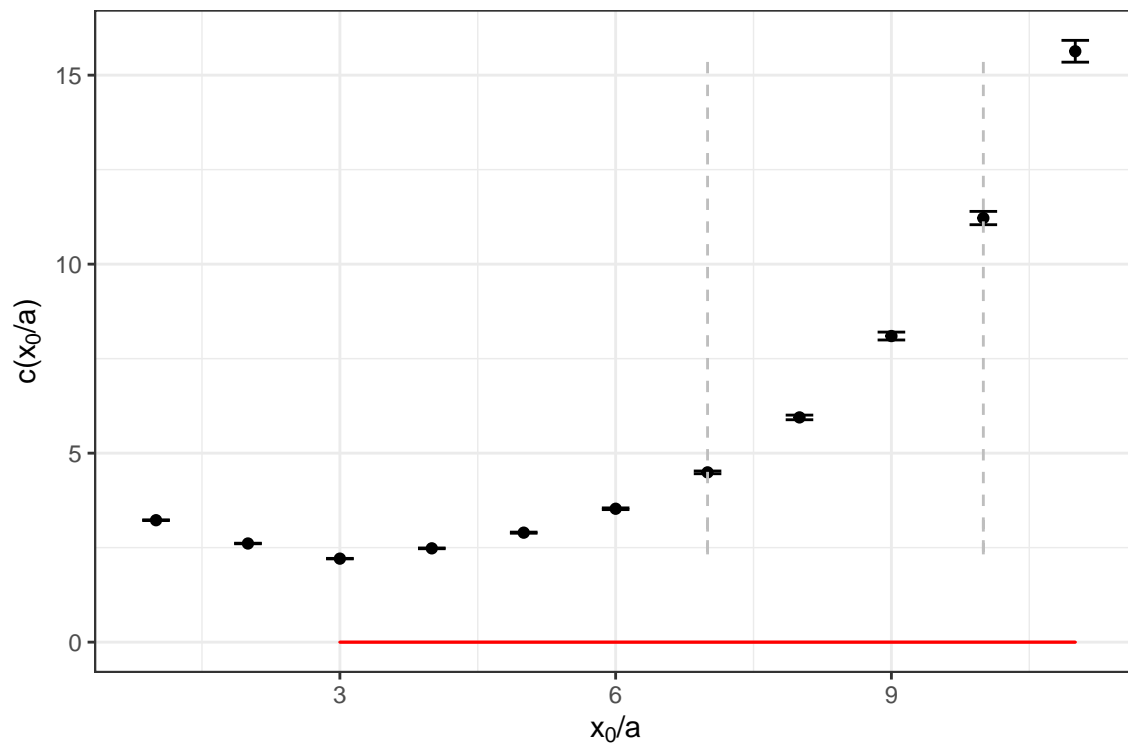
index  $n= 2$

fit:  $m_{eff} = 0.714658 \pm 0.006289$



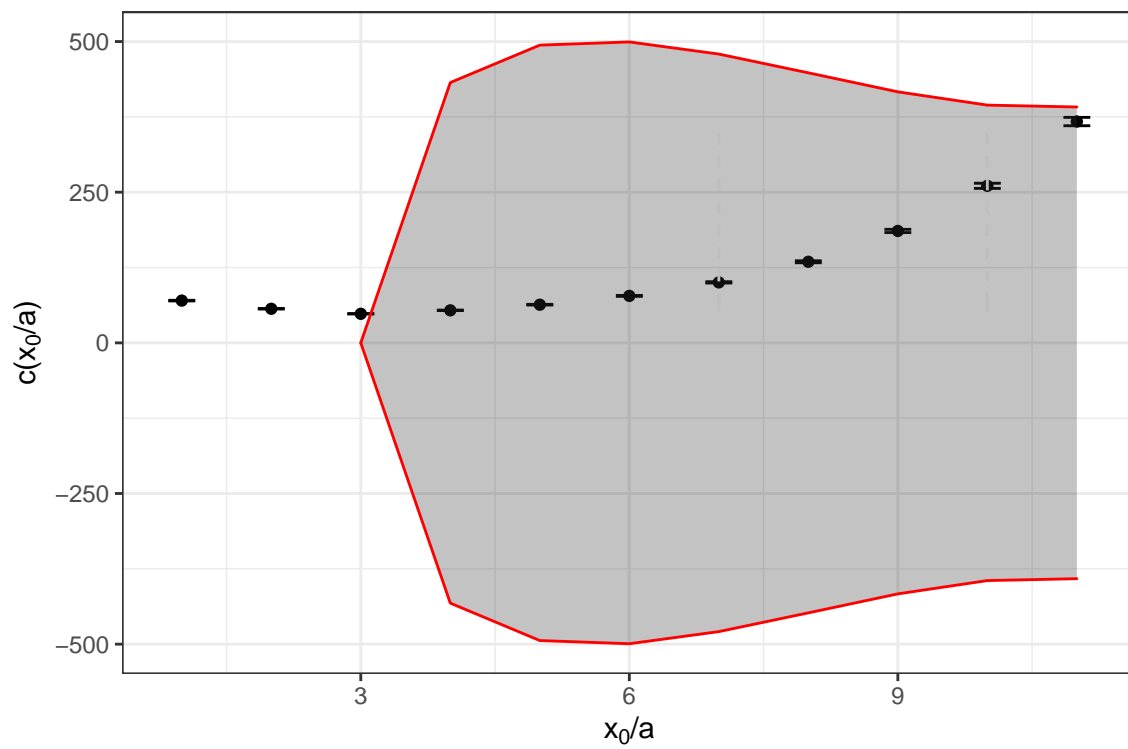
**C4\_BH** index  $n= 0$

fit:  $m_{eff} = -0.000035 \pm 0.000000$



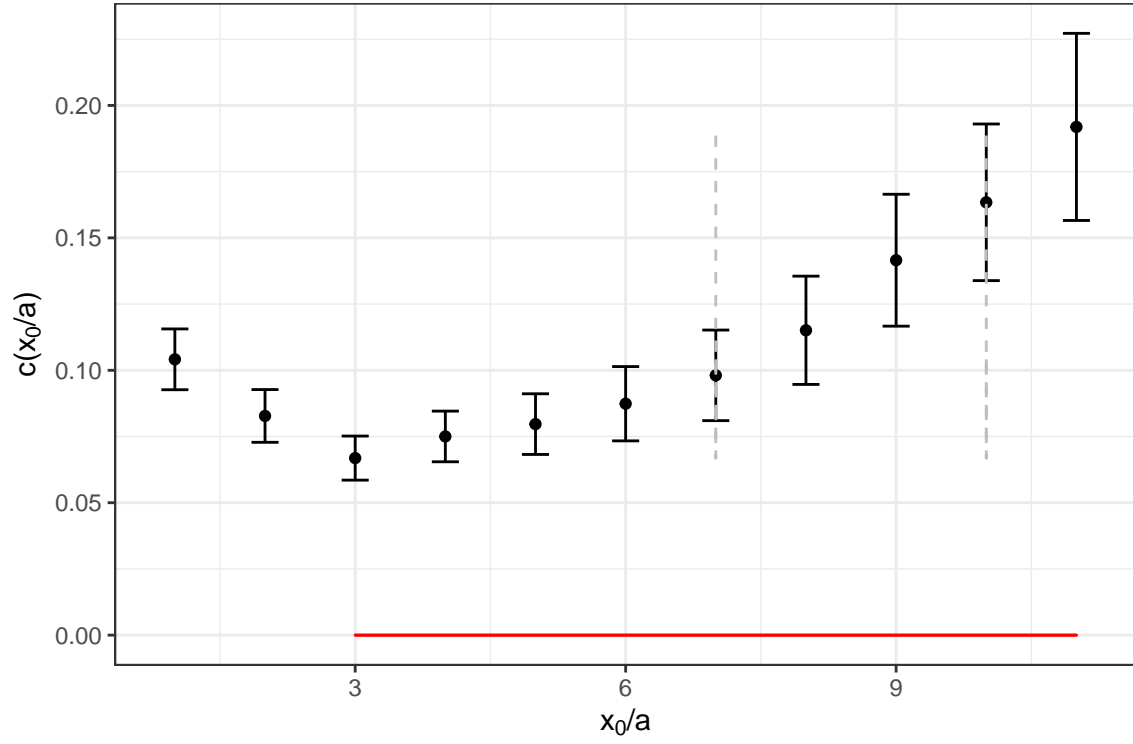
index  $n=1$

fit:  $m_{eff} = 3.790810 \pm 16.999300$



index  $n=2$

fit:  $m_{eff} = 0.000014 \pm 0.000001$

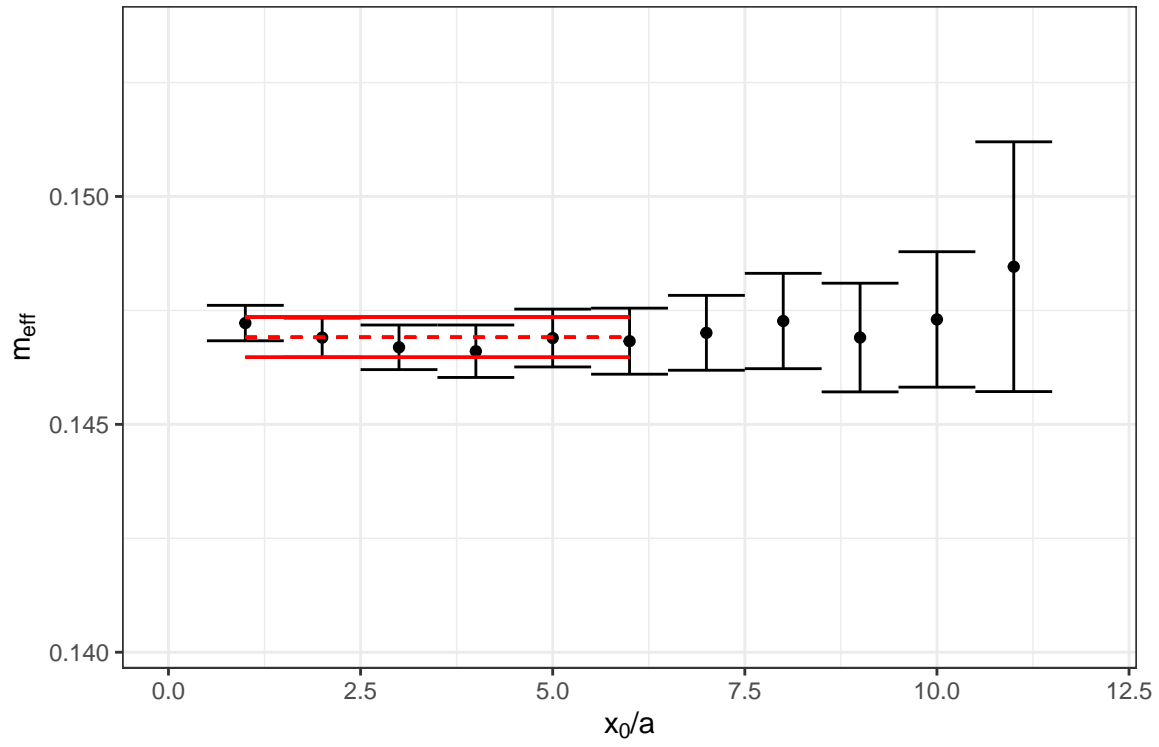


**T 24 L 10**

$m_0^2 = -4.95$   $m_1^2 = -4.9$   $\lambda_0^2 = 2.5$   $\lambda_1^2 = 2.5$   $\mu^2 = 5$   $g^2 = 0$  replica = 0

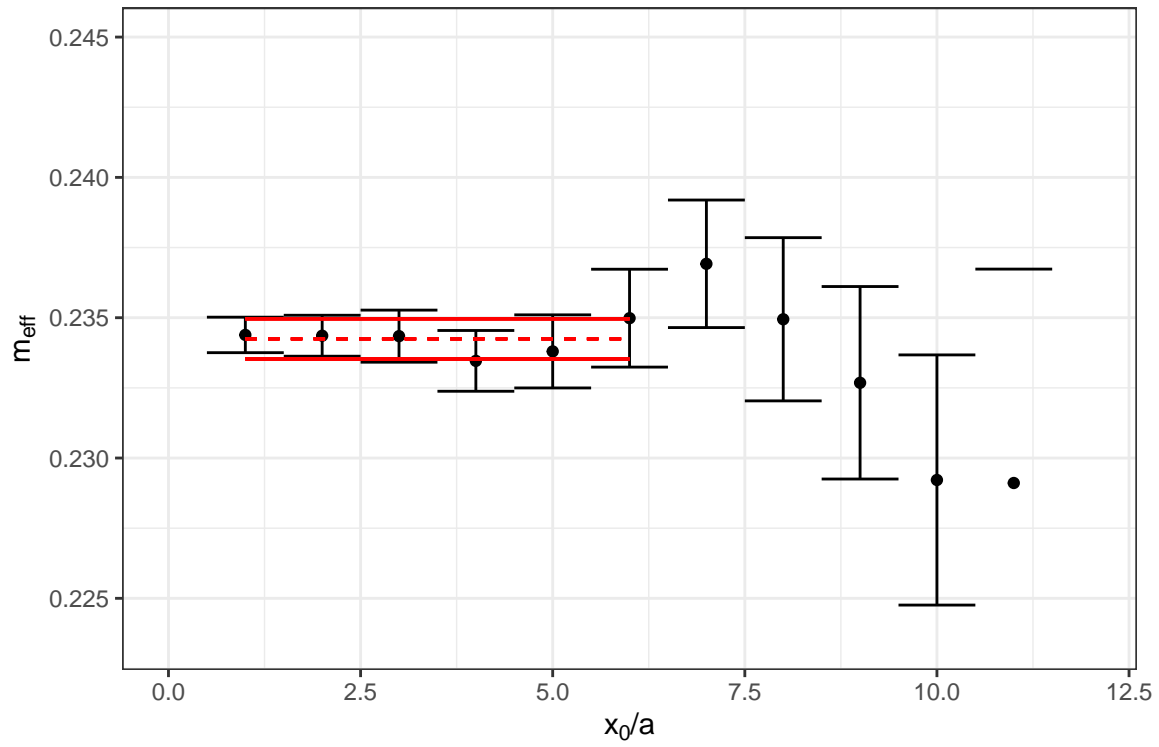
**Mass** index n= 0

fit:  $m_{eff} = 0.146911 \pm 0.000440$



index  $n=1$

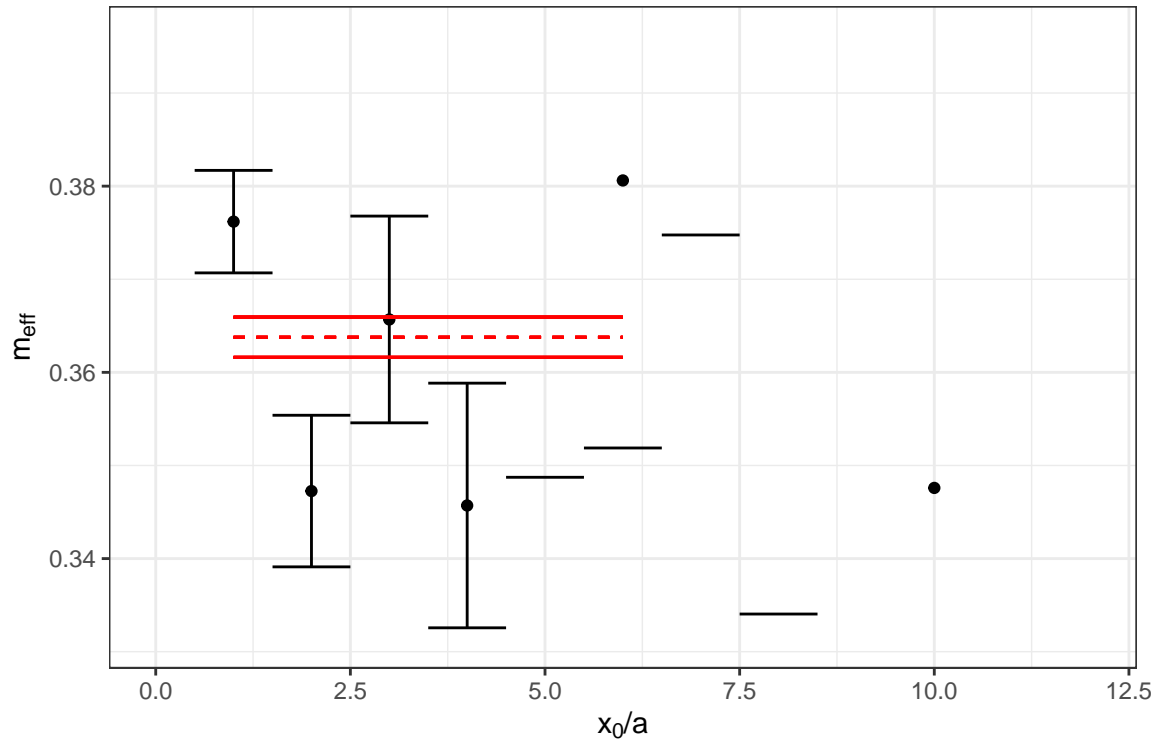
fit:  $m_{\text{eff}} = 0.234246 \pm 0.000715$



**Two particle energy** index  $n=0$

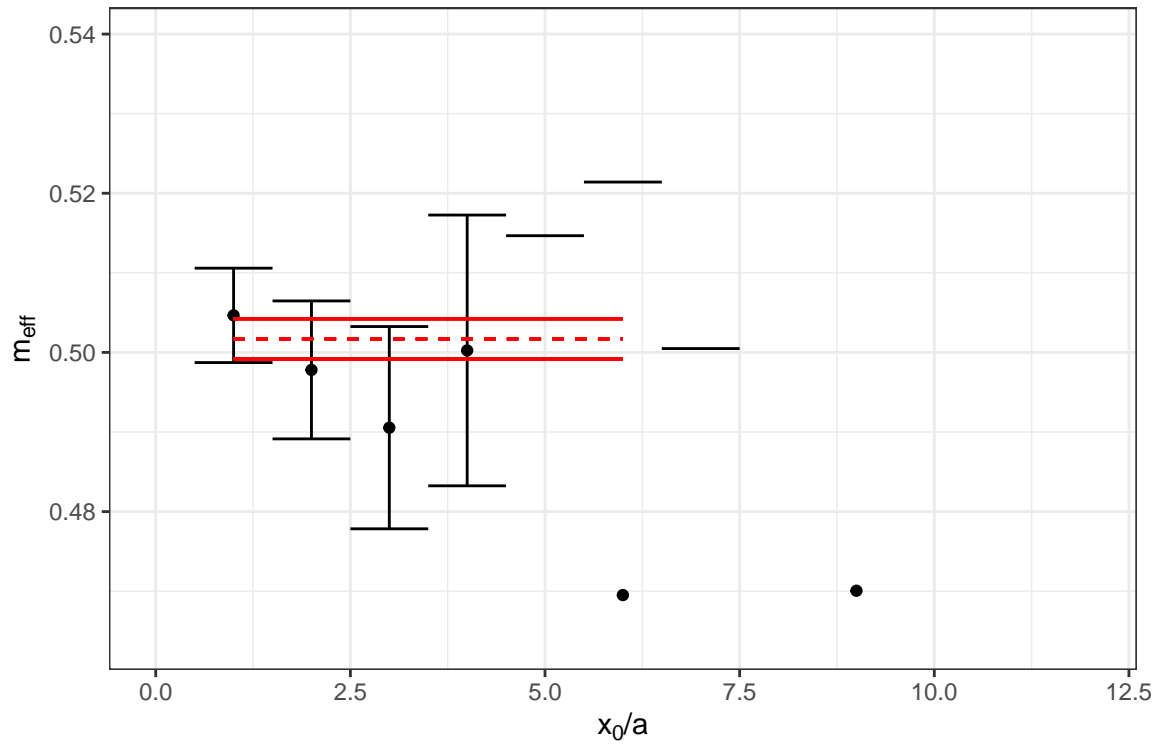
fit:  $m_{\text{eff}} = 0.363771 \pm 0.002154$





