8-2真题

1.（89-3）已知 $z=a^{\sqrt{x^2-y^2}}$，其中 $a>0,a\neq1$，求 dz.

$\frac{\partial z}{\partial x}=a^{\sqrt{x^2-y^2}}\cdot\ln a\cdot\frac{1}{2}\frac{1}{\sqrt{x^2-y^2}}\cdot(2x)=\frac{xz\ln a}{\sqrt{x^2-y^2}}$

$\frac{\partial z}{\partial y}=a^{\sqrt{x^2-y^2}}\cdot\ln a\cdot\frac{1}{2}\frac{1}{\sqrt{x^2-y^2}}\cdot(-2y)=-\frac{yz\ln a}{\sqrt{x^2-y^2}}$

$dz=\frac{z\ln a}{\sqrt{x^2-y^2}}(x\,dx-y\,dy)$

2.（94-1）设 $u=e^{-x}\sin\frac{x}{y}$，则 $\frac{\partial^2 u}{\partial x\partial y}$ 在点 $\left(2,\frac{1}{\pi}\right)$ 处的值为 $e^{-2}\pi^2$.

$\frac{\partial u}{\partial x}=-e^{-x}\sin\frac{x}{y}+e^{-x}\cos\frac{x}{y}\cdot\frac{1}{y}$ 代入 $x=2\Rightarrow\frac{\partial u}{\partial x}\Big|_{x=2}=-e^{-2}\sin\frac{2}{y}+e^{-2}\cos\frac{2}{y}\cdot\frac{1}{y}$

$\frac{\partial^2 u}{\partial x\partial y}=-e^{-2}\cos\frac{2}{y}\cdot\left(-\frac{2}{y^2}\right)+e^{-2}\sin\frac{2}{y}\cdot\frac{2}{y^3}-e^{-2}\cos\frac{2}{y}\cdot\frac{1}{y^2}$ 代入 $\frac{1}{\pi}\Rightarrow$ 原式 $=e^{-2}\pi^2$

3.（98-1）设 $z=\frac{1}{x}f(xy)+y\varphi(x+y)$，$f,\varphi$ 具有二阶连续导数，则 $\frac{\partial^2 z}{\partial x\partial y}=$ $yf''(xy)+\varphi'(x+y)+y\varphi''(x+y)$.

$\frac{\partial z}{\partial x}=\frac{-f(xy)}{x^2}+\frac{1}{x}f'(xy)\cdot y+y\varphi'(x+y)$

$\frac{\partial^2 z}{\partial x\partial y}=-\frac{1}{x^2}f'(xy)\cdot x+\frac{1}{x}[f''(xy)\cdot xy+f'(xy)]+\varphi'(x+y)+y\varphi''(x+y)$

4.（00-1）设 $z=f\left(xy,\frac{x}{y}\right)+g\left(\frac{y}{x}\right)$，其中 f 具有二阶连续偏导数，g 具有二阶连续导数，求 $\frac{\partial^2 z}{\partial x\partial y}$.

$\frac{\partial z}{\partial x}=f_1'\cdot y+f_2'\cdot\frac{1}{y}+g'\cdot\frac{-y}{x^2}$

$\frac{\partial^2 z}{\partial x\partial y}=f_1'+\left(f_{11}''\cdot x+f_{12}''\cdot\left(-\frac{x}{y^2}\right)\right)y+f_2'\cdot\frac{-1}{y^2}+\frac{1}{y}\left(f_{21}''x+f_{22}''\cdot\left(-\frac{x}{y^2}\right)\right)+g'\cdot\left(-\frac{1}{x^2}\right)+g''\cdot\frac{1}{x}\cdot\frac{-y}{x^2}=f_1'-\frac{1}{y^2}f_2'-\frac{x}{y^3}f_{22}''+xyf_{11}''-\frac{g'}{x^2}-g''\frac{y}{x^3}$

5.（03-3）设 $f(u,v)$ 具有二阶连续偏导数，且满足 $\frac{\partial^2 f}{\partial u^2}+\frac{\partial^2 f}{\partial v^2}=1$，又 $g(x,y)=f\left[xy,\frac{1}{2}(x^2-y^2)\right]$，求 $\frac{\partial^2 g}{\partial x^2}+\frac{\partial^2 g}{\partial y^2}$.

$\frac{\partial g}{\partial x}=f_1'\cdot y+f_2'\cdot\frac{1}{2}(2x)$

$\frac{\partial^2 g}{\partial x^2}=[f_{11}''\cdot y+f_{12}''\cdot\frac{1}{2}(2x)]y+f_2'+[f_{21}''\cdot y+f_{22}''\cdot x]x$

同理求出 $\frac{\partial^2 g}{\partial x^2}+\frac{\partial^2 g}{\partial y^2}=(x^2+y^2)\underbrace{(f_{11}''+f_{22}'')}_{\frac{\partial^2 f}{\partial u^2}+\frac{\partial^2 f}{\partial v^2}=1}=x^2+y^2$

6.（05-1;2）设函数

$$u(x,y)=\varphi(x+y)+\varphi(x-y)+\int_{x-y}^{x+y}\psi(t)dt,$$

其中函数 φ 具有二阶导数，ψ 具有一阶导数，则必有

B

（A）$\frac{\partial^2 u}{\partial x^2}=-\frac{\partial^2 u}{\partial y^2}$.　（B）$\frac{\partial^2 u}{\partial x^2}=\frac{\partial^2 u}{\partial y^2}$.

