Q No.	**PAPER NAME** Uncertainty Modelling for Intelligent Systems **Q.SETTER INITIALS** JL	Marks
1(a)(i)	P1: $P(W)=1$ and $P(\emptyset)=0$, P2: If $A\cap B=\emptyset$ then $P(A\cup B)=P(A)+P(B)$.	2
1(a)(ii)	Since $A\subseteq B$ then $B=A\cup(B\cap A^c)$ where A and $B\cap A^c$ do not overlap. Hence by P2 $P(B)=P(A)+P(B\cap A^c)\geq P(A)$.	3
1(b)(i)	By Bayes theorem $P(H_i\vert E)=\frac{P(E\vert H_i)P(H_i)}{P(E)}$ and since $P(E)$ is constant for each H_i we have that $P(H_i\vert E)\propto P(E\vert H_i)P(H_i)$.	2
	Now $P(E\vert H_1)P(H_1)=0.8\times 0.2=0.16$, $P(E\vert H_2)P(H_2)=0.6\times 0.7=0.42$ and $P(E\vert H_3)P(H_3)=0.5\times 0.1=0.05$.	1
	Hence, H_2 is most probable give E.	1
1(b)(ii)	By the theorem of total probability $P(E)=P(E\vert H_1)P(H_1)+P(E\vert H_1^c)(1-P(H_1))=0.8\times 0.2+0.3.8=0.4$.	2
	Hence, by Bayes theorem $P(H_1\vert E)=\frac{0.8\times 0.2}{0.4}=0.4$	1
1(c)(i)	The gain for Bet 1 is $S(1-p)\chi_A(w^*)-Sp(1-\chi_A(w^*))=S((1-p)\chi_A(w^*)-p(1-\chi_A(w^*)))=S(\chi_A(w^*)-p\chi_A(w^*)-p+p\chi_A(w^*))=S(\chi_A(w^*)-p)$	3
	The gain for Bet 2 is $-S(1-p)\chi_A(w^*)+Sp(1-\chi_A(w^*))=-$Bet 1 gain $=-S(\chi_A(w^*)-p)$	1
1(c)(ii)	For odds p' then Bet 1 gain is $S(\chi_A(w^*)-p')\geq S(\chi_A(w^*)-p)=$ Bet 1 gain for odds p	2
	Also, the gain for Bet 2 given odds p' is $-S(\chi_A(w^*)-p')\leq -S(\chi_A(w^*)-p)=$ Bet 2 gain for odds p	1
	Hence, if the agent chooses Bet 1 when offered odd p then she should also choose Bet 1 when offered odd p' since in the latter case the Bet 1 gain will be higher and the Bet 2 gain lower than in the former case.	1

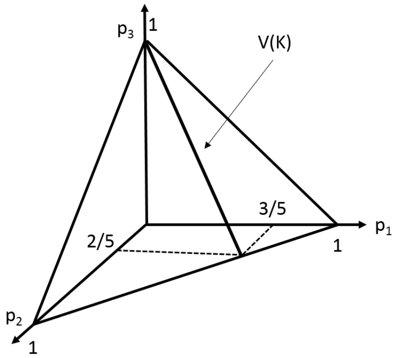
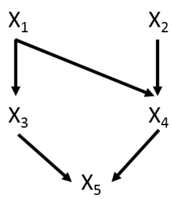
Continued on next page...

Q No.	**PAPER NAME** Uncertainty Modelling for Intelligent Systems **Q.SETTER INITIALS** JL	Marks
2(a)(i)	$\{A,S,C\}:0.2,\ \{S,C\}:0.4,\ \{S\}:0.3,\ \{A,C\}:0.1.$	4
2(a)(ii)	$Bel(\{S,C\}) = m(\{S,C\}) + m(\{S\}) = 0.7,\ Pl(\{S,C\}) = 1.$	2
2(b)(i)	$P(A\|m) = \sum_{B\subseteq W} P(A\|B)m(B)$	2
2(b)(ii)	$P(A\|m) = \sum_{B\subseteq W} P(A\|B)m(B) = \sum_{B\subseteq A} P(A\|B)m(B) + \sum_{B\nsubseteq A} P(A\|B)m(B)$	2
	$= \sum_{B\subseteq A} m(B) + \sum_{B\nsubseteq A} P(A\|B)m(B)$	1
	$= Bel(A) + \sum_{B\nsubseteq A} P(A\|B)m(B) \geq Bel(A)$	1
	Similarly, $P(A^c\|m) \geq Bel(A^c)$ and hence $P(A\|m) = P(A^c\|m) \leq 1 - Bel(A^c) = Pl(A)$	1
2(c)	(table below)	2