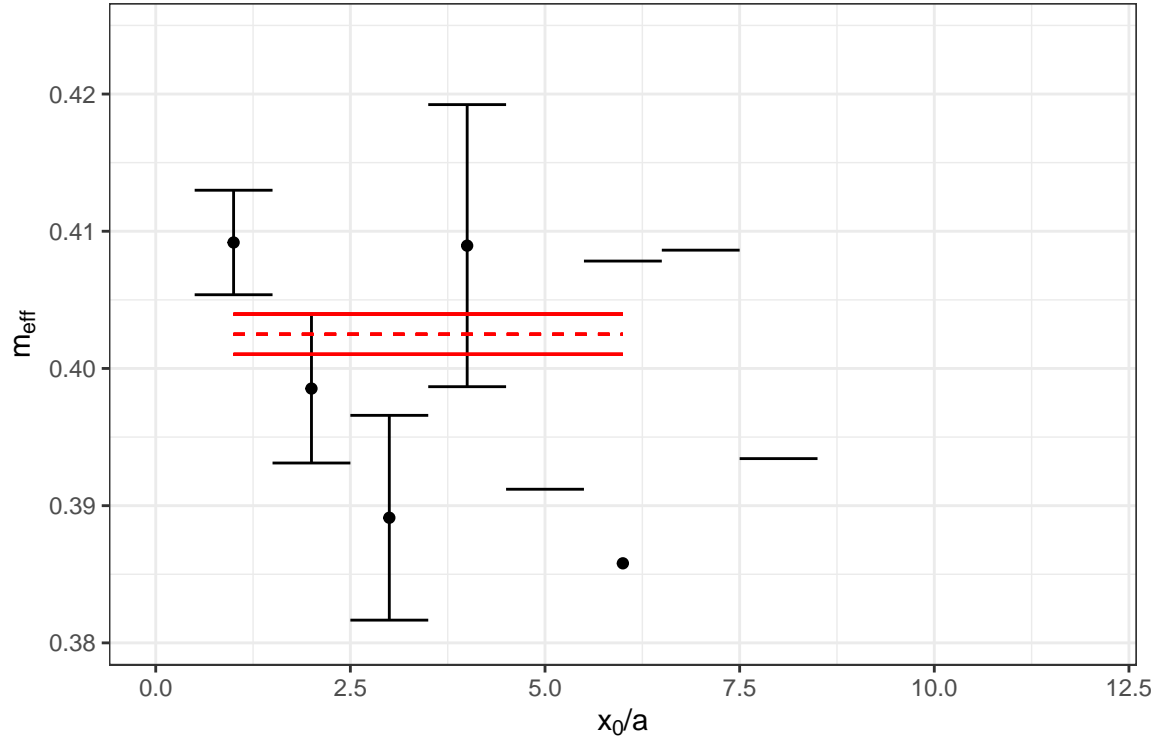
index n= 1

fit:  $m_{\text{eff}} = 0.501705 \pm 0.002520$



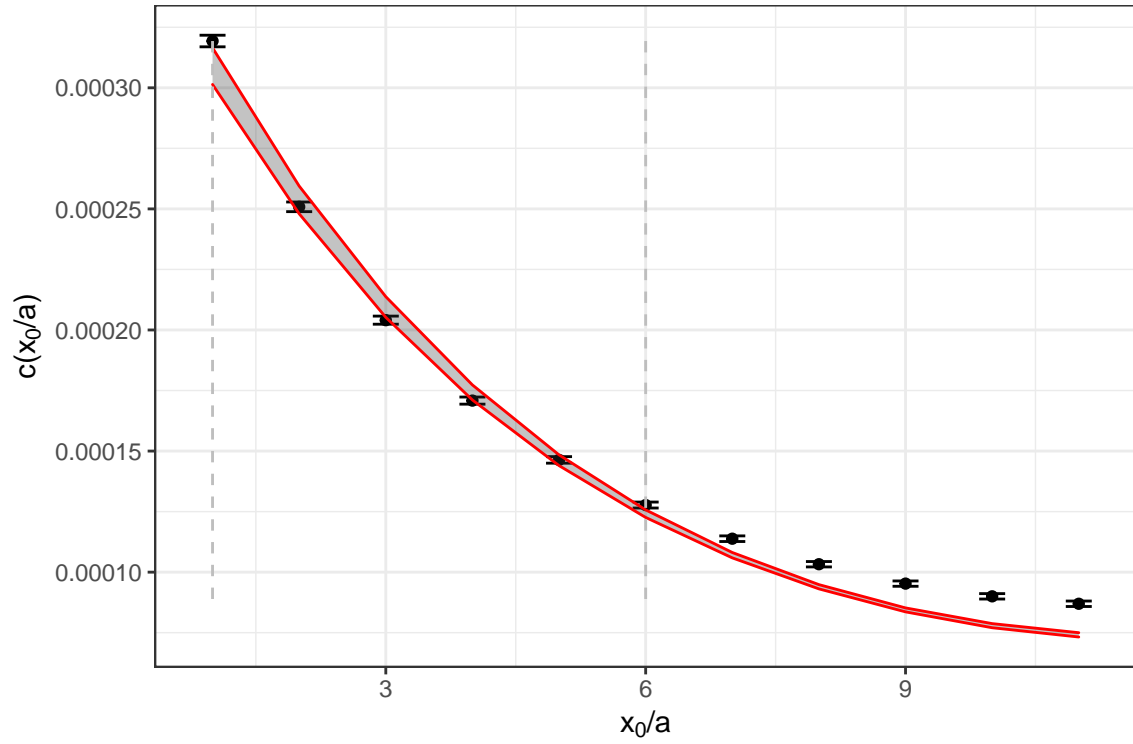
index n= 2

fit:  $m_{\text{eff}} = 0.402493 \pm 0.001462$



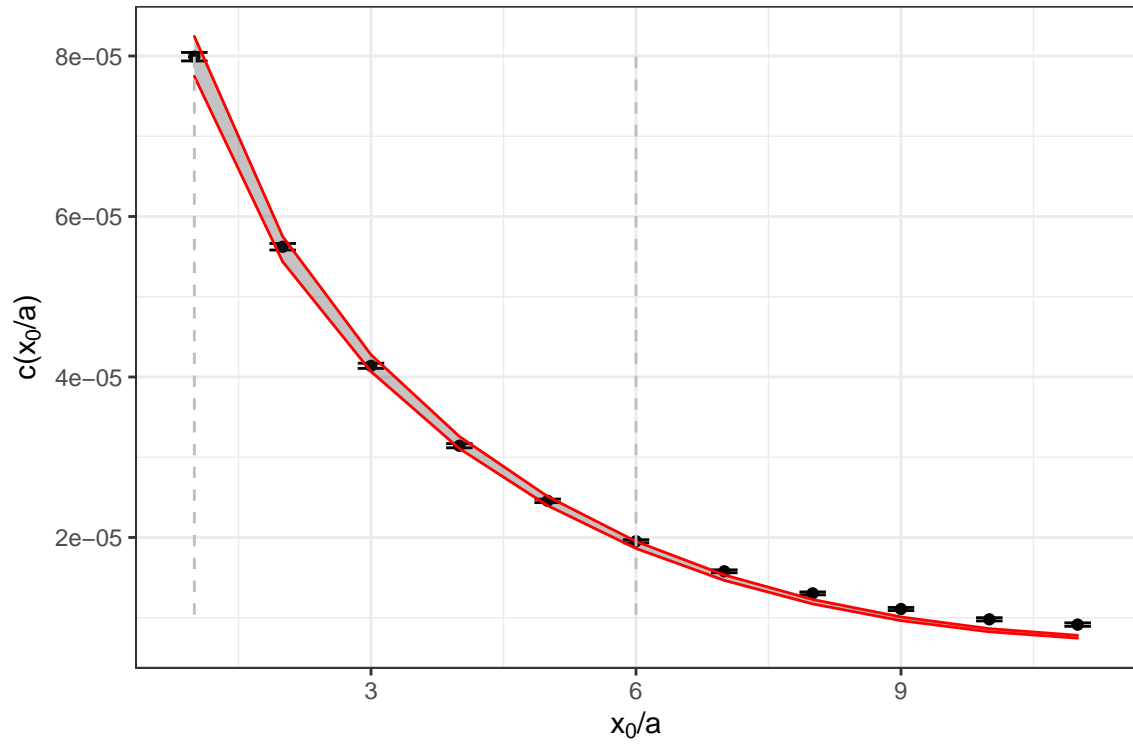
Three particle energy index  $n=0$

fit:  $m_{\text{eff}} = -0.001516 \pm 0.000856$



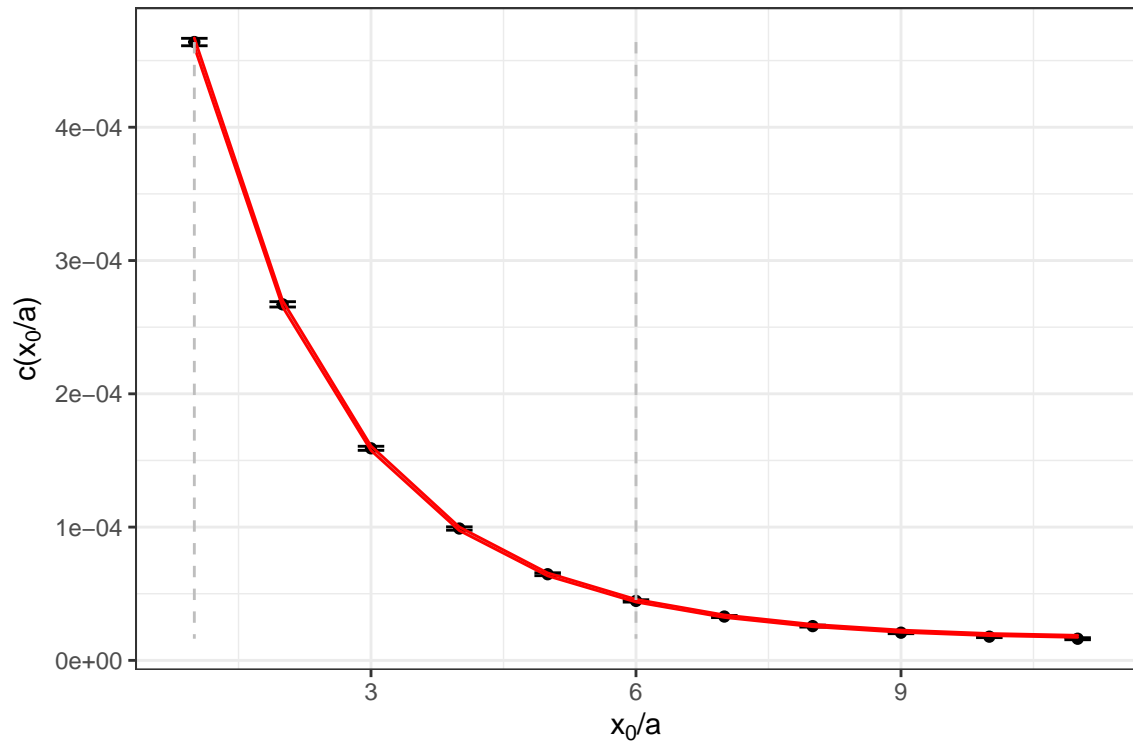
index  $n=1$

fit:  $m_{\text{eff}} = 1.480982 \pm 0.090273$



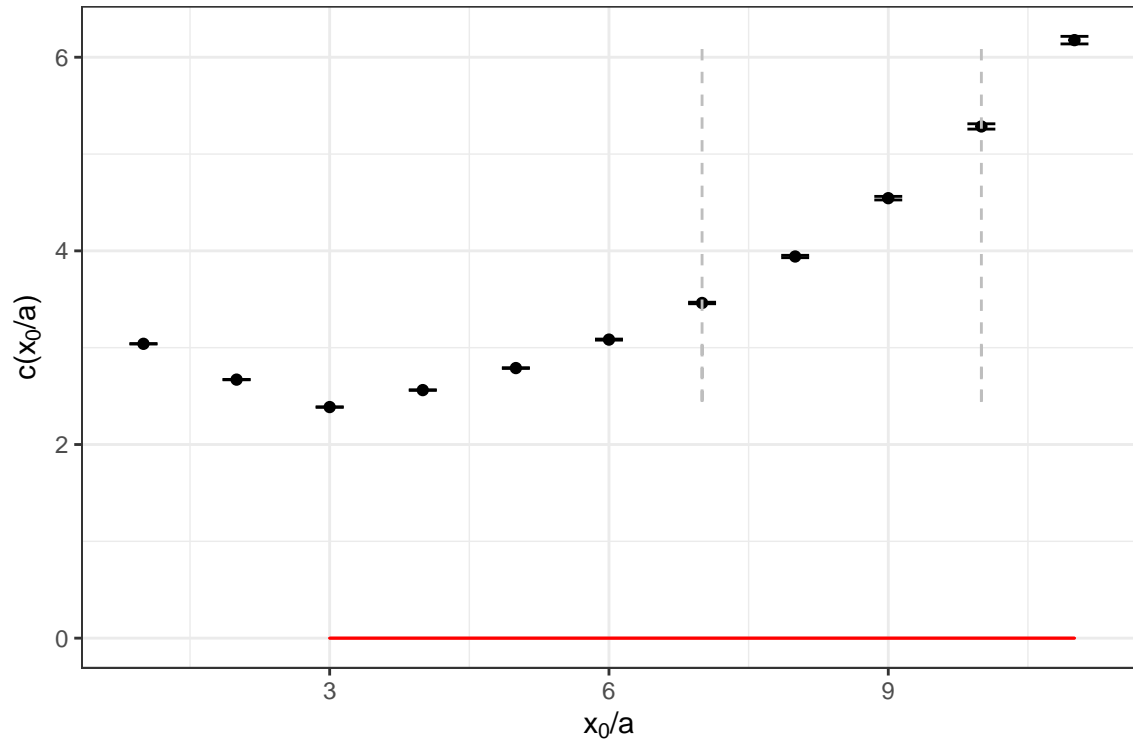
index  $n= 2$

fit:  $m_{eff} = 0.616006 \pm 0.004034$



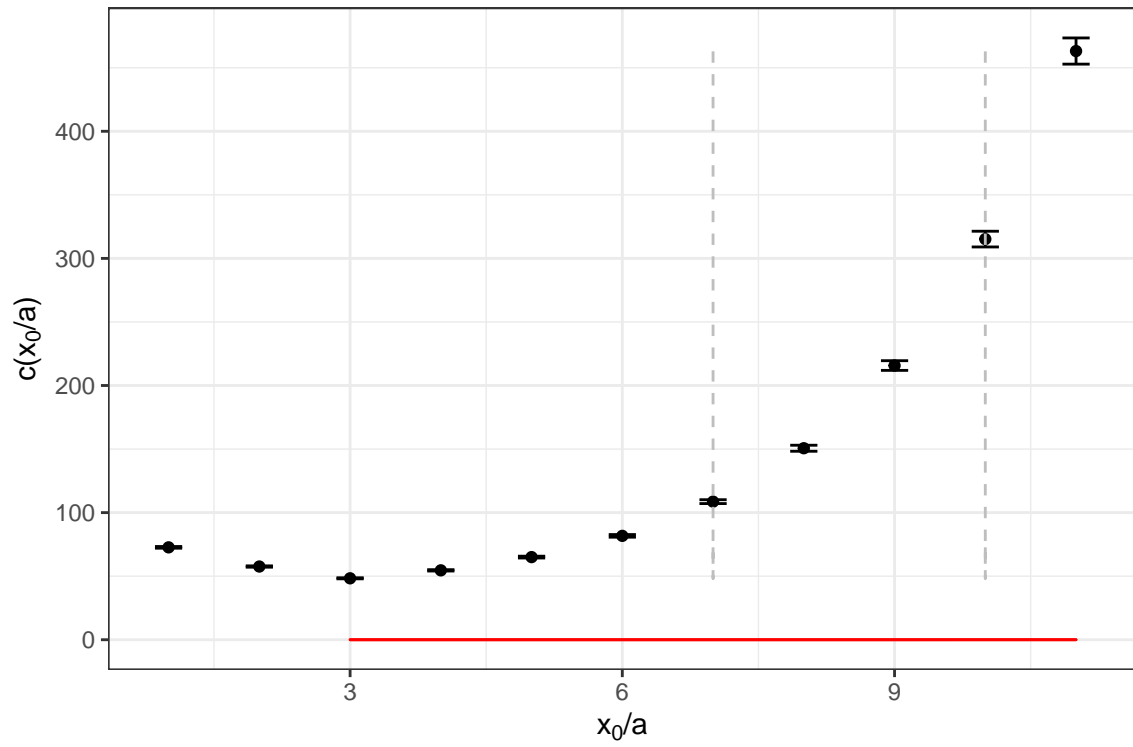
**C4\_BH** index  $n= 0$

fit:  $m_{eff} = -0.000000 \pm 0.000000$



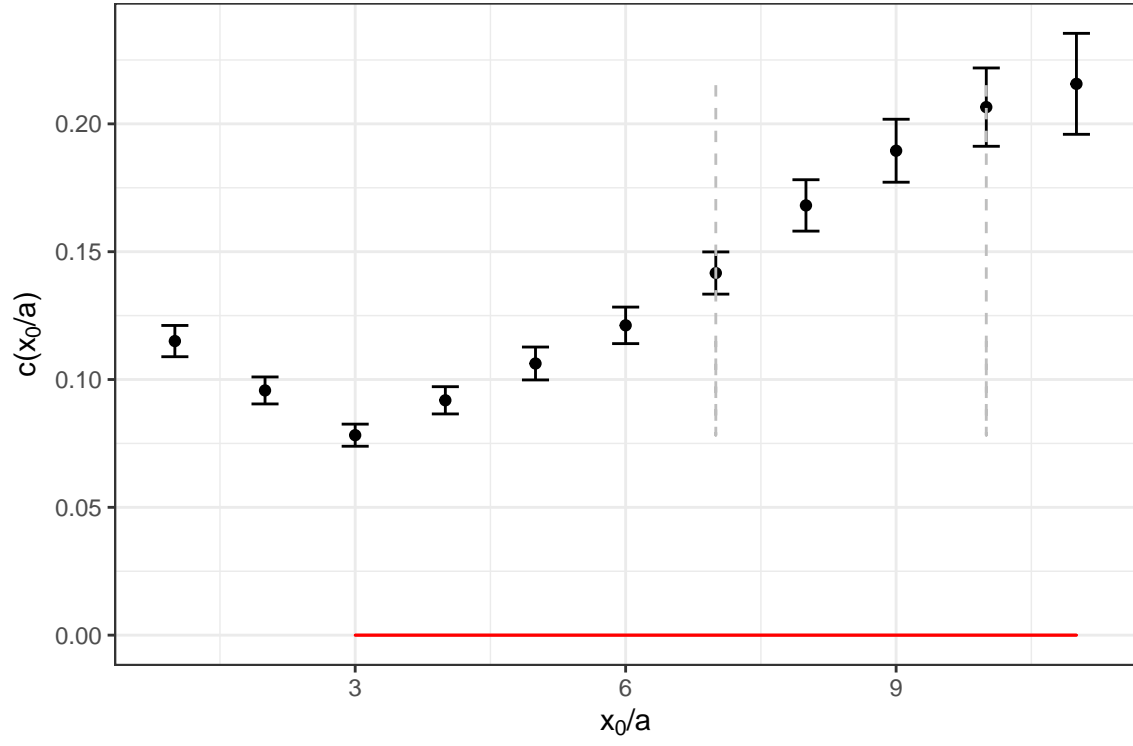
index  $n=1$

fit:  $m_{eff} = 3.661866 \pm 0.002463$



index  $n=2$

fit:  $m_{eff} = -0.000013 \pm 0.000002$

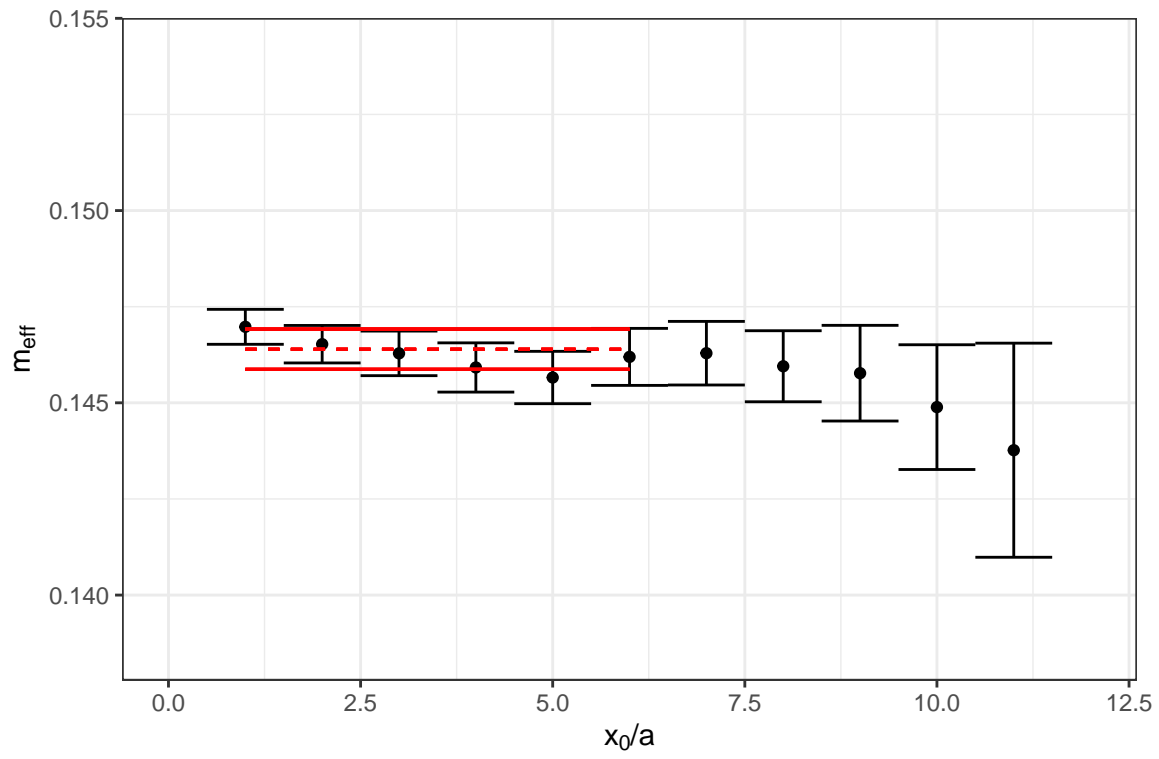


**T 24 L 10**

$m_0^2 = -4.95$   $m_1^2 = -4.9$   $\lambda_0^2 = 2.5$   $\lambda_1^2 = 2.5$   $\mu^2 = 5$   $g^2 = 0$  replica = 1

