Performance Analysis

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Introduction

In HPC we care about performance.

It's easy to make a code go faster, but how do you know when it's really performing **well**?

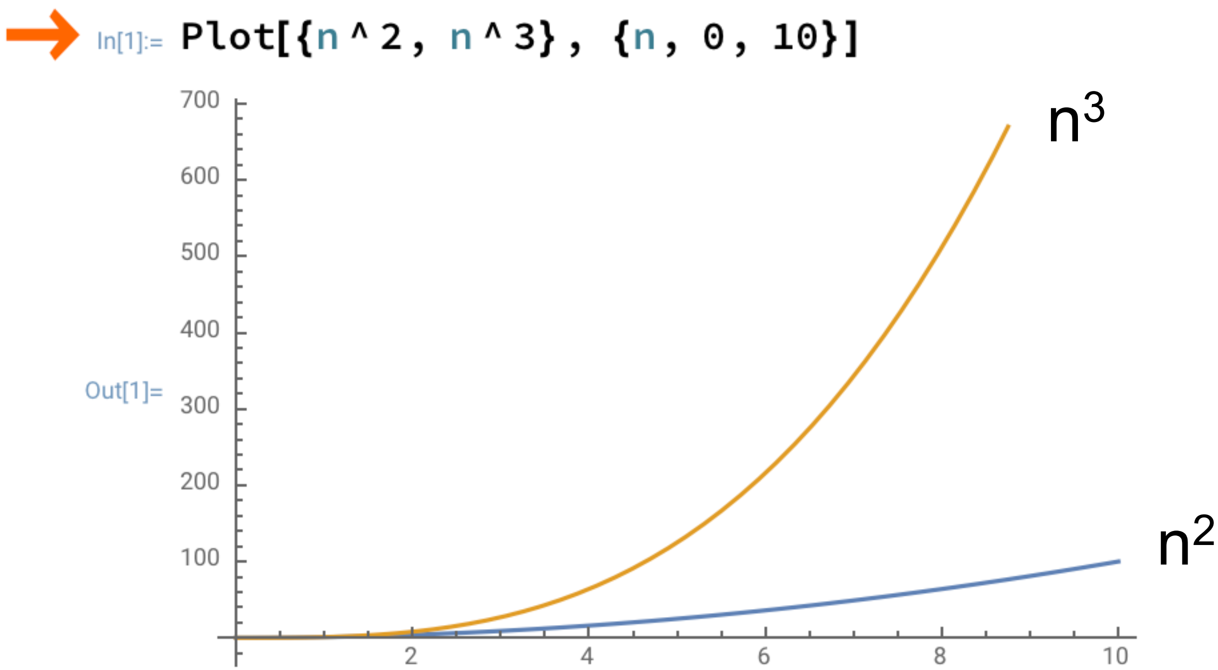
We need ***performance analysis***

Performance analysis

- Understand the characteristics of the algorithm
  - What is the overall algorithmic complexity?
    - For compute?
    - For data movement?

- E.g. vector-vector and vector-matrix operations are O(n) for both compute and data movement, but matrix-matrix multiply is $O(n^3)$ compute to $O(n^2)$ data movement

$O(n^2)$ vs $O(n^3)$



- Implication: matrix-matrix multiply becomes compute-bound at big enough *n*, but all vector-vector or vector-matrix operations remain memory bandwidth bound

What is your rate limiting factor?

- Most HPC codes are
  memory bandwidth bound
- A few are **compute bound**
- Other possibilities:
  - Network bound (e.g. MPI communication)
  - I/O bound (e.g. writing to disk)
  - Memory latency bound
  - Memory capacity bound
  - …

Which one am I?

- Try to reason about this on paper first
- Remember: floating point and integer operations are generally very cheap
  - O(1) cycle for most int / float / double operations
  - Exceptions: divide, transcendentals (sin, cos), exp, log
  - Load/store 2-3 times slower in the best case (hitting L1 cache), can be much slower if missing the cache
- You could simply count bytes loaded and stored vs. floating point operations…

Example: Jacobi

```
for (row = 0; row < N; row++) {
  dot = 0.0;                                    O(N)
  for (col = 0; col < N; col++) {
    if (row != col)                             O(N2)
      dot += A[row + col*N] * x[col];
  }
  xtmp[row] =
    (b[row] - dot) / A[row + row*N];
}
```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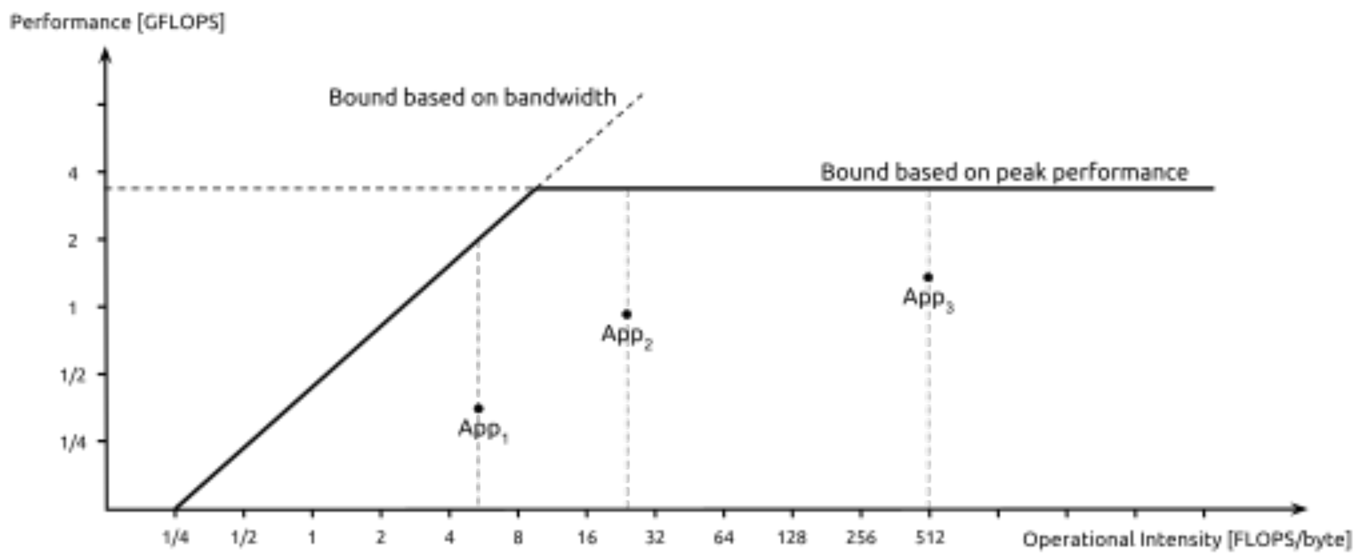
Analysis Process

1. Think about the algorithm and select the best one you can
   – E.g. $O(n\log n)$ beats $O(n^2)$ etc.
2. Establish the rate limiting factor
   – E.g. memory bandwidth bound or compute bound
3. Optimise primarily for the rate limiting factor
4. Analyse the results to establish where to focus your efforts next
   – There are tools that can help you – profilers etc.

