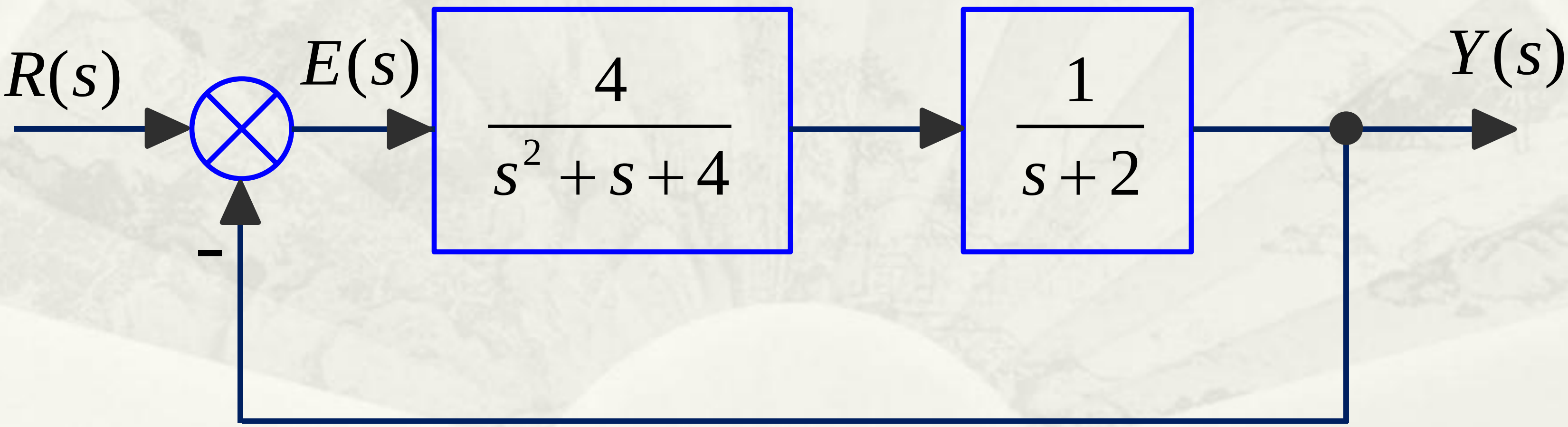
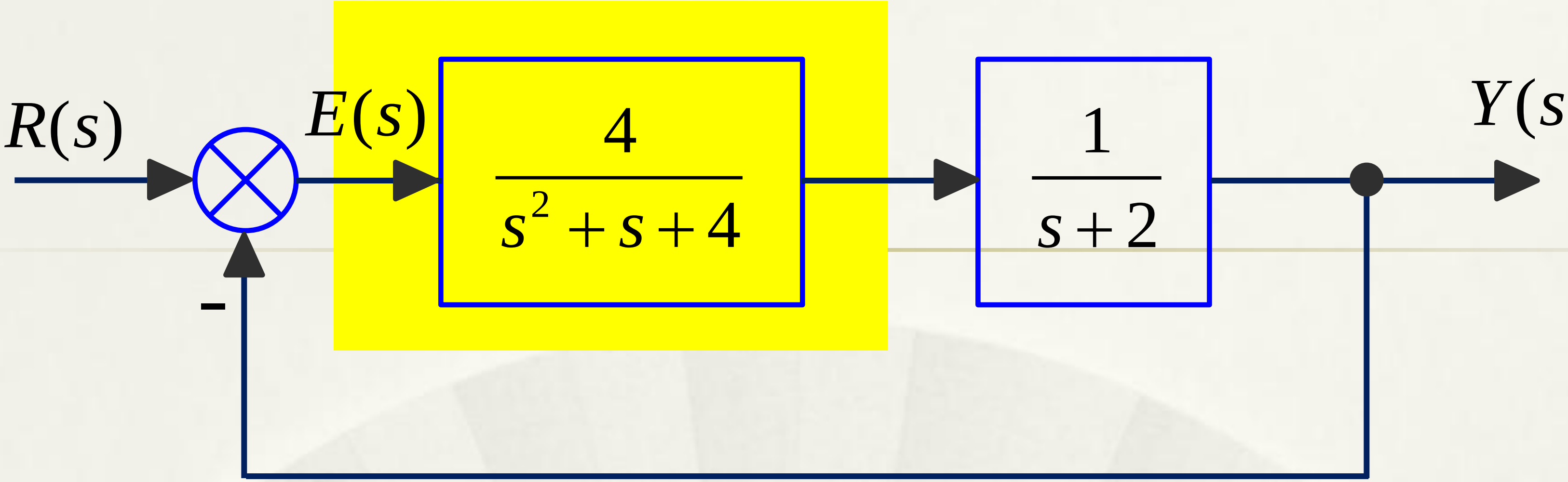