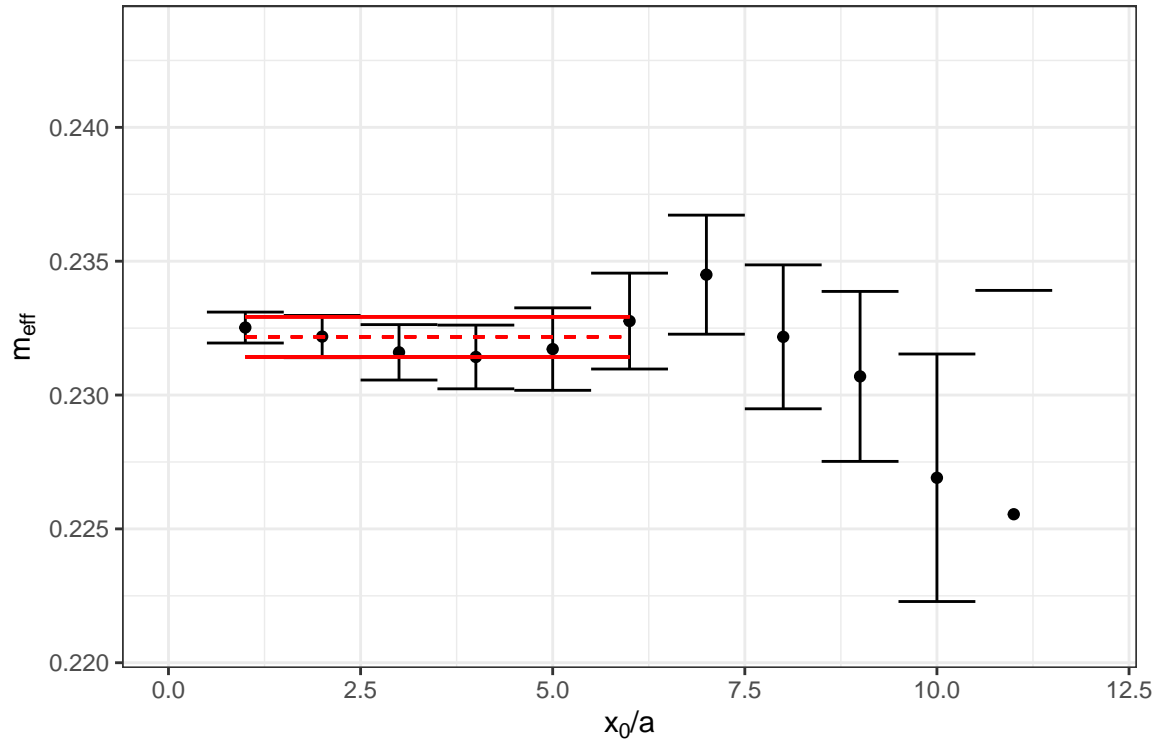
**Mass** index n= 0

fit:  $m_{eff} = 0.146393 \pm 0.000522$



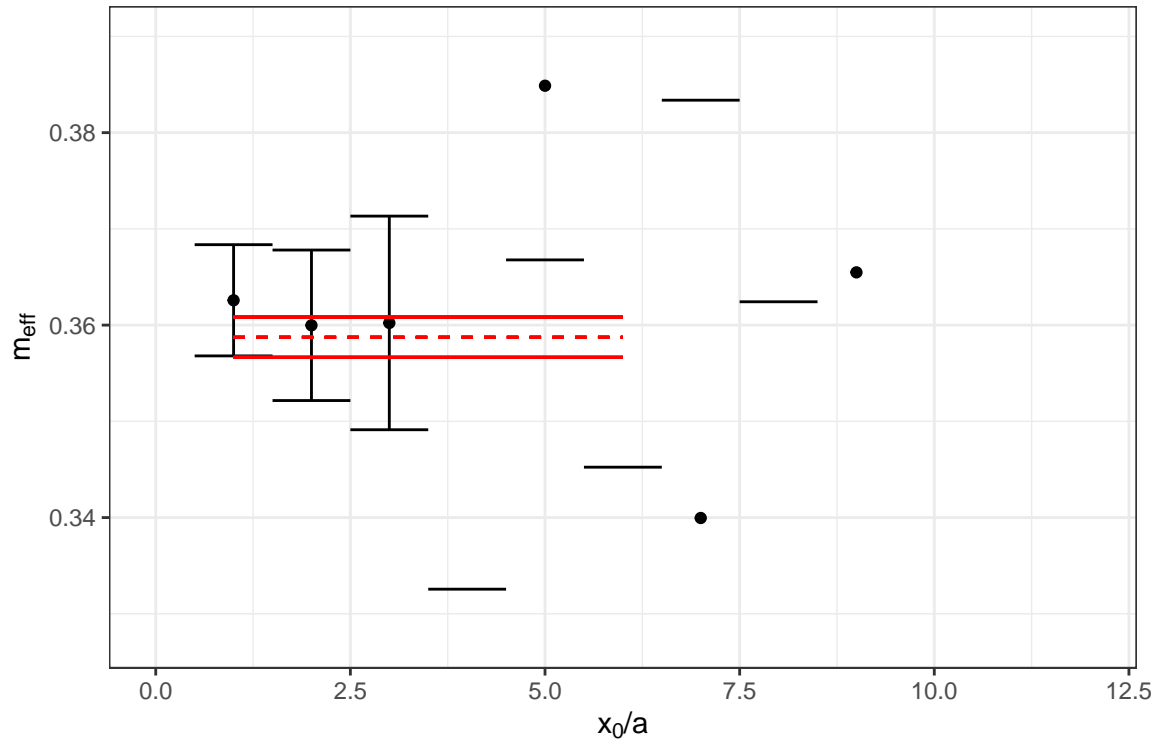
index  $n=1$

fit:  $m_{\text{eff}} = 0.232174 \pm 0.000749$



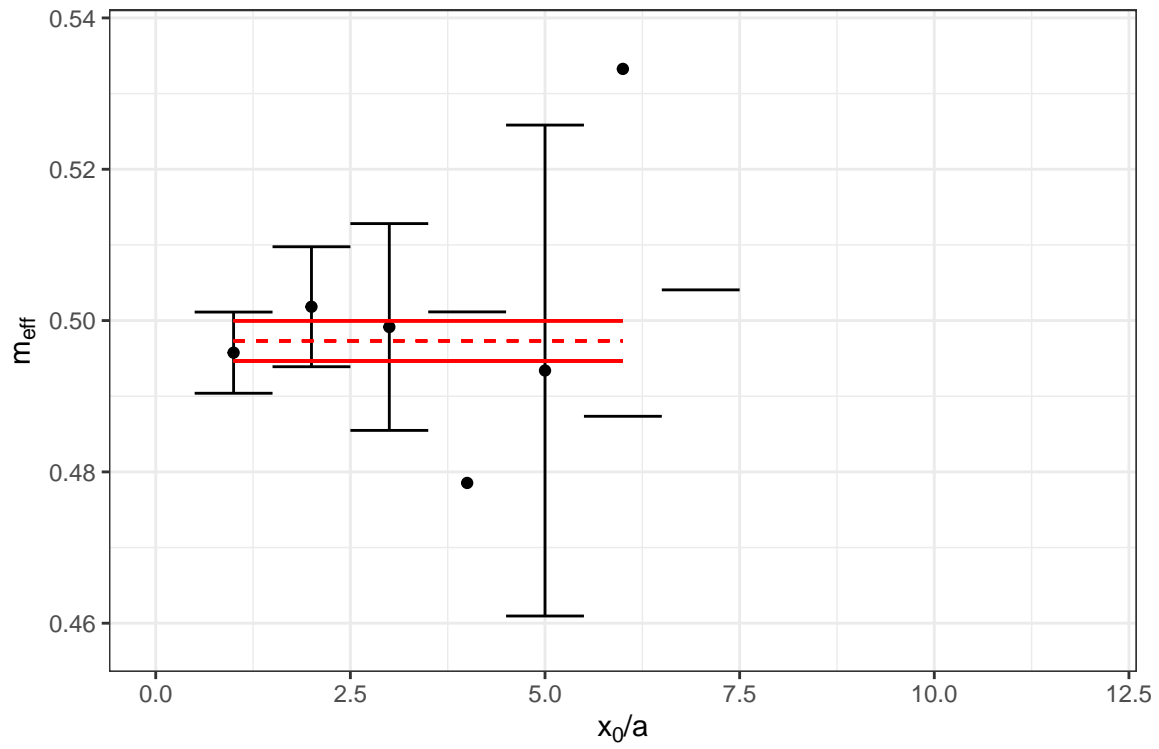
**Two particle energy** index  $n=0$

fit:  $m_{\text{eff}} = 0.358738 \pm 0.002084$



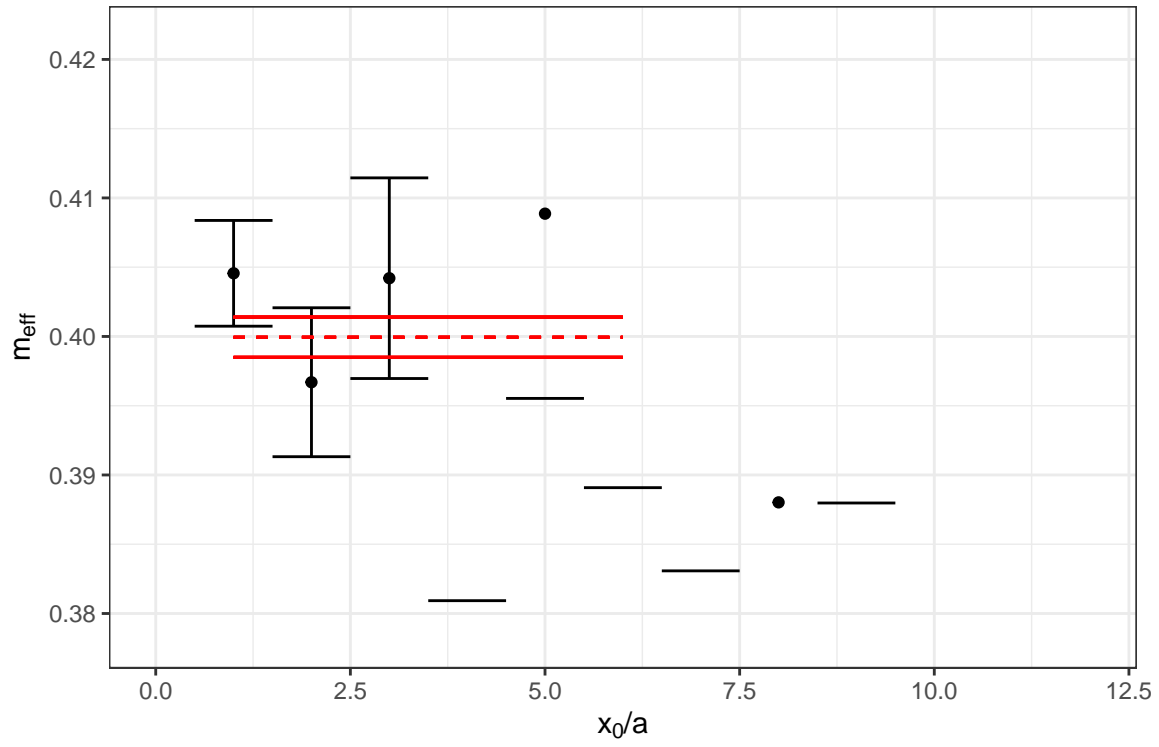
index  $n=1$

fit:  $m_{\text{eff}} = 0.497320 \pm 0.002651$



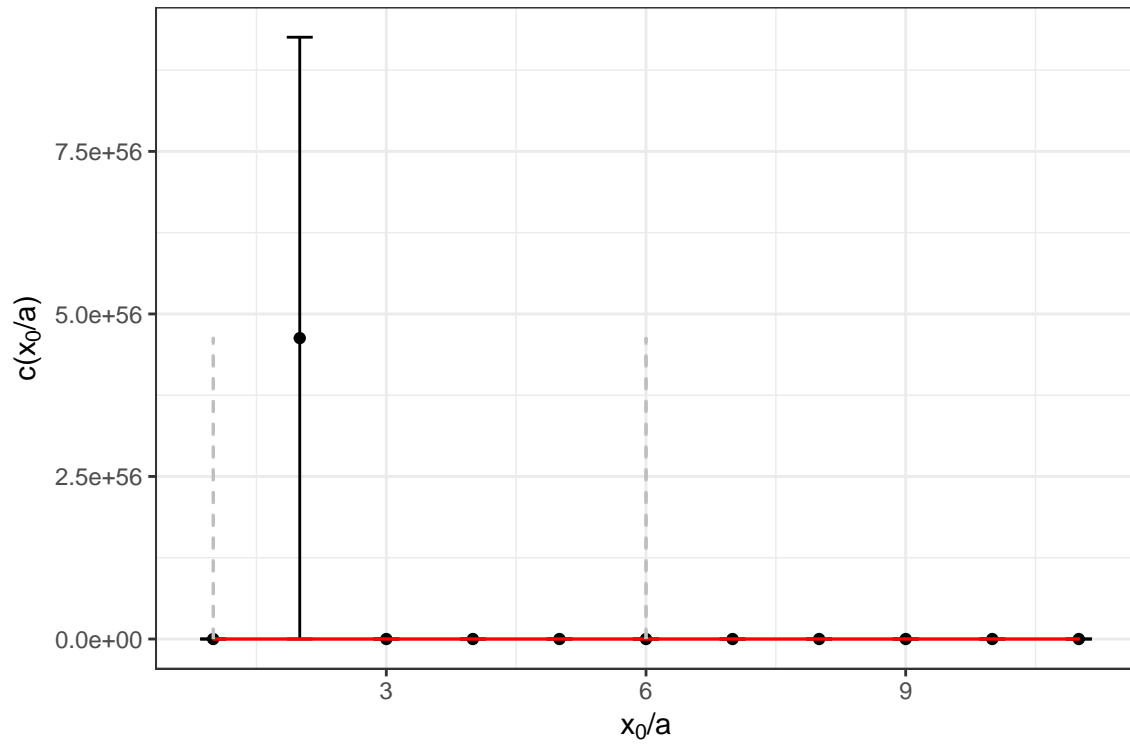
index  $n=2$

fit:  $m_{\text{eff}} = 0.399942 \pm 0.001448$



Three particle energy index n= 0

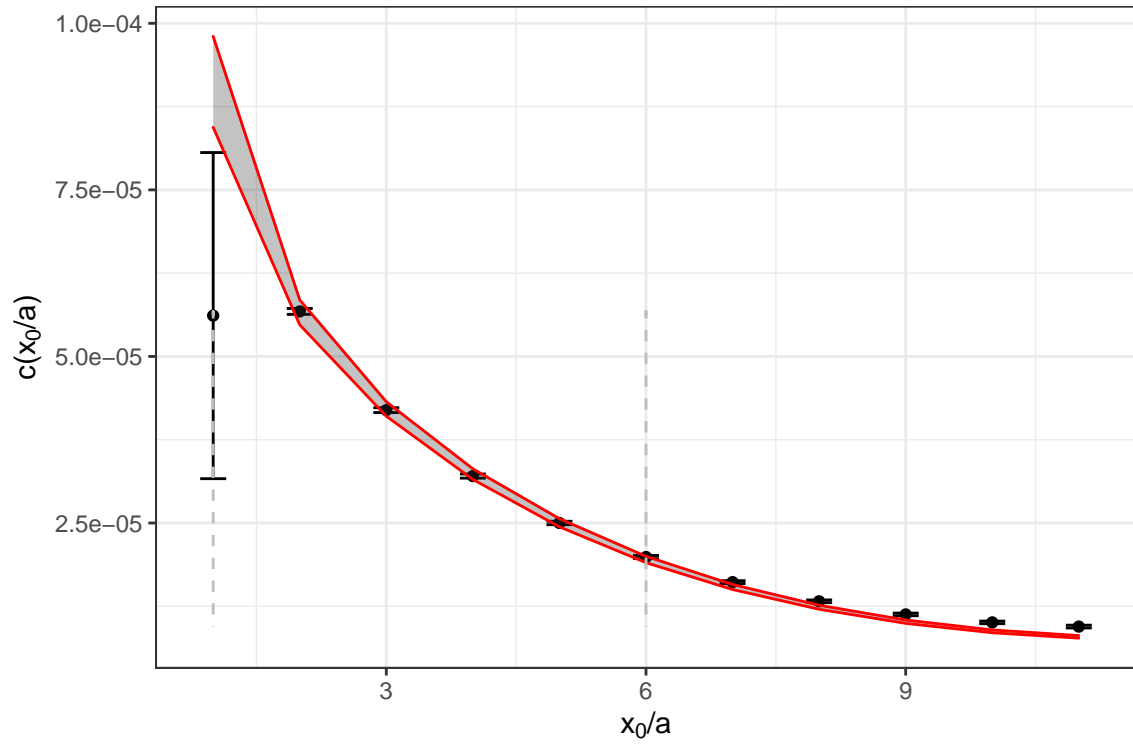
fit:  $m_{\text{eff}} = 0.246088 \pm 0.000157$



index n= 1

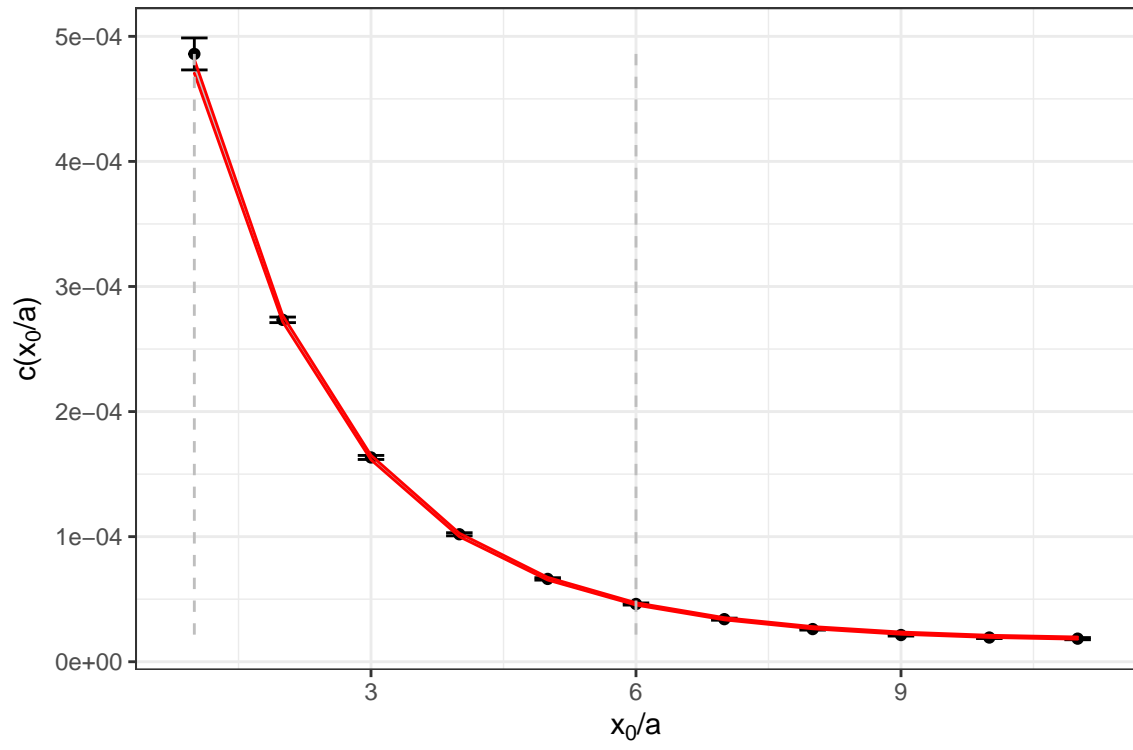
fit:  $m_{\text{eff}} = 2.220848 \pm 0.031745$





