Some underlying foundations for RL

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Google Brain

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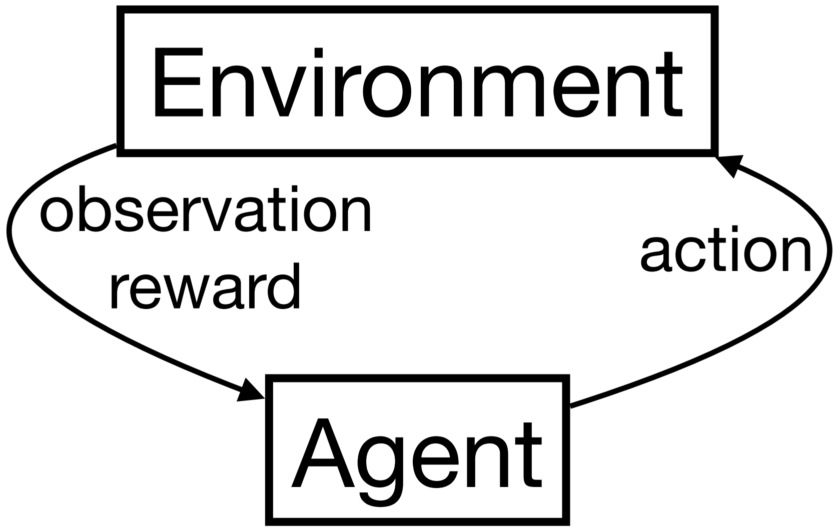
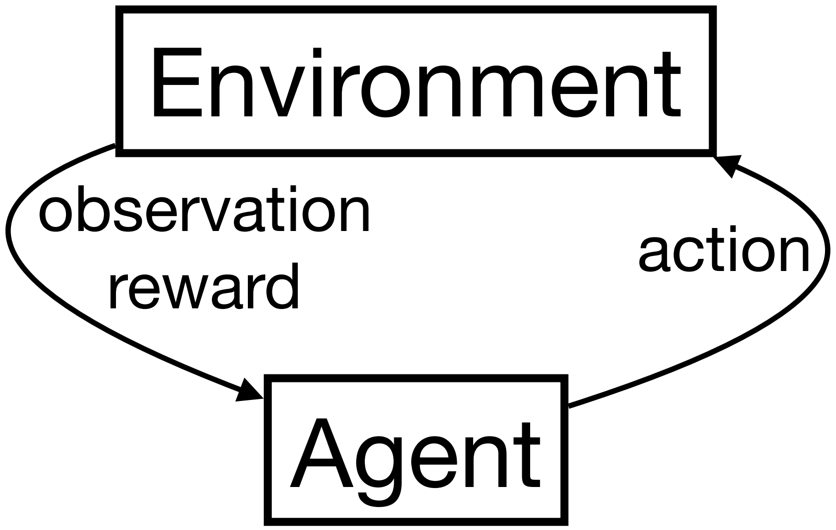
amii

The RL problem



The RL problem



1. multi-agent interaction ➡ non-stationarity
2. partial observability ➡ construct memory
3. exploration ➡ explore/exploit
4. sequential decisions ➡ temporal credit assignment
5. exploitation ➡ policy optimization

Optimizing one step decision making

Batch policy optimization

Batch policy optimization

Given data

	a_1	a_2	$\dots$	a_n
x_1		r_1		
x_2				r_2
x_3	r_3			
x_4			r_4	
x_5	r_5			
x_6		r_6		
$\vdots$			$r_{\vdots}$	
x_m				r_m

Optimize policy $\pi : X \to \Delta^n$ to maximize expected reward on **test** contexts

$$\pi(a \,|\, x) = e^{q(x)_a - F(q(x))}$$

$$F(q(x)) = \log \sum_a e^{q(x)_a}$$

$q : X \to \Re^n$ neural network

Batch policy optimization

Given data

$$
\begin{array}{c|cccc|}
 & a_1 & a_2 & \dots & a_n \\
\hline
x_1 & & r_1 & & \\
x_2 & & & & r_2 \\
x_3 & r_3 & & & \\
x_4 & & & r_4 & \\
x_5 & r_5 & & & \\
x_6 & & r_6 & & \\
\vdots & & & r_{:} & \\
x_m & & & & r_m \\
\hline
\end{array}
$$

Three key issues

1. generalization
2. optimization
3. missing data

Optimize policy $\pi : X \to \Delta^n$ to maximize expected reward on **test** contexts

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Batch policy optimization

Given data

$$
\begin{array}{c|cccc|}
 & a_1 & a_2 & \dots & a_n \\
\hline
x_1 & & r_1, \beta_1 & & \\
x_2 & & & & r_2, \beta_2 \\
x_3 & r_3, \beta_3 & & & \\
x_4 & & & r_4, \beta_4 & \\
x_5 & r_5, \beta_5 & & & \\
x_6 & & r_6, \beta_6 & & \\
\vdots & & & r_:, \beta_: & \\
x_m & & & & r_m, \beta_m \\
\hline
\end{array}
$$

Isn't this a solved problem?

We know how to do this, right?

Default answer

- maximize importance corrected expected reward
- (assume have proposal probabilities)

$$\max \sum_i \frac{\pi(a_i \mid x_i)}{\beta_i} r_i$$

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	a_1	a_2	$\dots$	a_n
x_1		r_1		
x_2				r_2
x_3	r_3			
x_4			r_4	
x_5	r_5			
x_6		r_6		
$\vdots$			$r_{\vdots}$	
x_m				r_m

Three key issues	Importance corrected expected reward
1. generalization	okay
2. optimization	👎 ✘
3. missing data	👎 ✘

Optimize policy $\pi : X \to \Delta^n$ to maximize expected reward on **test** contexts

$$\pi(a\,|\,x) = e^{q(x)_a - F(q(x))}$$

$$F(q(x)) = \log \sum_a e^{q(x)_a}$$

$q : X \to \Re^n$ neural network

Optimization objectives

Given data

$$
\begin{array}{c|cccc|}
 & a_1 & a_2 & \dots & a_n \\
\hline
x_1 & & r_1 & & \\
x_2 & & & & r_2 \\
x_3 & r_3 & & & \\
x_4 & & & r_4 & \\
x_5 & r_5 & & & \\
x_6 & & r_6 & & \\
\vdots & & & r_{\vdots} & \\
x_m & & & & r_m \\
\hline
\end{array}
$$

Optimize policy $\pi : X \to \Delta^n$ to maximize expected reward on **test** contexts

$$\pi(a \,|\, x) = e^{q(x)_a - F(q(x))}$$

$$F(q(x)) = \log \sum_a e^{q(x)_a}$$

$q : X \to \mathfrak{R}^n$ neural network

Optimization objectives

Now assume given **complete** data

$$
\begin{array}{c|cccc|}
 & a_1 & a_2 & \dots & a_n \\
\hline
x_1 & r_{11} & r_{12} & r_{1\dots} & r_{1n} \\
x_2 & r_{21} & r_{22} & r_{2\dots} & r_{2n} \\
x_3 & r_{31} & r_{32} & r_{3\dots} & r_{3n} \\
x_4 & r_{41} & r_{42} & r_{4\dots} & r_{4n} \\
x_5 & r_{51} & r_{52} & r_{5\dots} & r_{5n} \\
x_6 & r_{61} & r_{62} & r_{6\dots} & r_{6n} \\
\vdots & r_{:1} & r_{:2} & r_{:\dots} & r_{:n} \\
x_m & r_{m1} & r_{m2} & r_{m\dots} & r_{mn} \\
\hline
\end{array}
$$

Optimize policy $\pi : X \to \Delta^n$ to maximize expected reward on **test** contexts

$$\pi(a \mid x) = e^{q(x)_a - F(q(x))}$$

$$F(q(x)) = \log \sum_a e^{q(x)_a}$$

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Optimization objectives

Now assume given **complete** data

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x_3	r_{31}	r_{32}	$r_{3\ldots}$	r_{3n}
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$\vdots$	$r_{:1}$	$r_{:2}$	$r_{:\ldots}$	$r_{:n}$
x_m	r_{m1}	r_{m2}	$r_{m\ldots}$	r_{mn}

Target objective

- expected reward: $\max \sum_i \mathbf{r}_i \cdot \boldsymbol{\pi}(x_i)$

Done, right?
Not so fast …

This objective has serious problems

- actually trying to solve: $\max \sum_i \mathbf{r}_i \cdot \mathbf{f}(q(x_i))$
- plateaus everywhere
- can have **exponentially many** local maxima
- nearly impossible to reach a global optima

Also: you already know not to train this way!

