Q1. A group of United Nations climate experts are asked to assess the UK's progress towards meeting its 2050 CO_2 emissions target in terms of the following three possible outcomes. If the UK continues with its current policies then either:

A: The UK will have reduced CO_2 emissions by more than the target by 2050.

B: The UK will have reduced CO_2 emissions so as to meet the target by 2050.

C: The UK will have reduced CO_2 emissions by less than the target by 2050.

The group reports that 10% of their members are completely uncertain, 30% say that the UK will not reduce emissions by more than the target, 20% say that the UK will reduce emissions by less than the target and 40% say that the UK will meet that emissions target.

(a) Taking the set of possible worlds to be $W = \{A, B, C\}$, give the mass assignment summarising these responses.

(4 marks)

(b) If the UK's CO_2 emissions are reduced by less than the target then there will be a penalty fine. This is represented by a random variable $X : \{A, B, C\} \to \{0, -1\}$ such that $X(A) = X(B) = 0$ and $X(C) = -1$. Use the mass assignment from part (a) to determine the belief and plausibility that $X = -1$.

(3 marks)

(c) Assuming that there is a uniform prior probability distribution on the outcomes A, B and C, determine the posterior distribution conditional on the mass assignment from part (a).

(6 marks)

(d) Two different independent panels of experts also provide assessments about the UK's CO_2 emissions target in the form of the following mass assignments:

$$m_1 := \{A\} : 0.01,\ \{C\} : 0.99 \text{ and } m_2 := \{A\} : 0.01,\ \{B\} : 0.99$$

(i) Use Dempster's rule to obtain a combined mass assignment $m_1 \oplus m_2$.

(3 marks)

(ii) Discuss the result you obtain and explain why it might serve as a criticism of Dempster's rule of combination.

(4 marks)

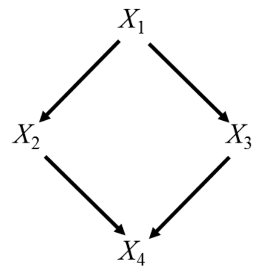

Figure 1: Bayesian network for question Q2(a)

Q2. (a) Consider the Bayesian network in figure 1 involving binary variables $X_1, \ldots, X_4$ each taking values in $\{0, 1\}$. Given the conditional independence assumptions encoded in this network together with the following probability values;

$$P(X_1 = 1) = 0.6, P(X_2 = 1|X_1 = 1) = 0.8, P(X_2 = 1|X_1 = 0) = 0.5,$$
$$P(X_3 = 1|X_1 = 1) = 0.4, P(X_3 = 1|X_1 = 0) = 0.1,$$
$$P(X_4 = 1|X_2 = 1, X_3 = 1) = 0.9, P(X_4 = 1|X_2 = 1, X_3 = 0) = 0.7,$$
$$P(X_4 = 1|X_2 = 0, X_3 = 1) = 0.1, P(X_4 = 1|X_2 = 0, X_3 = 0) = 0.5$$

determine the value of $P(X_1 = 1, X_4 = 1)$.

(6 marks)

(b) Explain what is meant by a Dutch book.

(2 marks)

Show that accepting both of the following bets for proposition A, stake $S > 0$ and odds $\frac{1}{2} < p < 1$ corresponds to a Dutch book;

- Gain $S(1 - p)$ if A is true and lose Sp if A is false.
- Gain $S(1 - p)$ if A^c is true and lose Sp if A^c is false.

(4 marks)

(c) Let $W = \{w_1, w_2, w_3, w_4\}$ and consider the propositions $A = \{w_1, w_2\}$ and $B = \{w_1, w_3\}$. Assuming that it is only known that $P(A) - P(B) = 0.25$ use the maximum entropy inference process in order to determine an exact probability distribution on W and evaluate $P(A)$ and $P(B)$.

(8 marks)

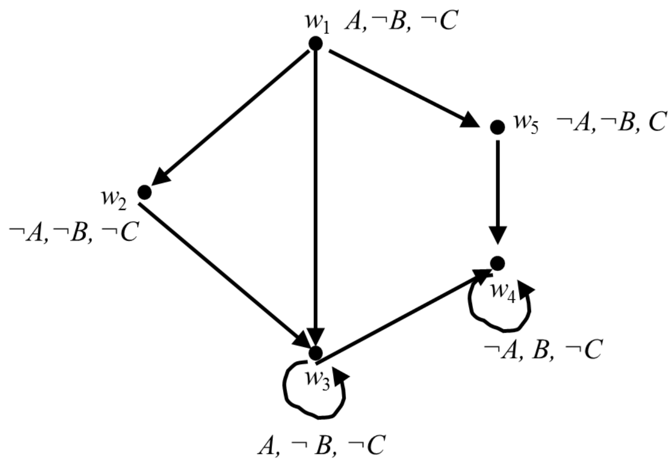

Figure 2: Kripke for question Q3(e)

Q3. (a) State the axioms of a possibility measure.

(2 marks)

(b) For any $A \subseteq W$ show that if $Nec(A) > 0$ then $Pos(A) = 1$.

(5 marks)

(c) Suppose that we come across an alien die with six sides marked P, Q, R, S, T and U. After an initial study we estimate that P, Q, R each have the same possibility. Furthermore, S, T, U also all have the same possibility which is 4 times that of P, Q, R. Give the possibility distribution representing this information, and determine its underlying mass assignment.

(6 marks)

(d) In modal logic, give the definition of a Kripke interpretation.

(2 marks)

(e) Consider the Kripke interpretation shown in figure 2 for possible worlds $W = \{w_1, w_2, w_3, w_4, w_5\}$ and propositions A, B and C. For this interpretation state the worlds in which the following modal logic formulae are true:

(i) $\Box \neg B$

(2 marks)

(ii) $\Diamond A \wedge \Box \neg B$

(3 marks)

Q4. (a) State the axioms of a T-conorm.

(4 marks)

(b) Show that the dual T-conorm of the T-norm $T_\lambda(x,y) = \max(0, (1+\lambda)(x+y-1) - \lambda xy)$ for $\lambda > -1$ is $S_\lambda(x,y) = \min(1, x+y+\lambda xy)$

(4 marks)

(c) The Dombi Family of T-norms is defined by:

$$T_\lambda(x,y) = \left[1 + \left[\left(\frac{1}{x} - 1\right)^\lambda + \left(\frac{1}{y} - 1\right)^\lambda\right]^{\frac{1}{\lambda}}\right]^{-1} \quad \text{for } \lambda > 0$$

Show that no member of this family satisfies the law of non-contradiction.

(6 marks)

(d) Let the function $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x,y) = \frac{x}{y}$. Use the α-cut method to evaluate $f(\tilde{A}, \tilde{B})$ where:

$$\tilde{A} = 1/1 + 2/0.6 + 3/0.4 \text{ and } \tilde{B} = 5/1 + 6/0.8 + 7/0.7$$

(6 marks)