（C）$\frac{\partial^2 u}{\partial x\partial y}=\frac{\partial^2 u}{\partial y^2}$.　（D）$\frac{\partial^2 u}{\partial x\partial y}=\frac{\partial^2 u}{\partial x^2}$.

$\frac{\partial u}{\partial x}=\varphi'(x+y)+\varphi'(x-y)+\psi(x+y)-\psi(x-y)$

$\frac{\partial^2 u}{\partial x^2}=\varphi''(x+y)+\varphi''(x-y)+\psi'(x+y)-\psi'(x-y)$

$\frac{\partial u}{\partial y}=\varphi'(x+y)-\varphi'(x-y)+\psi(x+y)+\psi(x-y)$

$\frac{\partial^2 u}{\partial y^2}=\varphi''(x+y)+\varphi''(x-y)+\psi'(x+y)-\psi'(x-y)$

7.（06-3）设函数 $f(u)$ 可微，且 $f'(0)=\frac{1}{2}$，则 $z=f(4x^2-y^2)$ 在点 $(1,2)$ 处的全微分

$dz\big|_{(1,2)}=$ $4dx-2dy$

采用一种方法：$\frac{\partial z}{\partial x}=f'(4x^2-y^2)\cdot 8x$

$\frac{\partial z}{\partial y}=f'(4x^2-y^2)\cdot(-2y)$

$4x^2-y^2=0$

8.（07-2）已知函数 $f(u)$ 具有二阶导数，且 $f'(0)=1$，函数 $y=y(x)$ 由方程 $y-xe^{y-1}=1$ 所确定，设 $z=f(\ln y-\sin x)$，求 $\frac{dz}{dx}\Big|_{x=0}$，$\frac{d^2z}{dx^2}\Big|_{x=0}$

$\frac{dz}{dx}=f'(\ln y-\sin x)\cdot(\frac{1}{y}\cdot y'-\cos x)$，由于 $y-xe^{y-1}=1$，所以 $y'-e^{y-1}-xe^{y-1}y'=0$

$\Rightarrow y'=\frac{e^{y-1}}{1-xe^{y-1}}$，代入 $x=0, y=1$，$\Rightarrow y'=1$，$\frac{dz}{dx}\Big|_{x=0}=0$；$y''=\frac{e^{y-1}y'(1-xe^{y-1})-e^{y-1}(-e^{y-1}-xe^{y-1}y')}{(1-xe^{y-1})^2}$

令 $\frac{d^2z}{dx^2}=f''(\ln y-\sin x)(\frac{1}{y}y'-\cos x)^2+f'(\ln y-\sin x)(-\frac{1}{y^2}\cdot y'^2+\frac{1}{y}y''+\sin x)$ 所以 $\frac{d^2z}{dx^2}\Big|_{x=0}=1$

9.（01-3）设 $u=f(x,y,z)$ 有连续的一阶偏导数，又函数 $y=y(x)$ 及 $z=z(x)$ 分别由下列两式确定：

$$e^{xy}-xy=2 \text{ 和 } e^x=\int_0^{x-z}\frac{\sin t}{t}dt\text{，求 }\frac{du}{dx}.$$

$e^{xy}-xy=2$ 得 $\frac{dy}{dx}=-\frac{F_x}{F_y}=-\frac{y}{x}$

$e^x=\frac{\sin(x-z)}{x-z}\cdot(1-\frac{dz}{dx})\Rightarrow\frac{dz}{dx}=\frac{(x-z)e^x-\sin(x-z)}{\sin(x-z)}$

$\frac{du}{dx}=f_1'+f_2'\cdot\frac{dy}{dx}+f_3'\cdot\frac{dz}{dx}=f_1'-\frac{y}{x}f_2'+[1-\frac{(x-z)e^x}{\sin(x-z)}]f_3'$

10.（04-2）设函数 $z=z(x,y)$ 由方程 $z=e^{2x-3z}+2y$ 确定，则 $3\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=$ 2.

$\frac{\partial z}{\partial x}=e^{2x-3z}(2-3\frac{\partial z}{\partial x})$

$\Rightarrow\frac{\partial z}{\partial x}=\frac{2e^{2x-3z}}{1+3e^{2x-3z}}$

$\frac{\partial z}{\partial y}=e^{2x-3z}(-3\frac{\partial z}{\partial y})+2$

$\Rightarrow\frac{\partial z}{\partial y}=\frac{2}{1+3e^{2x-3x}}$

$\Rightarrow 3\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=2$

11.（04-3）函数 $f(u,v)$ 由关系式 $f[xg(y),y]=x+g(y)$ 确定，其中函数 $g(y)$ 可微，且

$g(y)\neq 0$，则 $\dfrac{\partial^2 f}{\partial u\partial v}=$ $-\dfrac{g'(v)}{g^2(v)}$ ____.