Q No.	**PAPER NAME** Uncertainty Modelling for Intelligent Systems **Q.SETTER INITIALS** JL	Marks
1 (a)	$m = \{i\} : 0.1,\ \{d,s\} : 0.2,\ \{d,i\} : 0.1,\ \{d,s,i\} : 0.6$	4
1 (b)	$Bel(\{d,i\}) = m(\{d,i\}) + m(\{i\}) = 0.1 + 0.1 = 0.2$	1
	$Pl(\{d,i\}) = m(\{d,i,s\}) + m(\{d,i\}) + m(\{d,s\}) + m(\{i\}) = 1$	1
1 (c)	$P(i\|m) = \frac{m(\{d,i,s\})}{3} + \frac{m(\{d,i\})}{2} + \frac{m(\{i\})}{1} = \frac{0.6}{3} + \frac{0.1}{2} + 0.1 = 0.35$	2
	$P(d\|m) = \frac{m(\{d,i,s\})}{3} + \frac{m(\{d,i\})}{2} + \frac{m(\{d,s\})}{2} = \frac{0.6}{3} + \frac{0.1}{2} + \frac{0.2}{2} = 0.35$	2
	$P(s\|m) = \frac{m(\{d,i,s\})}{3} + \frac{m(\{d,s\})}{2} = \frac{0.6}{3} + \frac{0.2}{2} = 0.3$	2
1 (d)	The mass function from the second survey is $m_2 = \{d,s\} : 0.2,\ \{d,i\} : 0.4, \{d,s,i\} : 0.4$	1

$m_1 \oplus m_2$	$\{i\} : 0.1$	$\{d,s\} : 0.2$	$\{d,i\} : 0.1$	$\{d,i,s\} : 0.6$
$\{d,s\} : 0.2$	$\emptyset : 0.02$	$\{d,s\} : 0.04$	$\{d\} : 0.02$	$\{d,s\} : 0.12$
$\{d,i\} : 0.4$	$\{i\} : 0.04$	$\{d\} : 0.08$	$\{d,i\} : 0.04$	$\{d,i\} : 0.24$
$\{d,s,i\} : 0.4$	$\{i\} : 0.04$	$\{d,s\} : 0.08$	$\{d,i\} : 0.04$	$\{d,i,s\} : 0.24$

(2 marks)

	$m_1 \oplus m_2 = \{d,s\} : \frac{0.04+0.12+0.08}{0.98} = 0.24490,\ \{d,i\} : \frac{0.04+0.24+0.04}{0.98} = 0.32653$ $\{i\} : \frac{0.04+0.04}{0.98} = 0.08163,\ \{d\} : \frac{0.02+0.08}{0.98} = 0.10204, \{d,i,s\} : \frac{0.24}{0.98} = 0.24450$	3
1 (e)	$Bel(\{d,i\}) = m(\{d,i\}) + m(\{d\}) + m(\{i\}) = 0.32653 + 0.10204 + 0.08163 = 0.5102$	1
	$Pl(\{d,i\}) = m(\{d,i,s\}) + m(\{d,s\}) + m(\{d,i\}) + m(\{d\}) + m(\{i\}) = 1$	
	Hence, there is no change to the plausibility which remains at 1, but there is a substantial increase in belief from 0.2 to 0.5102.	1