Optimize policy $\pi : X \to \Delta^n$ to maximize expected reward on **test** contexts

$$\pi(a\,|\,x) = e^{q(x)_a - F(q(x))}$$

$$F(q(x)) = \log \sum_a e^{q(x)_a}$$

$q : X \to \Re^n$ neural network

Optimization objectives

Special case: **supervised classification**

$$
\begin{array}{c|cccc|}
 & a_1 & a_2 & \ldots & a_n \\
\hline
x_1 & 0 & 1 & 0 & 0 \\
x_2 & 0 & 0 & 0 & 1 \\
x_3 & 1 & 0 & 0 & 0 \\
x_4 & 0 & 0 & 1 & 0 \\
x_5 & 1 & 0 & 0 & 0 \\
x_6 & 0 & 1 & 0 & 0 \\
\vdots & 0 & 0 & 1 & 0 \\
x_m & 0 & 0 & 0 & 1 \\
\hline
\end{array}
$$

Optimize policy $\pi : X \to \Delta^n$ to maximize expected **accuracy** on **test** contexts

$$\pi(a \,|\, x) = e^{q(x)_a - F(q(x))}$$

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Optimization objectives

Special case: **supervised classification**

$$
\begin{array}{c|cccc|}
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x_1 & 0 & 1 & 0 & 0 \\
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\vdots & 0 & 0 & 1 & 0 \\
x_m & 0 & 0 & 0 & 1 \\
\hline
\end{array}
$$

Target objective

- expected **accuracy**: $\max \sum_i \mathbf{r}_i \cdot \boldsymbol{\pi}(x_i)$

But you have never trained with this objective

Instead, you used a **surrogate objective**

maximum likelihood

$$\max \sum_i \mathbf{r}_i \cdot \log \boldsymbol{\pi}(x_i)$$

What's going on?

- $\mathbf{r}_i \cdot \boldsymbol{\pi}(x_i)$ is differentiable, that's not the issue
- training with $\mathbf{r}_i \cdot \log \boldsymbol{\pi}(x_i)$ actually achieves better values of $\mathbf{r}_i \cdot \boldsymbol{\pi}(x_i)$ on the training data

Optimize policy $\pi : X \to \Delta^n$ to maximize expected **accuracy** on **test** contexts

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Optimization objectives

Special case: **supervised classification**

$$\begin{array}{c|cccc|} & a_1 & a_2 & \dots & a_n \\ \hline x_1 & 0 & 1 & 0 & 0 \\ x_2 & 0 & 0 & 0 & 1 \\ x_3 & 1 & 0 & 0 & 0 \\ x_4 & 0 & 0 & 1 & 0 \\ x_5 & 1 & 0 & 0 & 0 \\ x_6 & 0 & 1 & 0 & 0 \\ \vdots & 0 & 0 & 1 & 0 \\ x_m & 0 & 0 & 0 & 1 \\ \hline \end{array}$$

Why?

- expected accuracy: $\max \sum_i \mathbf{r}_i \cdot \boldsymbol{\pi}(x_i)$
- maximum likelihood: $\max \sum_i \mathbf{r}_i \cdot \log \boldsymbol{\pi}(x_i)$

Useful properties of maximum likelihood

- $\mathbf{r}_i \cdot \log \boldsymbol{\pi}(x_i)$ is concave in $\mathbf{q}(x_i)$
- it is also calibrated w.r.t. $\mathbf{r}_i \cdot \boldsymbol{\pi}(x_i)$:

$$\forall \epsilon > 0 \exists \delta > 0 \;\; \mathbf{r} \cdot \log \boldsymbol{\pi}^* - \mathbf{r} \cdot \log \boldsymbol{\pi} < \delta \Rightarrow \mathbf{r} \cdot \boldsymbol{\pi}^* - \mathbf{r} \cdot \boldsymbol{\pi} < \epsilon$$

Optimize policy $\pi : X \to \Delta^n$ to maximize expected **accuracy** on **test** contexts

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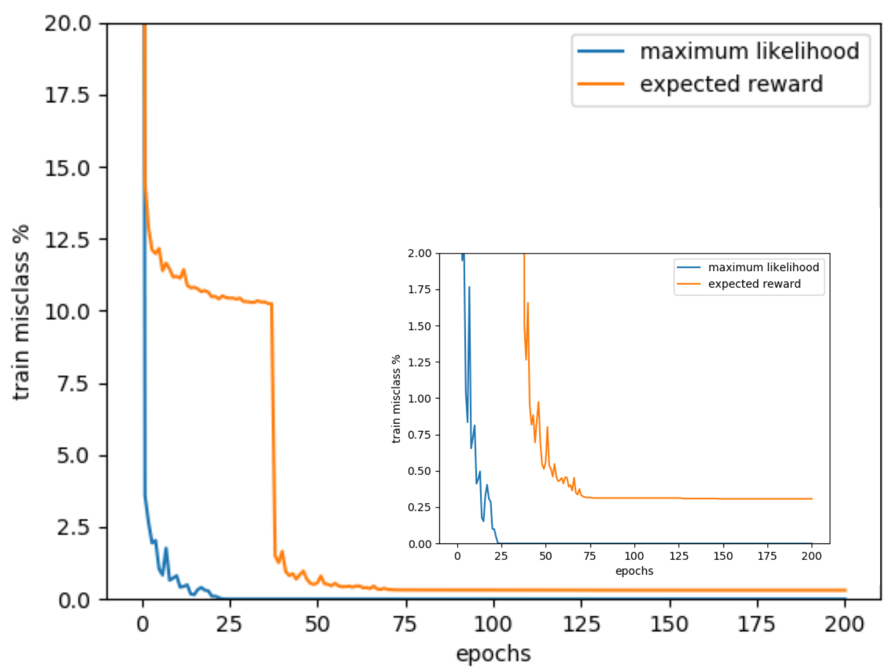
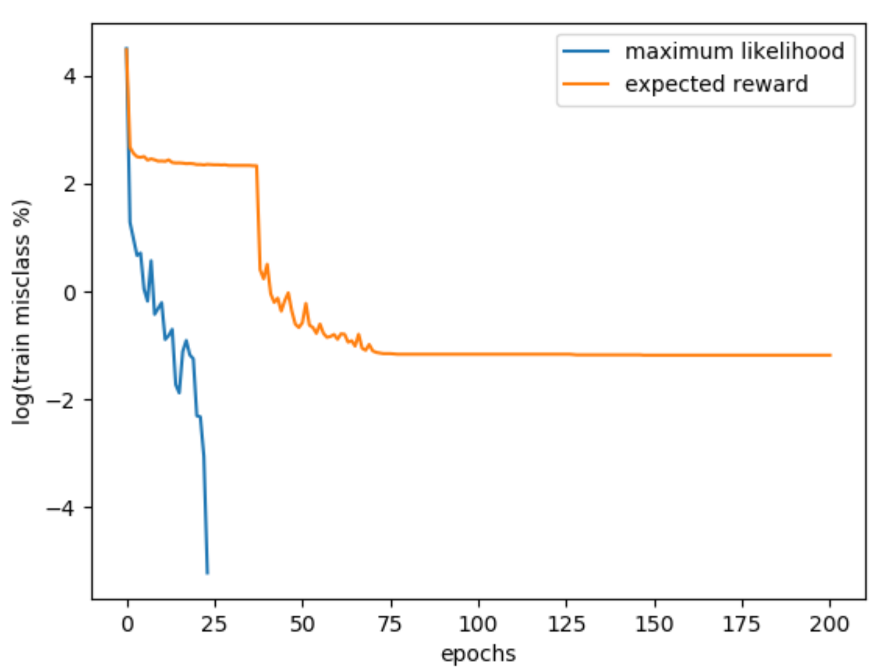
Optimization objectives