Roofline model

- A useful conceptual tool to establish whether compute bound or memory bandwidth bound



Samuel Williams, Andrew Waterman, and David Patterson. Roofline: an insightful visual performance model for multicore architectures. *Commun. ACM* 52, 4 (April 2009), 65-76. DOI: https://doi.org/10.1145/1498765.1498785

Some definitions

- **Operational Intensity (OI)**
  - Operations per byte of memory traffic
  - An operation could be floating point, integer …
  - Traffic is measured at main memory (DRAM)
    - i.e. the number of bytes transferred between the last level cache and memory, rather than between the processor and the caches
  - Also known as “arithmetic” or “computational” intensity

- Example:
  - **double a[], b[], c[];   a[i] += b[i] * c[i];**
  - 24 bytes loaded, 8 bytes stored, 2 operations (+, *)
  - → Operational intensity of (ops)/(bytes) = 2/32 = 1/16

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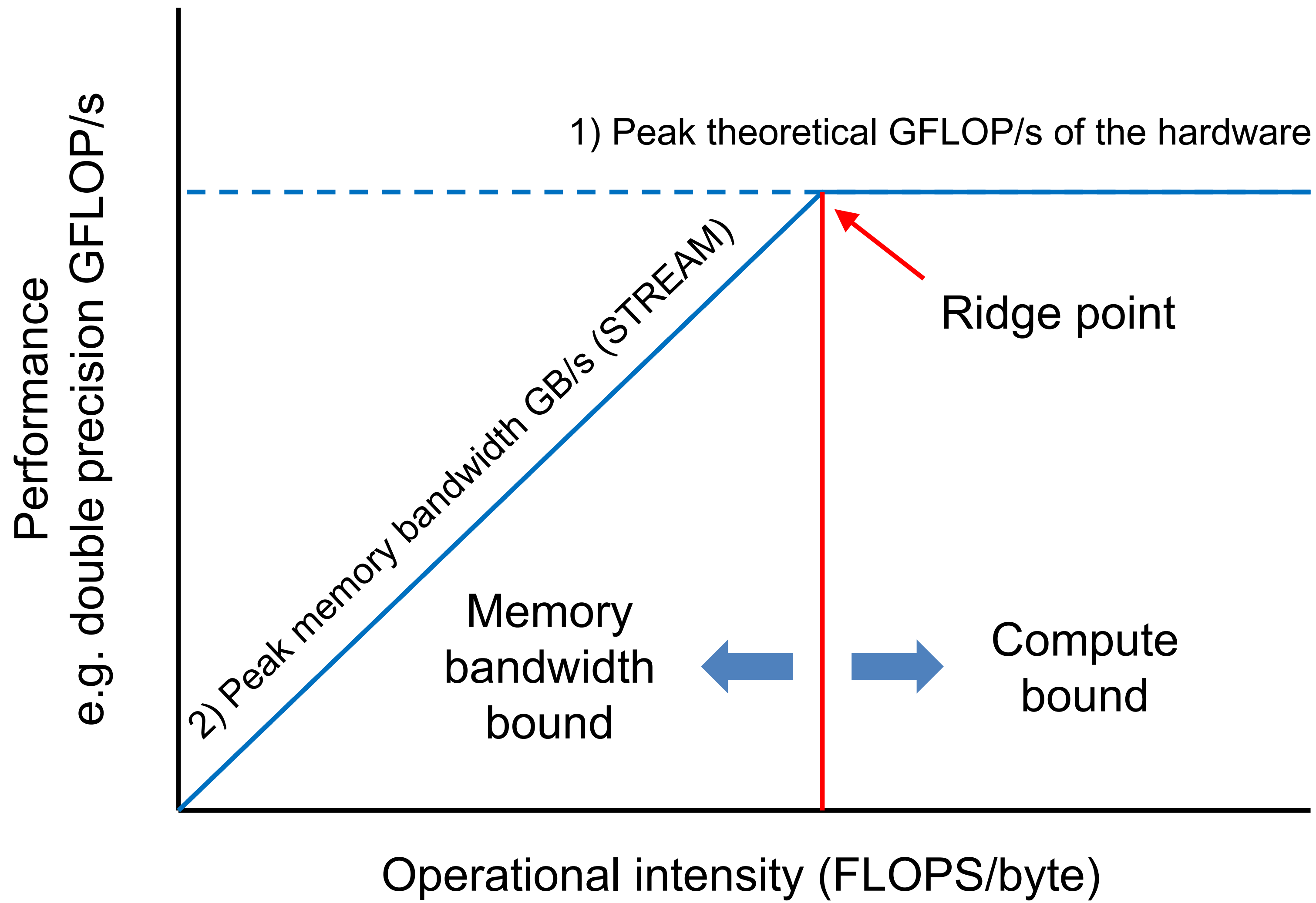
A roofline graph



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Roofline definition

$$\text{Attainable GFLOP/s} = \mathit{min} \begin{cases} \text{Peak floating-point performance} \\ \text{Peak memory bandwidth} \times \text{Operational intensity} \end{cases}$$

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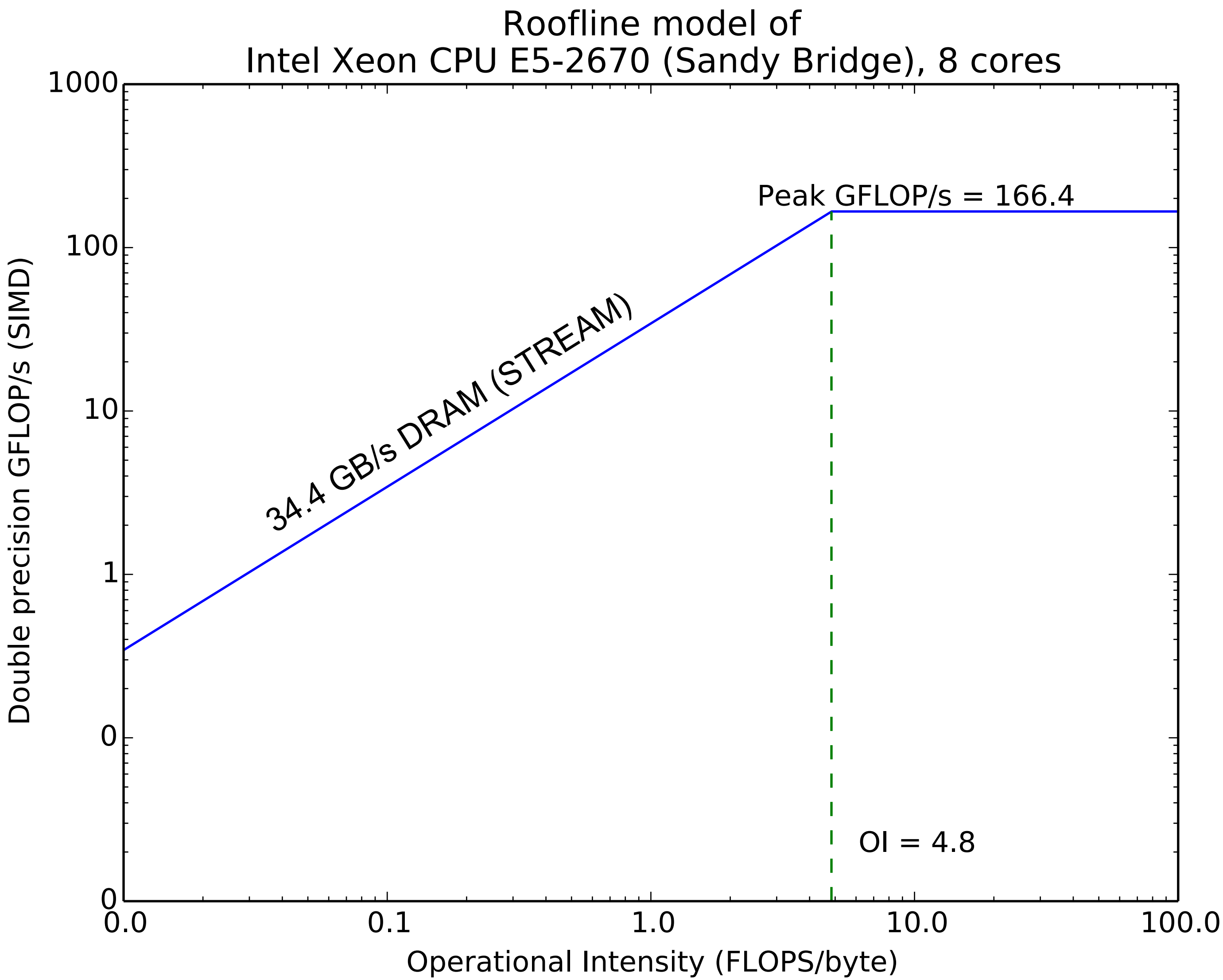
The original definition is slightly dated

