# Solutions 2: Dempster Shafer Theory and Fuzzy Set Theory

**1 (i)** Beginning with singleton sets we have that:

$$Bel(\{Bill\}) = m(\{Bill\}) = 0.2,\ \ Bel(\{Edd\}) = m(\{Edd\}) = 0.2 \text{ and } Bel(\{Jim\}) = m(\{Jim\}) = 0$$

Then for sets with 2 elements:

$$Bel(\{Bill, Edd\}) = 0.3 = m(\{Bill, Edd\}) + m(\{Bill\}) + m(\{Edd\}) = m(\{Bill, Edd\}) + 0.2$$
$$\Rightarrow m(\{Bill, Edd\}) = 0.3 - 0.2 = 0.1$$
$$Bel(\{Edd, Jim\}) = 0.2 = m(\{Edd, Jim\}) + m(\{Edd\}) + m(\{Jim\}) = m(\{Edd, Jim\})$$
$$Bel(\{Bill, Jim\}) = 0.3 = m(\{Bill, Jim\}) + m(\{Bill\}) + m(\{Jim\}) = m(\{Bill, Jim\}) + 0.2$$
$$\Rightarrow m(\{Bill, Jim\}) = 0.3 - 0.2 = 0.1$$

Finally, for the set with 3 elements:

$$m(\{Bill, Edd, Jim\}) = 1 - 0.1 - 0.2 - 0.1 - 0.2 - 0 - 0 = 0.4$$

**1 (ii)**

$$P_u(Bill|m) = \frac{m(\{Bill, Edd, Jim\})}{3} + \frac{m(\{Bill, Jim\})}{2} + \frac{m(\{Bill, Edd\})}{2} + m(\{Bill\})$$
$$= \frac{0.4}{4} + \frac{0.1}{2} + \frac{0.1}{2} + 0.2 = 0.43333$$

$$P_u(Edd|m) = \frac{m(\{Bill, Edd, Jim\})}{3} + \frac{m(\{Bill, Edd\})}{2} + \frac{m(\{Edd, Jim\})}{2} + m(\{Edd\})$$
$$\frac{0.4}{4} + \frac{0.1}{2} + \frac{0.2}{2} + 0 = 0.28333$$

$$P_u(Jim|m) = \frac{m(\{Bill, Edd, Jim\})}{3} + \frac{m(\{Bill, Jim\})}{2} + \frac{m(\{Edd, Jim\})}{2} + m(\{Jim\})$$
$$= \frac{0.4}{4} + \frac{0.1}{2} + \frac{0.2}{2} + 0 = 0.28333$$

**2** For $A \subseteq W$

$$Pl(A) = \sum_{B:A\cap B\neq\emptyset} m(B) = 1 - \sum_{B:A\cap B=\emptyset} m(B) = 1 - \sum_{B:B\subseteq A^c} m(B) = 1 - Bel(A^c)$$

**3**

| $m_1 \oplus m_2$ | $\{B,F,M,E\} : 0.4$ | $\{B,F,M\} : 0.3$ | $\{F,M\} : 0.1$ | $\{B,E\} : 0.2$ |
|---|---|---|---|---|
| $\{B,F,M,E\} : 0.1$ | $\{B,F,M,E\} : 0.04$ | $\{B,F,M\} : 0.03$ | $\{F,M\} : 0.01$ | $\{B,E\} : 0.02$ |
| $\{B,E\} : 0.2$ | $\{B,E\} : 0.08$ | $\{B\} : 0.06$ | $\emptyset : 0.02$ | $\{B,E\} : 0.04$ |
| $\{F\} : 0.7$ | $\{F\} : 0.28$ | $\{F\} : 0.21$ | $\{F\} : 0.07$ | $\emptyset : 0.14$ |

Hence,

$$m_1 \oplus m_2 := \{B,F,M,E\} : \frac{0.04}{1-0.16} = \frac{0.04}{0.84} = 0.047619$$
$$\{B,F,M\} : \frac{0.03}{0.84} = 0.0357142$$
$$\{F,M\} : \frac{0.01}{0.84} = 0.0119047$$
$$\{B,E\} : \frac{0.02+0.08+0.04}{0.84} = 0.1666666$$
$$\{B\} : \frac{0.06}{0.84} = 0.0714286$$
$$\{F\} : \frac{0.28+0.21+0.07}{0.84} = 0.666666$$

**4** The dual of t-norm $T$ is given by:

$$S(x,y) = 1 - T(1-x, 1-y)$$

Hence, for $T = \max(0, x+y-1)$ we have that:

$$S(x,y) = 1 - \max(0, 1-x+1-y-1) = 1 - \max(0, 1-x-y) = \min(1-0, 1-(1-x-y)) = \min(1, x+y)$$

**5** The $\alpha$-cuts of $\tilde{A}$ and $\tilde{B}$ are:

$$\tilde{A}_\alpha = \begin{cases} \{0,1,2\} : \alpha \in (0,0.6] \\ \{1,2\} : \alpha \in (0.6,0.7] \\ \{1\} : \alpha \in (0.7,1] \end{cases} \quad \text{and } \tilde{B}_\alpha = \begin{cases} \{2,3,4\} : \alpha \in (0,0.4] \\ \{4,3\} : \alpha \in (0.4,0.5] \\ \{4\} : \alpha \in (0.5,1] \end{cases}$$

| | $\{0,1,2\} : (0,0.6]$ | $\{1,2\} : (0.6,0.7]$ | $\{1\} : (0.7,1]$ |
|---|---|---|---|
| $\{2,3,4\} : (0,0.4]$ | $\{0,2,4,3\} : (0.0.4]$ | $\emptyset$ | $\emptyset$ |
| $\{4,3\} : (0.4,0.5)$ | $\{0,4,2,3\} : (0.4,0.5]$ | $\emptyset$ | $\emptyset$ |
| $\{4\} : (0.5,1]$ | $\{0,4,2\} : (0.5,0.6]$ | $\{4,2\} : (0.6,0.7]$ | $\{4\} : (0.7,1]$ |

Hence,

$$(\tilde{(A)} \times \tilde{B} \mod 6)_\alpha = \begin{cases} \{0,2,3,4\} : \alpha \in (0,0.5] \\ \{0,2,4\} : \alpha \in (0.5,0.6] \\ \{2,4\} : \alpha \in (0.6,0.7] \\ \{4\} : \alpha \in (0.7,1] \end{cases}$$

Therefore,

$$\tilde{(A)} \times \tilde{B} \mod 6 = 4/1 + 2/0.7 + 0/0.6 + 3/0.5$$