Misclassification error on MNIST training data





Optimization objectives

Back to **general** rewards

$$
\begin{array}{c|cccc|}
 & a_1 & a_2 & \dots & a_n \\
\hline
x_1 & r_{11} & r_{12} & r_{1\dots} & r_{1n} \\
x_2 & r_{21} & r_{22} & r_{2\dots} & r_{2n} \\
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\vdots & r_{:1} & r_{:2} & r_{:\dots} & r_{:n} \\
x_m & r_{m1} & r_{m2} & r_{m\dots} & r_{mn} \\
\hline
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x_m & r_{m1} & r_{m2} & r_{m\dots} & r_{mn} \\
\hline
\end{array}
$$

Target objective

- expected reward: $\max \sum_i \mathbf{r}_i \cdot \boldsymbol{\pi}(x_i)$

"cost sensitive classification"

Calibrated surrogates exist for $\mathbf{r}_i \cdot \boldsymbol{\pi}(x_i)$ (Pires et al. ICML-2013)

Interesting alternative

- entropy regularized expected reward

$$\max \sum_i \mathbf{r}_i \cdot \boldsymbol{\pi}(x_i) - \tau \boldsymbol{\pi}(x_i) \cdot \log \boldsymbol{\pi}(x_i)$$

Optimize policy $\pi : X \to \Delta^n$ to maximize expected reward on **test** contexts

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Optimization objectives

Back to **general** rewards

	a_1	a_2	$\dots$	a_n
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$\vdots$	$r_{:1}$	$r_{:2}$	$r_{:\dots}$	$r_{:n}$
x_m	r_{m1}	r_{m2}	$r_{m\dots}$	r_{mn}

Optimize policy $\pi : X \to \Delta^n$

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Entropy regularized expected reward

$$\arg\max \mathbf{r}\cdot\boldsymbol{\pi} - \tau\boldsymbol{\pi}\cdot\log\boldsymbol{\pi}$$
$$= \arg\min \tau F(\mathbf{r}/\tau) - \mathbf{r}\cdot\boldsymbol{\pi} + \tau F^*(\boldsymbol{\pi})$$
$$= \arg\min F(\mathbf{r}/\tau) - \mathbf{r}\cdot\boldsymbol{\pi}/\tau + F^*(\boldsymbol{\pi})$$
$$= \arg\min \mathbf{KL}(\boldsymbol{\pi}\|\mathbf{p}) \text{ where } \mathbf{p} = e^{\mathbf{r}/\tau - F(\mathbf{r}/\tau)}$$

Suggests a natural surrogate

$$\arg\min \mathbf{KL}(\mathbf{p}\|\boldsymbol{\pi}) = \arg\min F(\mathbf{q}) - \mathbf{q}\cdot\mathbf{p}$$

- convex in $\mathbf{q}$

Let $F^*(\pi) = \pi\cdot\log\pi$

Optimization objectives

Back to **general** rewards

$$\begin{array}{c|cccc|}
 & a_1 & a_2 & \dots & a_n \\
\hline
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\hline
\end{array}$$

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$q : X \to \mathfrak{R}^n$ neural network

Comparison to maximum likelihood

before $-\mathbf{r} \cdot \log \boldsymbol{\pi} = F(\mathbf{q}) - \mathbf{q} \cdot \mathbf{r}$

now **KL**$(\mathbf{p} \| \boldsymbol{\pi}) \equiv F(\mathbf{q}) - \mathbf{q} \cdot \mathbf{p}$

If $\mathbf{r} = \mathbf{1}_a$ is an indicator

- become equivalent as $\tau \to 0$
- $\lim_{\tau \to 0} \mathbf{p} = \mathbf{r} = \mathbf{1}_a$
- but $\tau > 0$ gives soft targets for **KL**

  "label smoothing"

  improves generalization in practice

Optimization objectives

Back to **general** rewards

	a_1	a_2	$\dots$	a_n
x_1	r_{11}	r_{12}	$r_{1\dots}$	r_{1n}
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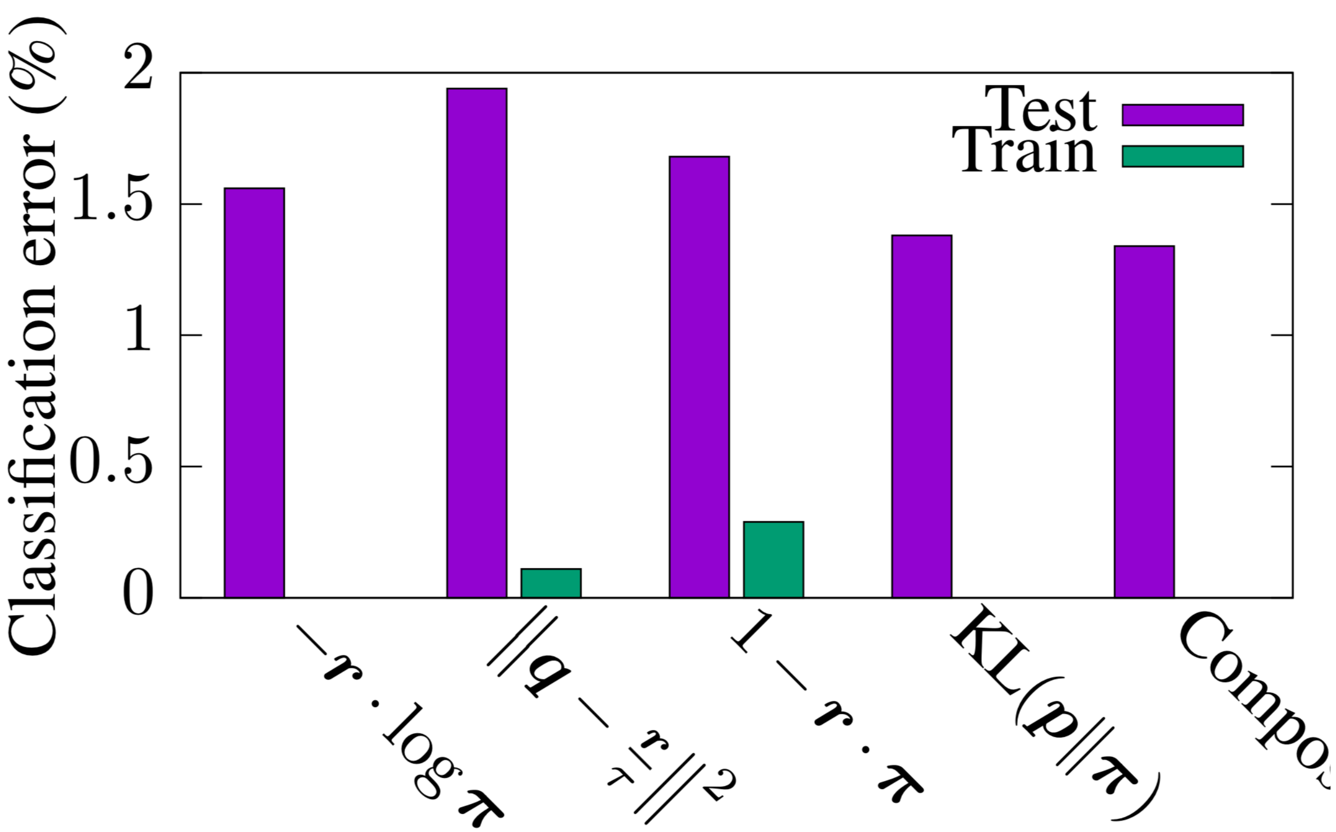
A convex, calibrated upper bound