- It didn't model in-cache behaviour
- Vague about SIMD and multi-core

- "The *x*-coordinate of the ridge point is the minimum operational intensity required to achieve maximum performance."
  - But this implies that performance is all about FLOP/s, which it isn't
  - E.g. if you have a memory bandwidth bound code, performance is all about moving bytes. FLOP/s might be much less important

Roofline for BCp3



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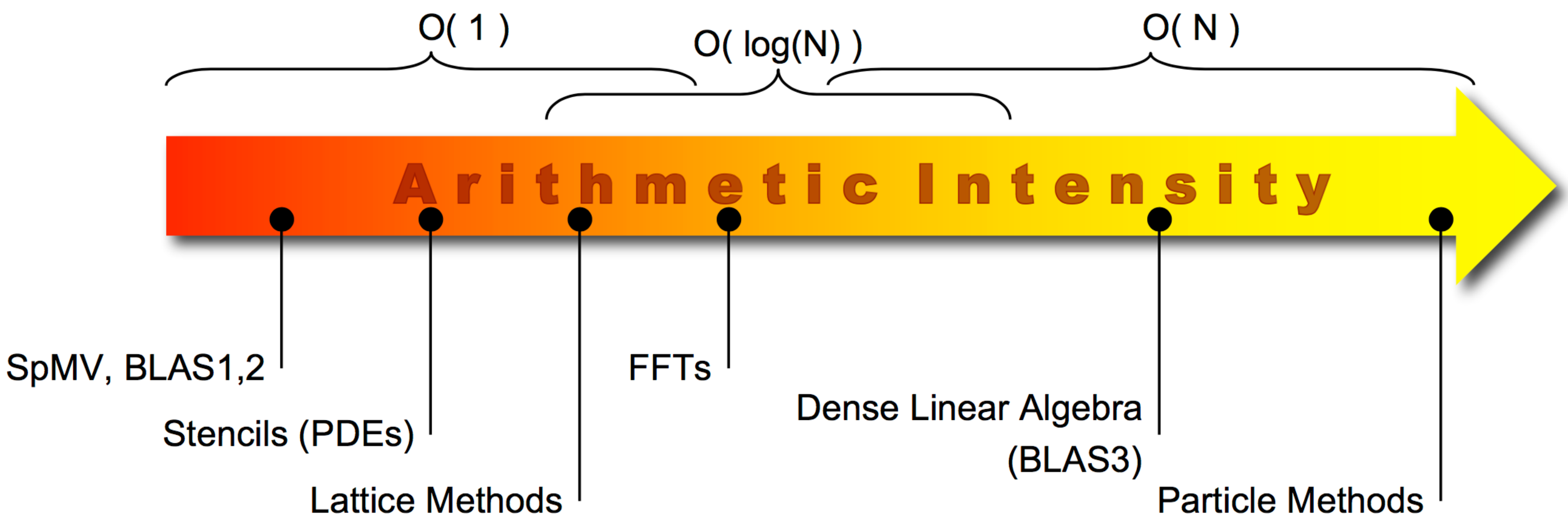
Operational (Arithmetic) Intensity



- **True Operational Intensity (OI) ~ Total FLOPS / Total DRAM Bytes**
  – Constant with respect to problem size for many problems of interest
  – Ultimately limited by unavoidable traffic to DRAM
  – Reduced by conflict or capacity cache misses

Jacobi Operational Intensity

```
for (row = 0; row < N; row++) {
  dot = 0.0;                                    O(N)
  for (col = 0; col < N; col++) {
    if (row != col)                             O(N2)
      dot += A[row + col*N] * x[col];
  }
  xtmp[row] =
    (b[row] - dot) / A[row + row*N];
}
```

$OI_{fp64} = 2/16 = 0.125$
$OI_{fp32} = 2/8 \quad = 0.25$

When are you doing well?

- Be near your limit, whatever that is

- **For memory bandwidth bound codes:**
  - Aim to achieve a good fraction of measured STREAM bandwidth
    - STREAM is a simple memory bandwidth benchmark. It represents the maximum bandwidth you could ever achieve. Typically get to within 85% of the theoretical peak memory bandwidth of the hardware

When are you doing well?

- Be near your limit, whatever that is

- **For compute bound codes:**
  - Aim to achieve a good fraction of the theoretical peak floating point performance of the hardware
    - Is that for scalar or SIMD?
    - Single or double precision? (or even “half” now)