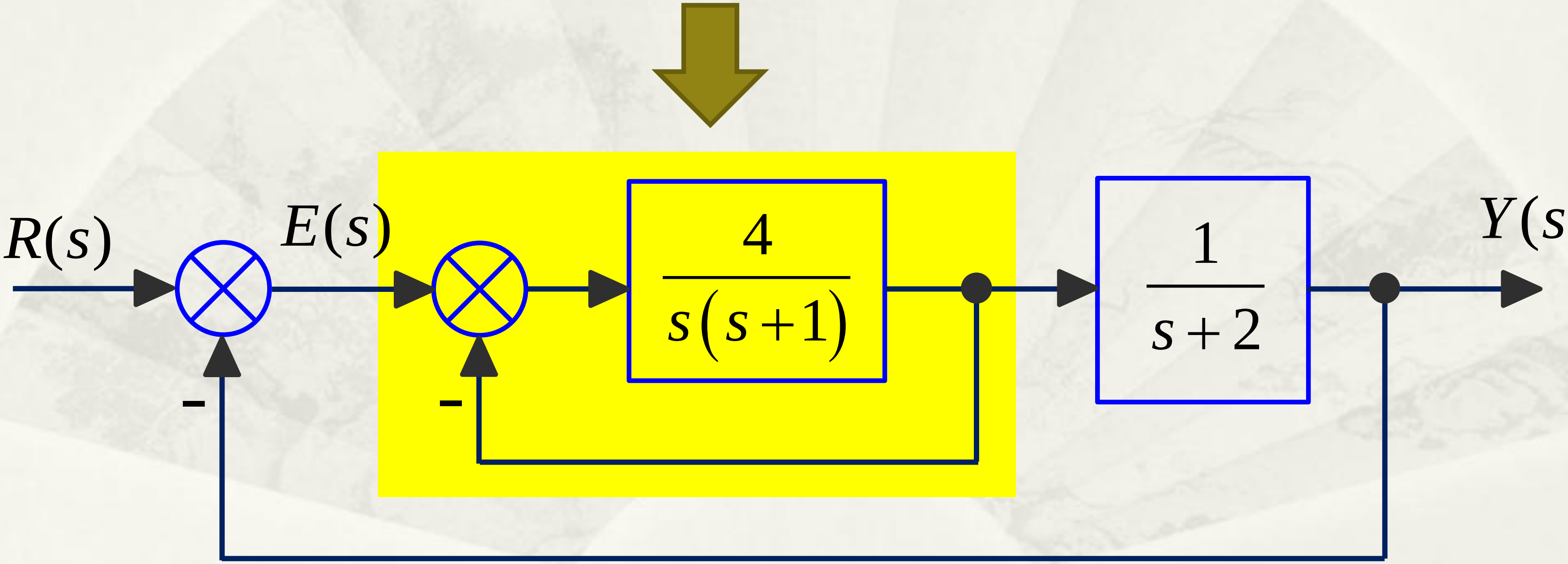