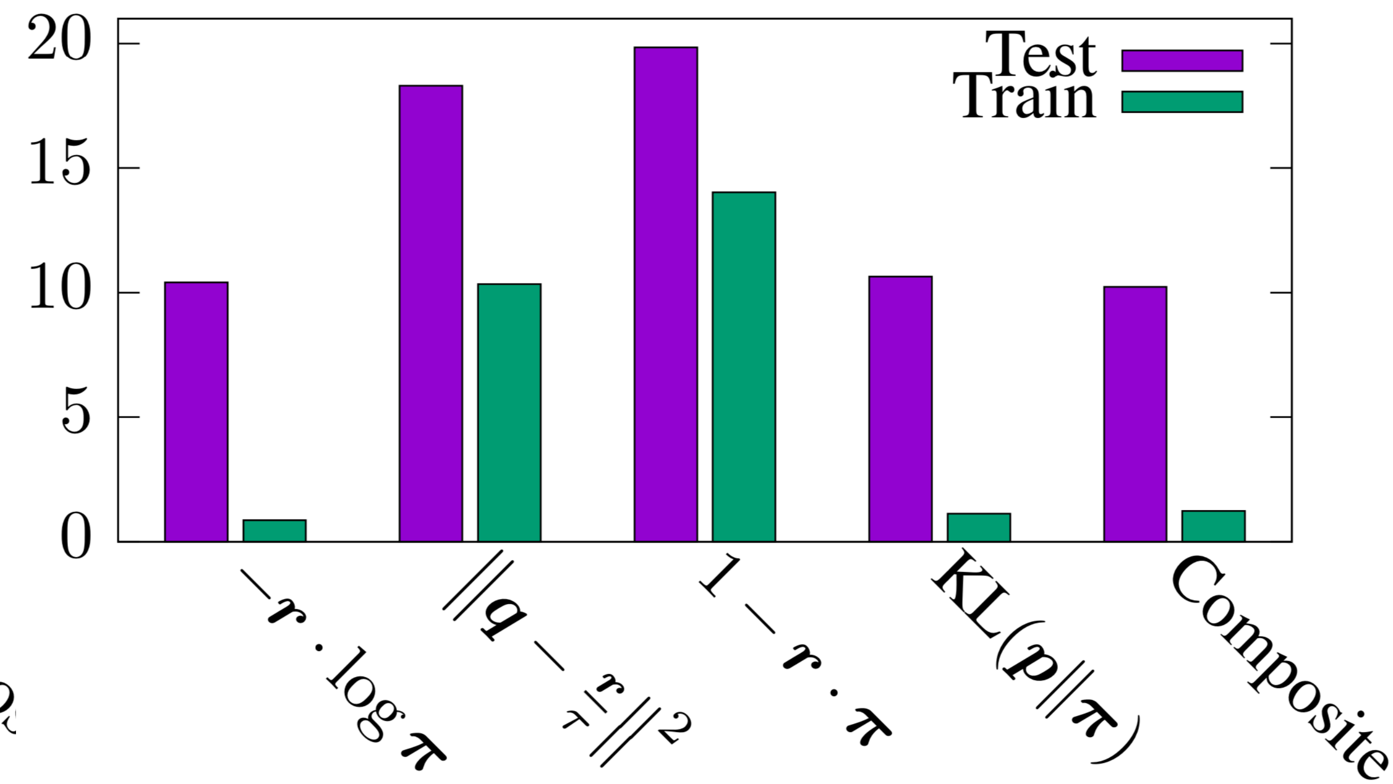
$$\mathbf{KL}(\boldsymbol{\pi} \| \mathbf{p}) \leq \mathbf{KL}(\mathbf{p} \| \boldsymbol{\pi}) + \frac{\tau}{4} \|\mathbf{r}/\tau - \mathbf{q}\|^2$$

Optimization objectives

MNIST



CIFAR10



Batch policy optimization

	a_1	a_2	$\ldots$	a_n
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$\vdots$	$r_{:1}$	$r_{:2}$	$r_{:\ldots}$	$r_{:n}$
x_m	r_{m1}	r_{m2}	$r_{m\ldots}$	r_{mn}

Three key issues

1. generalization
2. optimization
3. missing data

training objective
$\neq$
target objective

Batch policy optimization

$$
\begin{array}{c|cccc|}
 & a_1 & a_2 & \dots & a_n \\
\hline
x_1 & & r_1 & & \\
x_2 & & & & r_2 \\
x_3 & r_3 & & & \\
x_4 & & & r_4 & \\
x_5 & r_5 & & & \\
x_6 & & r_6 & & \\
\vdots & & & r_{\vdots} & \\
x_m & & & & r_m \\
\hline
\end{array}
$$

Three key issues

1. generalization
2. optimization
3. missing data

Supervised vs reinforcement learning

supervised classification

$$
\begin{array}{c|cccc|}
 & a_1 & a_2 & \dots & a_n \\
\hline
x_1 & r_{11} & r_{12} & r_{1\dots} & r_{1n} \\
x_2 & r_{21} & r_{22} & r_{2\dots} & r_{2n} \\
x_3 & r_{31} & r_{32} & r_{3\dots} & r_{3n} \\
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x_m & r_{m1} & r_{m2} & r_{m\dots} & r_{mn} \\
\hline
\end{array}
$$

batch policy optimization

$$
\begin{array}{c|cccc|}
 & a_1 & a_2 & \dots & a_n \\
\hline
x_1 & & r_1 & & \\
x_2 & & & & r_2 \\
x_3 & r_3 & & & \\
x_4 & & & r_4 & \\
x_5 & r_5 & & & \\
x_6 & & r_6 & & \\
\vdots & & & r_: & \\
x_m & & & & r_m \\
\hline
\end{array}
$$

Optimize policy $\pi : X \rightarrow \Delta^n$ to maximize expected reward on **test** contexts

key difference is missing data

Missing data inference

How to handle missing data?

$$
\begin{array}{c|cccc|}
 & a_1 & a_2 & \dots & a_n \\
\hline
x_1 & & r_1 & & \\
x_2 & & & & r_2 \\
x_3 & r_3 & & & \\
x_4 & & & r_4 & \\
x_5 & r_5 & & & \\
x_6 & & r_6 & & \\
\vdots & & & r_{\vdots} & \\
x_m & & & & r_m \\
\hline
\end{array}
$$