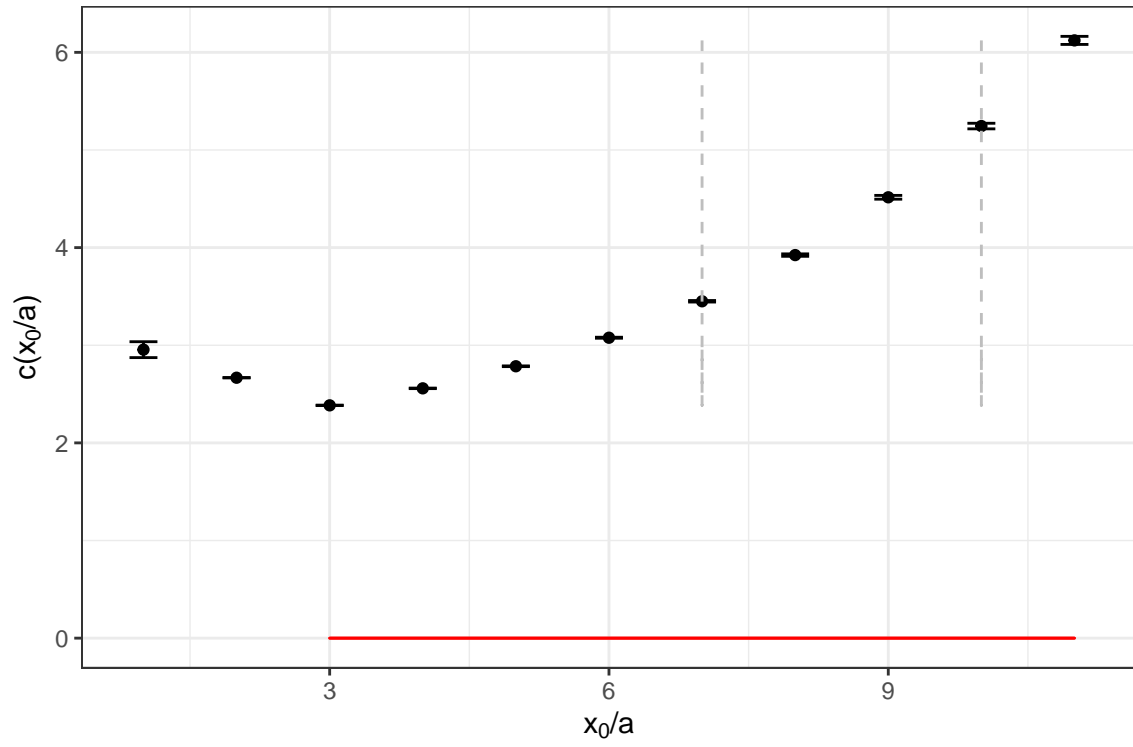
index  $n= 2$

fit:  $m_{eff} = 0.618762 \pm 0.010830$



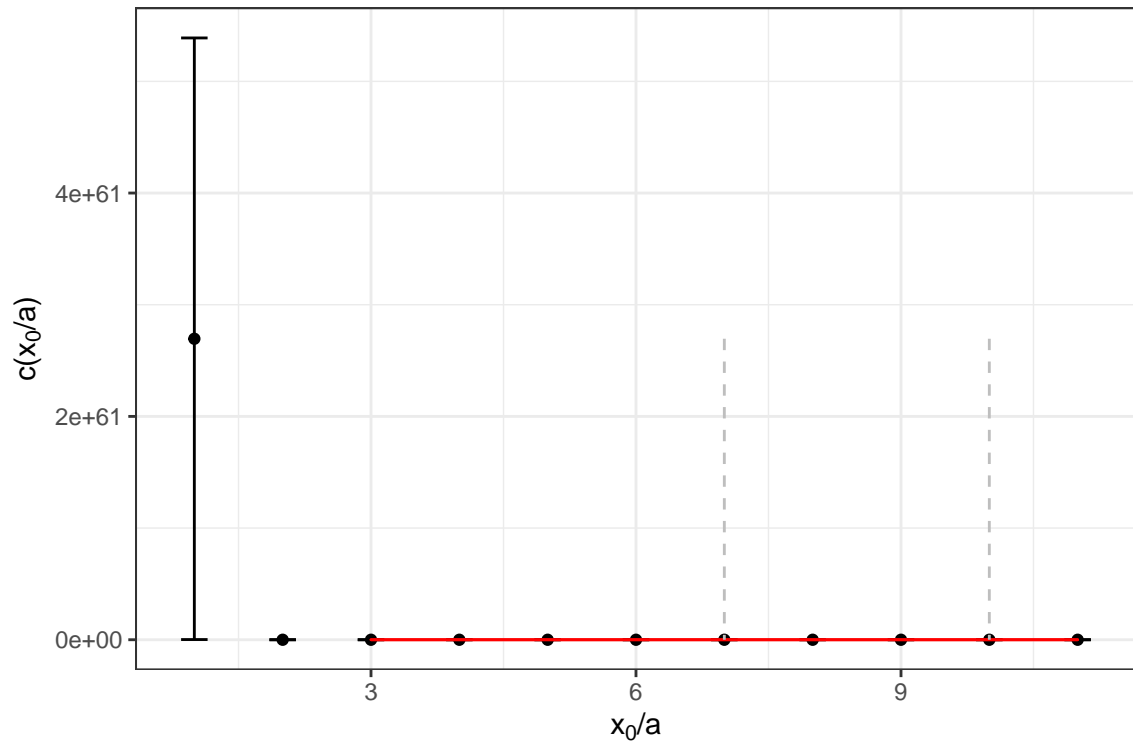
**C4\_BH** index  $n= 0$

fit:  $m_{eff} = 0.000001 \pm 0.000000$



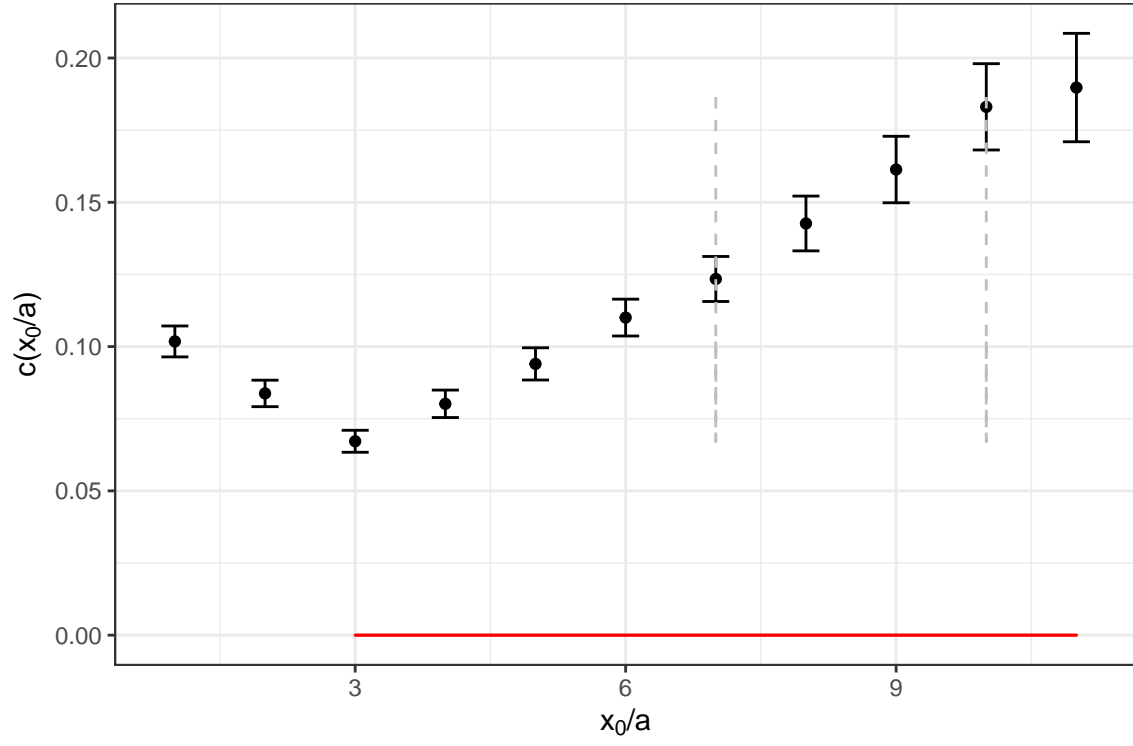
index  $n=1$

fit:  $m_{eff} = 0.000040 \pm 0.000001$



index  $n=2$

fit:  $m_{eff} = 0.000072 \pm 0.000003$

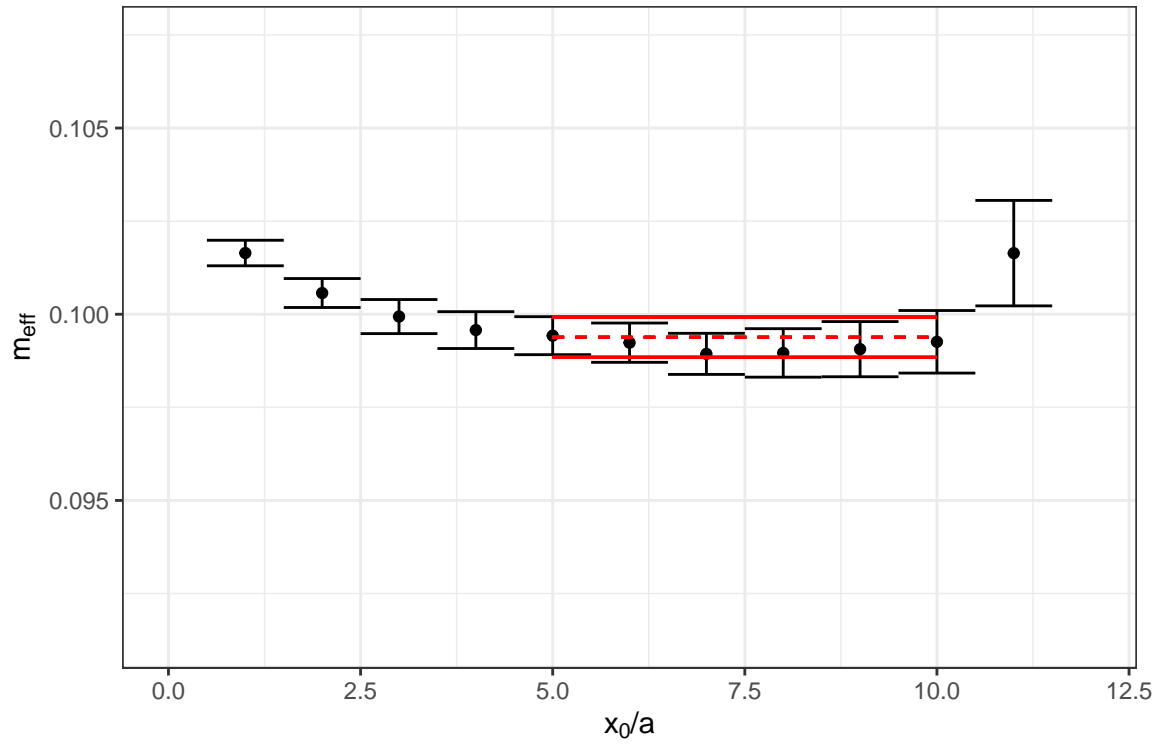


**T 24 L 10**

$m_0^2 = -4.98$   $m_1^2 = -4.9$   $\lambda_0^2 = 2.5$   $\lambda_1^2 = 2.5$   $\mu^2 = 5$   $g^2 = 0$  replica = 0