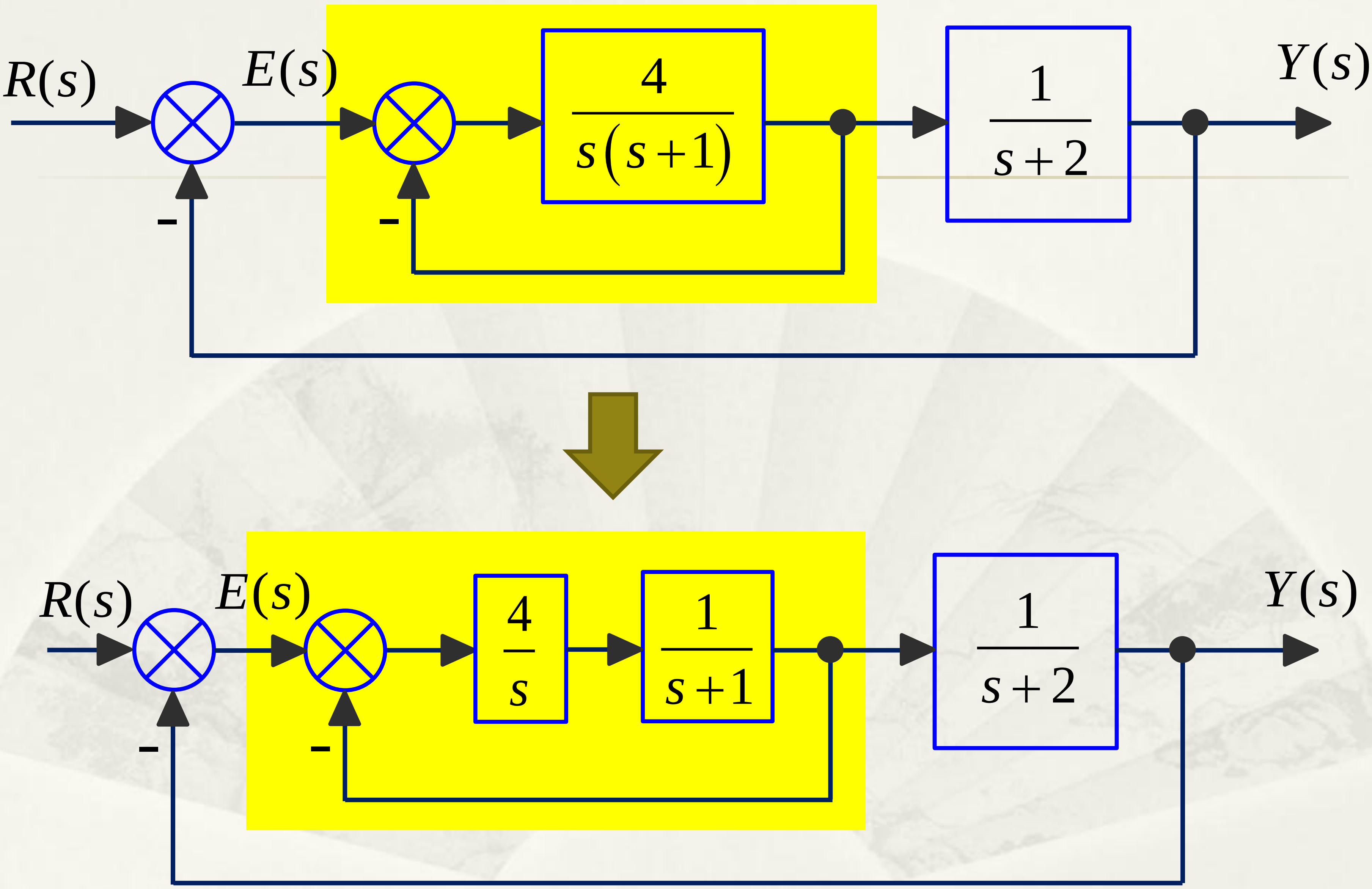
如何根据含有高阶环节的方块图建立状态空间表达式？