Optimize policy $\pi : X \to \Delta^n$

Missing data inference

$$
\begin{array}{c|cccc|}
 & a_1 & a_2 & \ldots & a_n \\
\hline
x_1 & q_{11} & r_1 & q_{1\ldots} & q_{1n} \\
x_2 & q_{21} & q_{22} & q_{2\ldots} & r_2 \\
x_3 & r_3 & q_{32} & q_{3\ldots} & q_{3n} \\
x_4 & q_{41} & q_{42} & r_4 & q_{4n} \\
x_5 & r_5 & q_{52} & q_{5\ldots} & q_{5n} \\
x_6 & q_{61} & r_6 & q_{6\ldots} & q_{6n} \\
\vdots & q_{:1} & q_{:2} & r_{:} & q_{:n} \\
x_m & q_{m1} & q_{m2} & q_{m\ldots} & r_m \\
\hline
\end{array}
$$

Simple idea

imputation

- fill in guesses for missing values
- reduce to fully observed case

Might sound naive

- but this is actually a dominant approach

Optimize policy $\pi : X \to \Delta^n$

Missing data inference

$$
\begin{array}{c|cccc|}
 & a_1 & a_2 & \dots & a_n \\
\hline
x_1 & 0 & \frac{r_1}{\beta_1} & 0 & 0 \\
x_2 & 0 & 0 & 0 & \frac{r_2}{\beta_2} \\
x_3 & \frac{r_3}{\beta_3} & 0 & 0 & 0 \\
x_4 & 0 & 0 & \frac{r_4}{\beta_4} & 0 \\
x_5 & \frac{r_5}{\beta_5} & 0 & 0 & 0 \\
x_6 & 0 & \frac{r_6}{\beta_6} & 0 & 0 \\
\vdots & 0 & 0 & \frac{r_:}{\beta_:} & 0 \\
x_m & 0 & 0 & 0 & \frac{r_m}{\beta_m} \\
\hline
\end{array}
$$

Optimize policy $\pi : X \to \Delta^n$

Example

importance corrected expected reward

$$\max \sum_i \frac{\pi(a_i \mid x_i)}{\beta_i} r_i$$

where β are proposal probabilities from behavior strategy

We already know this is a poor objective but what about missing data inference?

Equivalent to $\max \hat{\mathbf{r}} \cdot \boldsymbol{\pi}$ using

$$\hat{\mathbf{r}}_i = \mathbf{1}_{a_i} \frac{r_i}{\beta_i}$$

That is

- exaggerate observed values by $1/\beta_i$
- fill in all unobserved values with 0

Missing data inference

$$
\begin{array}{c|cccc|}
 & a_1 & a_2 & \dots & a_n \\
\hline
x_1 & 0 & \frac{r_1}{\beta_1} & 0 & 0 \\
x_2 & 0 & 0 & 0 & \frac{r_2}{\beta_2} \\
x_3 & \frac{r_3}{\beta_3} & 0 & 0 & 0 \\
x_4 & 0 & 0 & \frac{r_4}{\beta_4} & 0 \\
x_5 & \frac{r_5}{\beta_5} & 0 & 0 & 0 \\
x_6 & 0 & \frac{r_6}{\beta_6} & 0 & 0 \\
\vdots & 0 & 0 & \frac{r_:}{\beta_:} & 0 \\
x_m & 0 & 0 & 0 & \frac{r_m}{\beta_m} \\
\hline
\end{array}
$$

This is a pretty lame inference principle

- altering the data we do see
- to compensate for a bad guess about the data we don't see

But … its unbiased!

$$\mathbb{E}[\hat{\mathbf{r}} \,|\, x] = \sum_a \beta_a \mathbf{1}_a \frac{r_a}{\beta_a} = \sum_a \mathbf{1}_a r_a = \mathbf{r}$$

Optimize policy $\pi : X \to \Delta^n$

Missing data inference

$$
\begin{array}{c|cccc|}
 & a_1 & a_2 & \dots & a_n \\
\hline
x_1 & \tau q_{11} & \lambda(r_1 - \tau q_{12}) & \tau q_{1\dots} & \tau q_{1n} \\
x_2 & \tau q_{21} & \tau q_{22} & \tau q_{2\dots} & \lambda(r_2 - \tau q_{2n}) \\
x_3 & \lambda(r_3 - \tau q_{31}) & \tau q_{32} & \tau q_{3\dots} & \tau q_{3n} \\
x_4 & \tau q_{41} & \tau q_{42} & \lambda(r_4 - \tau q_{4\dots}) & \tau q_{4n} \\
x_5 & \lambda(r_5 - \tau q_{51}) & \tau q_{52} & \tau q_{5\dots} & \tau q_{5n} \\
x_6 & \tau q_{61} & \lambda(r_6 - \tau q_{62}) & \tau q_{6\dots} & \tau q_{6n} \\
\vdots & \tau q_{:1} & \tau q_{:2} & \lambda(r_{:} - \tau q_{:\dots}) & \tau q_{:n} \\
x_m & \tau q_{m1} & \tau q_{m2} & \tau q_{m\dots} & \lambda(r_m - \tau q_{mn}) \\
\hline
\end{array}
$$

Optimize policy $\pi : X \to \Delta^n$

Improvement

"douby robust estimation"

- instead of filling in with 0s
- fill in with guesses from a model $\mathbf{q}(x)$

$$\hat{\mathbf{r}} = \tau \mathbf{q} + \lambda \mathbf{1}_a (r - \tau q_a)$$

Also unbiased

- as long as $\lambda = 1/\beta_i$

but still alters observed data

Where should the model come from?

- could use a separate critic
- train via least squares, then optimize $\boldsymbol{\pi}$
- works okay, but not great

Note

- there is only one action value function for single-step decision making, $r(x, a)$
- actor-critic approaches trivialized

Missing data inference

	a_1	a_2	$\ldots$	a_n
x_1	τq_{11}	$\lambda(r_1 - \tau q_{12})$	$\tau q_{1\ldots}$	τq_{1n}
x_2	τq_{21}	τq_{22}	$\tau q_{2\ldots}$	$\lambda(r_2 - \tau q_{2n})$
x_3	$\lambda(r_3 - \tau q_{31})$	τq_{32}	$\tau q_{3\ldots}$	τq_{3n}
x_4	τq_{41}	τq_{42}	$\lambda(r_4 - \tau q_{4\ldots})$	τq_{4n}
x_5	$\lambda(r_5 - \tau q_{51})$	τq_{52}	$\tau q_{5\ldots}$	τq_{5n}
x_6	τq_{61}	$\lambda(r_6 - \tau q_{62})$	$\tau q_{6\ldots}$	τq_{6n}
$\vdots$	$\tau q_{:1}$	$\tau q_{:2}$	$\lambda(r_: - \tau q_{:\ldots})$	$\tau q_{:n}$
x_m	τq_{m1}	τq_{m2}	$\tau q_{m\ldots}$	$\lambda(r_m - \tau q_{mn})$

Optimize policy $\pi : X \to \Delta^n$

Unified approach

- actor and critic are same model
- $\boldsymbol{\pi} = e^{\mathbf{q} - F(\mathbf{q})}$ where $F(\mathbf{q}) = \log \mathbf{1} \cdot e^{\mathbf{q}}$
- use logits $\tau \mathbf{q}(x)$ to predict rewards

