Q No.			
1(a)	$m := \{A, B, C\} : 0,1, \{B, C\} : 0.3, \{C\} : 0.2, \{B\} : 0.4$	4	
1(b)	$Bel(X = -1) = Bel(\{C\}) = m(\{C\}) = 0.2$	1	
	$Pl(X = -1) = Pl(\{C\}) = m(\{C\}) + m(\{B, C\}) + m(\{A, B, C\}) = 0.2 + 0.3 + 0.1 = 0.6$	2	
1(c)	$P(A\|m) = \frac{0.1}{3} = 0.03333$	2	
	$P(B\|m) = \frac{0.1}{3} + \frac{0.3}{2} + 0.4 = 0.58333$	2	
	$P(C\|m) = \frac{0.1}{3} + \frac{0.3}{2}) + 0.2 = 0.38333$	2	
1 (d)	$\begin{array}{\|c\|\|c\|c\|} \hline m_1/m_2 & \{A\} : 0.01 & \{C\} : 0.99 \\ \hline \{A\} : 0.01 & \{A\} : 0.0001 & \emptyset : 0.099 \\ \hline \{B\} : 0.99 & \emptyset : 0.099 & \emptyset : 0.9801 \\ \hline \end{array}$	2	
	$m_1 \oplus m_2(A) = \frac{0.0001}{0.0001} = 1$	1	
	Both groups of experts agree that outcome A is extremely unlikely and yet Dempster's rule results is a mass function in which A is absolutely certain. This seems to be highly counter-intuitive. The problem seems to result from the normalisation process in which mass associated with the empty set is uniformly reallocated to the other sets with non-zero mass. In the case when the sources are highly inconsistent this can lead to anomalous results.	4	

Q No.			
2(a)	$P(X_1 = 1, X_4 = 1) = P(X_1 = 1)\sum_{X_2=0}^{1}\sum_{X_3=0}^{1} P(X_2\|X_1 = 1)P(X_3\|X_1 = 1)P(X_4 = 1\|X_2, X_3)$	1	
	$= P(X_1 = 1) \times [P(X_2 = 1\|X_= 1)P(X_3 = 1\|X_1 = 1)P(X_4 = 1\|X_2 = 1, X_3 = 1)$	2	
	$+P(X_2 = 1\|X_1 = 1)P(X_3 = 0\|X_1 = 1)P(X_4 = 1\|X_2 = 1, X_3 = 0)$		
	$+P(X_2 = 0\|X_1 = 1)P(X_3 = 1\|X_1 = 1)P(X_4 = 1\|X_2 = 0, X_3 = 1)$		
	$+P(X_2 = 0\|X_1 = 1)P(X_3 = 0\|X_1 = 1)P(X_4 = 1\|X_2 = 0, X_3 = 0)]$		
	$= 0.6 \times [0.8 \times 0.4 \times 0.9 + 0.8 \times 0.6 \times 0.7 + 0.2 \times 0.4 \times 0.1 + 0.2 \times 0.6 \times 0.5]$	2	
	$= 0.6 \times [0.288 + 0.336 + 0.032 + 0.06] = 0.4476$	1	
2(b)	A Dutch book is a sequence of bets the outcome of which is a sure loss no matter what the actual state of the world.	2	
	The gain from the two bets is: $S(\chi_A(w^*) - p) + S(\chi_{A^c}(w^*) - p)$ where w^* is the actual state of the world. Simplifying gives $S(\chi_A(w^*) + \chi_{A^c}(w * 1) - 2p) = S(1 - 2p) < 0$ since $S > 0$ and $p > \frac{1}{2}$.	4	
2(c)	Let $P(w_i) = p_i$ then we obtain $(p_1 + p_2) - (p_1 + p_3) = 0.25 \Rightarrow p_3 = p_2 - 0.25$. Also $p_4 = 1 - p_1 - p_2 - p_3$ and substituting gives $p_4 = 1.25 - p_1 - 2p_2$.	2	
	Hence, $H = -p_1 \log_2(p_1) - p_2 \log_2(p_2) - (p_2 - 0.25)\log_2(p_2 - 0.25) - (1.25 - p_1 - 2p_2)\log_2(1.25 - p_1 - 2p_2) \Rightarrow \frac{\partial H}{\partial p_1} = -log_2(p_1) + \log_2(1.25 - p_1 - 2p_2)$ and $\frac{\partial H}{\partial p_1} = 0 \Rightarrow p_1 = 1.25 - p_1 - p_2 \Rightarrow p_1 + p_2 = 0.625$	2	
	$\frac{\partial H}{\partial p_2} = -\log_2(p_2) - \log_2(p_2 - 0.25) + 2\log_2(1.25 - p_1 - 2p_2)$, $\frac{\partial H}{\partial p_2} = 0 \Rightarrow p_2(p_2 - 0.25) = (1.25 - p_1 - 2p_2)^2$ substituting $p_1 = 0.625 - p_2$ gives $p_2(p_2 - 0.25) = (0.625 - p_2)^2 \Rightarrow p_2 = 0.390625$.	2	
	From this we obtain $p_1 = 0.234375, P(A) = 0.234375 + 0.39025 = 0.625$	1	
	Also $p_3 = p_2 - 0.25 = 0.390625 - 0.25 = 0.140625$ and hence $P(B) = 0.234375 + 0.140625 = 0.375$	1	

