

$$1. (97-2) \int_0^{+\infty} \frac{dx}{x^2+4x+8} = \frac{\frac{\pi}{8}}{8}.$$

$$\int_0^{+\infty} \frac{d(x+2)}{(x+2)^2+4} = \frac{1}{2} \arctan \frac{x+2}{2} \Big|_0^{+\infty} \\ = \frac{\pi}{4} - \frac{\pi}{8}$$

$$2. (99-2) \text{ 计算 } \int_1^{+\infty} \frac{\arctan x}{x^2} dx.$$

$$- \int_1^{+\infty} \arctan x d \frac{1}{x} \\ = - \left(\frac{\arctan x}{x} \Big|_0^{+\infty} - \int_1^{+\infty} \frac{1}{x(1+x^2)} dx \right) = - \left(-\frac{\pi}{4} - \frac{1}{2} \int_1^{+\infty} \frac{dx^2}{x^2(1+x^2)} \right) \\ = \frac{\pi}{4} + \frac{1}{2} \ln(x^2) \Big|_1^{+\infty} - \frac{1}{2} \ln(x^2+1) \Big|_1^{+\infty} = \frac{\pi}{4} + \frac{1}{2} \ln 2$$

$$3. (02-1) \int_e^{+\infty} \frac{dx}{x \ln^2 x} = \frac{1}{1}.$$

$$\int_e^{+\infty} \frac{d(\ln x)}{\ln^2 x} = \frac{-1}{\ln x} \Big|_e^{+\infty} = 1$$

$$4. (04-2) \int_1^{+\infty} \frac{dx}{x\sqrt{x^2-1}} = \frac{\frac{\pi}{2}}{2}. \quad \text{令 } x = \sec x$$

$$\int_0^{\frac{\pi}{2}} \frac{\sec x \cdot \tan x}{\sec x \cdot \tan x} dx = \frac{\pi}{2}$$

$$5. (06-2) \text{ 广义积分 } \int_0^{+\infty} \frac{xdx}{(1+x^2)^2} = \frac{1}{2}.$$

$$\frac{1}{2} \int_0^{+\infty} \frac{d(x^2+1)}{(1+x^2)^2} = \frac{1}{2} \frac{-1}{1+x^2} \Big|_0^{+\infty}$$

6. (87-3) 下列广义积分收敛的是

(A) $\int_e^{+\infty} \frac{\ln x}{x} dx.$

(B) $\int_e^{+\infty} \frac{dx}{x \ln x}.$

(C) $\int_e^{+\infty} \frac{dx}{x(\ln x)^2}.$

(D) $\int_e^{+\infty} \frac{dx}{x\sqrt{\ln x}}.$

A. $\frac{\ln x}{\frac{1}{x}} = \ln x \rightarrow \infty$

B. $\int_e^{+\infty} \frac{d(\ln x)}{\ln x} = \ln |\ln x| \Big|_e^{+\infty} \rightarrow \infty$

C. \checkmark

D. $\int_e^{+\infty} \frac{d(\ln x)}{\sqrt{\ln x}} = 2(\ln x)^{\frac{1}{2}} \rightarrow \infty$

A. 7. (95-3) 下列广义积分发散的是

(A) $\int_{-1}^1 \frac{1}{\sin x} dx.$

(B) $\int_{-1}^1 \frac{1}{\sqrt{1-x^2}} dx.$

(C) $\int_0^{+\infty} e^{-x^2} dx.$

(D) $\int_2^{+\infty} \frac{1}{x \ln^2 x} dx.$ 收.