【例】某系统的方块图如下，试建立其状态空间表达式。

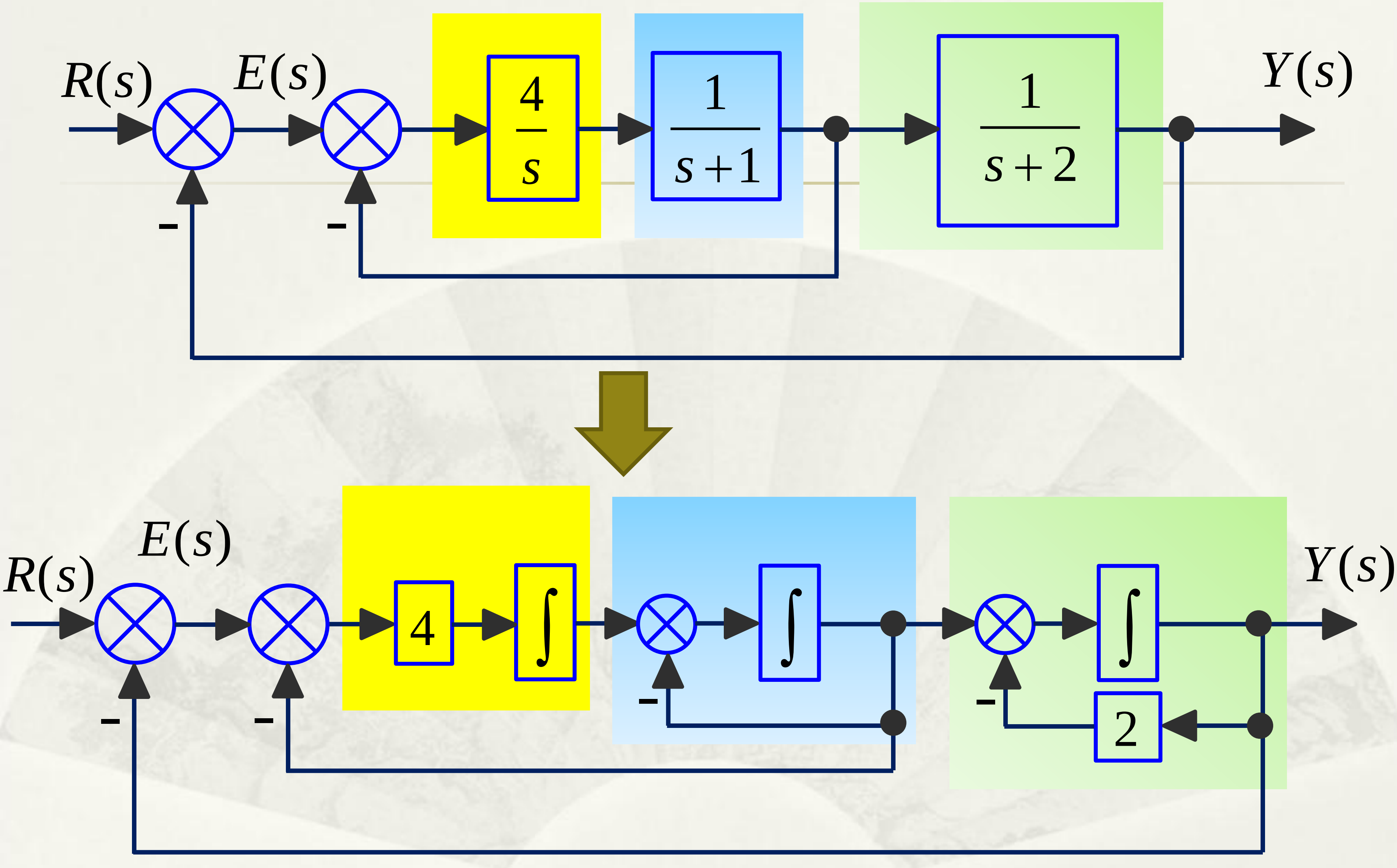


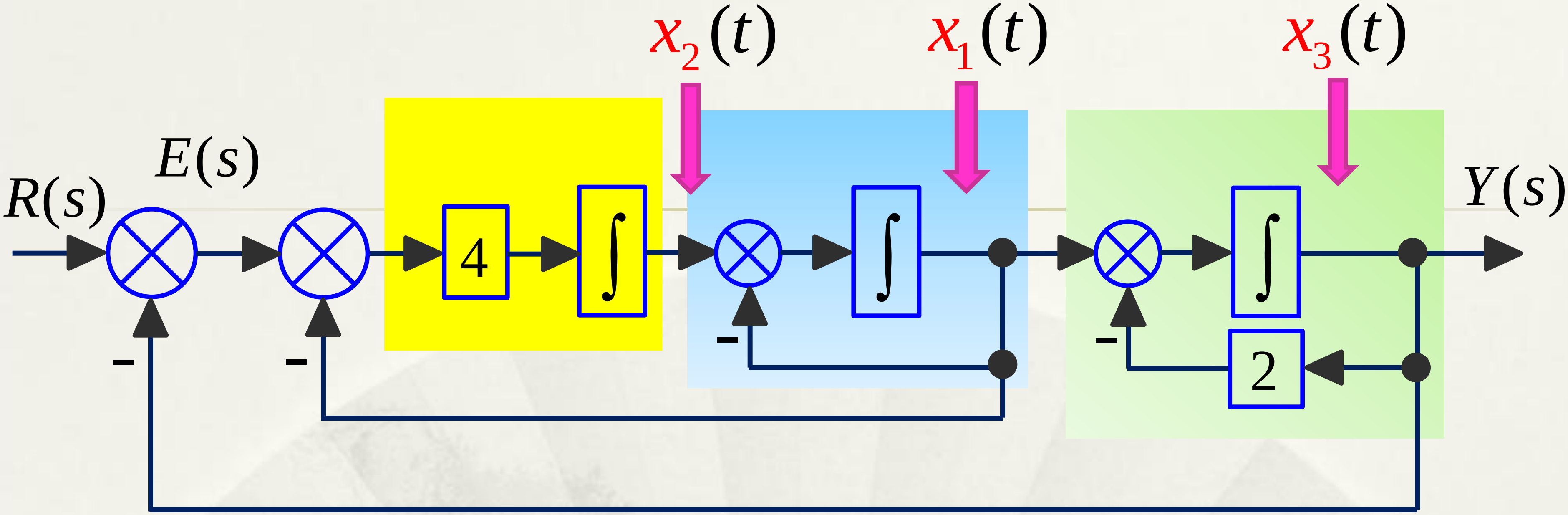

R(s)
E(s)
4 / (s^2 + s + 4)
1 / (s + 2)
Y(s)
-


R(s)
E(s)
4 / (s(s+1))
1 / (s + 2)
Y(s)
-
-



$R(s)$
$E(s)$
$\frac{4}{s(s+1)}$
$\frac{1}{s+2}$
$Y(s)$
-
-
$R(s)$
$E(s)$
$\frac{4}{s}$
$\frac{1}{s+1}$
