

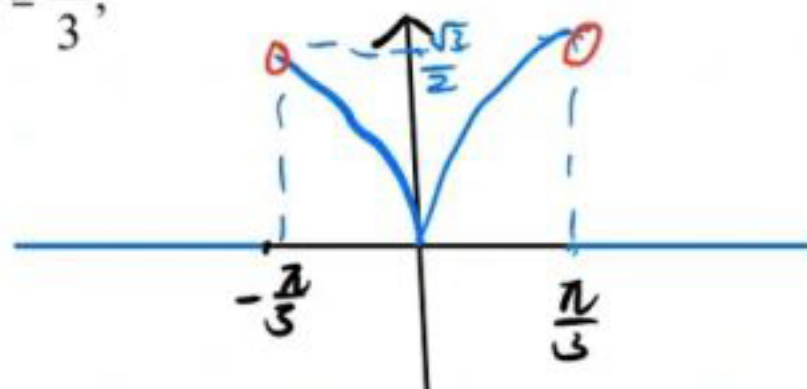
1. 设  $f(x) = \begin{cases} 0, & x \leq 0, \\ x, & x > 0, \end{cases}$   $g(x) = \begin{cases} 0, & x \leq 0, \\ -x^2, & x > 0, \end{cases}$  求  $f[f(x)], g[g(x)], f[g(x)], g[f(x)]$ .

$$\begin{aligned} x \leq 0 &\Rightarrow f[f(x)] = 0 & x \leq 0 &\Rightarrow f[g(x)] = 0 & x \leq 0 &\Rightarrow g[g(x)] = 0 & x \leq 0 &\Rightarrow g[f(x)] = 0 \\ x > 0 &\Rightarrow f[f(x)] = x & x > 0 &\Rightarrow f[g(x)] = 0 & x > 0 &\Rightarrow g[g(x)] = 0 & x > 0 &\Rightarrow g[f(x)] = -x^2 \\ \therefore f[f(x)] &= \begin{cases} 0, & x \leq 0, \\ x, & x > 0, \end{cases} & \therefore f[g(x)] &= 0 & g[g(x)] &= 0 & \therefore g[f(x)] &= \begin{cases} 0, & x \leq 0, \\ -x^2, & x > 0, \end{cases} \end{aligned}$$

2. 设  $\varphi(x) = \begin{cases} |\sin x|, & |x| < \frac{\pi}{3}, \\ 0, & |x| \geq \frac{\pi}{3}, \end{cases}$  求  $\varphi(\frac{\pi}{6}), \varphi(\frac{\pi}{4}), \varphi(-\frac{\pi}{4}), \varphi(-2)$ , 并作出函数  $y = \varphi(x)$  的图形.



$$\begin{aligned} \varphi(\frac{\pi}{6}) &= \frac{1}{2} \\ \varphi(\frac{\pi}{4}) &= \frac{\sqrt{2}}{2} \\ \varphi(-\frac{\pi}{4}) &= \frac{\sqrt{2}}{2} \\ \varphi(-2) &= 0 \end{aligned}$$



3. 下列函数中哪些是偶函数, 哪些是奇函数, 哪些既非偶函数又非奇函数?

(1)  $y = x^2(1-x^2)$ ; 偶

(2)  $y = 3x^2 - x^3$ ; 非奇非偶

(3)  $y = \frac{1-x^2}{1+x^2}$ ; 偶

(4)  $y = x(x-1)(x+1)$ ; 奇

(5)  $y = \sin x - \cos x + 1$ ;  $\sin(-x) + \cos(-x) + 1$  非奇非偶

(6)  $y = \frac{a^x + a^{-x}}{2}$ ; 偶

4. 下列各函数中哪些是周期函数? 对于周期函数, 指出其周期:

(1)  $y = \cos(x-2)$ ;  $2\pi \Rightarrow T = 2\pi$

(2)  $y = \cos 4x$ ;  $\frac{\pi}{2}$

(3)  $y = 1 + \sin \pi x$ ; 2

(4)  $y = x \cos x$ ; 非周期

(5)  $y = \sin^2 x$ ;  $\frac{1 - \cos 2x}{2}$   $\pi$   
 $\cos 2x$   
 $= 1 - 2\sin^2 x$



$$\begin{aligned} (1+x)y &= 1-x \\ y+xy+x &= 1 \\ x(y+1) &= 1-y \\ x &= \frac{1-y}{1+y} \end{aligned}$$

5. 求下列函数的反函数:

(1)  $y = \sqrt[3]{x+1}$ ;  $x+1 = y^3$   
 $x = y^3 - 1$   
 即  $y = \sqrt[3]{x-1}$

