Q1. In a far away country the region of Scotalonia is about to take part in a referendum concerning its future. The referendum will offer three possible choices; status quo(s), devolution (d) and independence (i). A series of interviews carried out on a sample of the Scotalonian population reveals the following voting intentions: 10% say that they will vote for independence, 20% say that they will not vote for independence, 10% say that they will not vote for the status quo, and 60% say that they are undecided how to vote.

(a) Write down a mass function to represent this information.

(4 marks)

(b) Find the belief and plausibility that the vote will be against the status quo.

(2 marks)

(c) Assuming a uniform prior distribution on the outcome, evaluation the posterior distribution conditional on the information provided by the survey i.e. determine the pignistic distribution of the mass function in part (a).

(6 marks)

A second independent survey is carried out with the following results: 20% say that they will not vote for independence, 40% say that they will not vote for the status quo, and 40% say that they are undecided.

(d) Use Dempster's rule to combine the information from the two surveys.

(6 marks)

(e) Describe how this new evidence has changed the belief and plausibility of a vote against the status quo.

(2 marks)

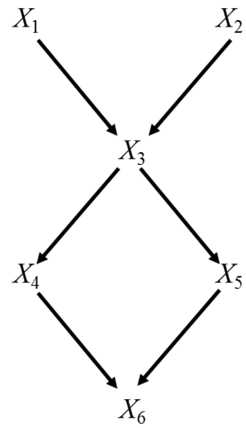

Figure 1: Bayesian network for question 2(c)

Q2. (a) A particular gene G has been linked to hair loss in men. It is known that 30% of men with gene G become bald. However, 10% of men without gene G also become bald. Finally, it is estimated that between 40% and 60% of men have gene G. Given that Bill is bald determine lower and upper bounds on the probability that he has gene G

(6 marks)

(b) Suppose that two genes A and B have been linked to a certain medical condition. In this context there are 4 possible states of the world:

- w_1 in which both A and B are present.
- w_2 in which A is present but not B.
- w_3 in which B is present but not A.
- w_4 in which neither A nor B are present.

Give the knowledge base:

$$P(A|B) = \frac{1}{4}$$

Use the maximum entropy inference process to determine a probability value of an individual having neither gene.

(9 marks)

(c) Given the Bayesian network shown in figure 1 write down an expression for the joint probability distribution on $X_1, X_2, X_3, X_4, X_5, X_6$.

(2 marks)

If $X_1, X_2, X_3, X_4, X_5, X_6$ are all binary variables then determine how many probability values must be specified in order to completely evaluate this joint distribution.

(2 marks)

How many probability values would be required to determine the joint distribution if the network were totally connected?

(1 mark)

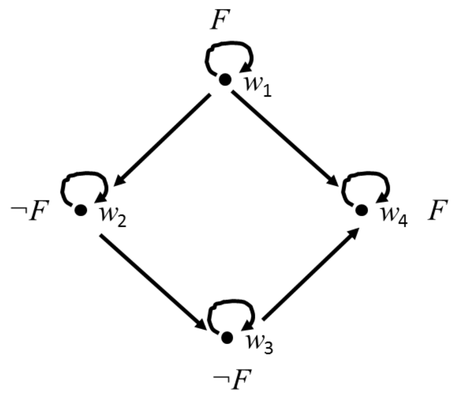

Figure 2: Kripke interpretation for question 3(d)

Q3. (a) State the axioms of a possibility measure $Pos : 2^W \to [0,1]$ and use these to prove that for any set $A \subseteq W$, either $Pos(A) = 1$ or $Pos(A^c) = 1$.

(3 marks)

(b) A lecture course has 4 people registered on it. This means that the number of attendees at a given lecture is in the set $W = \{0, 1, 2, 3, 4\}$. Suppose we have the following possibility distribution defined over W:

$$\pi(0) = 1,\ \pi(1) = 0.6, \pi(2) = 0.5,\ \pi(3) = 0.2,\ \pi(4) = 0.1$$

(i) Determine the underlying mass function for this possibility distribution.

(5 marks)

(ii) Use this mass function in order to evaluate the necessity measure that two or less people will attend the lecture.

(2 marks)

(c) In modal logic, give the definition of a Kripke interpretation.

(2 marks)

(d) Consider the Kripke interpretation in figure 2 showing the relation between possible worlds $W = \{w_1, w_2, w_3, w_4\}$ and indicating for each world whether a formula F or its negation $\neg F$ holds. Give the set of worlds in which the following modal logic sentences are true:

(i) $\Box F$

(1 mark)

(ii) $\Diamond F$

(1 mark)

(iii) $\Box\Diamond F$

(2 marks)

turn over/Qu. continues . . .

(e) Prove that for any Kripke interpretation $\Box(F \to G) \models \Box F \to \Box G$.

(4 marks)

Q4. (a) Using an example briefly explain what is meant by a Sorites paradox.

(4 marks)

(b) Show that the dual T-conorm of the T-norm $T(x,y) = \frac{xy}{x+y-xy}$ is given by $S(x,y) = \frac{x+y-2xy}{1-xy}$.

(4 marks)

(c) Prove that there is no T-norm satisfying both idempotence and the law of non-contradiction.

(6 marks)

(d) Let the function $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x,y) = 2x - y^2$. Use the α-cut method to evaluate $f(\tilde{A}, \tilde{B})$ where:

$$\tilde{A} = 4/1 + 5/0.4 + 6/0.2 \text{ and } \tilde{B} = 1/1 + 2/0.7 + 3/0.5$$

(6 marks)