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Q No.	**PAPER NAME** Uncertainty Modelling for Intelligent Systems **Q.SETTER INITIALS** JL	Marks
2 (a)	$P(B\|G) = 0.3$ and $P(B\|G^c) = 0.1$	1
	By Bayes theorem $P(G\|B) = \frac{P(B\|G)P(G)}{P(B)} = \frac{P(B\|G)P(G)}{P(B\|G)P(G)+P(B\|G^c)(1-P(G))}$	2
	$= \frac{0.3P(G)}{0.3P(G)+0.1(1-P(G))} = \frac{0.3P(G)}{0.2P(G)+0.1}$	1
	The upper bound is when $P(G) = 0.6$ which gives $\frac{0.3(0.6)}{0.2(0.6)+0.1} = 0.8182$	1
	The lower bound is when $P(G) = 0.4$ which gives $\frac{0.3(0.4)}{0.2(0.4)+0.1} = 0.6667$	1
2 (b)	Let $p(w_i) = p_i$ then from the knowledge base $p_1 = \frac{1}{4}(p_1 + p_3)$ which implies $p_3 = 3p_1$	2
	Hence, $H = -p_1 \log_2(p_1) - p_2 \log_2(p_2) - 3p_1 \log_2(3p_1) - (1 - 4p_1 - p_2) \log_2(1 - 4p_1 - p_2)$	1
	$\frac{\partial H}{\partial p_2} = 0 \Rightarrow -\log_2(p_2) + \log_2(1 - 4p_1 - p_2) = 0 \Rightarrow p_2 = 1 - 4p_1 - p_2$	1
	$\frac{\partial H}{\partial p_1} = 0 \Rightarrow -\log_2(p_1) - 3\log_2(3p_1) + 4\log_2(1 - 4p_1 - p_2) = 0$	2
	$\Rightarrow p_1(3p_1)^3 = (1 - 4p_1 - p_2)^4 \Rightarrow 3^3 p_1^4 = p_2^4 \Rightarrow p_2 = 3^{\frac{3}{4}} p_1$	
	Substituting gives $3^{\frac{3}{4}} p_1 = 1 - 4p_1 - 3^{\frac{3}{4}} p_1 \Rightarrow p_1 = \frac{1}{2(3^{\frac{3}{4}})+4}$	1
	$P(G^c \cap H^c) = p_4 = p_2 = 3^{\frac{3}{4}} p_1$	1
	$p_2 = \frac{3^{\frac{3}{4}}}{2(3^{\frac{3}{4}})+4} = 0.26633$	1
2 (c)	$P(X_1, X_2, X_3, X_4, X_5, X_6) = P(X_1)P(X_2)P(X_3\|X_1, X_2)P(X_4\|X_3)P(X_5\|X_3)P(X_6\|X_4, X_5)$	2
	$1 + 1 + 4 + 2 + 2 + 4 = 14$	2
	$2^6 - 1 = 63$	1

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Q No.	**PAPER NAME** Uncertainty Modelling for Intelligent Systems **Q.SETTER INITIALS** JL	Marks
3 (a)	**1)** $Pos(\emptyset) = 0,\ Pos(W) = 1$**, 2)** $Pos(A \cup B) = \max(Pos(A), Pos(B))$	2
	$1 = Pos(W) = Pos(A \cup A^c) = \max(Pos(A), Pos(A^c))$	1
3 (b)(i)	$m := \{0,1,2,3,4\} : 0.1,\ \{0,1,2,3\} : 0.1,\ \{0,1,2\} : 0.3,\ \{0,1\} : 0.1,\ \{0\} : 0.4$	5
3 (b) (ii)	$Nec(\{0,1,2\}) = m(\{0,1,2\}) + m(\{0,1\}) + m(\{0\}) = 0.4 + 0.1 + 0.3 = 0.8$	2
3 (c)	A Kripke interpretation is a pair $< W, E >$ where W is a set of possible worlds and $E : W^2 \to \{0,1\}$ is a relation on W.	2
3 (d)(i)	$\{w_4\}$	1
3 (d) (ii)	$\{w_1, w_3, w_4\}$	1
3 (d) (iii)	$\{w_3, w_4\}$	2
3 (e)	If $w \models \Box(F \to G)$ then for all w' where $E(w,w') = 1$, $w' \models F \to G$	1
	Now suppose $w \not\models \Box F \to \Box G$ then $w \models \Box F$ and $w \not\models \Box G$	1
	Hence, there exists $w' \in W$ such that $E(w,w') = 1$ and $w' \models F$ and $w' \not\models G$	1
	Therefore, $w' \not\models F \to G$ which is a contradiction.	1