$\frac{1}{s+2}$
$Y(s)$
-
-


R(s)
E(s)
4/s
1/(s+1)
1/(s+2)
Y(s)
-
-
R(s)
E(s)
4
∫
∫
∫
2
Y(s)
-
-
-
-



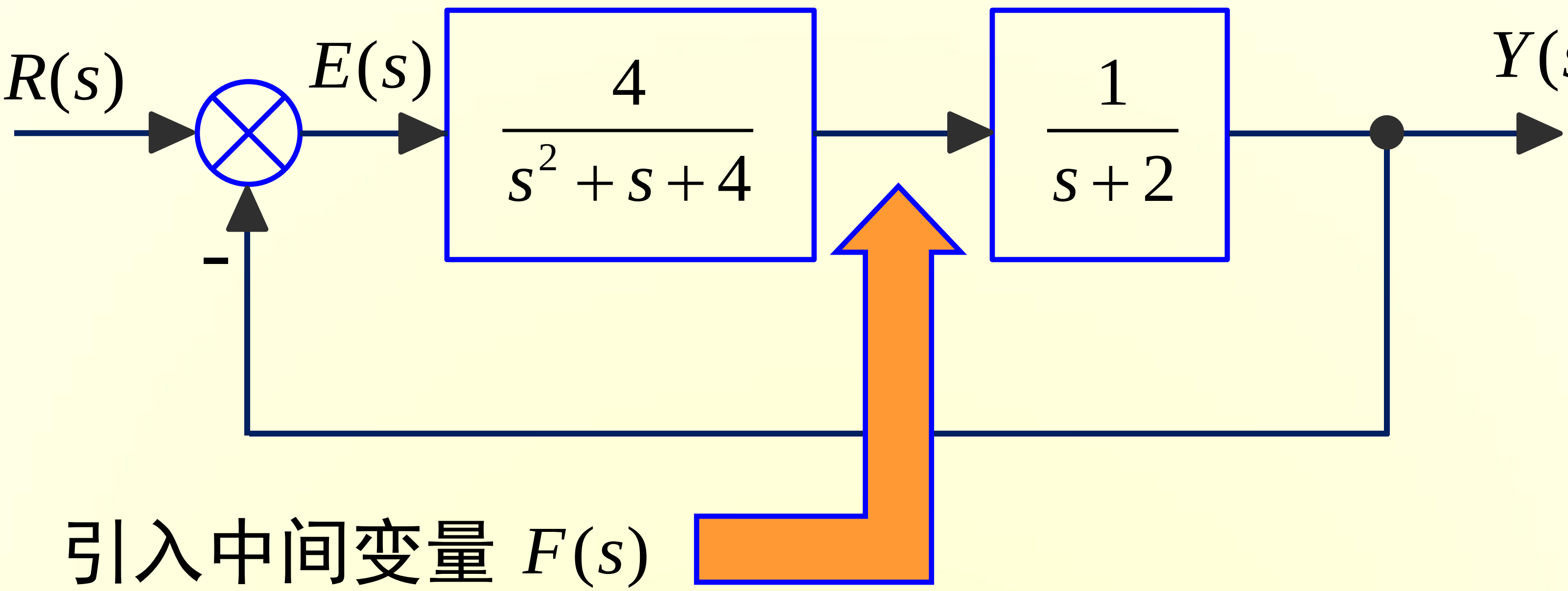
$$
\begin{cases}
\dot{x}_1(t) = -x_1(t) + x_2(t) \\
\dot{x}_2(t) = 4\left[r(t) - x_3(t) - x_1(t)\right] \\
\dot{x}_3(t) = -2x_3(t) + x_1(t) \\
y(t) = x_3(t)
\end{cases}
$$