m_1/m_2	$\{w_1,w_2,w_4\}:0.3$	$\{w_4\}:0.5$	$\{w_2,w_3\}:0.2$
$\{w_1,w_2\}:0.4$	$\{w_1,w_2\}:0.12$	$\emptyset:0.2$	$\{w_2\}:0.08$
$\{w_1,w_3\}:0.4$	$\{w_1\}:0.12$	$\emptyset:0.2$	$\{w_3\}:0.08$
$\{w_3,w_4\}:0.2$	$\{w_4\}:0.06$	$\{w_4\}:0.1$	$\{w_3\}:0.04$

	$m_1 \oplus m_2 := \{w_1,w_2\}:\frac{0.12}{0.6} = 0.2,\ \{w_1\}:\frac{0.12}{0.6} = 0.2, \{w_2\}:\frac{0.08}{0.6} = 0.1333,\ \{w_3\}:\frac{0.08+0.04}{0.6} = 0.2,\ \{w_4\}:\frac{0.1+0.06}{0.6} = 0.2667$	1
2(c)(ii)	$m \oplus m_B(C) = \frac{\sum_{D:B\cap D=C} m(D)}{1-\sum_{D:D\cap B=\emptyset} m(D)} = \frac{\sum_{D:B\cap D=C} m(D)}{\sum_{D:D\cap B\neq\emptyset} m(D)} = \frac{\sum_{D:B\cap D=C} m(D)}{Pl(B)}$	2
	Hence, $Pl(A\|B) = \frac{\sum_{C:C\cap A\neq\emptyset}\sum_{D:B\cap D=C} m(D)}{Pl(B)} = \frac{\sum_{D:D\cap(A\cap B)\neq\emptyset} m(D)}{Pl(B)} = \frac{Pl(A\cap B)}{Pl(B)}$	2

Continued on next page...

Q No.	**PAPER NAME** Uncertainty Modelling for Intelligent Systems **Q.SETTER INITIALS** JL	Marks
3(a)(i)	Let $P(r) = p_1,\ P(g) = p_2, P(b) = p_3$. Then $K = P(r \mid \{r, g\}) = 0.6$	2
		2
3(a)(ii)	Entropy is given by $H = -\frac{3}{2}p_2 \log_2(\frac{3}{2}p_2) - p_2 \log_2(p_2) - (1 - \frac{5}{2}p_2)\log_2(1 - \frac{5}{2}p_2)$	3
	$\frac{dH}{dp_2} = -\frac{3}{2}\log_2(\frac{3}{2}p_2) - \log_2(p_2) + \frac{5}{2}\log_2(1 - \frac{5}{2}p_2)$	1
	$\frac{dH}{dp_2} = 0 \Rightarrow \log_2((\frac{3}{2}p_2)^{\frac{3}{2}}p_2) = \log_2((1 - \frac{5}{2}p_2)^{\frac{5}{2}}) \Rightarrow (\frac{3}{2})^{\frac{3}{2}}p_2^{\frac{5}{2}} = (1 - \frac{5}{2}p_2)^{\frac{5}{2}}$	1
	$\Rightarrow (\frac{3}{2})^{\frac{3}{5}}p_2 = (1 - \frac{5}{2}p_2) \Rightarrow p_2 = \frac{1}{(\frac{3}{2})^{\frac{3}{5}} + \frac{5}{2}} = 0.26487$	1
3(b)(i)	$P(X_1, \ldots, X_n) = P(X_1)\frac{P(X_1, X_2)}{P(X_1)} \ldots P(X_1, \ldots X_{n-1})\frac{P(X_1, \ldots, X_n)}{P(X_1, \ldots X_{n-1})} = \prod_{i=1}^{n} P(X_i \mid X_1, \ldots X_{i-1}) = \prod_{i=1}^{n} P(X_i \mid \Pi(X_i))$	3
3(b)(ii)	Total values $1 + \sum_{i=2}^{n} 2^{i-1} = 1 + \sum_{i=0}^{n-2} 2^i = 1 + \frac{1(1-2^{n-1})}{1-2}$ (geometric series $a = 1$, $r = 2$) $= 2^{n-1}$	3
3 (b) (iii)		4

Continued on next page...