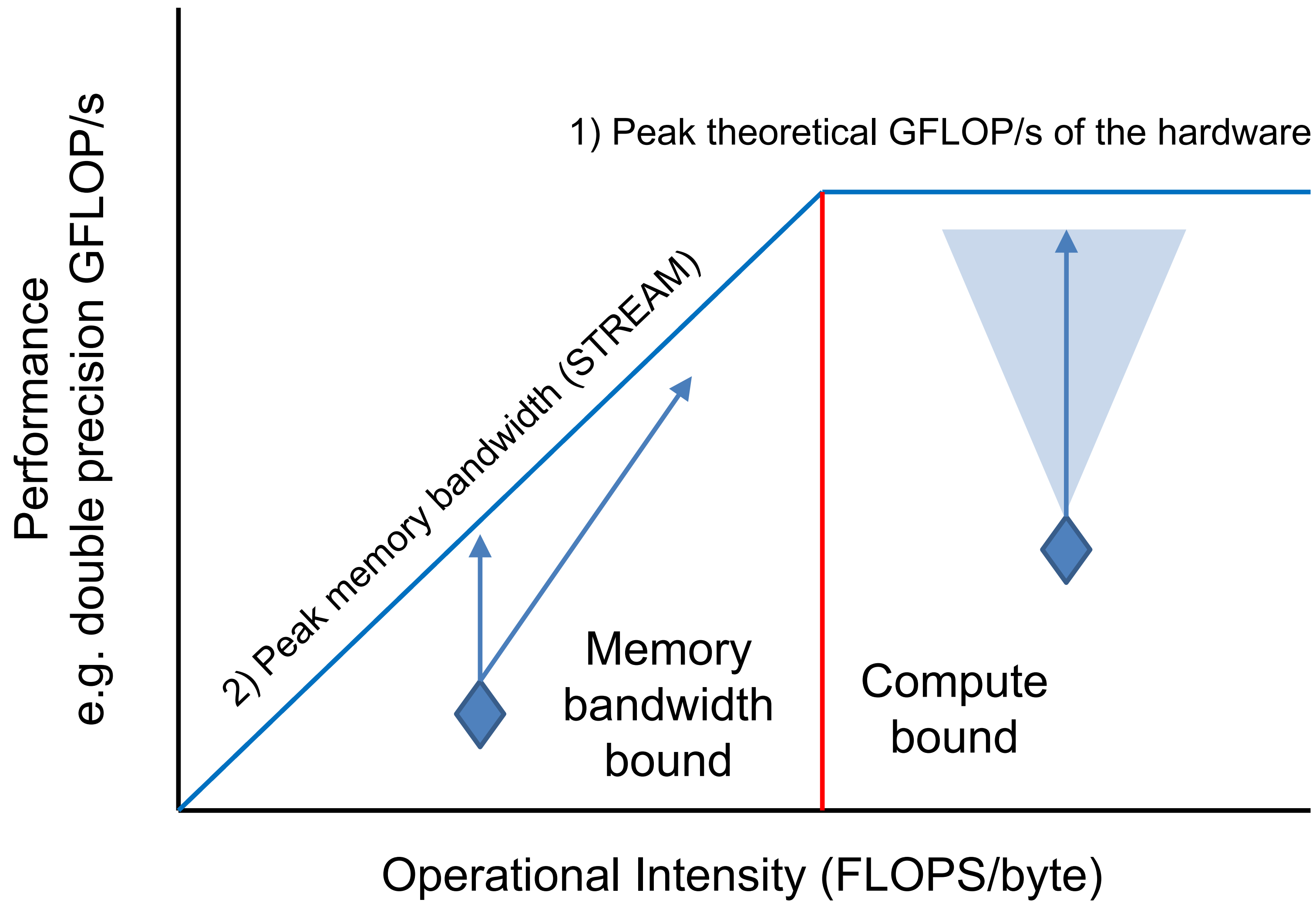
Use the right roofline

The following have big effects:

- Single core vs multi-core
- Scalar vs SIMD
- Single precision vs double precision

Use the right version of the graph for you

Improving performance as seen through the Roofline model



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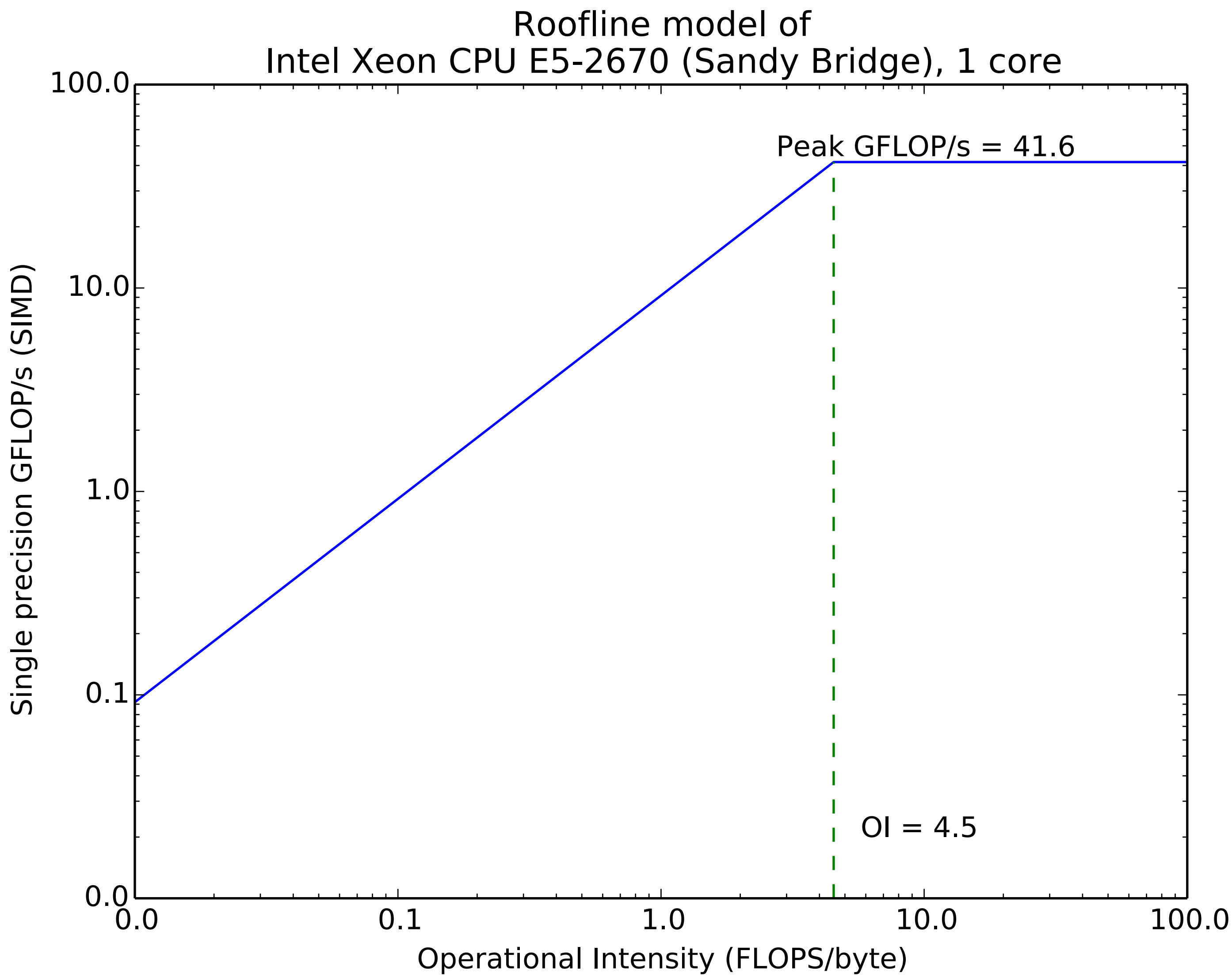
For the scalar optimisation exercise