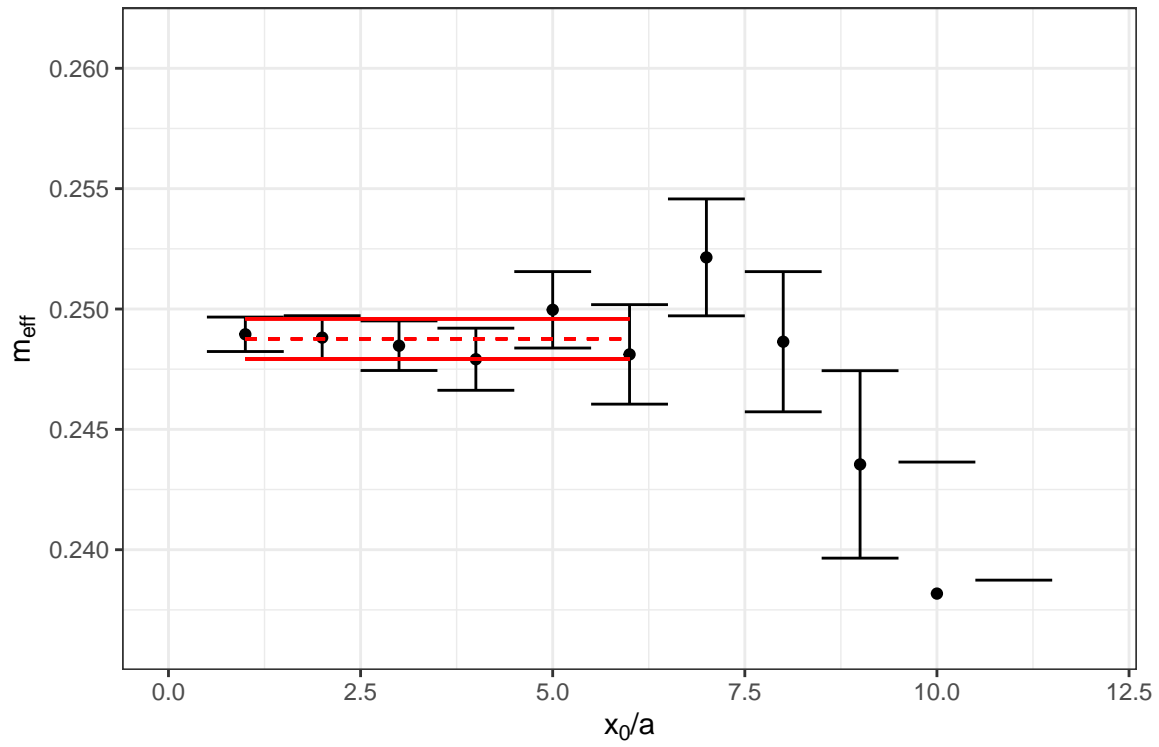
**Mass** index n= 0

fit:  $m_{eff} = 0.099382 \pm 0.000539$



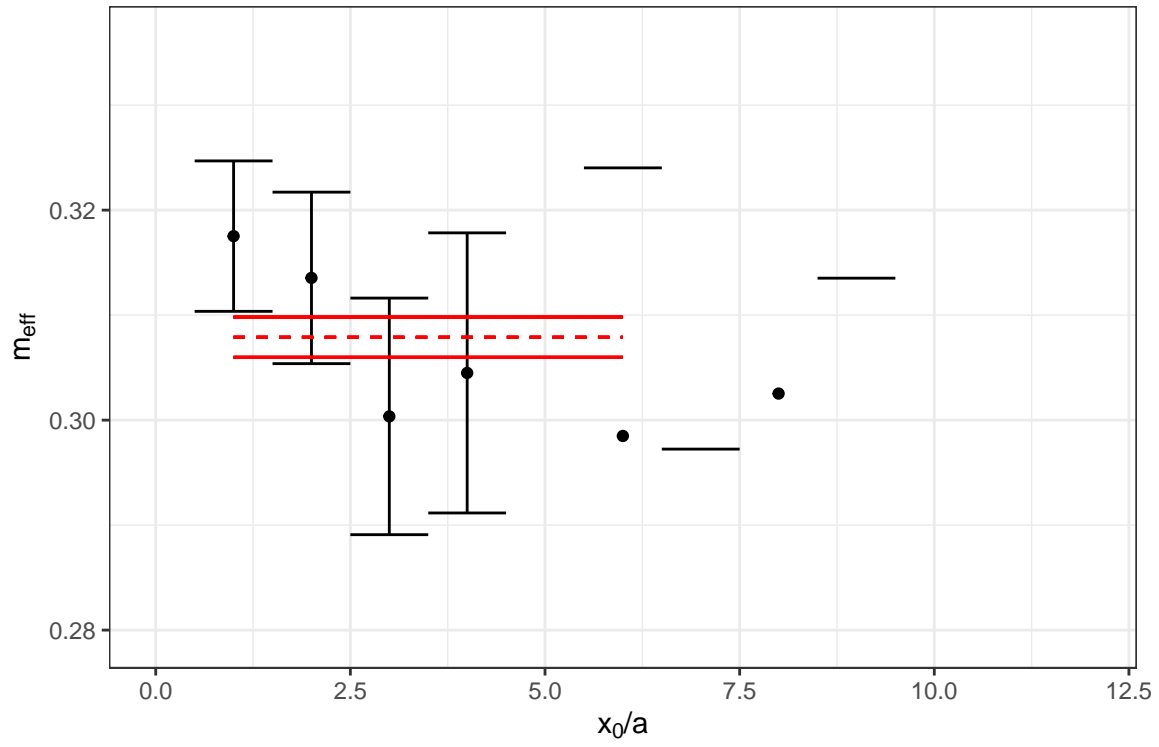
index  $n=1$

fit:  $m_{\text{eff}} = 0.248760 \pm 0.000833$



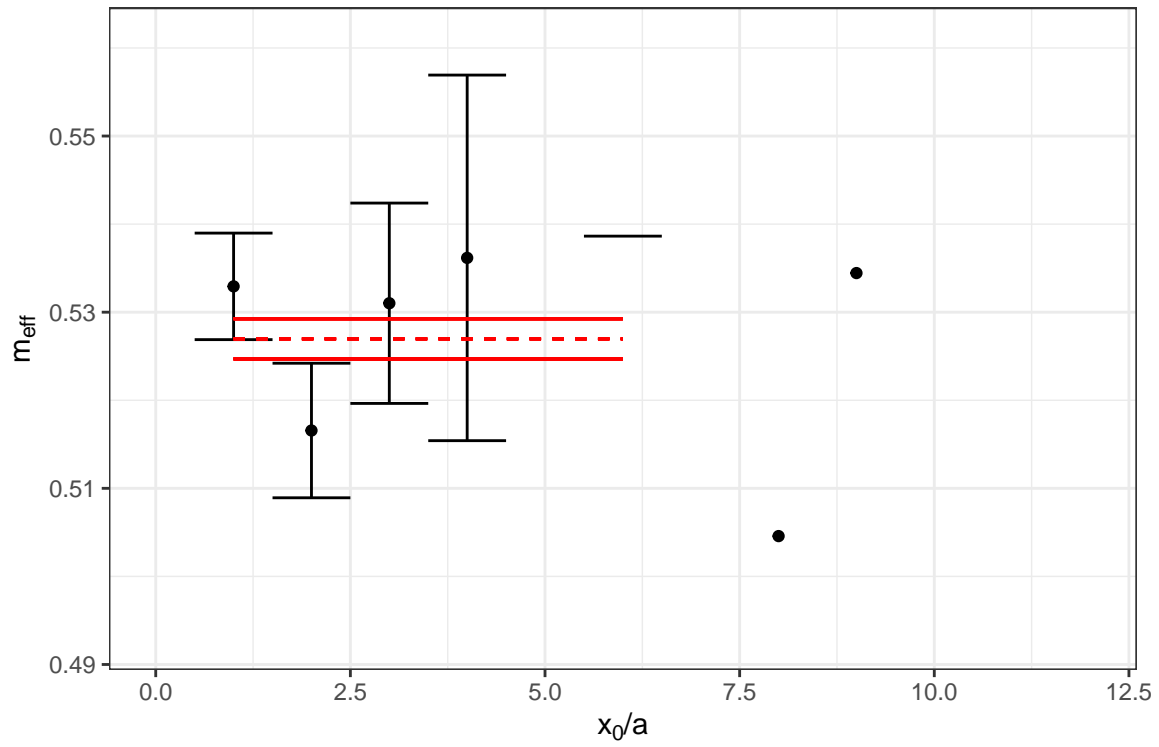
**Two particle energy** index  $n=0$

fit:  $m_{\text{eff}} = 0.307891 \pm 0.001911$



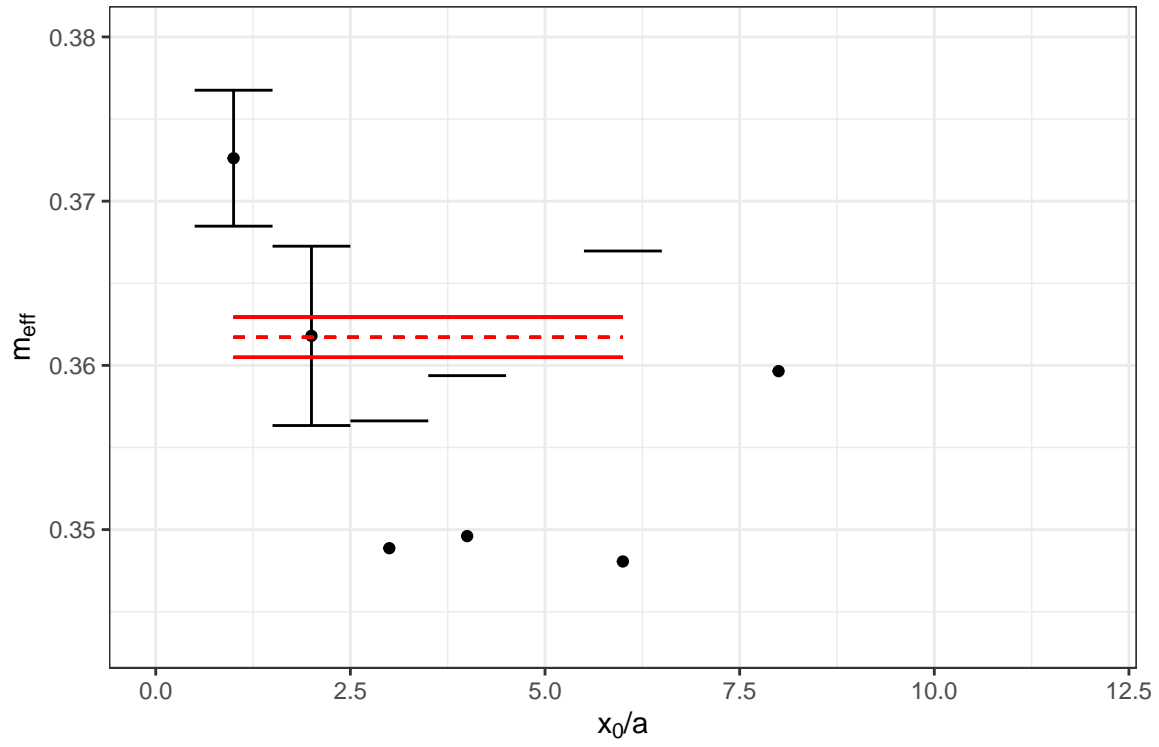
index n= 1

fit:  $m_{\text{eff}} = 0.526973 \pm 0.002278$



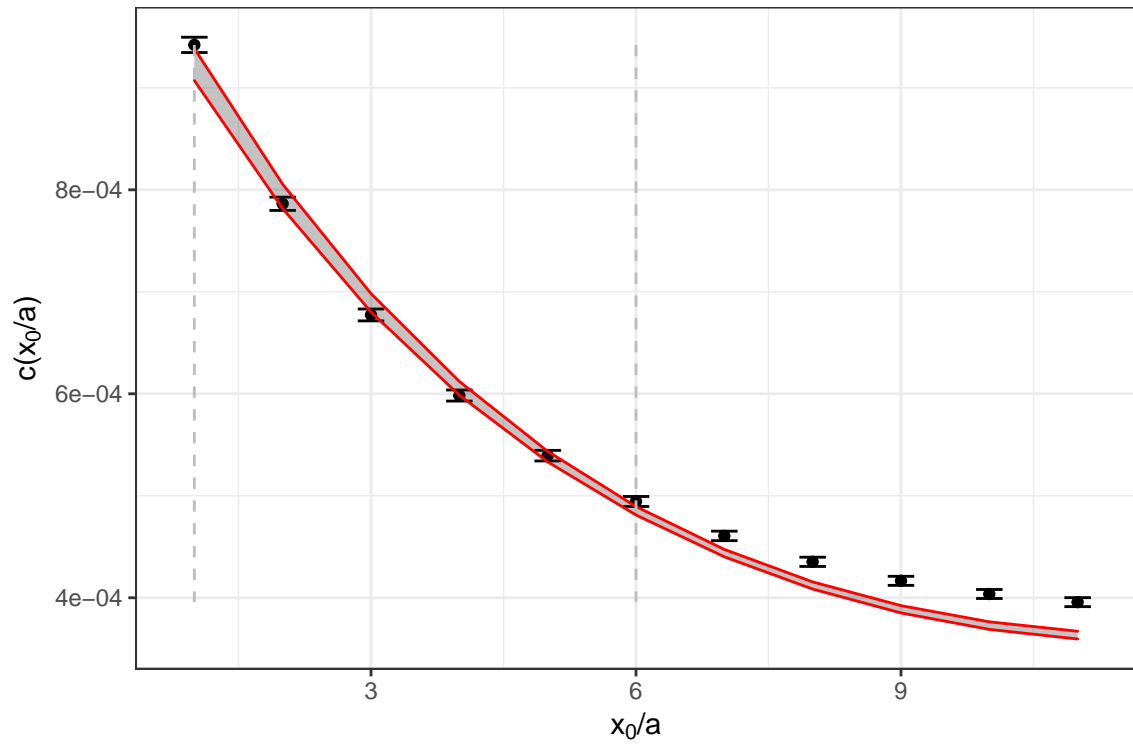
index n= 2

fit:  $m_{\text{eff}} = 0.361713 \pm 0.001221$



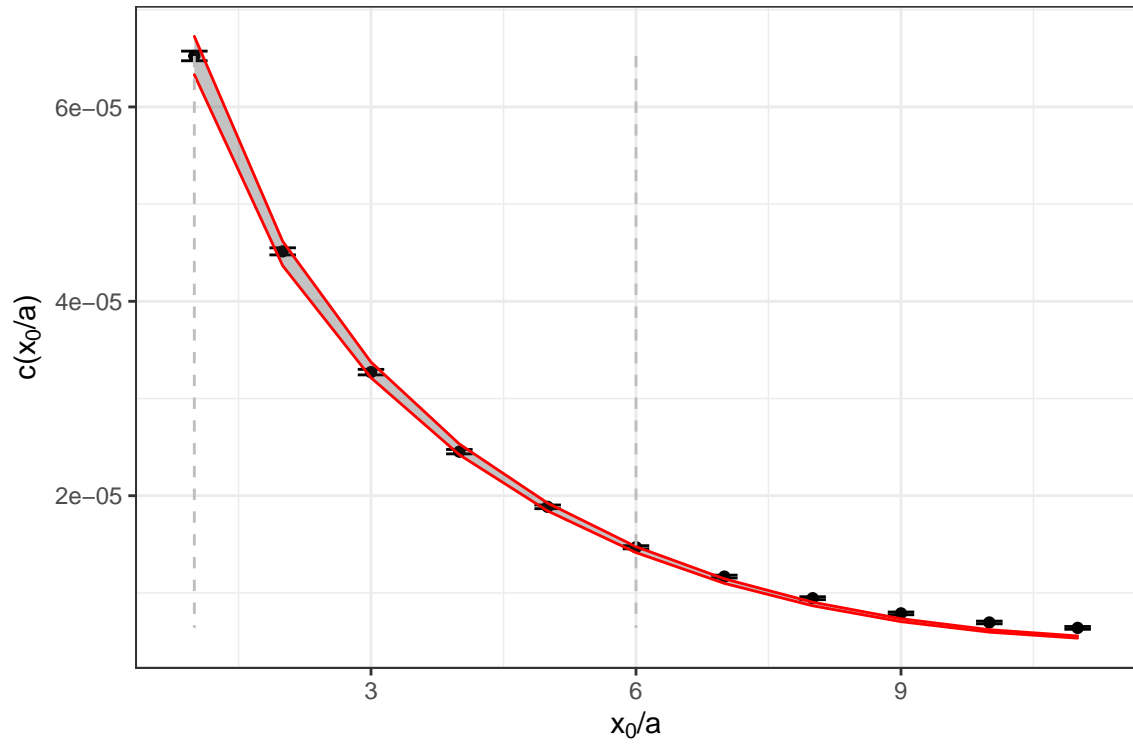
Three particle energy index  $n=0$

fit:  $m_{\text{eff}} = 0.000320 \pm 0.000405$



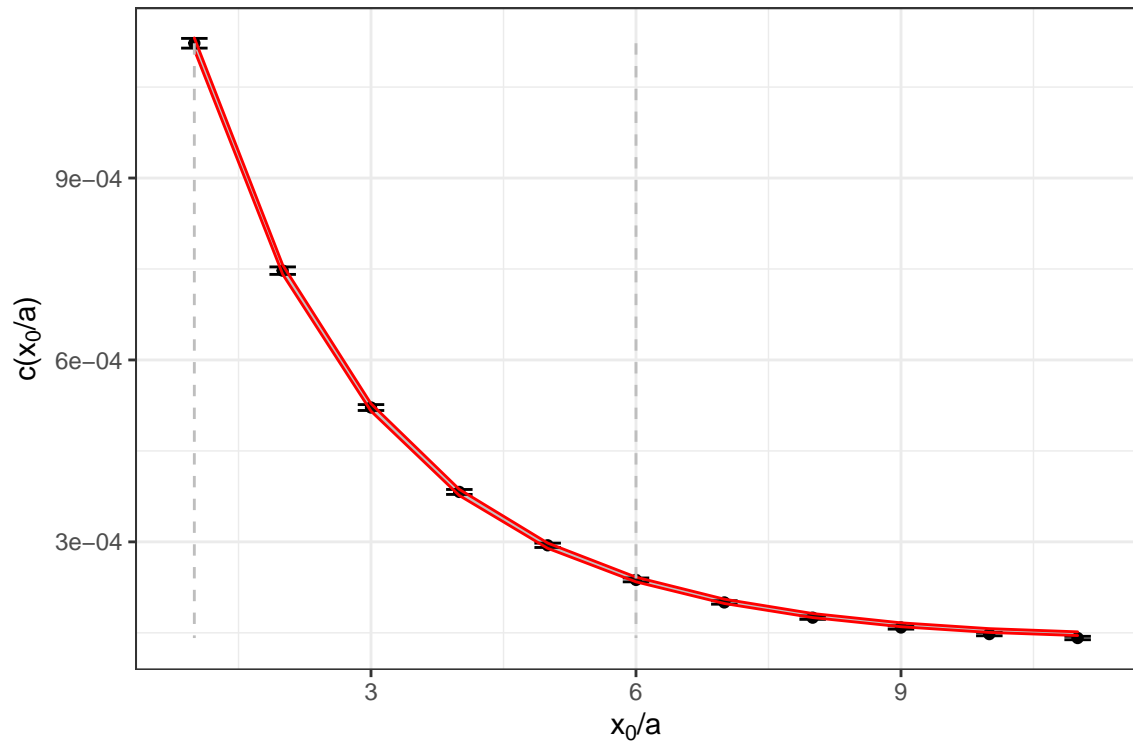
index  $n=1$

fit:  $m_{\text{eff}} = 1.372215 \pm 0.090647$



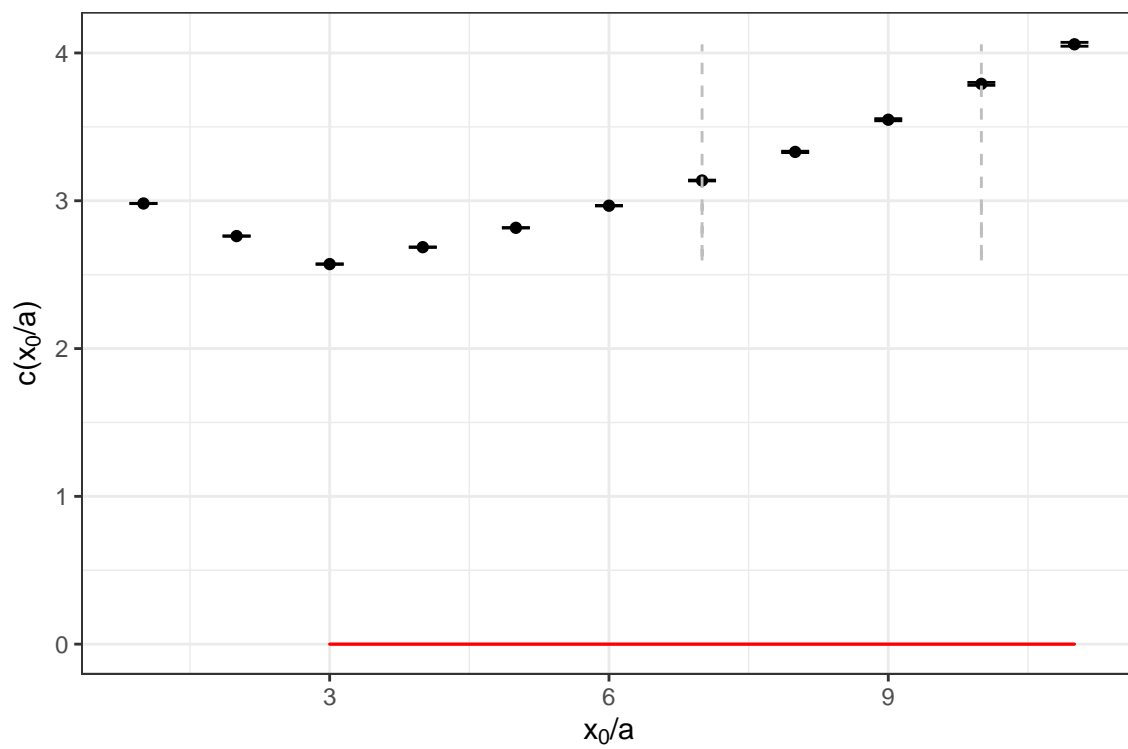
index  $n=2$

fit:  $m_{eff} = 0.523029 \pm 0.003161$



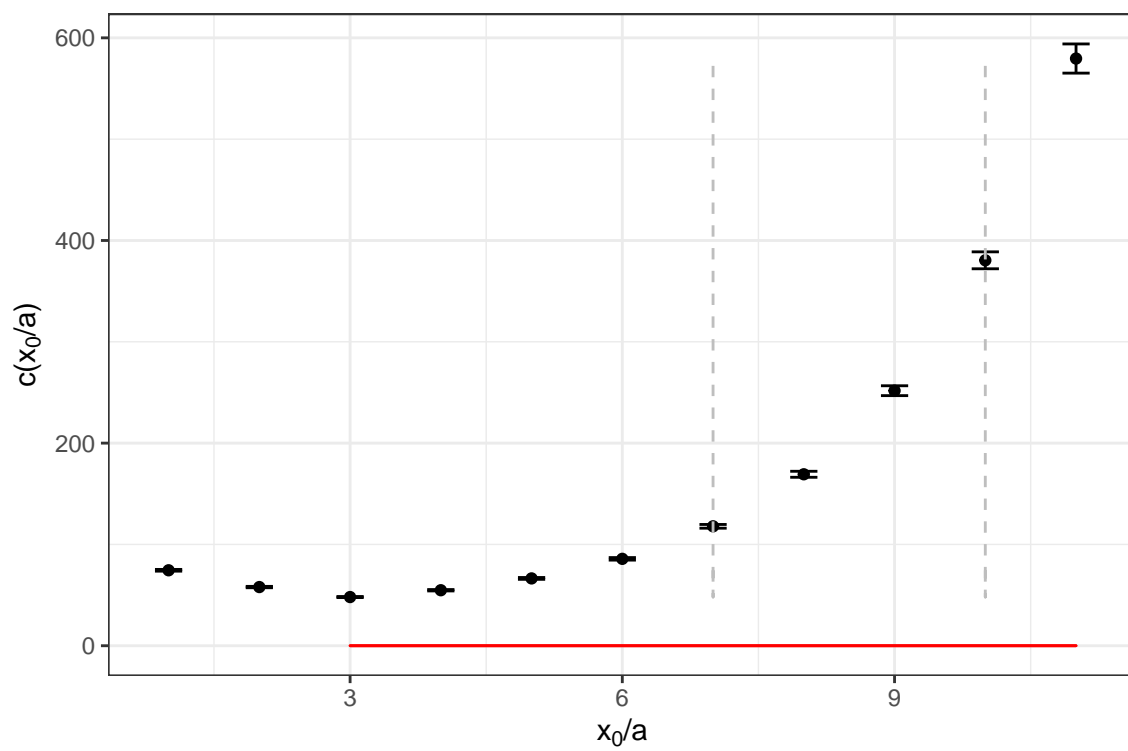
**C4\_BH** index  $n=0$

fit:  $m_{eff} = -0.000000 \pm 0.000000$



index  $n=1$

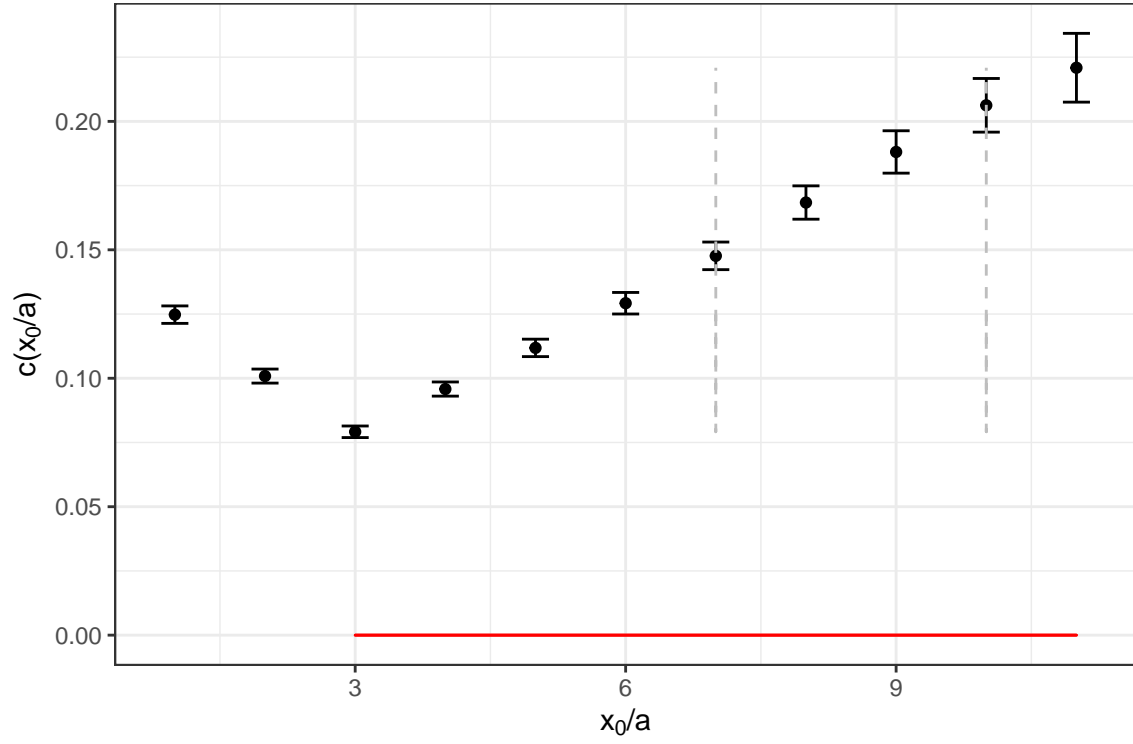
fit:  $m_{eff} = 0.000003 \pm 0.000000$



index  $n=2$

fit:  $m_{eff} = -0.000001 \pm 0.000001$



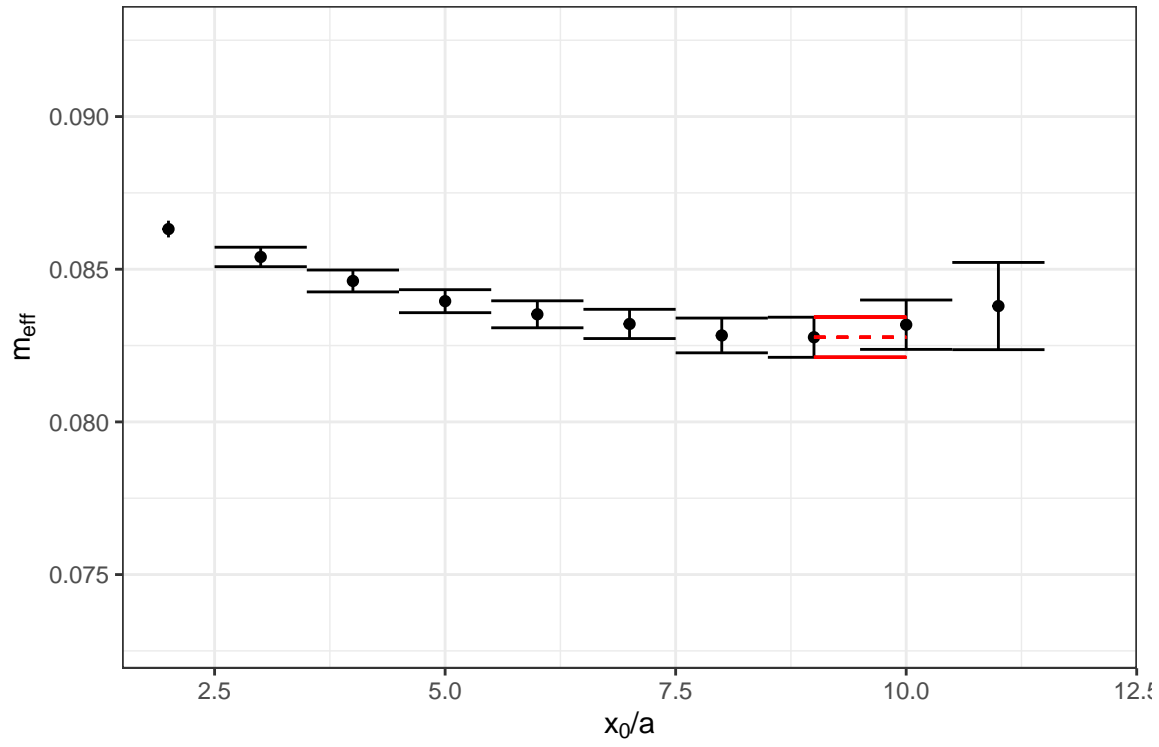


**T 24 L 10**

$m_0^2 = -4.99$   $m_1^2 = -4.9$   $\lambda_0^2 = 2.5$   $\lambda_1^2 = 2.5$   $\mu^2 = 5$   $g^2 = 0$  replica = 0