(2)  $y = \frac{1-x}{1+x}$ ; 即  $y = \frac{1-x}{1+x}$

(3)  $y = \frac{ax+b}{cx+d} (ad-bc \neq 0)$

(4)  $y = 2 \sin 3x (-\frac{\pi}{6} \leq x \leq \frac{\pi}{6})$

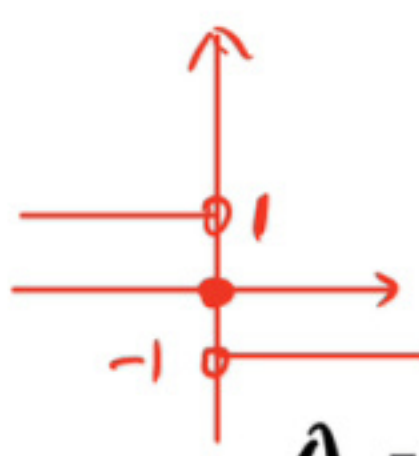
$$\begin{aligned} (cx+d)y &= ax+b \\ cyx - ax &= b-dy \\ x &= \frac{b-dy}{cy-a} \end{aligned}$$

即  $y = \frac{ax+b}{cx+d}$  的反函数为  $y = \frac{b-dx}{cx-a}$

$$\begin{aligned} \frac{y}{2} &= \sin 3x \\ 3x &= \arcsin \frac{y}{2} \\ x &= \frac{1}{3} \arcsin \frac{y}{2} \quad (-2 \leq y \leq 2) \end{aligned}$$

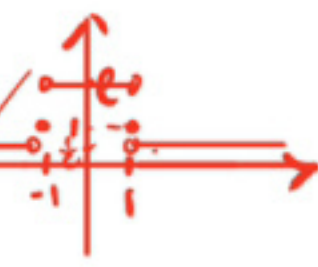
$\therefore y = 2 \sin 3x (-\frac{\pi}{6} \leq x \leq \frac{\pi}{6})$  的反函数为  $y = \frac{1}{3} \arcsin \frac{x}{2}$

6. 设  $f(x) = \begin{cases} 1, & |x| < 1, \\ 0, & |x| = 1, \\ -1, & |x| > 1, \end{cases}$   $g(x) = e^x$ , 求  $f[g(x)]$  和  $g[f(x)]$ , 并作出这两个函数的图形.



$$\begin{aligned} f[g(x)] &= \begin{cases} 1, & x < 0 \\ 0, & x = 0 \\ -1, & x > 0 \end{cases} \\ g[f(x)] &= \begin{cases} e, & |x| < 1 \\ 1, & |x| = 1 \\ e^{-1}, & |x| > 1 \end{cases} \end{aligned}$$

$$\begin{aligned} |x| < 1, & g[f(x)] = e \\ |x| = 1, & g[f(x)] = 1 \\ |x| > 1, & g[f(x)] = e^{-1} \end{aligned}$$



7. 设函数  $f(x) = x \cdot \arctan x \cdot e^{\cos x}$ , 则  $f(x)$  是

- (A) 偶函数. (B) 有界函数. (C) 周期函数. (D) 单调函数.



$$e^{\cos(-x)} = e^{\cos x}$$

8. 设函数  $f(x) = \begin{cases} 0, & |x| \leq 1, \\ 1, & |x| > 1, \end{cases}$  则  $f[f(x)] =$

$$\begin{aligned} |x| \leq 1 &\Rightarrow 0 \\ |x| > 1 &\Rightarrow 1 \Rightarrow 0 \end{aligned}$$

$\because f(x) = \arcsin x$  值域为  $[-\frac{\pi}{2}, \frac{\pi}{2}]$   
 $\therefore f[f(x)] = \frac{\pi}{2} - x^2$  的值域为  $[-\frac{\pi}{2}, \frac{\pi}{2}]$   
 $\frac{\pi}{2} - x^2 \in [-\frac{\pi}{2}, \frac{\pi}{2}]$   
 $\therefore x \in [-\sqrt{\pi}, \sqrt{\pi}]$

9. 设  $f(x) = \arcsin x$ ,  $f[\varphi(x)] = \frac{\pi}{2} - x^2$ , 则  $\varphi(x) = \sin(\frac{\pi}{2} - x^2)$ ; 其定义域为  $[-\sqrt{\pi}, \sqrt{\pi}]$

$$\begin{aligned} \arcsin \varphi(x) &= \frac{\pi}{2} - x^2 \\ \varphi(x) &= \sin(\frac{\pi}{2} - x^2) \end{aligned}$$

10. (1) 幂函数:  $y = x, y = x^2, y = \sqrt{x}, y = x^3, y = \sqrt[3]{x}, y = \frac{1}{x}$

(2) 指数函数:  $y = a^x (a > 0 \text{ 且 } a \neq 1)$ .

(3) 对数函数:  $y = \log_a x (a > 0 \text{ 且 } a \neq 1)$ .

(4) 三角函数:  $y = \sin x, y = \cos x, y = \tan x, y = \cot x$ .

(5) 反三角函数:  $y = \arcsin x, y = \arccos x, y = \arctan x, y = \text{arc cot } x$ .