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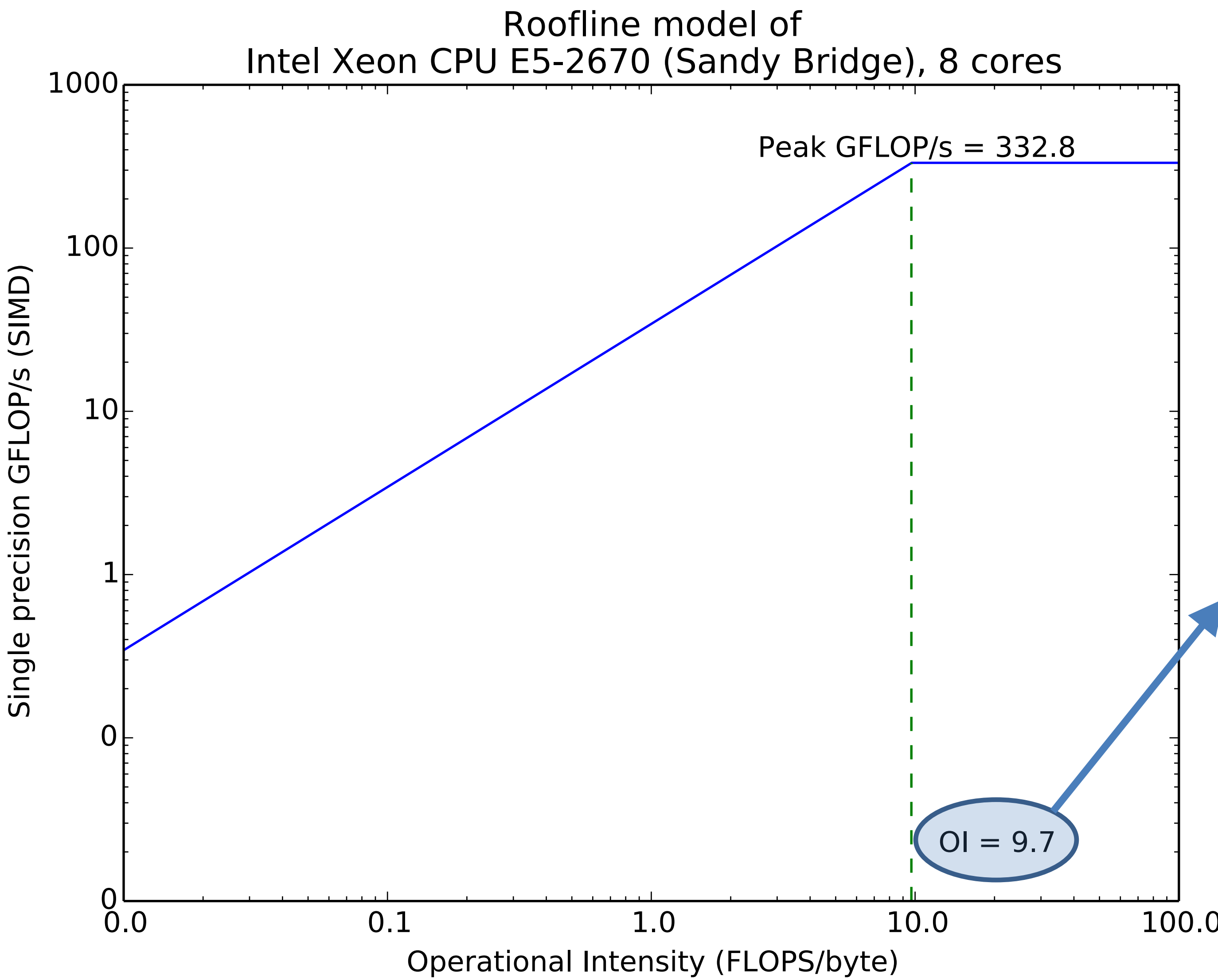
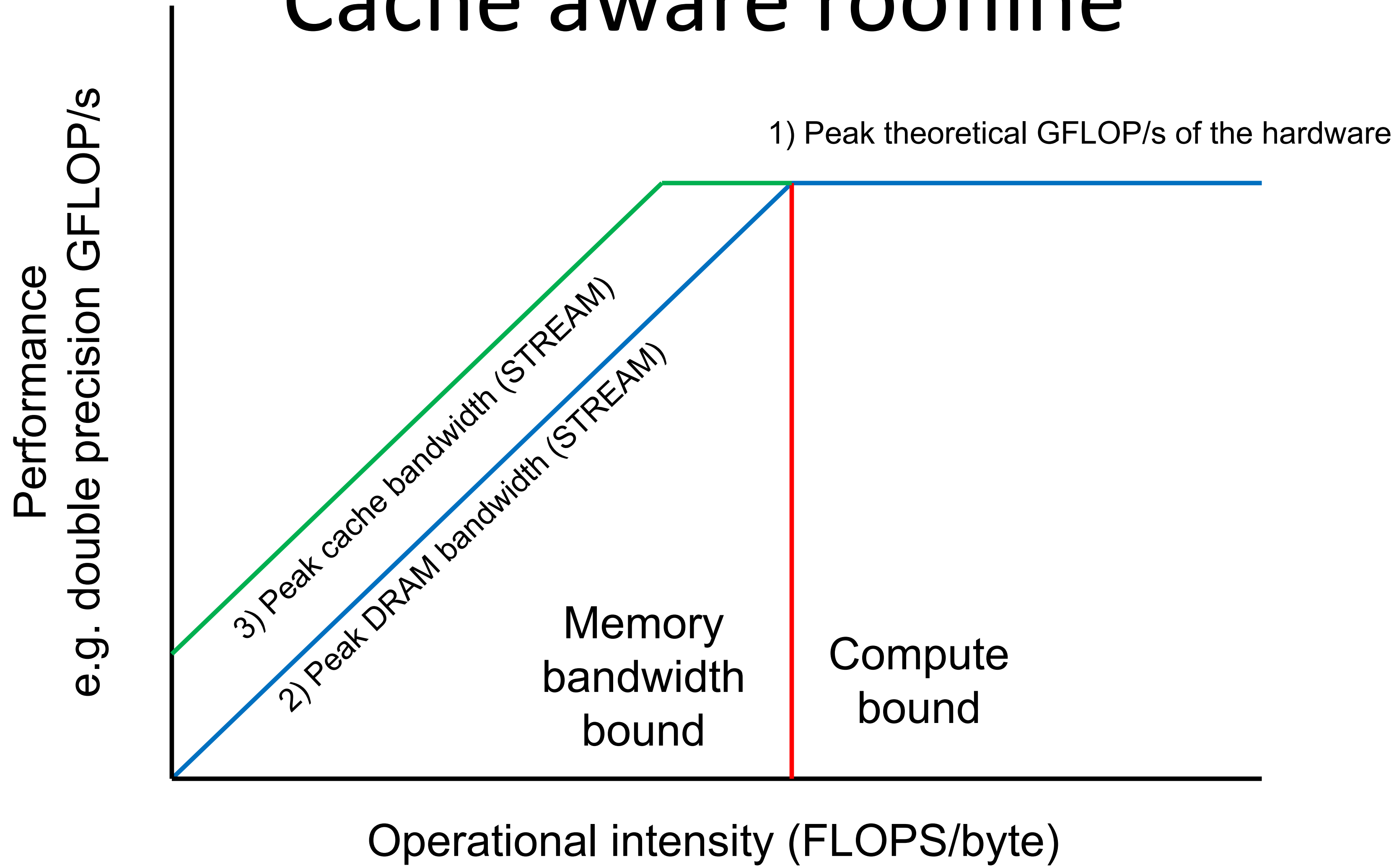
Multicore raises the height of the roof



Single core OI was 4.5. Now 8 times as many core sharing the bandwidth. So why isn't this 8*4.5 = 36?