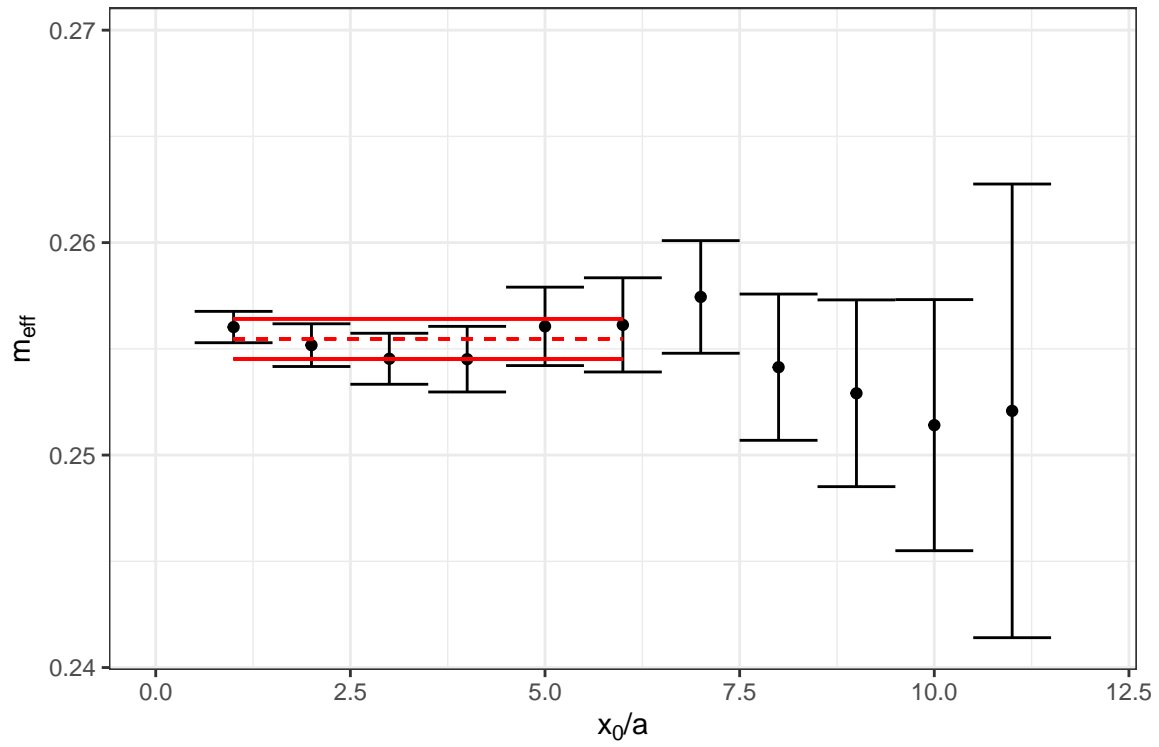
**Mass** index n= 0

fit:  $m_{eff} = 0.082772 \pm 0.000657$



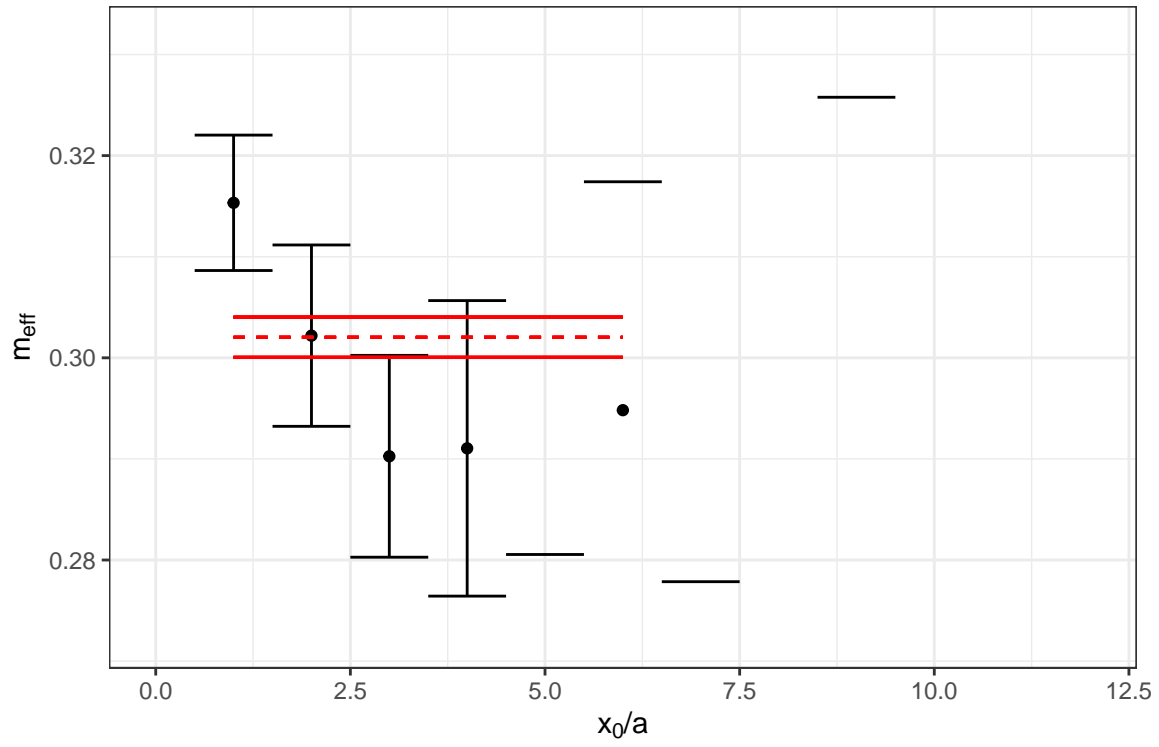
index  $n=1$

fit:  $m_{eff} = 0.255471 \pm 0.000944$



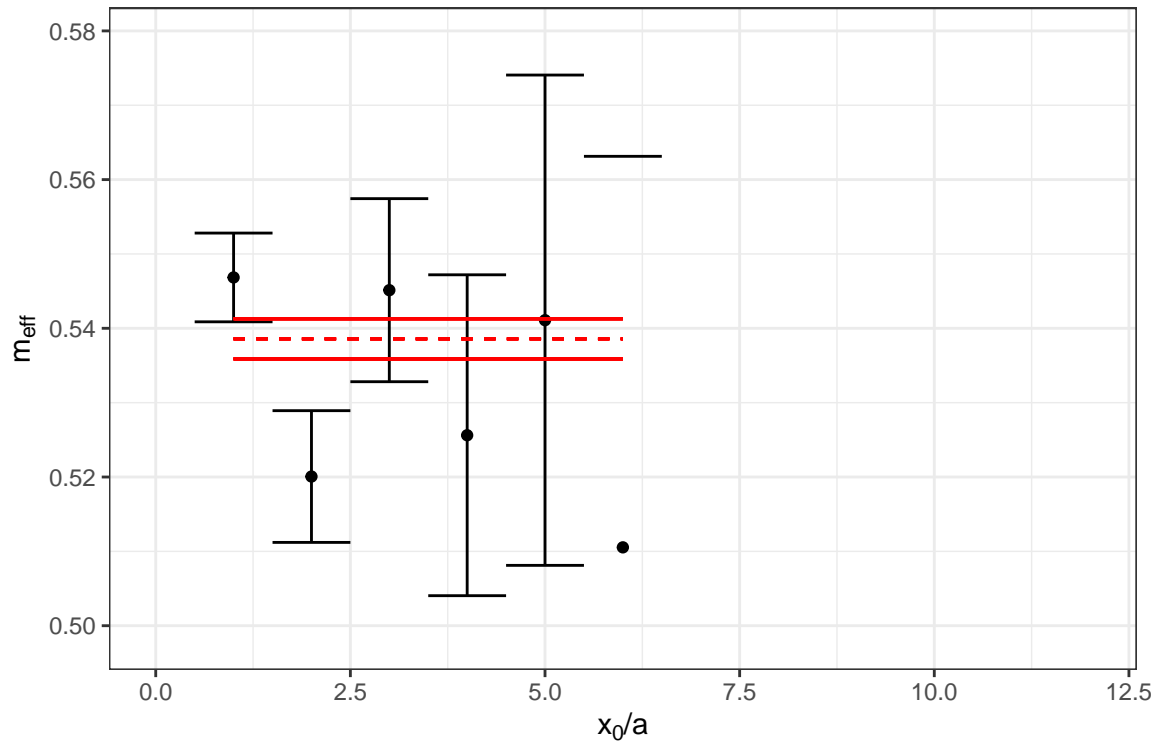
**Two particle energy** index  $n=0$

fit:  $m_{eff} = 0.302034 \pm 0.001984$



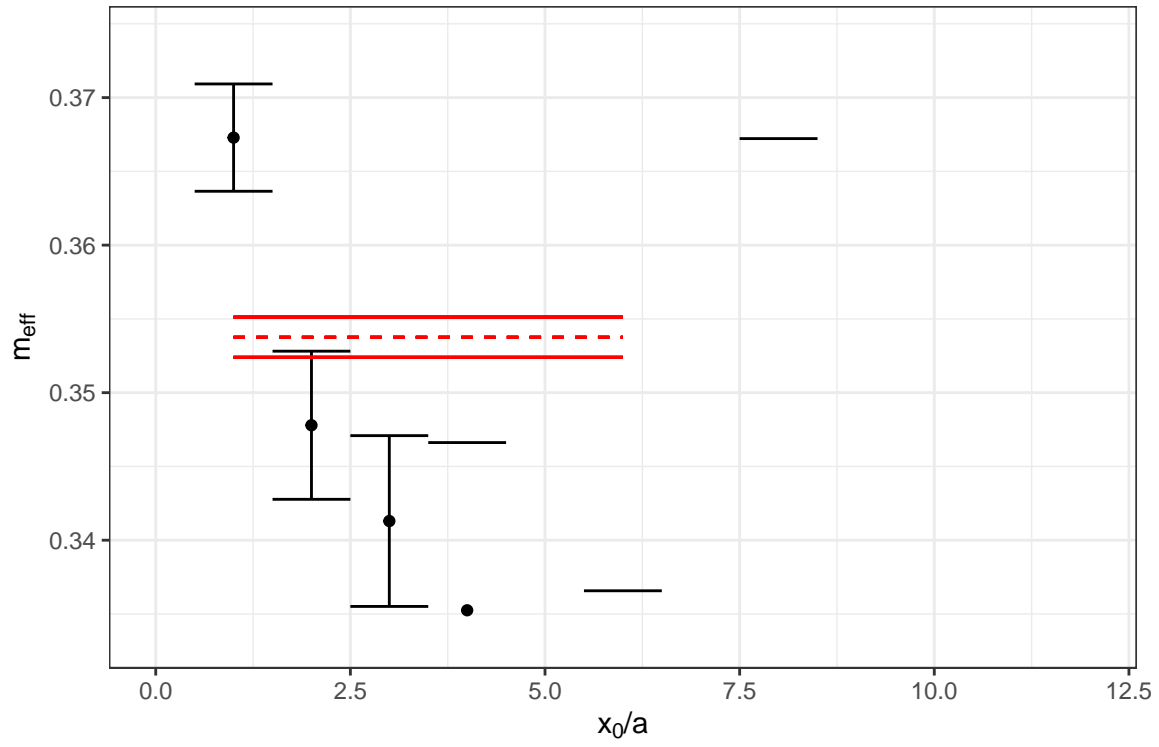
index  $n=1$

fit:  $m_{\text{eff}} = 0.538582 \pm 0.002698$



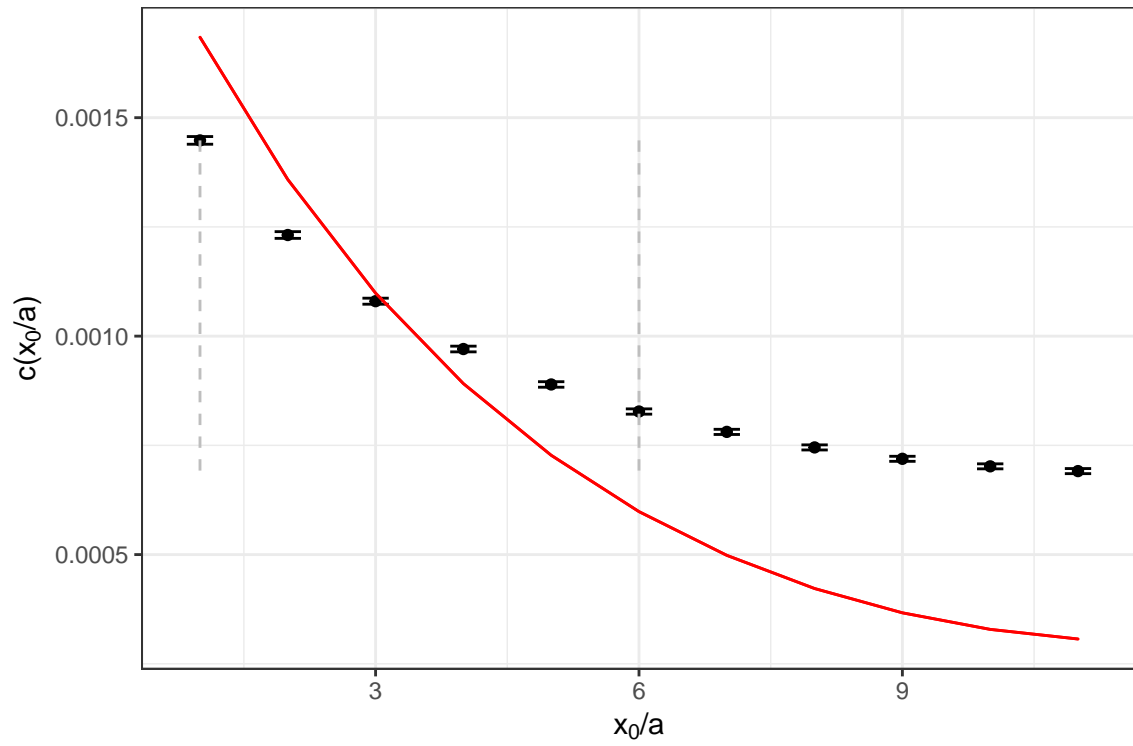
index  $n=2$

fit:  $m_{\text{eff}} = 0.353756 \pm 0.001360$



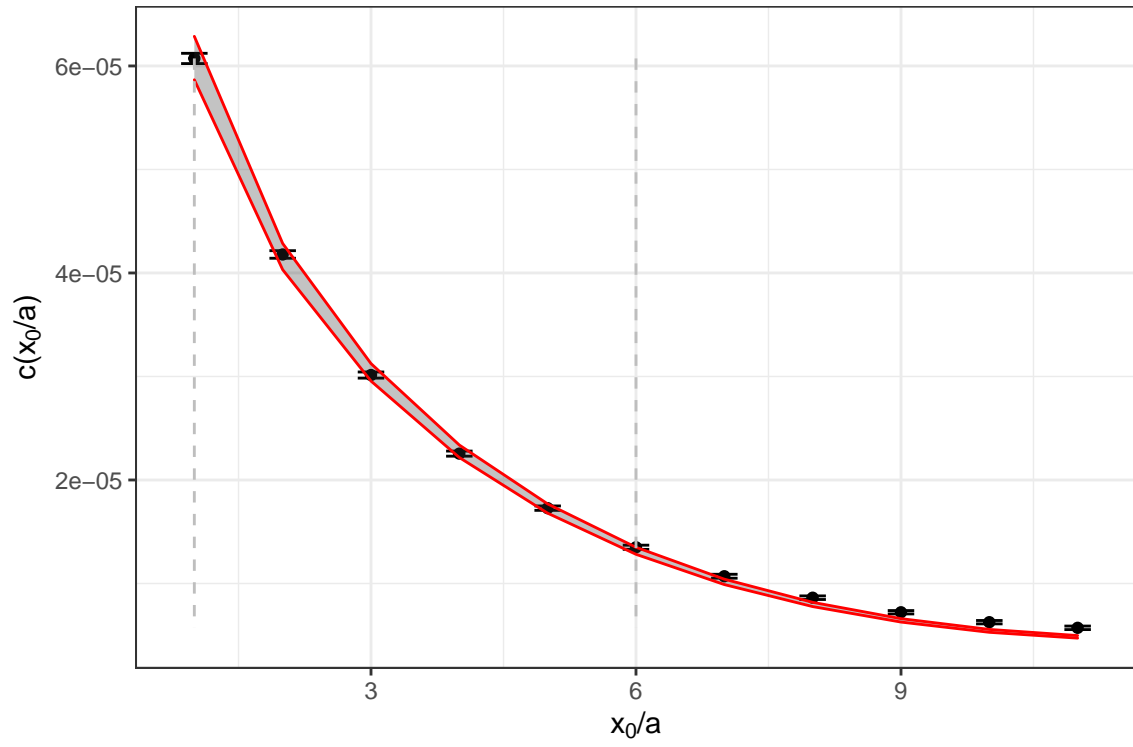
Three particle energy index  $n=0$

fit:  $m_{\text{eff}} = 6.289068 \pm 0.000000$



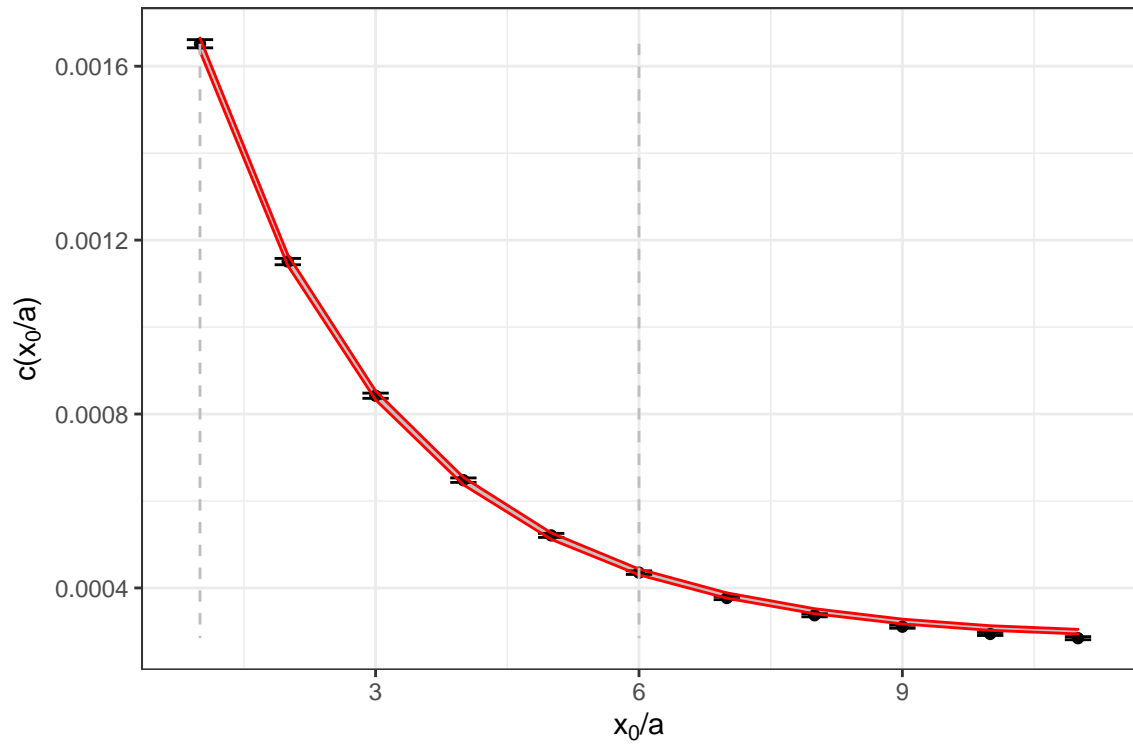
index  $n=1$

fit:  $m_{\text{eff}} = 1.442154 \pm 0.058761$



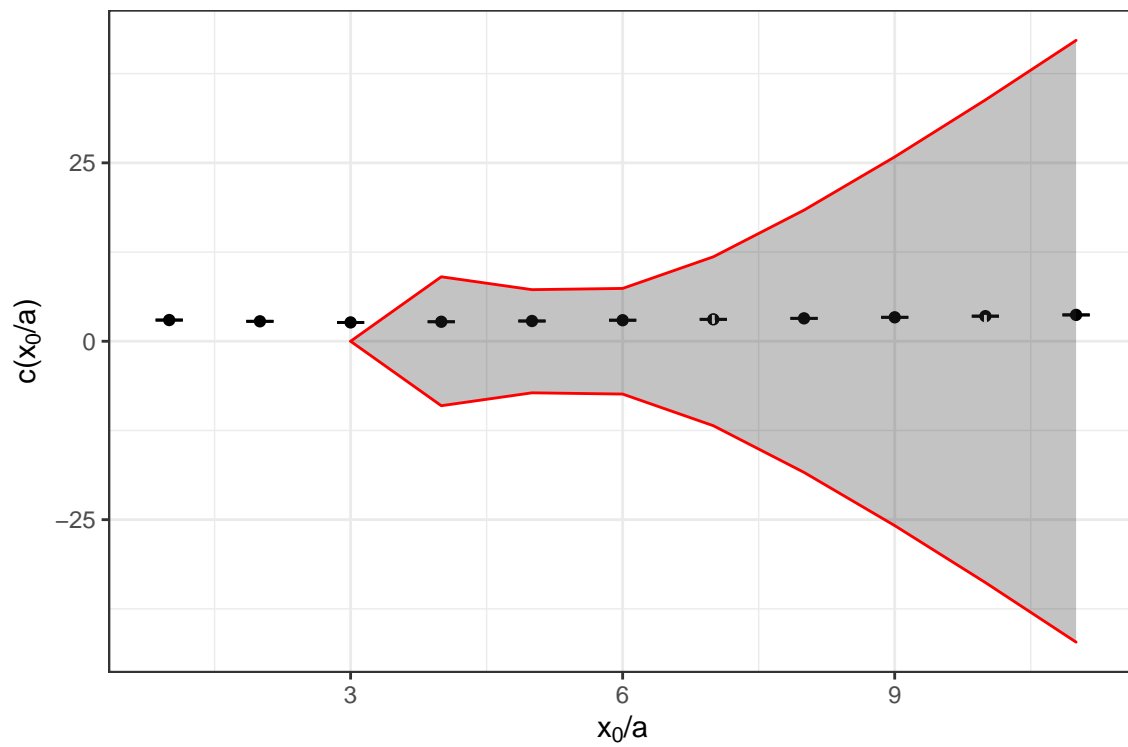
index  $n=2$

fit:  $m_{eff} = 0.496624 \pm 0.002531$



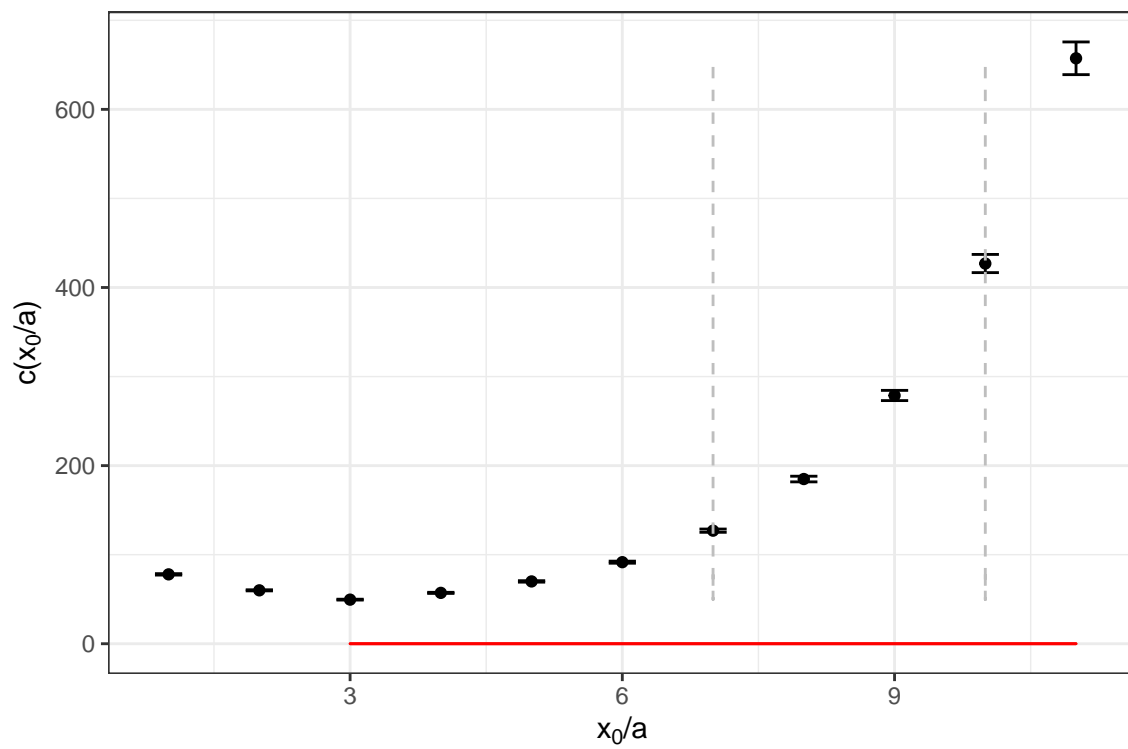
**C4\_BH** index  $n=0$

fit:  $m_{eff} = 6.160731 \pm 21.607296$



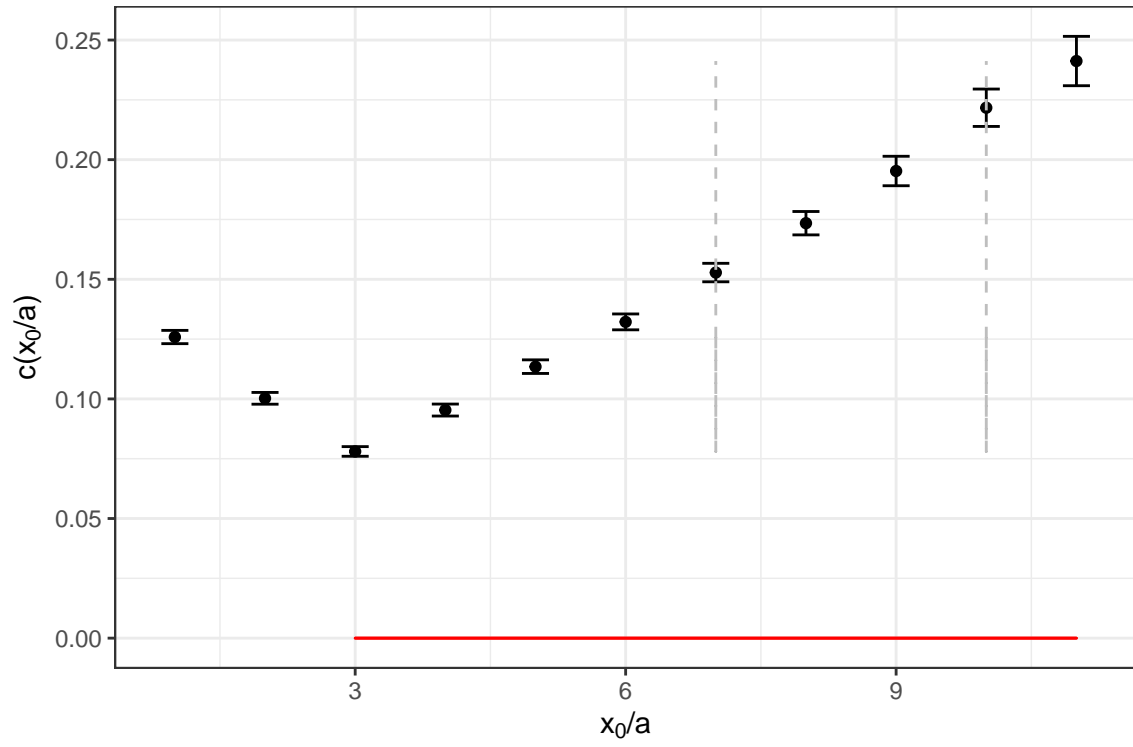
index  $n=1$

fit:  $m_{eff} = -0.000214 \pm 0.000001$



index  $n=2$

fit:  $m_{eff} = -0.000001 \pm 0.000000$

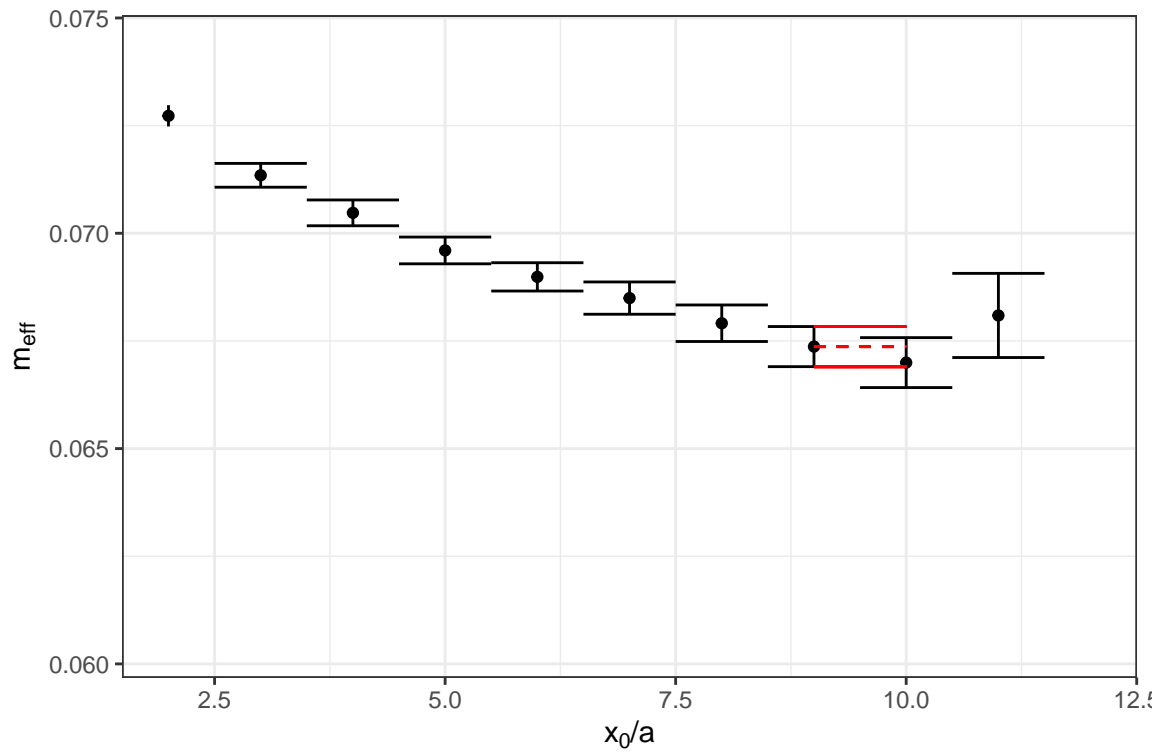


**T 24 L 10**

$m_0^2 = -5$   $m_1^2 = -4.9$   $\lambda_0^2 = 2.5$   $\lambda_1^2 = 2.5$   $\mu^2 = 5$   $g^2 = 0$  replica = 0

