

第9年作业:

5-2 解: 极点配置希望

$$|sI - (A+BK)| = (s+10)(s^2+2s+4) = s^3+12s^2+24s+40$$

$$\begin{vmatrix} s & -1 & 0 \\ 0 & s+1 & -1 \\ -10k_0 & 10k_1 & s+10k_2 \end{vmatrix} = (s^2+s)(s+10-10k_2) - 10k_0 + s(-10k_1)$$

$$\Rightarrow K_0 = -4 \quad K_1 = -1.2 \quad K_2 = -0.1$$

所以状态反馈矩阵为 $K = [-4, -1.2, -0.1]$

状态控制律为 $u = Kx + v$

5-6 解: 1) $|sI - A| = s^2(s^2-1)$

由于特征矩阵存在正的特征根, 所以系统不稳定

2) 由于 $\text{rank}(M) = 4$, 故可知系统能够镇定。不妨假设要求闭环极点为 $-1, -2, -2.5, -4$

$$|sI - (A+BK)| = \begin{vmatrix} s & -1 & 0 & 0 \\ -k_0 & s-k_1 & 1-k_2 & -k_3 \\ 0 & 0 & s & -1 \\ k_0 & k_1 & k_2-1 & s+k_3 \end{vmatrix} = (s+1)(s+2)(s+2.5)(s+4)$$

$$= s^4 + 9.5s^3 + 31.5s^2 + 43s + 20$$

可得出 $K = [2 \quad 4.3 \quad 44.5 \quad 13.8]$

状态反馈控制律为 $u = Kx + v$

5-7 解: 设 $W_1(s) = W_0(s)W_d(s)$

$$u_d(s) = W_0(s) + W_d(s)$$

$$W_1(s)^{-1} = \frac{1}{\frac{1}{s(s+1)} - \frac{1}{s(s+1)(s+2)}} \begin{bmatrix} \frac{1}{s} & \frac{-1}{s+2} \\ \frac{-1}{s(s+1)} & \frac{1}{s+1} \end{bmatrix} = \begin{bmatrix} s+2 & -s \\ -\frac{(s+2)}{s+1} & \frac{s(s+2)}{s+1} \end{bmatrix}$$

$$W_d(s) = \begin{bmatrix} \frac{s+2}{(s+1)^2} & \frac{-s}{(s+2)^2} \\ \frac{-(s+2)}{(s+1)^2} & \frac{s}{(s+1)(s+2)} \end{bmatrix}$$

5-10. 解:

$\because N = \begin{bmatrix} c \\ ca \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ 满秩, 系统能观, 可构造观测器

引入反馈阵 $G = \begin{bmatrix} g_1 \\ g_2 \end{bmatrix}$, 使得观测器特征多项式等于期望特征式:

$$|sI - (A - GC)| = \begin{vmatrix} s + g_1 & -1 \\ g_2 & s \end{vmatrix} = (s + r)(s + 2r)$$

$$s^2 + g_1 s + g_2 = s^2 + 3rs + 2r^2 \Rightarrow g_1 = 3r, g_2 = 2r^2 \Rightarrow G = \begin{bmatrix} 3r \\ 2r^2 \end{bmatrix}$$

观测器方程为 $\dot{\hat{x}} = (A - GC)\hat{x} + bu + Gy$

$$= \begin{bmatrix} -3r & 1 \\ -2r^2 & 0 \end{bmatrix} \hat{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} 3r \\ 2r^2 \end{bmatrix} y$$

5-12. 解: $\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = u \\ y = x_1 \end{cases} \Rightarrow \begin{cases} \dot{\hat{x}} = \hat{x} \\ \hat{x}_2 = x_3 \\ \hat{x}_3 = u \end{cases} \Rightarrow \begin{cases} \dot{y} = [1 \ 0] \begin{bmatrix} \hat{x}_2 \\ \hat{x}_3 \end{bmatrix} \quad \text{输出方程} \\ \begin{bmatrix} \hat{x}_2 \\ \hat{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{x}_2 \\ \hat{x}_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \quad \text{状态方程} \end{cases}$

对 x_2, x_3 构造降维观测器, 期望极点在 $-4, -5$, 则

$$|sI - (A' - GC')| = |sI - (\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} [1 \ 0])| = (s + 4)(s + 5) \Rightarrow \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} = \begin{bmatrix} 9 \\ 20 \end{bmatrix}$$

$$\begin{cases} \begin{bmatrix} \dot{\hat{x}}_2 \\ \dot{\hat{x}}_3 \end{bmatrix} = (\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 9 \\ 20 \end{bmatrix} [1 \ 0]) \begin{bmatrix} \hat{x}_2 \\ \hat{x}_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} 9 \\ 20 \end{bmatrix} y \end{cases}$$

$$\begin{bmatrix} \dot{\hat{x}}_2 \\ \dot{\hat{x}}_3 \end{bmatrix} = \begin{bmatrix} w_2 \\ w_3 \end{bmatrix} + \begin{bmatrix} 9 \\ 20 \end{bmatrix} y$$

$$\hat{x} = \begin{bmatrix} y \\ \hat{x}_2 \\ \hat{x}_3 \end{bmatrix}$$

$$\Rightarrow \begin{cases} \begin{bmatrix} \dot{w}_2 \\ \dot{w}_3 \end{bmatrix} = \begin{bmatrix} -9 & 1 \\ -20 & 0 \end{bmatrix} \begin{bmatrix} w_2 \\ w_3 \end{bmatrix} + \begin{bmatrix} 9 \\ 20 \end{bmatrix} y + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \end{cases}$$

$$= \begin{bmatrix} -9 & 1 \\ 20 & 0 \end{bmatrix} \begin{bmatrix} w_2 \\ w_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} -6 \\ 120 \end{bmatrix} y$$

$$\hat{x} = \begin{bmatrix} y \\ \hat{x}_2 \\ \hat{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} w_2 \\ w_3 \end{bmatrix} + \begin{bmatrix} 9 \\ 20 \end{bmatrix} y$$