Exercise 2: Dempster Shafer Theory and Fuzzy Set Theory

1. The police have identified four individuals, Bill, Edd and Jim as possible suspects in a robbery. The current evidence has resulted in the police having the following D-S Belief function:

$$Bel(\{Bill, Edd, Jim\}) = 1,\ Bel(\{Bill, Edd\}) = 0.3,\ Bel(\{Edd, Jim\}) = 0.2$$
$$Bel(\{Bill, Jim\}) = 0.3,\ Bel(\{Bill\}) = 0.2,\ Bel(\{Edd\}) = Bel(\{Jim\}) = 0$$

(i) Determine the mass assignment on the subsets of $W = \{Bill, Edd, Jim\}$ which generated this belief function.

(ii) Assuming that the police began by believing that each of the three suspects was equally likely to have committed the crime (i.e. a uniform prior), compute the pignistic distribution of the mass assignment in part (i) to determine a posterior distribution given the current evidence.

2. Prove that $Pl(A) = 1 - Bel(A^c)$.

3. Suppose that one of the four students $W = \{Bill(B), Fred(F), Mary(M), Ethel(E)\}$ will win the prize for best mark in computer science. Two different lecturers provide mass functions representing the belief about which student will win. These are:

$$m_1 := \{B, F, M, E\} : 0.4, \{B, F, M\} : 0.3, \{F, M\} : 0.1, \{B, E\} : 0.2 \text{ and}$$
$$m_2 := \{B, F, M, E\} : 0.1, \{B, E\} : 0.2, \{F\} : 0.7$$

Use Dempster's rule to obtain a combined mass assignment $m_1 \oplus m_2$

4. Show that $S(x, y) = \min(1, x + y)$ is the dual co-norm of the t-norm $T(x, y) = \max(0, x + y - 1)$

5. Let $f : \{0, 1, 2, 3, 4, 5\}^2 \to \{0, 1, 2, 3, 4, 5\}$ be the function $f(x, y) = x \times y \mod 6$. By using the α-cut method to extend f to fuzzy sets determine $f(\tilde{A}, \tilde{B})$ where:

$$\tilde{A} = 0/0.6 + 1/1 + 2/0.7 \text{ and } \tilde{B} = 2/0.4 + 3/0.5 + 4/1$$