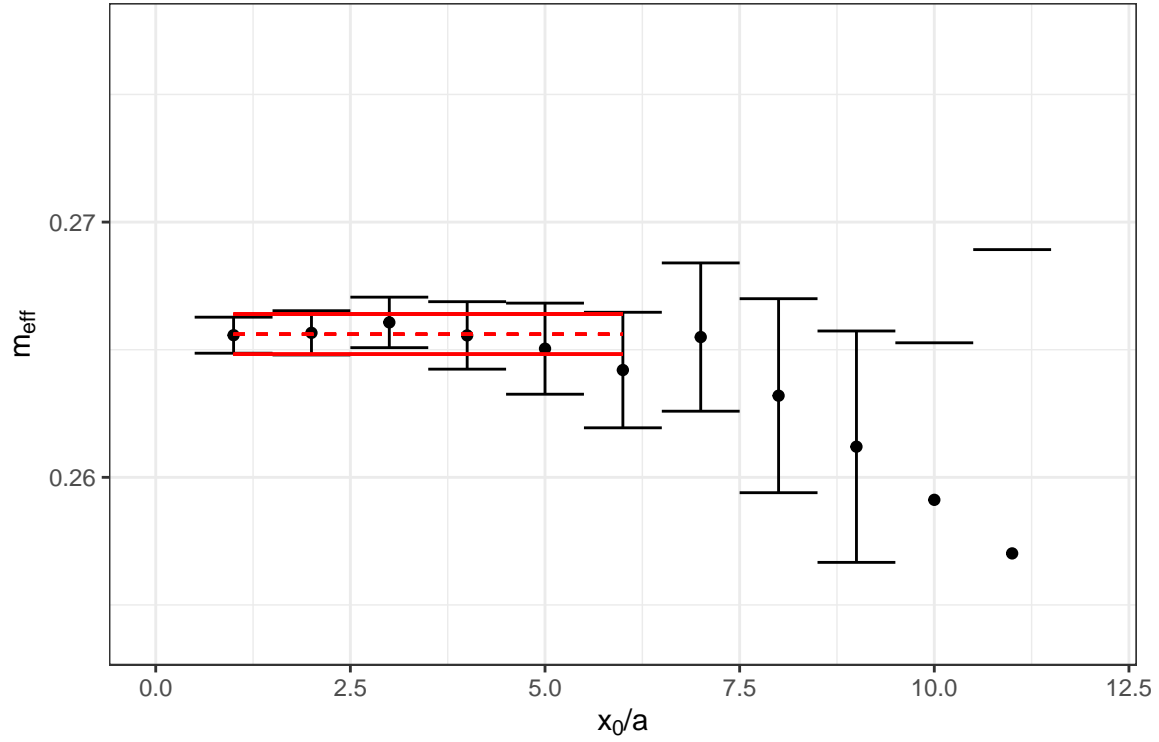
**Mass** index n= 0

fit:  $m_{eff} = 0.067369 \pm 0.000466$



index  $n=1$

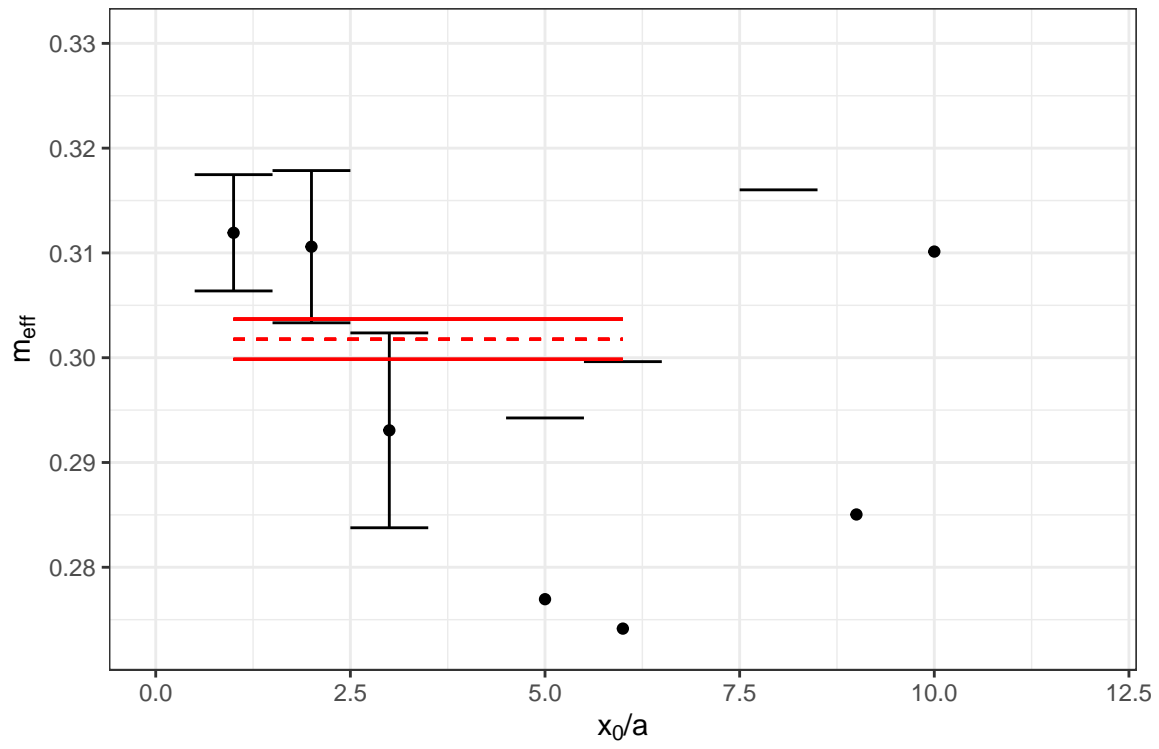
fit:  $m_{eff} = 0.265608 \pm 0.000786$



**Two particle energy** index  $n=0$

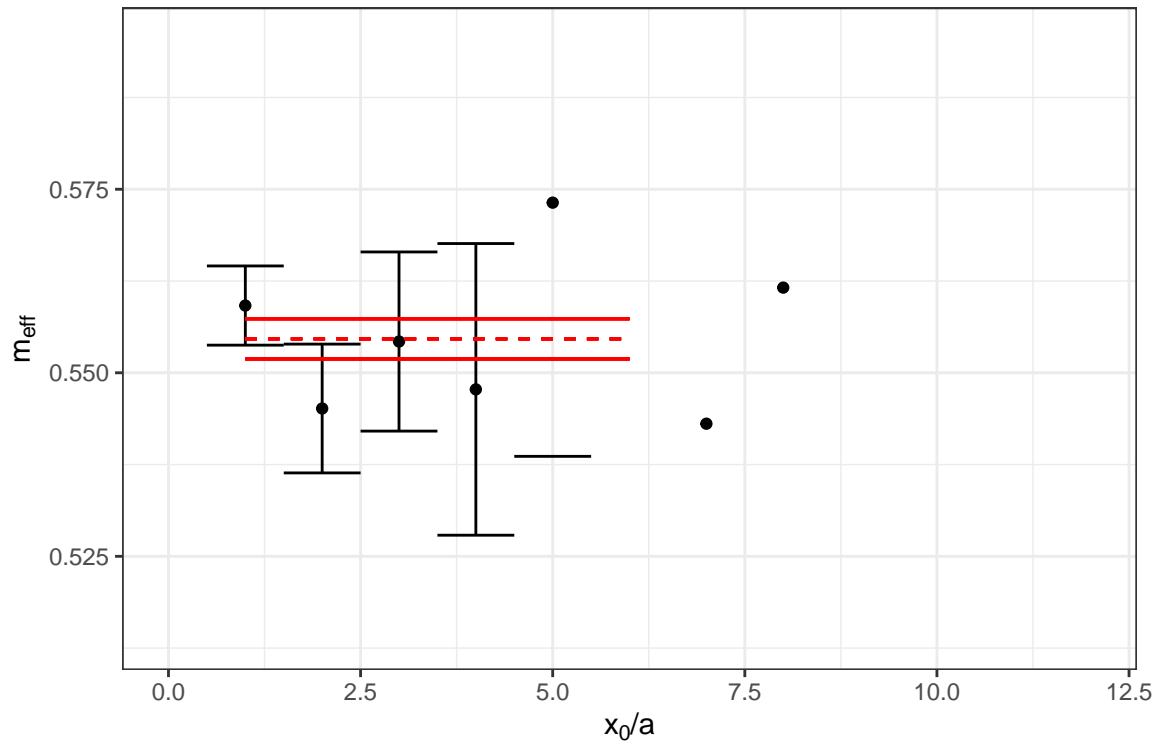
fit:  $m_{eff} = 0.301761 \pm 0.001914$





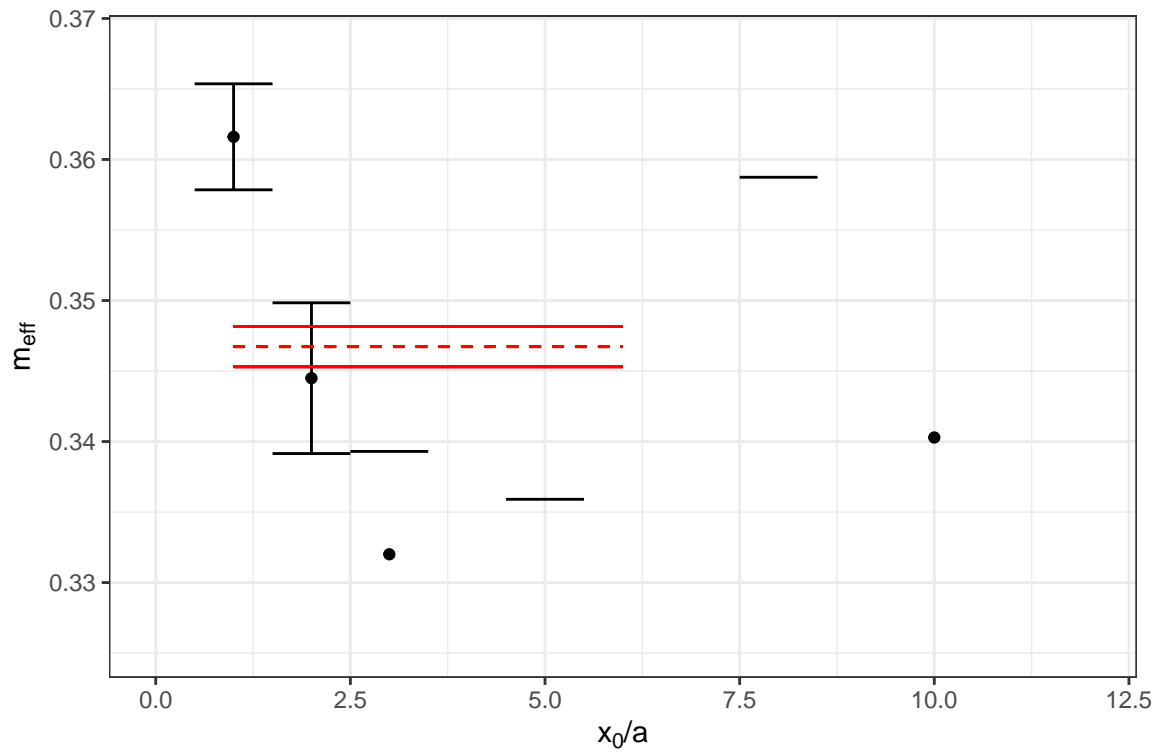
index n= 1

fit:  $m_{eff} = 0.554623 \pm 0.002732$



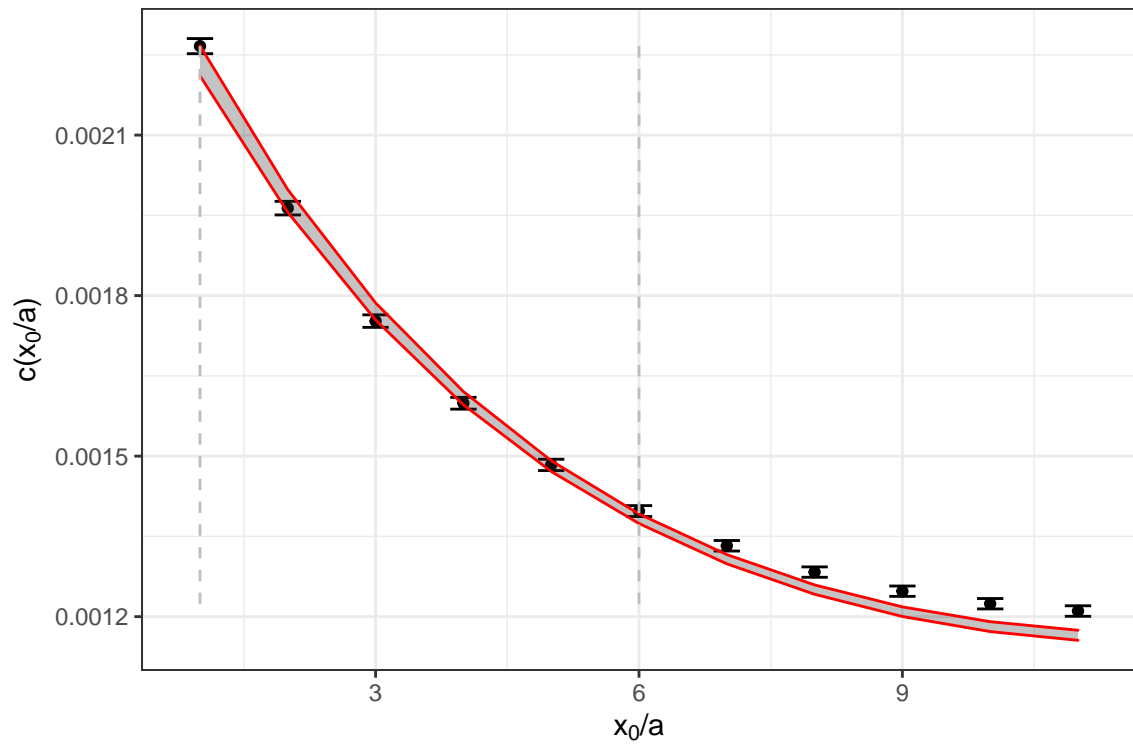
index n= 2

fit:  $m_{eff} = 0.346734 \pm 0.001422$



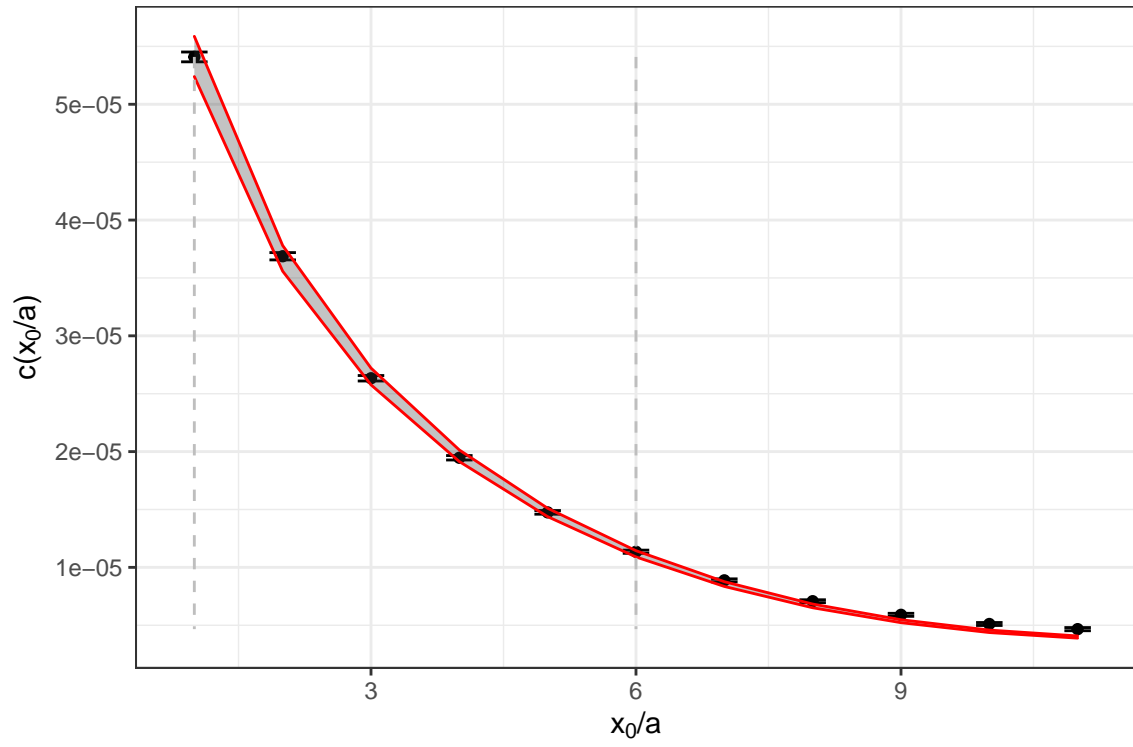
Three particle energy index  $n=0$

fit:  $m_{eff} = 0.000145 \pm 0.000218$



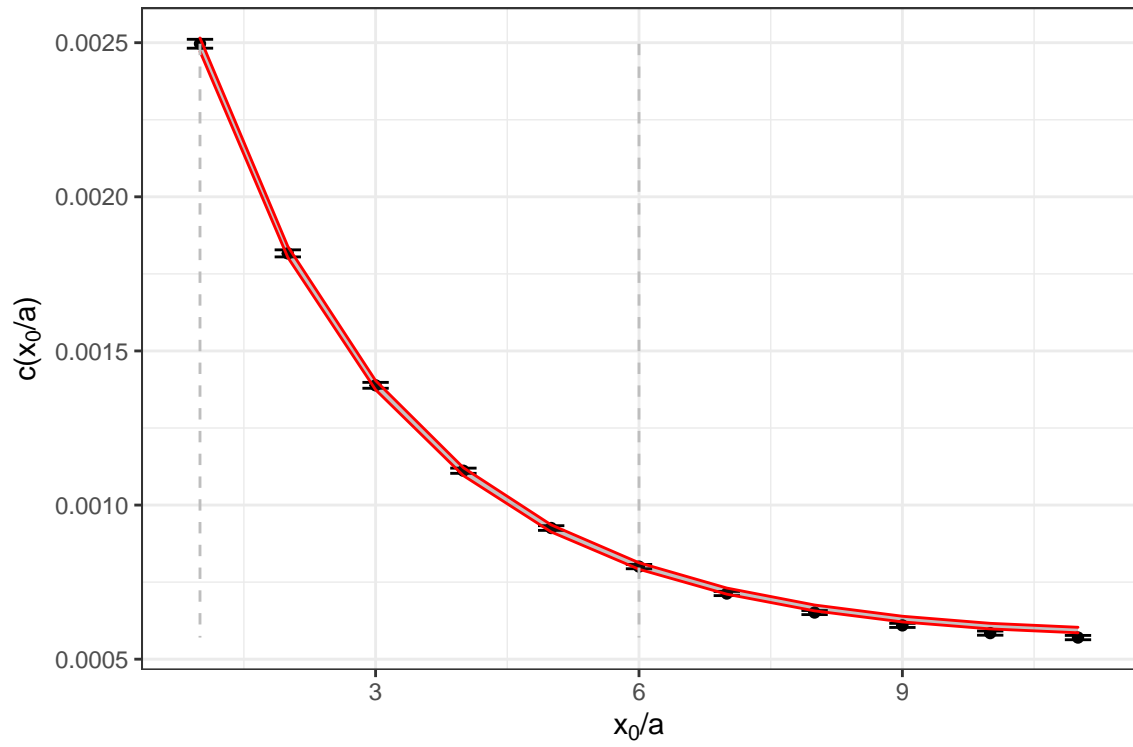
index  $n=1$

fit:  $m_{eff} = 1.308939 \pm 0.071829$



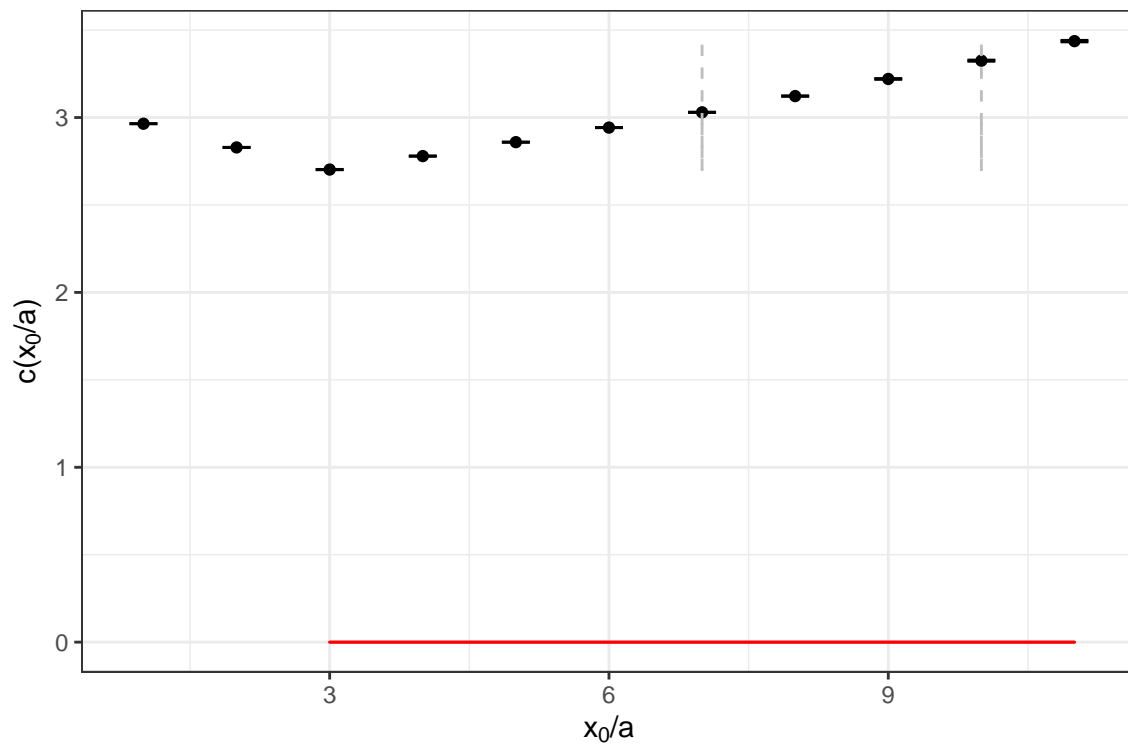
index  $n= 2$

fit:  $m_{eff} = 0.468042 \pm 0.002156$



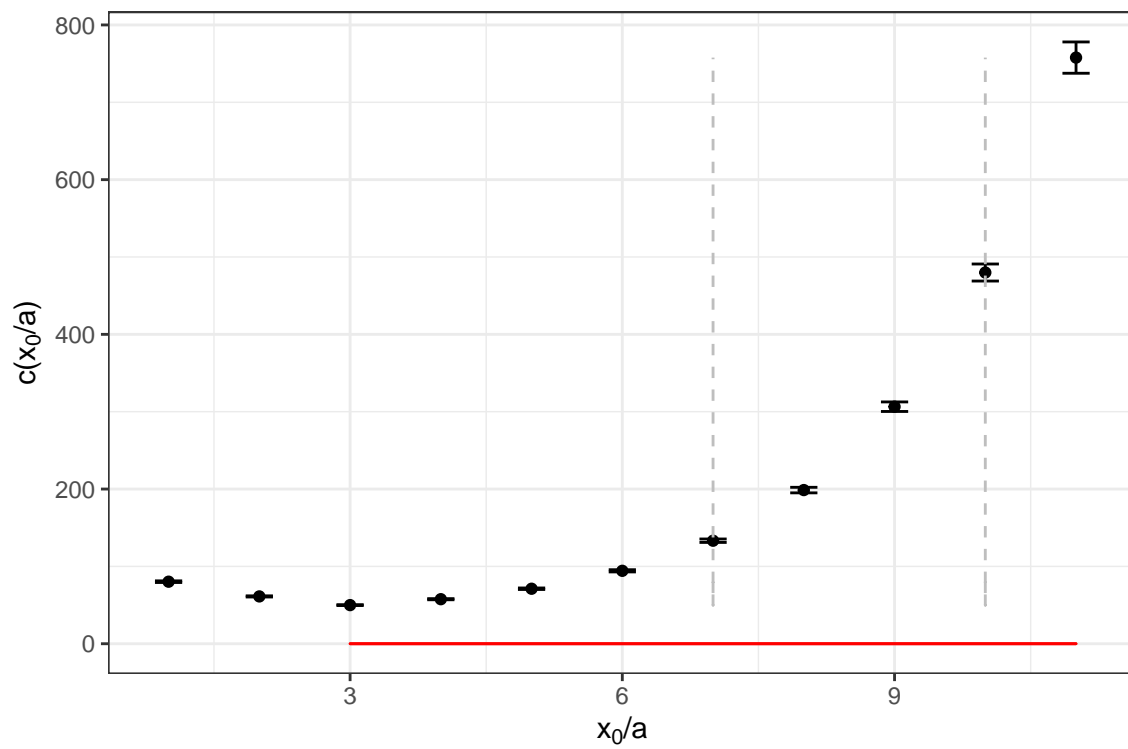
**C4\_BH** index  $n= 0$

fit:  $m_{eff} = -0.000000 \pm 0.000000$



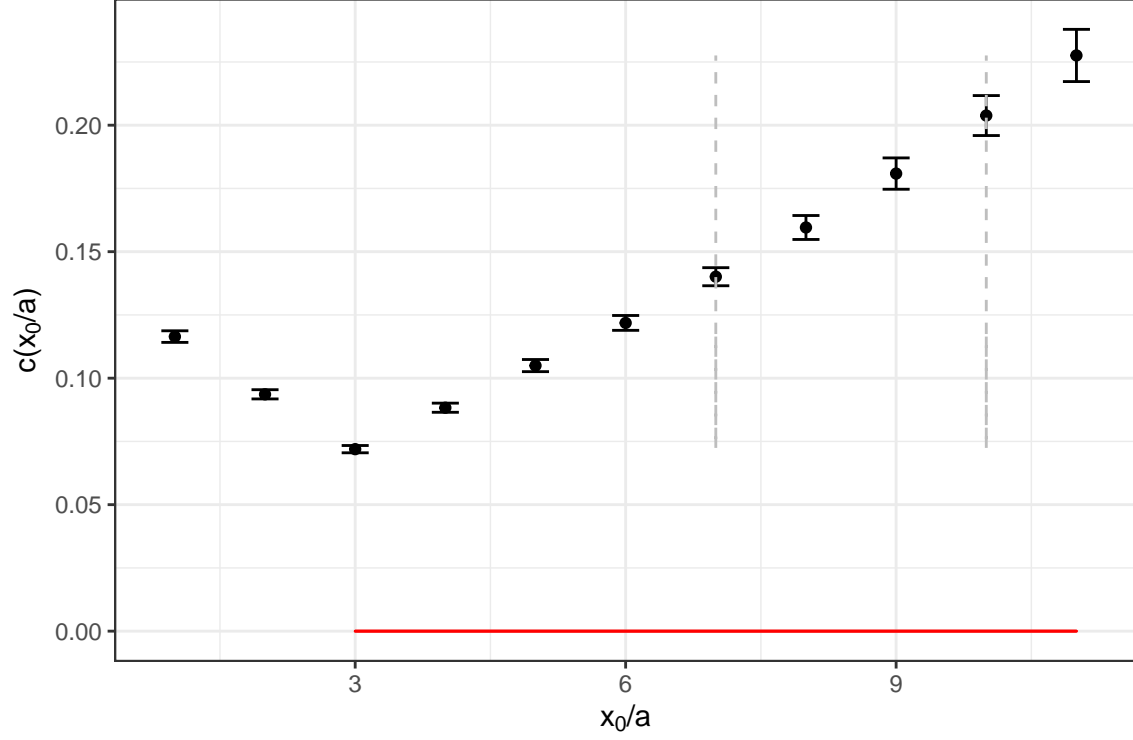
index  $n=1$

fit:  $m_{eff} = 0.000096 \pm 0.000003$



index  $n=2$

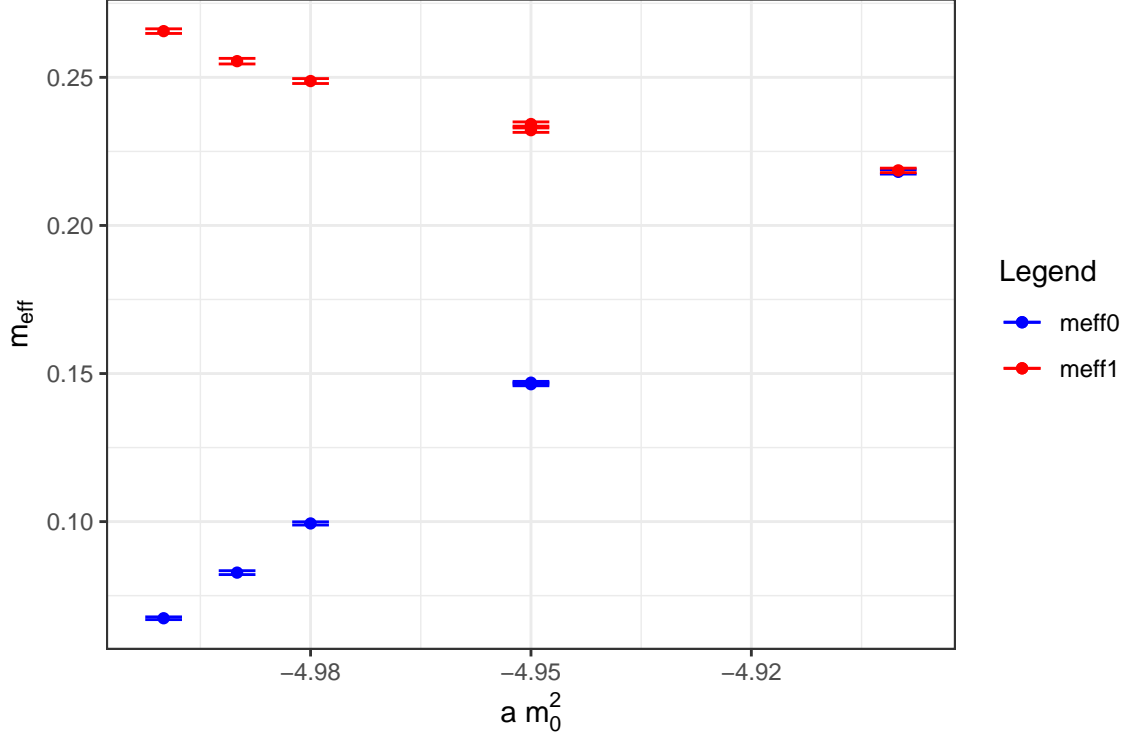
fit:  $m_{eff} = -0.000129 \pm 0.000001$



L	T	msq0	msq1	a_0	a_0err	a_1	a_1err	a_01	a_01err
10	24	-4.90	-4.9	-0.0000346	0.0000001	3.7908102	16.9993001	0.0000143	8.0e-07
10	24	-4.95	-4.9	-0.0000002	0.0000000	3.6618659	0.0024634	-0.0000127	2.0e-06
10	24	-4.95	-4.9	0.0000011	0.0000000	0.0000399	0.0000007	0.0000721	2.9e-06
10	24	-4.98	-4.9	-0.0000001	0.0000000	0.0000029	0.0000005	-0.0000008	5.0e-07
10	24	-4.99	-4.9	6.1607313	21.6072963	-0.0002143	0.0000007	-0.0000013	1.0e-07
10	24	-5.00	-4.9	-0.0000001	0.0000000	0.0000962	0.0000027	-0.0001294	9.0e-07

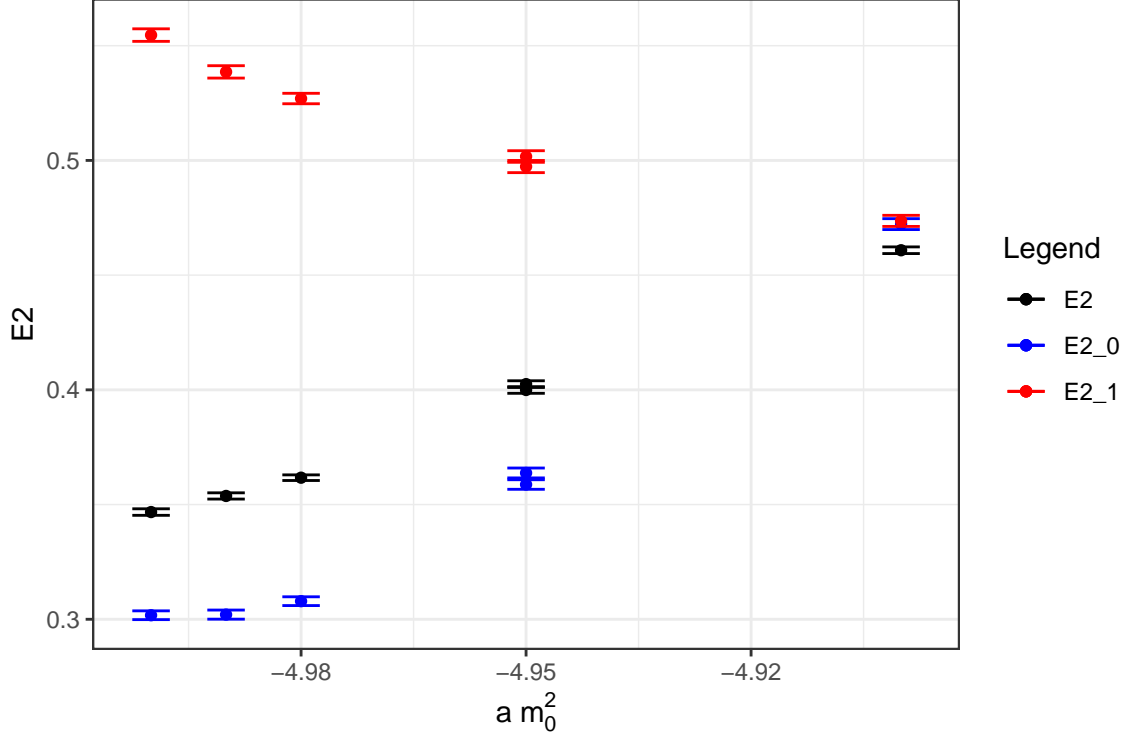
## Masses

L	T	msq0	msq1	meff0	Emeff0	meff1	Emeff1
10	24	-4.90	-4.9	0.2180975	0.0007541	0.2185971	0.0007512
10	24	-4.95	-4.9	0.1469108	0.0004401	0.2342464	0.0007148
10	24	-4.95	-4.9	0.1463931	0.0005217	0.2321742	0.0007494
10	24	-4.98	-4.9	0.0993824	0.0005387	0.2487601	0.0008334
10	24	-4.99	-4.9	0.0827720	0.0006570	0.2554706	0.0009441
10	24	-5.00	-4.9	0.0673686	0.0004658	0.2656081	0.0007859



## Two particle energy

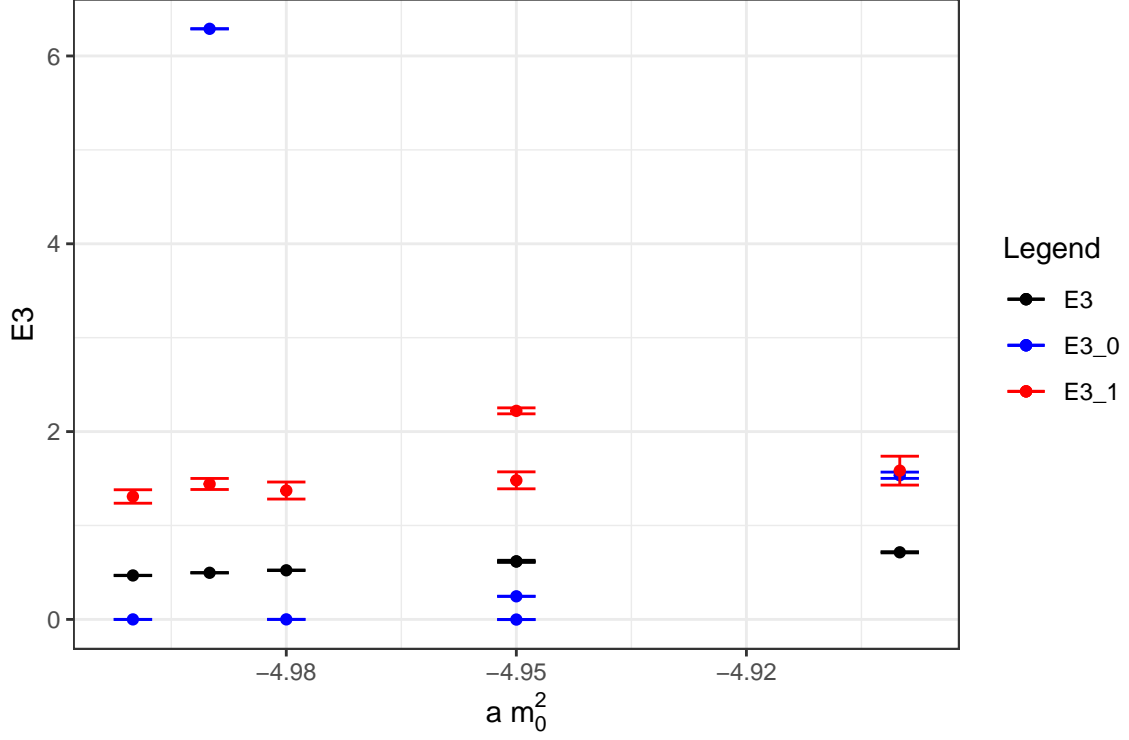
L	T	msq0	msq1	E2_0	E2_0err	E2_1	E2_1err	E2	E2err
10	24	-4.90	-4.9	0.4722318	0.0023788	0.4736475	0.0024145	0.4608479	0.0014562
10	24	-4.95	-4.9	0.3637708	0.0021544	0.5017052	0.0025205	0.4024927	0.0014620