Cache aware roofline



Ilic, Aleksandar, Frederico Pratas, and Leonel Sousa. "Cache-aware roofline model: Upgrading the loft." *IEEE Computer Architecture Letters* 13.1 (2014): 21-24.

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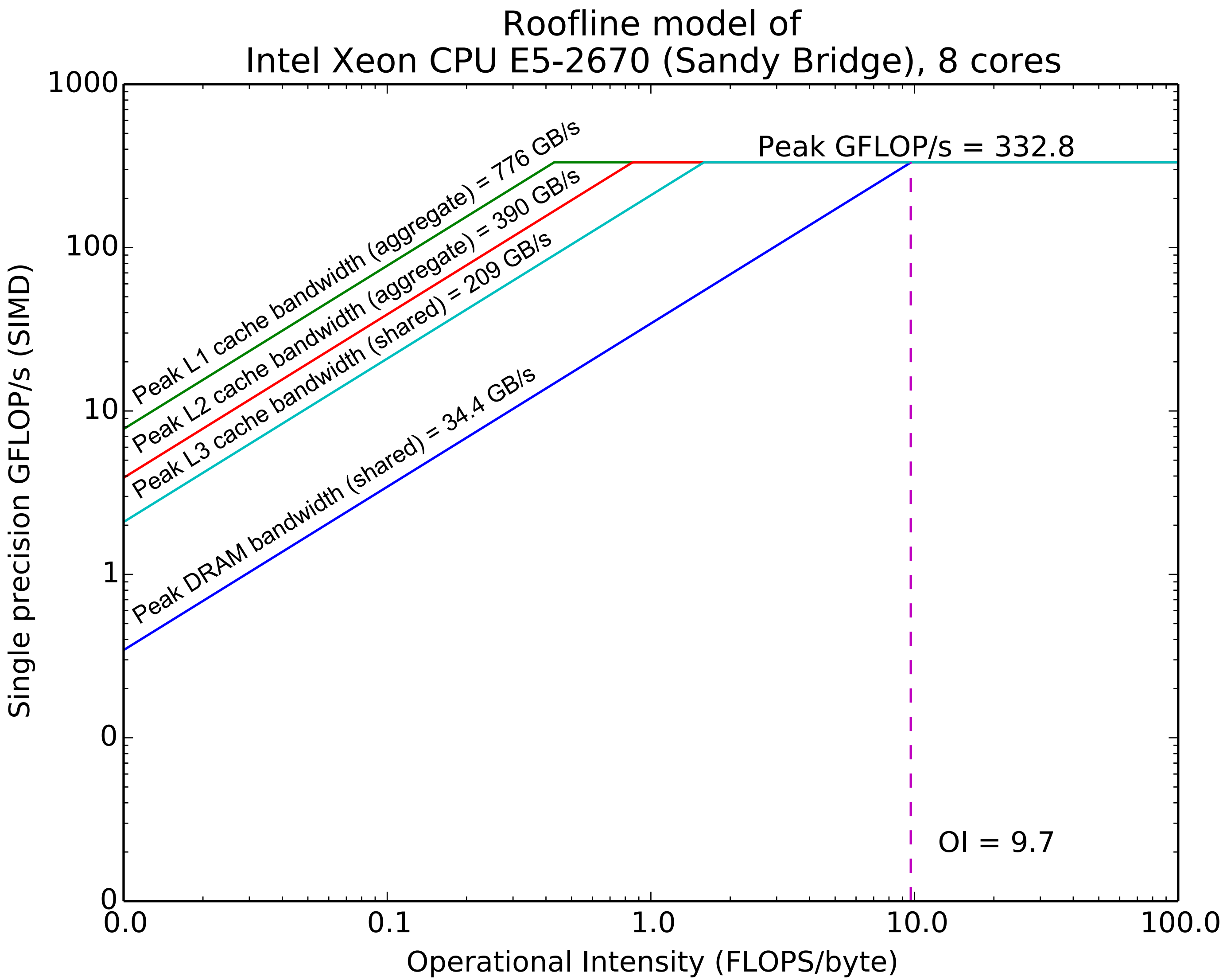
Cache aware roofline of BCp3



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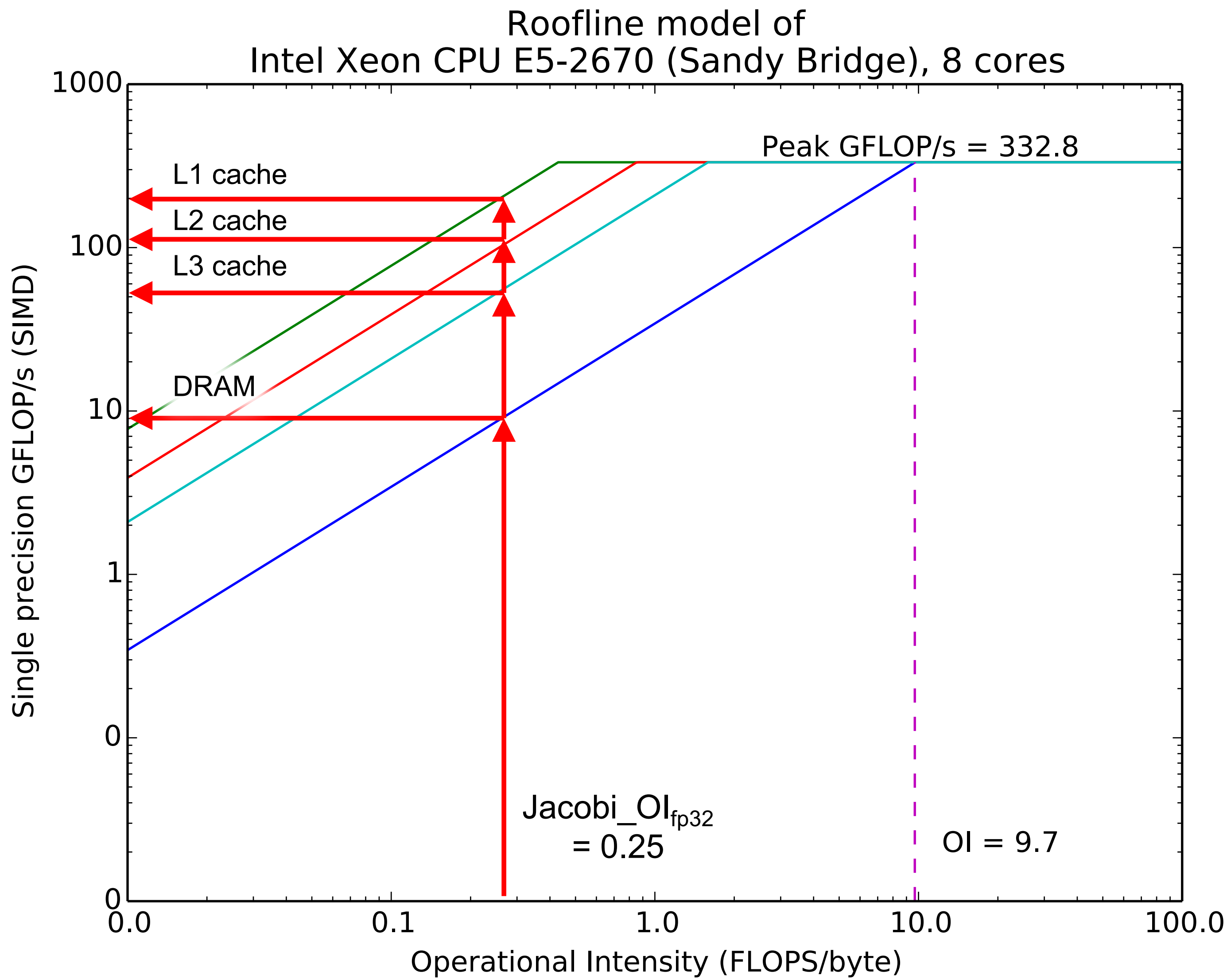
Cache aware roofline of BCp3