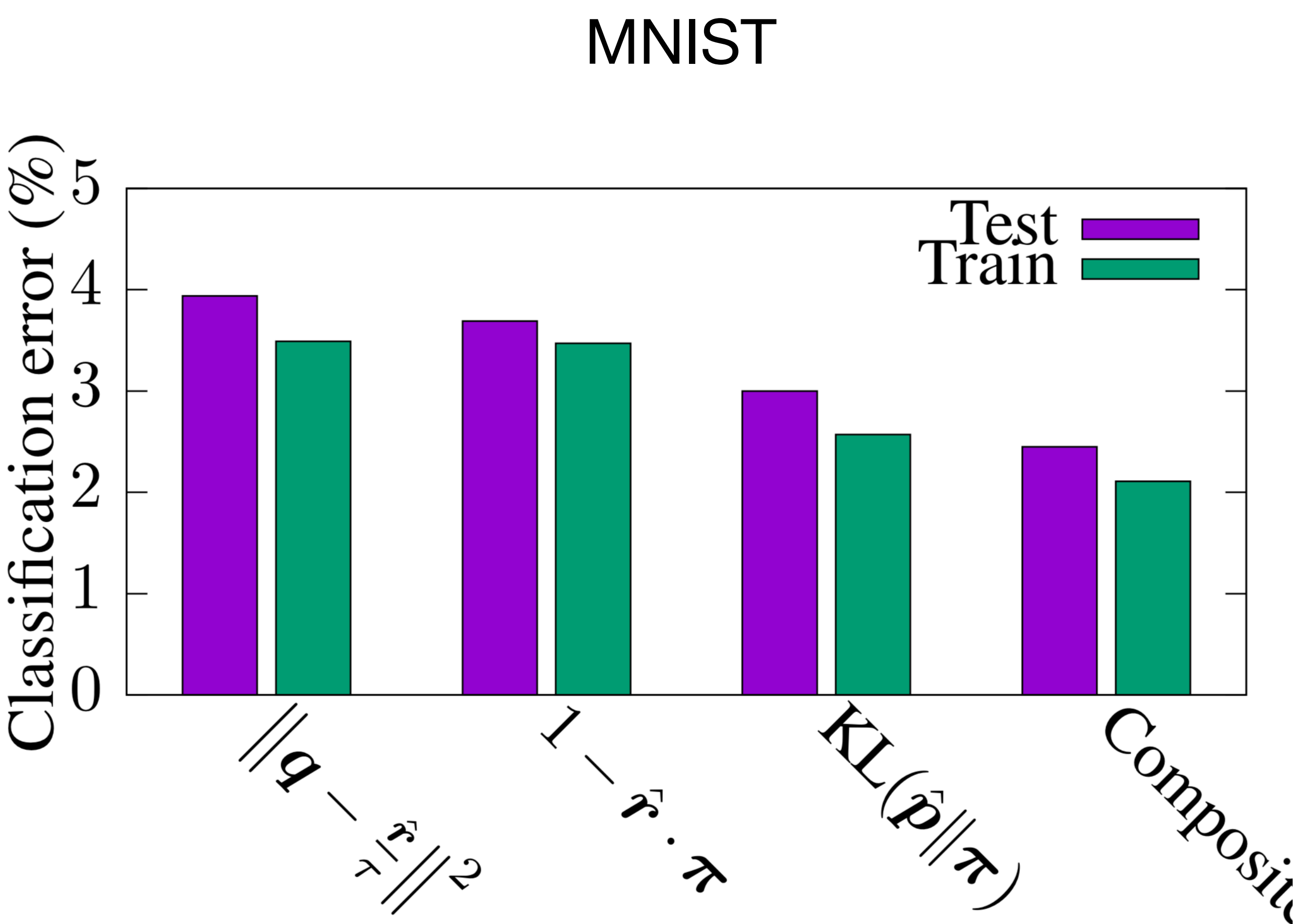
$$q(x, a) \approx \frac{r(x, a)}{\tau}$$

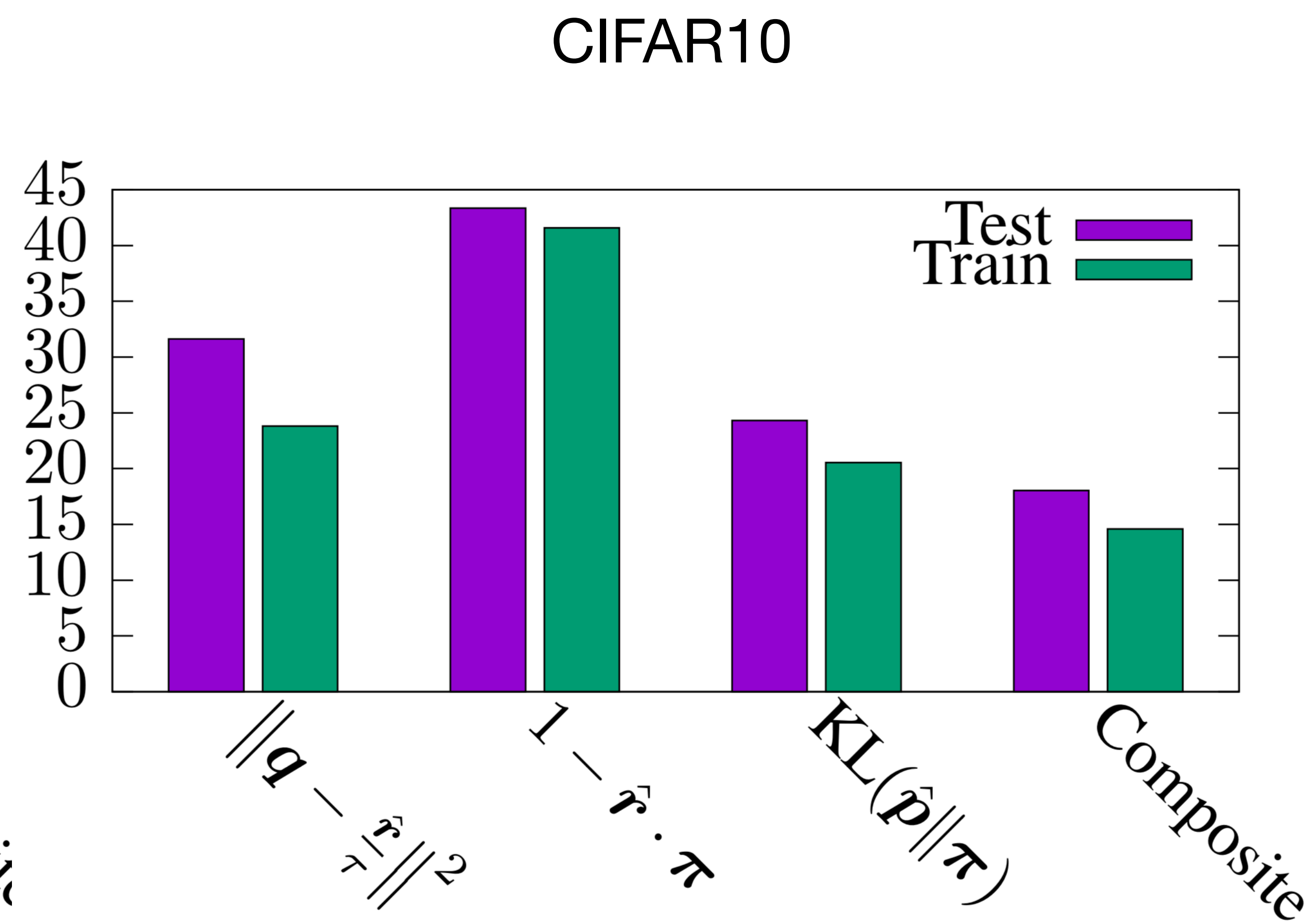
Can combine with previous objectives

- $\mathbf{KL}(\boldsymbol{\pi} \| \hat{\mathbf{p}})$ where $\hat{\mathbf{p}} = e^{\hat{\mathbf{r}}/\tau - F(\hat{\mathbf{r}}/\tau)}$
- $\mathbf{KL}(\hat{\mathbf{p}} \| \boldsymbol{\pi})$
- $\mathbf{KL}(\boldsymbol{\pi} \| \hat{\mathbf{p}}) \leq \mathbf{KL}(\hat{\mathbf{p}} \| \boldsymbol{\pi}) + \frac{\tau}{4} \| \hat{\mathbf{r}}/\tau - \mathbf{q} \|^2$