<table>
<tr><th>Q No.</th><th>PAPER NAME Uncertainty Modelling for Intelligent Systems Q.SETTER INITIALS JL</th><th>Marks</th></tr>
<tr><td>4(a)</td><td>$\tilde{A}_\alpha = \begin{cases} \{5,6,7\} : \alpha \in (0, 0.5] \\ \{5,6\} : \alpha \in (0.5, 0.8] \\ \{5\} : \alpha \in (0.8, 1] \end{cases}$ and $\tilde{B}_\alpha = \begin{cases} \{1,2,3\} : \alpha \in (0, 0.2] \\ \{1,2\} : \alpha \in (0.2, 0.5] \\ \{1\} : \alpha \in (0.5, 1] \end{cases}$</td><td>1</td></tr>
<tr><td></td><td><table>
<tr><th>$\frac{\tilde{A}_\alpha}{\tilde{B}_\alpha}$</th><th>$\{5,6,7\} : (0, 0.5]$</th><th>$\{5,6\} : (0.5, 0.8]$</th><th>$\{5\} : (0.8, 1]$</th></tr>
<tr><td>$\{1,2,3\} : (0, 0.2]$</td><td>$\{5,6,7,\frac{5}{2},3,\frac{7}{2},\frac{5}{3},2,\frac{7}{3}\} : (0, 0.2]$</td><td>$\emptyset$</td><td>$\emptyset$</td></tr>
<tr><td>$\{1,2\} : (0,2,0.5]$</td><td>$\{5,6,7,\frac{5}{2},3,\frac{7}{2}\} : (0.2, 0.5]$</td><td>$\emptyset$</td><td>$\emptyset$</td></tr>
<tr><td>$\{1\} : (0.5, 1]$</td><td>$\emptyset$</td><td>$\{5,6\} : (0.5, 0.8]$</td><td>$\{5\} : (0.8, 1]$</td></tr>
</table></td><td>2</td></tr>
<tr><td></td><td>$(\frac{\tilde{A}}{\tilde{B}})_\alpha = \begin{cases} \{5,6,7,\frac{5}{2},3,\frac{7}{2},\frac{5}{3},2,\frac{7}{3}\} : \alpha \in (0, 0.2] \\ \{5,6,7,\frac{5}{2},3,\frac{7}{2}\} : \alpha \in (0.2, 0.5] \\ \{5,6\} : \alpha \in (0.5, 0.8] \\ \{5\} : \alpha \in (0.8, 1] \end{cases}$</td><td>1</td></tr>
<tr><td></td><td>$\frac{\tilde{A}}{\tilde{B}} = 5/1 + 6/0.8 + 7/0.5 + \frac{5}{2}/0.5 + 3/0.5 + \frac{7}{2}/0.5 + \frac{5}{3}/0.2 + 2/0.2 + \frac{7}{3}/0.2$</td><td>1</td></tr>
<tr><td>4(b)</td><td>$\{\alpha : w \in \tilde{A}_\alpha \cap \tilde{B}_\alpha\} = \{\alpha : \chi_{\tilde{A}}(w) \geq \alpha, \chi_{\tilde{B}}(w) \geq \alpha\} = (0, \min(\chi_{\tilde{A}}(w), \chi_{\tilde{B}}(w))]$</td><td>2</td></tr>
<tr><td></td><td>Hence, $\sup\{\alpha : w \in \tilde{A}_\alpha \cap \tilde{B}_\alpha\} = \sup(0, \min(\chi_{\tilde{A}}(w), \chi_{\tilde{B}}(w))] = \min(\chi_{\tilde{A}}(w), \chi_{\tilde{B}}(w))$</td><td>2</td></tr>
<tr><td>4(c)</td><td>T1: $\forall x \in [0,1]\ T(x,1) = x$. T2: $\forall x, y, z \in [0,1]$, if $y \leq z$ then $T(x,y) \leq T(x,z)$. T3: $\forall x, y \in [0,1]$, $T(x,y) = T(y,x)$. T4: $\forall x, y, z \in [0,1]$, $T(x, T(y,z)) = T(T(x,y), z)$.</td><td>4</td></tr>
<tr><td>4(d)</td><td>Upper bound: $T(x,y) \leq T(x,1) = x$ by T1 and T2. Also $T(x,y) = T(y,x) \leq T(y,1) = y$ by T3,T2,T1. Hence, $T(x,y) \leq \min(x,y)$.</td><td>2</td></tr>
<tr><td></td><td>Lower bound: By T1 $T(x,1)$ and by T3 and T1 $T(1,y) = y$. Also, if $x, y < 1$ then $T(x,y) \geq 0$ as required.</td><td>2</td></tr>
<tr><td>4(e)</td><td>$\Box A$ is true in w_1 and w_3.</td><td>3</td></tr>
</table>