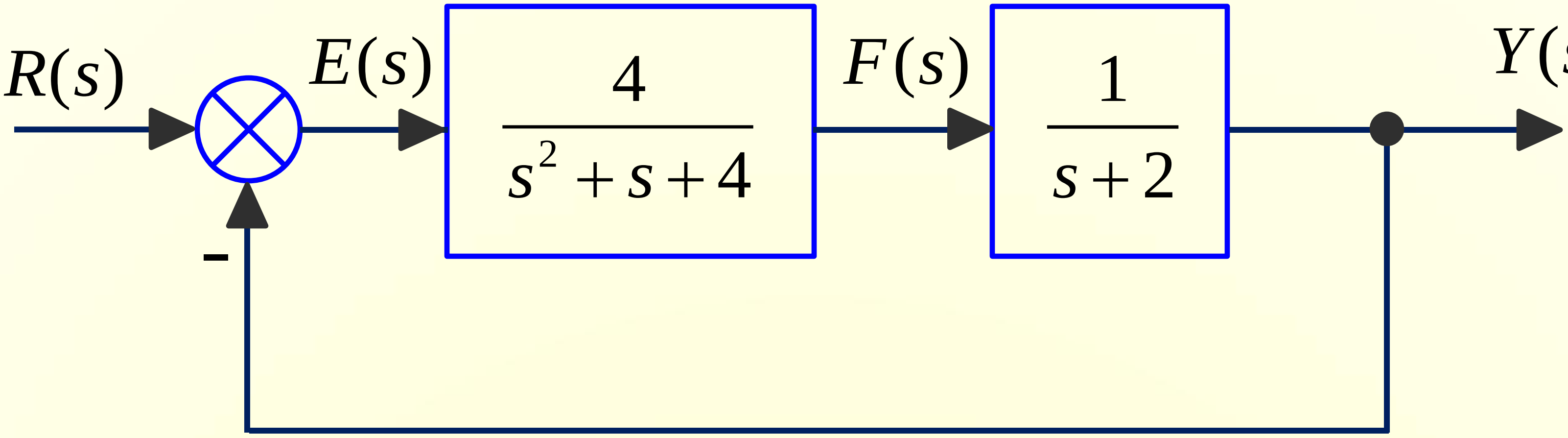
$$
\begin{cases}
\dot{x}_1(t) = -x_1(t) + x_2(t) \\
\dot{x}_2(t) = 4\left[r(t) - x_3(t) - x_1(t)\right] \\
\dot{x}_3(t) = -2x_3(t) + x_1(t) \\
y(t) = x_3(t)
\end{cases}
$$

$$
\begin{cases}
\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 \\ -4 & 0 & -4 \\ 1 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 4 \\ 0 \end{bmatrix} r(t) \\
y(t) = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}
\end{cases}
$$

高阶环节转换成对应的模拟结构图之过程比较复杂，所以在建立状态空间表达式的时候，也可以采用中间变量法。

【例】某系统的方块图如下，试建立其状态空间表达式。





$$\begin{cases} Y(s) = \dfrac{1}{s+2} F(s) \\ F(s) = \dfrac{4}{s^2+s+4}\left[R(s) - Y(s)\right] \end{cases}$$

$$\begin{cases} Y(s) = \dfrac{1}{s+2} F(s) \\[2ex] F(s) = \dfrac{4}{s^2+s+4} \left[R(s) - Y(s) \right] \end{cases}$$

$$\Downarrow$$

$$\begin{cases} sY(s) + 2Y(s) = F(s) \\[2ex] s^2 F(s) + sF(s) + 4F(s) = 4R(s) - 4Y(s) \end{cases}$$

$$\begin{cases} sY(s)+2Y(s)=F(s) \\ s^2F(s)+sF(s)+4F(s)=4R(s)-4Y(s) \end{cases}$$