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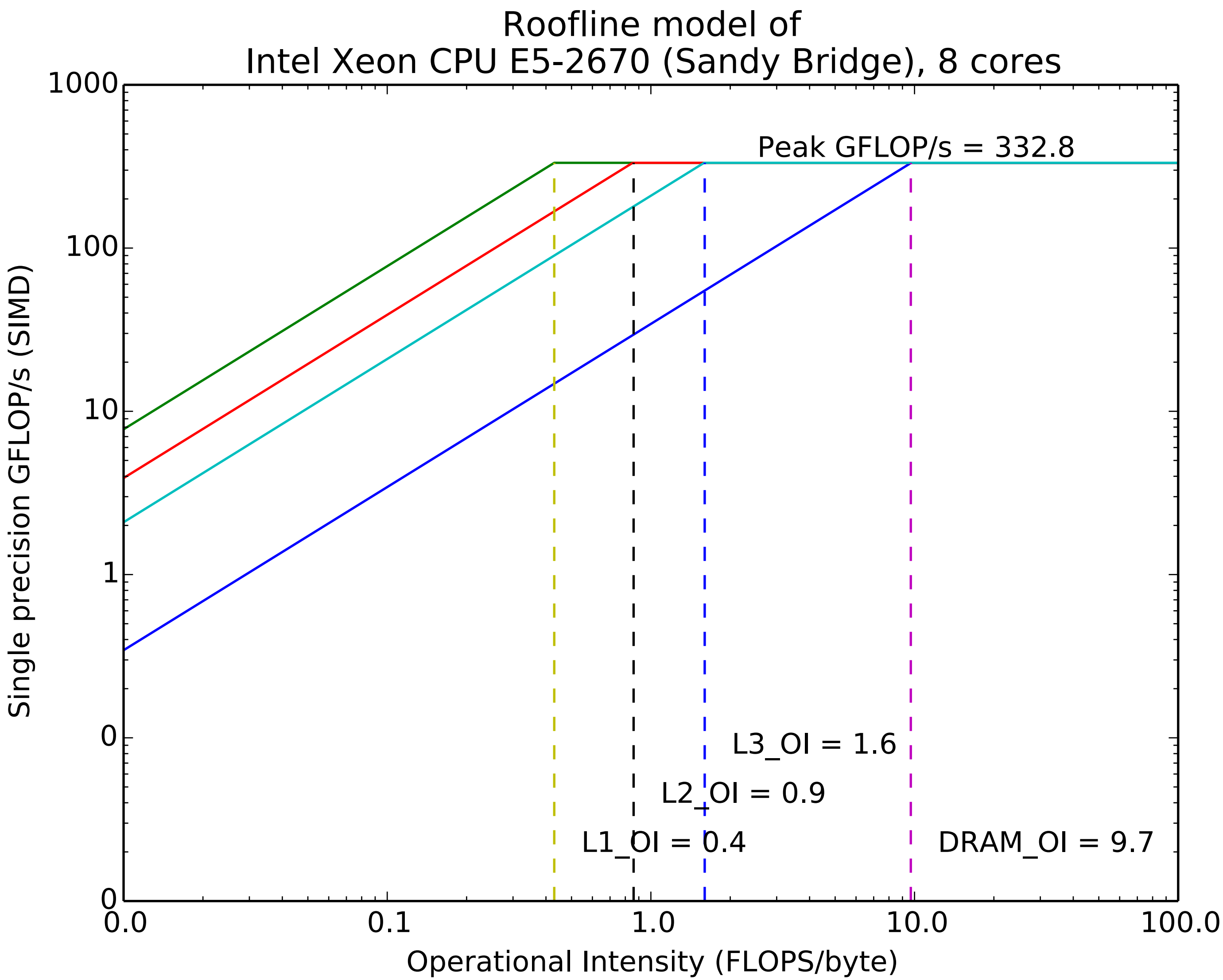
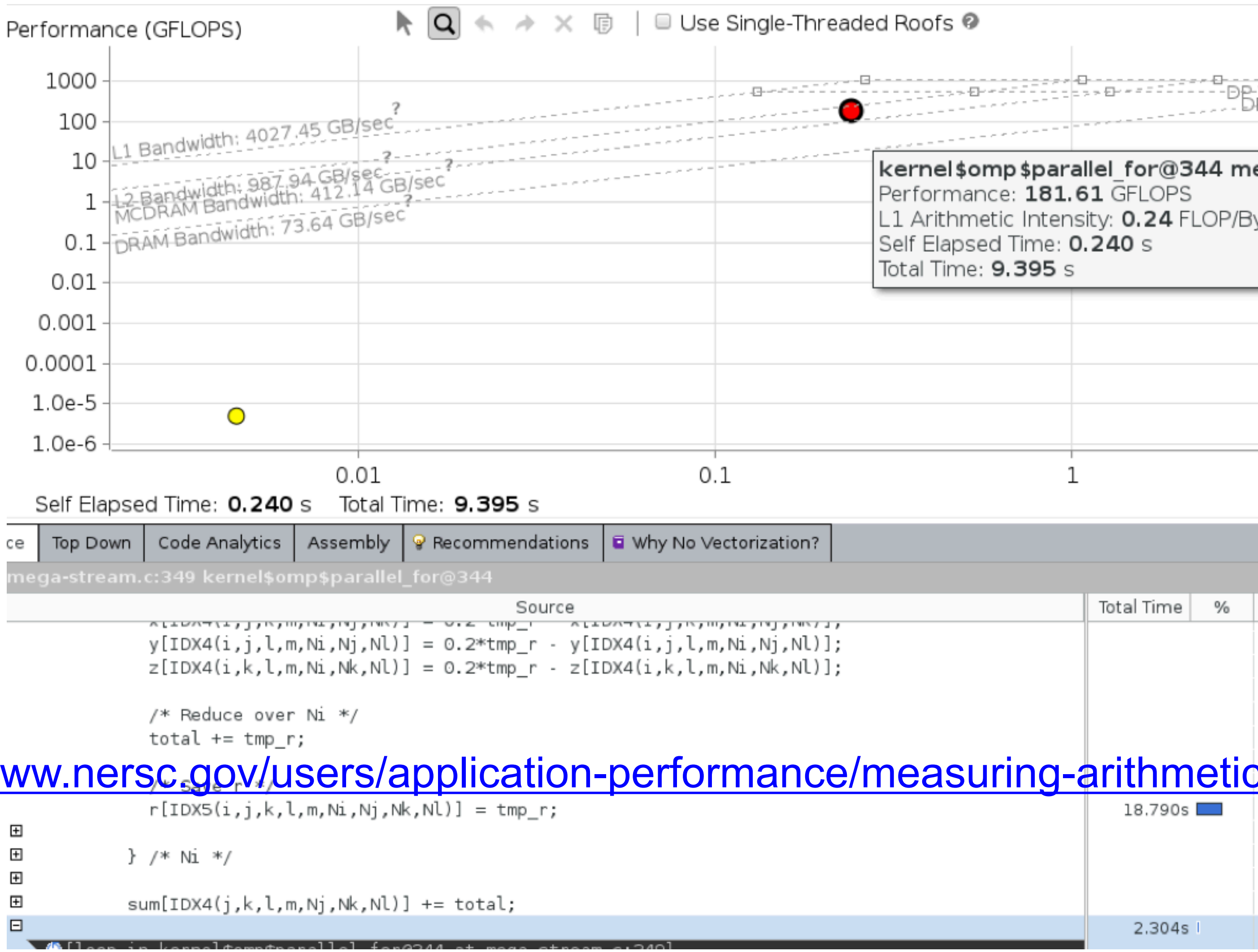
OIs for each level of the memory