Q No.			
3(a)	$Pos(W) = 1, Pos(\emptyset) = 0$ and $Pos(A \cup B) = \max(Pos(A), Pos(B))$	2	
3(b)	$Pos(A^c) = 1 - Nec(A)$ therefore $Nec(A) > 0 \Rightarrow Pos(A^c) < 1$	2	
	Now $1 = Pos(W) = Pos(A \cup A^c) = \max(Pos(A), Pos(A^c))$	2	
	Hence, since $Pos(A^c) < 1$ then $Pos(A) = 1$.	1	
3(c)	$\pi(S) = \pi(T) = \pi(U) = 4\pi(P) = 4\pi(Q) = 4\pi(R)$	2	
	But the max of all the possibilities is 1 so that $\pi(S) = \pi(T) = \pi(U) = 1$ and $\pi(P) = \pi(Q) = \pi(R) = \frac{1}{4}$	2	
	This has the mass function $m := \{P, Q, R, S, T, U\} : \frac{1}{4}, \{S, T, U\} : \frac{3}{4}$	2	
3(d)	A Kripke interpretation is tuple $< W, E >$ where W is a set of words and E is an accessibility relation between worlds.	2	
3(e)	w_1 and w_2	2	
	$\Diamond A$ is true in w_1, w_2, w_3	1	
	Hence $\Diamond A \wedge \Box \neg B$ is true in w_1, w_2.	2	

Q No.			
4(a)	**S1:** $\forall x \in [0,1]$ $S(x,0)=x$; **S2:** $\forall x,y,z \in [0,1]$ if $y \le z$ then $S(x,y) \le S(x,z)$; **S3:** $\forall x,y \in [0,1]$ $S(x,y)=S(y,x)$; **S4:** $\forall x,y,z \in [0,1]$ $S(x,S(y,z))=S(S(x,y),z)$.	4	
4 (b)	$S_\lambda(x,y) = 1 - T_\lambda(1-x,1-y) = 1 - \max(0,(1+\lambda)(1-x+1-y-1) - \lambda(1-x)(1-y))$	1	
	$= \min(1, 1-[(1+\lambda)(1-x-y) - \lambda(1-x)(1-y)]) = \min(1,1-[1-x-y-\lambda xy])$	2	
	$= \min(1, x+y+\lambda xy)$	1	
4(c)	For the law of non-contradiction we require that $T_\lambda(x,1-x)=0$ for all $x \in [0,1]$.	2	
	Now $T_\lambda(x,1-x) = \left[1 + \left[(\frac{1}{x}-1)^\lambda + (\frac{1}{1-x}-1)^\lambda\right]^{\frac{1}{\lambda}}\right]^{-1}$	2	
	Letting $x=\frac{1}{2}$ then $T_\lambda(x,1-x) = \frac{1}{1+2^{\frac{1}{\lambda}}} \neq 0$ for any λ	2	
4(c)	$\tilde{A}_\alpha = \{1,2,3\} : \alpha \in (0,0.4], \{1,2\} : \alpha \in (0.4,0.6], \{1\} : \alpha \in (0.6,1]$	1	
	$\tilde{B}_\alpha = \{5,6,7\} : \alpha \in (0,0.7], \{5,6\} : \alpha \in (0.7,0.8], \{5\} : \alpha \in (0.8,1]$	1	

$\frac{A_\alpha}{B_\alpha}$	$\{1,2,3\} : (0,0.4]$	$\{1,2\} : (0.4,0.6]$	$\{1\} : \alpha \in (0.6,1]$
$\{5,6,7\} : (0,0.7]$	$\{\frac{1}{5},\frac{2}{5},\frac{3}{5},\frac{1}{6},\frac{1}{3},\frac{1}{2},\frac{1}{7},\frac{2}{7},\frac{3}{7}\} : (0,0.4]$	$\{\frac{1}{5},\frac{2}{5},\frac{1}{6},\frac{1}{3},\frac{1}{7},\frac{2}{7}\} : (0.4,0.6]$	$\{\frac{1}{5},\frac{1}{6},\frac{1}{7}\} : (0.6,0.7]$
$\{5,6\} : (0.7,0.8]$	$\emptyset$	$\emptyset$	$\{\frac{1}{5},\frac{1}{6}\} : (0.7,0.8]$
$\{5\} : (0.8,1]$	$\emptyset$	$\emptyset$	$\{\frac{1}{5}\} : (0.8,1]$

2

	$f(\tilde{A},\tilde{B})_\alpha = \{\frac{1}{5},\frac{2}{5},\frac{3}{5},\frac{1}{6},\frac{1}{3},\frac{1}{2},\frac{1}{7},\frac{2}{7},\frac{3}{7}\} : \alpha \in (0,0.4], \{\frac{1}{5},\frac{2}{5},\frac{1}{6},\frac{1}{3},\frac{1}{7},\frac{2}{7}\} : \alpha \in (0.4,0.6], \{\frac{1}{5},\frac{1}{6},\frac{1}{7}\} : \alpha \in (0.6,0.7], \{\frac{1}{5},\frac{1}{6}\} : \alpha \in (0.7,0.8], \{\frac{1}{5}\} : \alpha \in (0.8,1]$	1	
	$f(\tilde{A},\tilde{B}) = \frac{1}{5}/1 + \frac{1}{6}/0.8 + \frac{1}{7}/0.7 + \frac{2}{5}/0.6 + \frac{1}{3}/0.6 + \frac{2}{7}/0.6 + \frac{3}{5}/0.4 + \frac{1}{2}/0.4 + \frac{3}{7}/0.4$	1	