1. (90-2) $\int_0^1 x\sqrt{1-x}dx =$ $\frac{4}{15}$.

解 $\underline{\underline{\sqrt{1-x}=t}}$ $\int_1^0 (1-t^2)t\cdot d(1-t^2) = \int_0^1 (1-t^2)2t^2 dt$

$= \frac{2}{3}t^3 - \frac{2}{5}t^5 \Big|_0^1 = \frac{4}{15}$

2. (91-2) 计算 $\int_1^4 \frac{dx}{x(1+\sqrt{x})}$.

解 $\underline{\underline{\sqrt{x}=t}}$ $\int_1^2 \frac{2t\,dt}{t^2(1+t)} = 2\int_1^2 (\frac{1}{t} - \frac{1}{1+t})dt = 2\ln t\Big|_1^2 - 2\ln(1+t)\Big|_1^2$

$= 4\ln 2 - 2\ln 3 = 2\ln\frac{4}{3}$

3. (92-2) 求 $\int_0^{\pi} \sqrt{1-\sin x}\,dx$.

解 $= \int_0^{\pi}\sqrt{\sin^2\frac{x}{2} - 2\sin\frac{x}{2}\cos\frac{x}{2} + \cos^2\frac{x}{2}} = \int_0^{\pi} \left|\cos\frac{x}{2} - \sin\frac{x}{2}\right| dx$

$= 2(\sin\frac{x}{2} + \cos\frac{x}{2})\Big|_0^{\frac{\pi}{2}} - 2(-\cos\frac{x}{2} - \sin\frac{x}{2})\Big|_{\frac{\pi}{2}}^{\pi} = 4(\sqrt{2}-1)$

4. (93-2) 求 $\int_0^{\frac{\pi}{4}} \frac{x}{1+\cos 2x}dx$. $\cos 2x = \frac{1-\tan^2 x}{1+\tan^2 x}$

解 $= \int_0^{\frac{\pi}{4}} \frac{1}{2}x\sec^2 x\,dx = \frac{1}{2}\int_0^{\frac{\pi}{4}} x\,d\tan x = \frac{1}{2}x\tan x\Big|_0^{\frac{\pi}{4}} + \frac{1}{2}\ln|\cos x|\Big|_0^{\frac{\pi}{4}}$

$= \frac{\pi}{8} + \frac{1}{2}\ln\frac{\sqrt{2}}{2}$

5. (96-2) 计算 $\int_0^{\ln 2} \sqrt{1-e^{-2x}}\,dx$.

解 $\underline{\underline{\sqrt{1-e^{-2x}}=t}}$ $\int_0^{\frac{\sqrt{3}}{2}} t\,d\frac{1}{2}\ln(1-t^2) = \int_0^{\frac{\sqrt{3}}{2}} \frac{t^2}{1-t^2} = -\frac{\sqrt{3}}{2} + \ln(2+\sqrt{3})$

6. (99-2) 函数 $y = \frac{x^2}{\sqrt{1-x^2}}$ 在区间 $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ 上的平均值为 $\frac{\sqrt{3}+1}{12}\pi$.

由积分中值定理:

$\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \frac{x^2\,dx}{\sqrt{1-x^2}} = f(\xi)\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right)$

解: $x=\sin t$ $= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin^2 t\cdot\cos t}{\cos t}dt = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{2}(1-\cos 2t)\,dt$

$= \frac{1}{2}t\Big|_{\frac{\pi}{6}}^{\frac{\pi}{3}} - \frac{1}{4}\sin 2t\Big|_{\frac{\pi}{6}}^{\frac{\pi}{3}}$ $\Rightarrow f(\xi) = \frac{\sqrt{3}+1}{12}\pi$

7.（07-1）$\int_1^2 \frac{1}{x^3} e^{\frac{1}{x}} dx =$ $\frac{1}{2}e^{\frac{1}{2}}$.

7. 解 $= \int_1^2 \frac{-1}{x} \, de^{\frac{1}{x}}$

$= \frac{-1}{x} e^{\frac{1}{x}} \Big|_1^2 + \int_1^2 e^{\frac{1}{x}} d\frac{1}{x}$

$= \frac{1}{2} e^{\frac{1}{2}}$

8.（89-3）已知函数 $f(x) = \begin{cases} x, & 0 \le x \le 1, \\ 2-x, & 1 < x \le 2. \end{cases}$ 计算下列各题：

（1）$S_0 = \int_0^2 f(x) e^{-x} dx$；

（2）$S_1 = \int_2^4 f(x-2) e^{-x} dx$；

（3）$S_n = \int_{2n}^{2n+2} f(x-2n) e^{-x} dx (n = 2, 3, \cdots)$；

（4）$S = \sum_{n=0}^{\infty} S_n$.

(1) 解 $= \int_0^1 x e^{-x} dx + \int_1^2 (2-x) e^{-x} dx$

$= (1-\frac{1}{e})^2$ (分部积分法)

(2) 解 $= \int_0^2 f(t) \cdot e^{-(2+t)} dt$

$= \int_0^1 t e^{-(2+t)} - \int_1^2 (2-t) e^{-(2+t)} dt$

$= (1-\frac{1}{e})^2 \cdot e^{-2}$

(3) 由上式可知，换元后，$e^{-x}$ 项变为 $e^{-2n} \cdot e^{-x}$；$e^{-2n}$ 为常数，$\Rightarrow S_n = S_0 \cdot e^{-2n}$

(4) $S = S_0 \sum_{n=0}^{\infty} (e^{-2})^n = S_0 \lim_{n\to\infty} \frac{1-e^{-2n}}{1-e^{-2}} = (1-\frac{1}{e})^2 \cdot \frac{1}{1-(\frac{1}{e})^2} = \frac{e-1}{e+1}$

9.（91-3）求定积分 $I = \int_{-1}^1 (2x + |x| + 1)^2 dx$.

解 $= \int_{-1}^0 (2x - x + 1)^2 dx + \int_0^1 (2x + x + 1)^2 dx$

$= \frac{1}{3}(x+1)^3 \Big|_{-1}^0 + \frac{1}{9}(3x+1)^3 \Big|_0^1 = \frac{22}{3}$

10.（96-2）$\int_{-1}^1 (x + \sqrt{1-x^2})^2 dx =$ $2$.

令 $x = \sin t$

$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\sin t + \cos t)^2 \cdot \cos t \, dt$

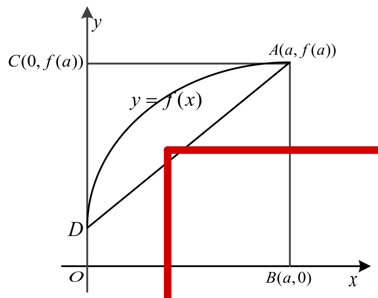
$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (1 + 2\sin t \cos t) \cos t \, dt$

$= \sin t \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} - 2\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2 t \, d\cos t$

$= 2 - 0 = 2$

C

11.（08-2;3）如图，曲线段的方程为 $y=f(x)$，函数 $f(x)$ 在区间 $[0,a]$ 上有连续的导数，则定积分 $\int_0^a xf'(x)\mathrm{d}x$ 等于




（A）曲边梯形 $ABOD$ 的面积.

（B）梯形 $ABOD$ 的面积.

（C）曲边三角形 $ACD$ 的面积.

（D）三角形 $ACD$ 的面积.

物理意义

11、$\int_0^a x\,df(x)$

$= xf(x)\Big|_0^a - \boxed{\int_0^a f(x)\,dx}$

$= af(a) - S_{ADOB}$

$= S_{\square ABCO}$