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Q No.	**PAPER NAME** Uncertainty Modelling for Intelligent Systems **Q.SETTER INITIALS** JL	Marks
4 (a)	Consider a man with a lot of hair on his head. One hair is plucked at a time until he is bald. It follows that there must be a hair pluck at which the man switches from being not bald to being bald. However, this is counter intuitive since the difference of one hair cannot induce a category change from not bald to bald. This or an alternative version of sorites.	4
4 (b)	$S(x,y) = 1 - T(1-x, 1-y) = 1 - \frac{(1-x)(1-y)}{(1-x)+(1-y)-(1-x)(1-y)}$	3
	$= 1 - \frac{(1-x)(1-y)}{1-xy} = \frac{x+y-2xy}{1-xy}$	1
4(c)	First we show that min is the only idempotent t-norm.	
	Suppose T is idempotent and w.l.o.g assume $x \leq y$.	1
	Then by idempotence and the axioms of a t-norm $x = T(x,x) \leq T(x,y) \leq T(x,1) = x$.	2
	Hence, $T(x,y) = x = \min(x,y)$.	1
	We now show by a counter example that min does not satisfy the law of non-contradiction.	
	Let $\chi_{\tilde{A}}(w) = 0.5$ and therefore $\chi_{\tilde{A}^c}(w) = 0.5$. Hence, $\chi_{\tilde{A}\cap\tilde{A}^c}(w) = \min(0.5, 0.5) = 0.5 \neq 0$.	2
4 (d)	$\tilde{A}_\alpha = \{4,5,6\} : \alpha \in [0, 0.2), \{4,5\} : \alpha \in (0.2, 0.4], \{4\} : \alpha \in (0.4, 1]$	1
	$\tilde{B}_\alpha = \{1,2,3\} : \alpha \in (0, 0.5], \{1,2\} : \alpha \in (0.5, 0.7], \{1\} : \alpha \in (0.7, 1]$	
	$2\tilde{A}_\alpha = \{8,10,12\} : \alpha \in [0, 0.2), \{8,10\} : \alpha \in (0.2, 0.4], \{8\} : \alpha \in (0.4, 1]$	1
	$\tilde{B}^2_\alpha = \{1,4,9\} : \alpha \in (0, 0.5], \{1,4\} : \alpha \in (0.5, 0.7], \{1\} : \alpha \in (0.7, 1]$	

$2\tilde{A}_\alpha - \tilde{B}^2_\alpha$	$\{8,10,12\} : [0, 0.2)$	$\{8,10\} : (0.2, 0.4]$	$\{8\} : (0.4, 1]$
$\{1,4,9\} : (0, 0.5]$	$\{7,4,-1,9,6,1,11,8,3\} : (0, 0.2]$	$\{7,9,4,6,-1,1\} : (0.2, 0.4]$	$\{7,4,-1\} : (0.4, 0.5]$
$\{1,4\} : (0.5, 0.7]$	$\emptyset$	$\emptyset$	$\{7,4\} : (0.5, 0.7]$
$\{1\} : (0.7, 1]$	$\emptyset$	$\emptyset$	$\{7\} : (0.7, 1]$

(Marks for table: 2)

	$f(\tilde{A}_\alpha, \tilde{B}_\alpha) = \{7,4,-1,9,6,1,11,8,3\} : \alpha \in (0, 0.2], \{7,9,4,6,-1,1\} : \alpha \in (0.2, 0.4], \{7,4,1\} : \alpha \in (0.4, 0.5], \{7,4\} : \alpha \in (0.5, 0.7], \{7\} : \alpha \in (0.7, 1]$	1
	$f(\tilde{A}, \tilde{B}) = 7/1 + 4/0.7 + 1/0.5 + 9/0.4 + 6/0.4 + -1/0.4 + 11/0.2 + 8/0.2 + 3/0.2$	1