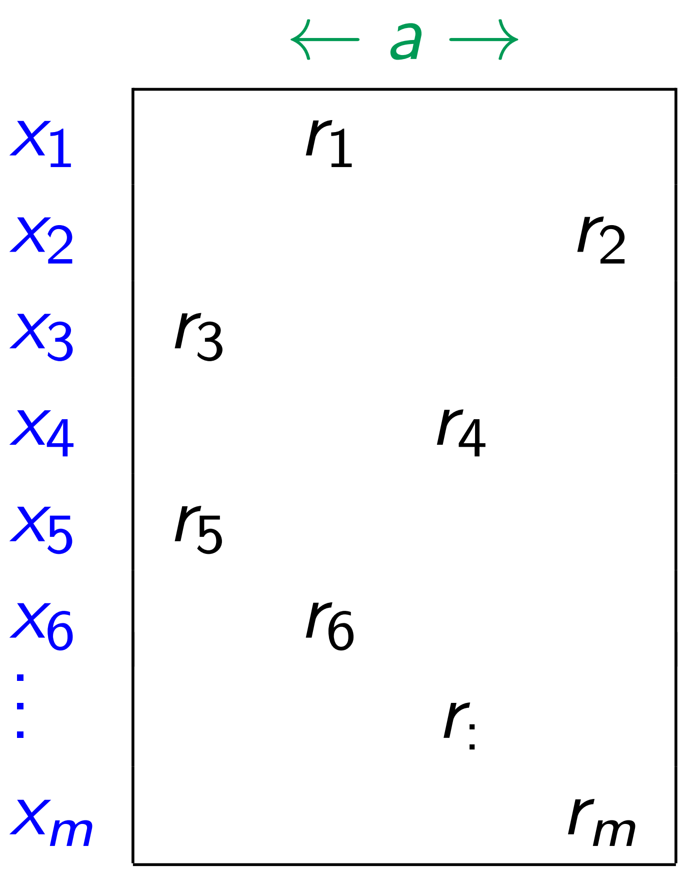
these are somewhat sensitive to ranking, unlike least squares

Missing data inference





Missing data inference



Even more principled approach

- back to first principles
- how do we reason about missing data in the rest of ML and statistics?

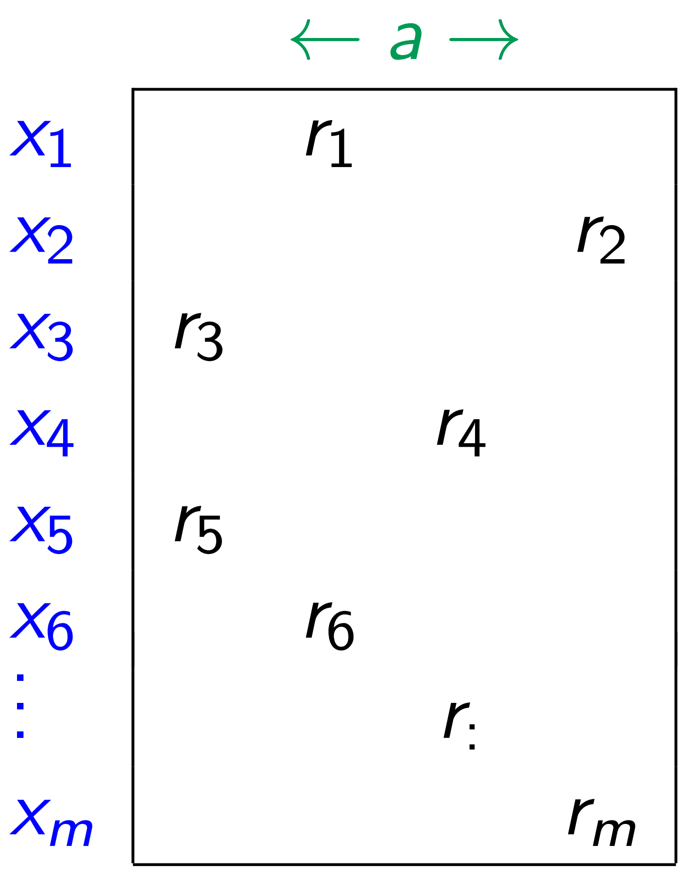
Optimize policy $\pi : X \to$ exp-family$(\mathfrak{R})$

$$\pi(a \mid x) = e^{q(x)_a - F(q(x))}$$

$$F(q(x)) = \log \int e^{q(x)_a} \mu(da)$$

$q : X \to \mathfrak{R}^k$ neural network

Missing data inference



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$q : X \to \mathfrak{R}^k$ neural network

Even more principled approach

- back to first principles
- how do we reason about missing data in the rest of ML and statistics?

Bayesian inference

- postulate a generative model of reward

$$q \to \xi \to r$$

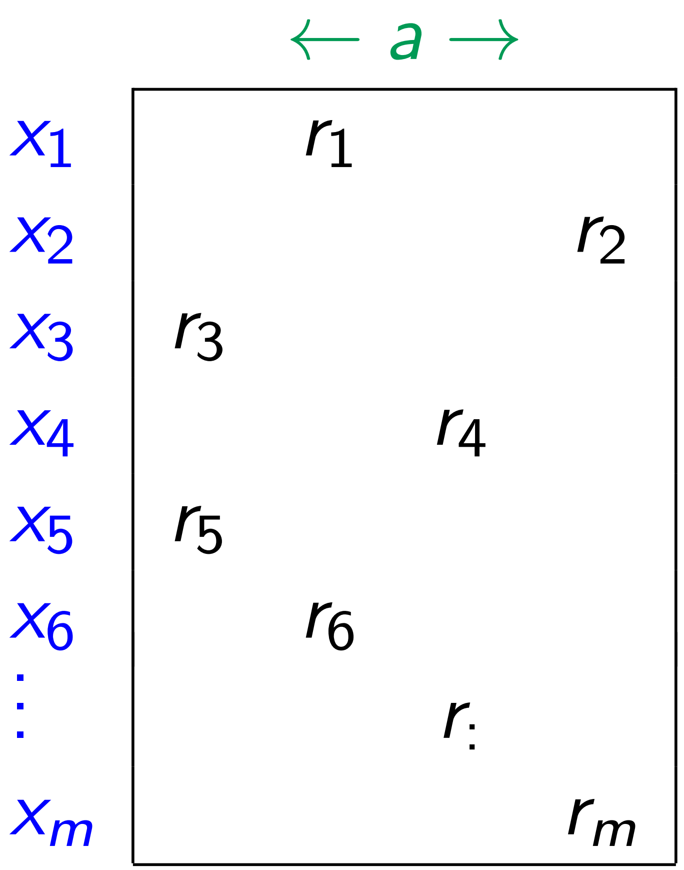
- **e.g. Gaussian**
  - prior $\xi \sim \mathcal{N}(q, Q)$
  - likelihood $r \,|\, a, \xi \sim \mathcal{N}(\phi(a) \cdot \xi, \sigma^2)$
  - posterior $\xi \,|\, r_0, a_0 \sim \mathcal{N}(\mu, C)$

$$\mu = C(\phi(a_0) r_0 \sigma^{-2} + Q^{-1} q)$$
$$C = (Q^{-1} + \sigma^{-2} \phi(a_0) \phi(a_0)^\top)^{-1}$$