10	24	-4.95	-4.9	0.3587376	0.0020845	0.4973199	0.0026512	0.3999422	0.0014481
10	24	-4.98	-4.9	0.3078913	0.0019105	0.5269728	0.0022776	0.3617127	0.0012214
10	24	-4.99	-4.9	0.3020339	0.0019841	0.5385825	0.0026980	0.3537561	0.0013596
10	24	-5.00	-4.9	0.3017607	0.0019144	0.5546231	0.0027325	0.3467344	0.0014224



### Three particle energy

L	T	msq0	msq1	E3_0	E3_0err	E3_1	E3_1err	E3	E3err
10	24	-4.90	-4.9	1.5351079	0.0334568	1.584059	0.1537696	0.7146585	0.0062890
10	24	-4.95	-4.9	-0.0015164	0.0008565	1.480982	0.0902729	0.6160061	0.0040340
10	24	-4.95	-4.9	0.2460881	0.0001568	2.220848	0.0317445	0.6187622	0.0108299
10	24	-4.98	-4.9	0.0003203	0.0004047	1.372215	0.0906470	0.5230289	0.0031609
10	24	-4.99	-4.9	6.2890679	0.0000000	1.442154	0.0587608	0.4966239	0.0025306
10	24	-5.00	-4.9	0.0001453	0.0002183	1.308939	0.0718295	0.4680424	0.0021564

Scale for 'colour' is already present. Adding another scale for 'colour', which will replace the existing scale.



to be compared with the result of the paper <https://arxiv.org/abs/1806.02367>

V1	V2	V3	V4	V5	V6	V7	V8	V9
L	T	nconf	ML	E2(L)	E3(L)	E2	E3	E3/ E2
4	24	18000	0.3634(16)	—	—	—	—	—
5	24	28000	0.3049(13)	0.6790(20)	1.1121(93)	0.0692(24)	0.1973(97)	2.85(12)
6	24	7500	0.2684(24)	0.5920(36)	0.962(16)	0.0552(46)	0.156(17)	2.83(26)
7	24	30000	0.2479(12)	0.5378(17)	0.8669(74)	0.0420(23)	0.1233(79)	2.93(17)
8	24	47000	0.2355(10)	0.5035(13)	0.8006(57)	0.0325(18)	0.0941(62)	2.90(17)
9	24	40000	0.2247(11)	0.4756(14)	0.7574(62)	0.0261(20)	0.0832(67)	3.19(24)
10	24	70000	0.21843(85)	0.4565(11)	0.7103(46)	0.0196(15)	0.0550(50)	2.80(23)
11	24	30000	0.2142(13)	0.4464(17)	0.6859(71)	0.0181(23)	0.0434(77)	2.40(37)
12	24	12000	0.2095(21)	0.4367(26)	0.672(11)	0.0177(37)	0.043(12)	2.43(60)
13	24	20000	0.2088(16)	0.4271(21)	0.6546(91)	0.0095(28)	0.0282(98)	2.97(97)
14	24	28000	0.2054(22)	0.4236(28)	0.650(13)	0.0127(38)	0.034(14)	2.64(96)
15	24	40000	0.2057(12)	0.4199(15)	0.6362(66)	0.0086(20)	0.0192(70)	2.23(72)
16	24	52000	0.2045(14)	0.4179(18)	0.6347(83)	0.0089(25)	0.0211(88)	2.37(88)
17	24	70000	0.20540(87)	0.4181(11)	0.6388(50)	0.0073(15)	0.0226(54)	3.11(71)
18	24	36000	0.2051(12)	0.4134(16)	0.6371(71)	0.0032(21)	0.0218(76)	6.8(4.0)
20	24	70000	0.20477(87)	0.4114(11)	0.6241(52)	0.0018(15)	0.0098(55)	5.4(4.1)
14	48	36000	0.20724(33)	0.42461(63)	0.6530(23)	0.01014(62)	0.0313(24)	3.09(20)
24	48	100000	0.20426(55)	0.4118(11)	0.6194(58)	0.0032(10)	0.0066(59)	2.0(1.7)