Example Intel Advisor roofline output



See:

http://www.nersc.gov/users/application-performance/measuring-arithmetic-intensity/

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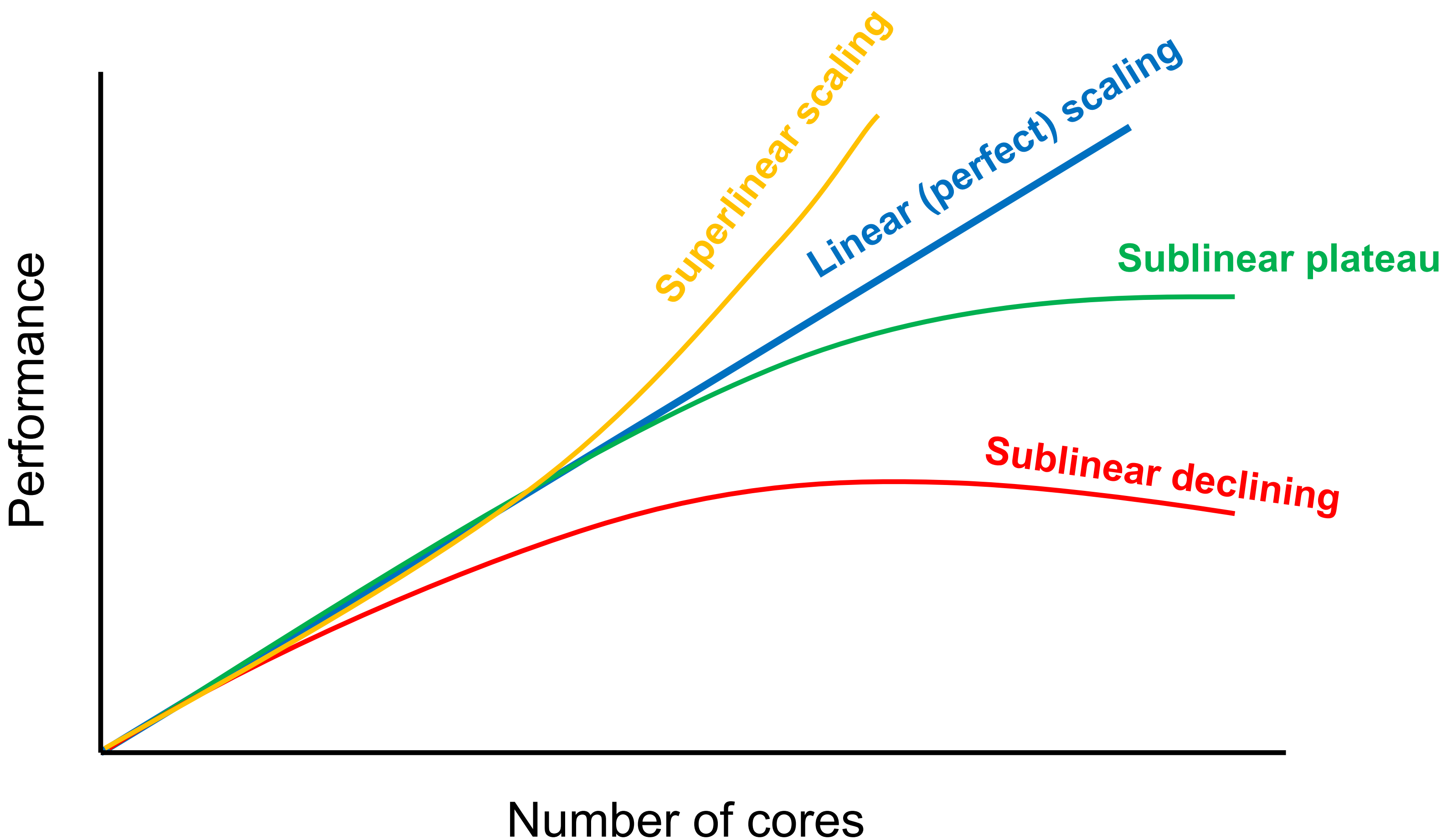
What else might we be interested in?

For parallel programs, such as OpenMP: scaling



Summary

- It's not enough to just make a program faster than it was
- You need to quantify how fast your program is, relative to what is possible
- The roofline model is one useful way to measure and assess what fraction of peak you are achieving
- Consider how close you are to linear scaling when going parallel

Further reading

- The original roofline paper:
  Samuel Williams, Andrew Waterman, and David Patterson. Roofline: an insightful visual performance model for multicore architectures. *Commun. ACM* 52, 4 (April 2009), 65-76. DOI: https://doi.org/10.1145/1498765.1498785
- The cache-aware extension to roofline:
  Ilic, Aleksandar, Frederico Pratas, and Leonel Sousa. "Cache-aware roofline model: Upgrading the loft." *IEEE Computer Architecture Letters* 13.1 (2014): 21-24.