  - predictive $r \,|\, a, r_0, a_0 \sim \mathcal{N}(\hat{\mu}, \hat{\sigma}^2)$

$$\hat{\mu} = \phi(a) \cdot \mu$$
$$\hat{\sigma}^2 = \sigma^2 + \phi(a_0)^\top C \phi(a_0)$$

Missing data inference



Optimize policy $\pi : X \to$ exp-family$(\mathfrak{R})$

$$\pi(a \,|\, x) = e^{q(x)_a - F(q(x))}$$

$$F(q(x)) = \log \int e^{q(x)_a} \mu(da)$$

$q : X \to \mathfrak{R}^k$ neural network

Empirical Bayes estimation

- optimize hyperparameters $\mathbf{q}$ (neural network)
- integrate out parameters ξ

Example

marginal likelihood

$$-\log p(r_0 \,|\, a_0, \mathbf{q})$$
$$= -\log \int p(r_0 \,|\, a_0, \xi) p(\xi \,|\, \mathbf{q}) \, d\xi$$
$$= \frac{1}{2\sigma^2}(\phi(a_0) \cdot q - r_0)^2 + \frac{1}{2} \log \sigma^2 + c$$

- essentially least squares regression

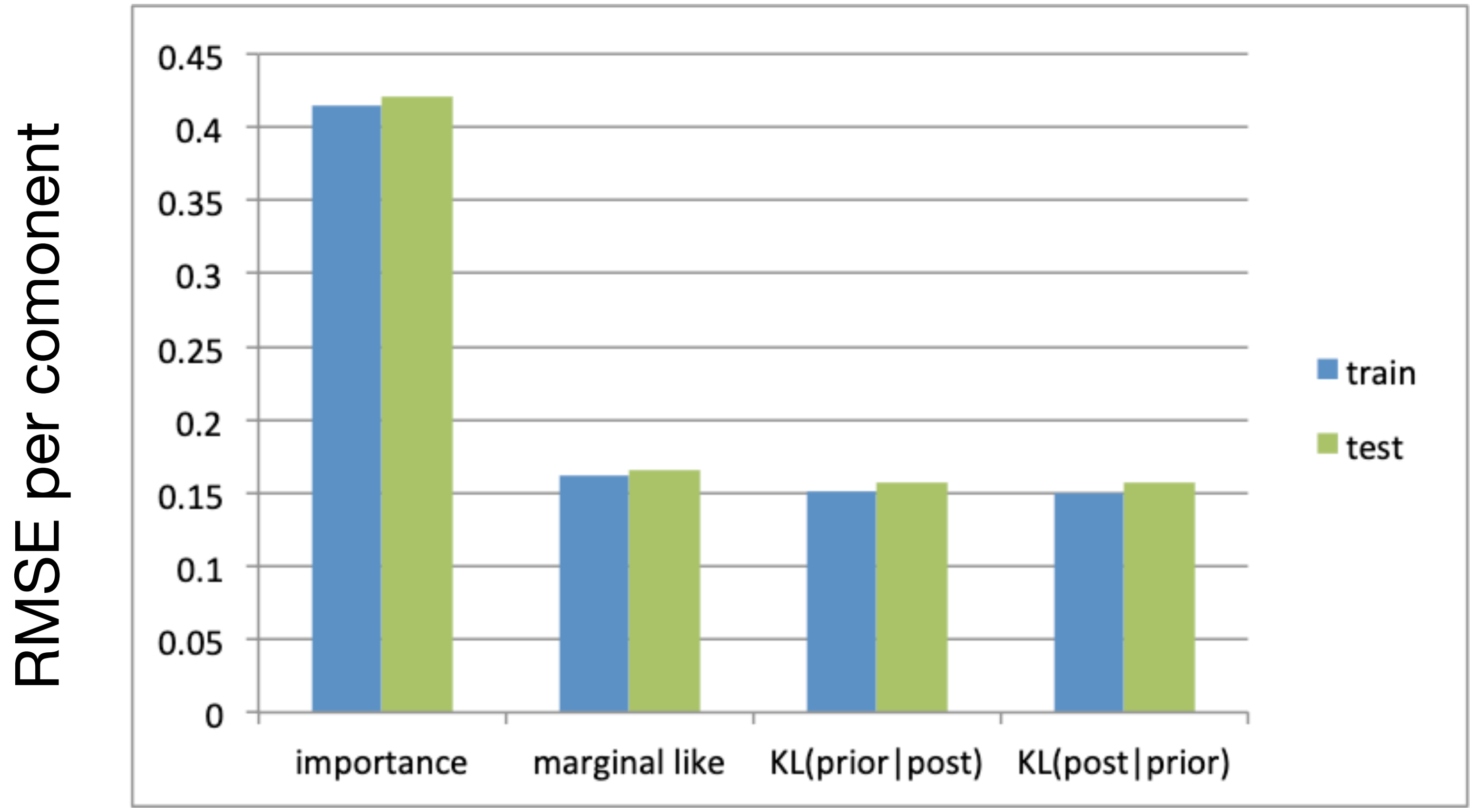
Can alternatively use surrogates

$\min$ **KL**(prior||posterior)

$\min$ **KL**(posterior||prior) $\approx \min I(\xi; r_0)$

Missing data inference

Sum of squared test error on continuous action MNIST ($a \in \Re^{10}$)



Batch policy optimization

$$
\begin{array}{c|cccc|}
 & a_1 & a_2 & \cdots & a_n \\
\hline
x_1 & & r_1 & & \\
x_2 & & & & r_2 \\
x_3 & r_3 & & & \\
x_4 & & & r_4 & \\
x_5 & r_5 & & & \\
x_6 & & r_6 & & \\
\vdots & & & r_{\vdots} & \\
x_m & & & & r_m \\
\hline
\end{array}
$$

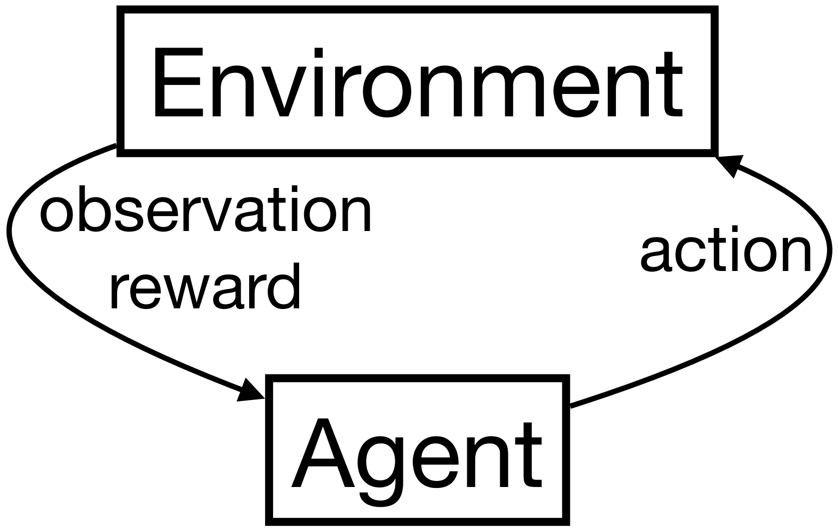
Three key issues

1. generalization
2. optimization
3. missing data

training objective $\neq$ target objective

classical methods still help

The RL problem



1. multi-agent interaction ➡ non-stationarity
2. partial observability ➡ construct memory
3. exploration ➡ explore/exploit
4. sequential decisions ➡ temporal credit assignment
5. exploitation ➡ policy optimization

4. Fundamentals of sequential decision making

approximate dynamic programming
- be afraid, be very afraid

policy optimization with simple value estimates
- surrogate objectives and missing data inference show promise

RL in the dual
- avoid the deadly triad
- new approaches to MCMC stationary distribution inference