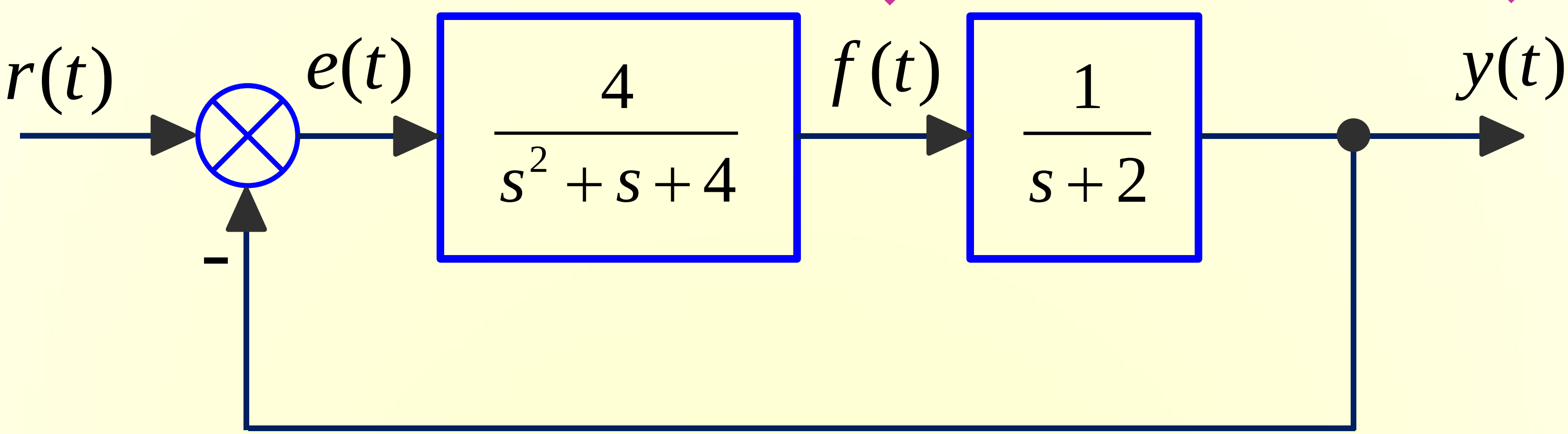
零初始条件下的拉氏反变换

$$\begin{cases} \dot{y}(t)+2y(t)=f(t) \\ \ddot{f}(t)+\dot{f}(t)+4f(t)=4r(t)-4y(t) \end{cases}$$

$$
\begin{cases}
\dot{y}(t)+2y(t)=f(t) \\
\ddot{f}(t)+\dot{f}(t)+4f(t)=4r(t)-4y(t)
\end{cases}
$$

$$x_2(t)=\dot{f}(t)$$

$x_1(t)$ $x_3(t)$



● 状态变量你可以有其他的选法哦！

$$\begin{cases} \dot{y}(t) + 2y(t) = f(t) \\ \ddot{f}(t) + \dot{f}(t) + 4f(t) = 4r(t) - 4y(t) \end{cases}$$

$$x_1(t) = f(t) \qquad x_2(t) = \dot{f}(t) \qquad x_3(t) = y(t)$$

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = -4x_1(t) - x_2(t) - 4x_3(t) + 4r(t) \\ \dot{x}_3(t) = x_1(t) - 2x_3(t) \\ y(t) = x_3(t) \end{cases}$$

$$
\begin{cases}
\dot{x}_1(t) = x_2(t) \\
\dot{x}_2(t) = -4x_1(t) - x_2(t) - 4x_3(t) + 4r(t) \\
\dot{x}_3(t) = x_1(t) - 2x_3(t) \\
y(t) = x_3(t)
\end{cases}
$$

$$
\begin{cases}
\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -4 & -1 & -4 \\ 1 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 4 \\ 0 \end{bmatrix} r(t) \\
y(t) = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}
\end{cases}
$$