Additional Probability Worksheet

Q1 Use the axioms for probability measures to prove the general additivity rule: For all $A, B \subseteq W$, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

Q2 Consider a wheel of fortune with three segments coloured red, green and blue. Suppose for $W = \{r, g, b\}$ you are offered the choice between Bet1 and Bet2 for the three propositions $\{r\}$, $\{g\}$ and $\{b\}$ with stake $S > 0$ and odds $p = 0.3$ in each case. Show that if your belief's are such that $\mu(\{r\}) = \mu(\{g\}) = \mu(\{b\}) = 0.5$ then you are vulnerable to accepting a Dutch book in this case.

Q3 For $W = \{w_1, w_2, w_3, w_4\}$ let $A = \{w_1, w_2\}$, $B = \{w_1, w_3\}$. Given the knowledge base $K = \{P(A) = \alpha, P(B) = \beta\}$ where $\alpha, \beta \in [0, 1]$, show that the maximum entropy distribution consistent with this knowledge base is such that $P(A \cap B) = \alpha \times \beta$.

Q4 Consider K as given in Q3 and show that the centre of mass distribution for K is such that $P(A \cap B) = \frac{\min(\alpha,\beta)+\max(0,\alpha+\beta-1)}{2}$.

Q5 For $W = \{w_1, w_2, w_3, w_4, w_5, w_6\}$ where $A = \{w_1, w_2\}$, $B = \{w_3, w_4\}$. Consider the knowledge base $K = \{P(A) = \alpha, P(B) = \beta\}$ where $0 \leqslant \alpha+\beta \leqslant 1$. Show that the maximum entropy distribution consistent with K is $P(w_1) = P(w_2) = \frac{\alpha}{2}$, $P(w_3) = P(w_4) = \frac{\beta}{2}$, $P(w_5) = P(w_6) = \frac{1-\alpha-\beta}{2}$.

Q6 For $W = \{w_1, w_2, w_3, w_4\}$ let $A = \{w_1, w_2\}$ and $B = \{w_1, w_3\}$. Let $K = \{P(A) = 5P(B)\}$ and find the centre of mass probability distribution for K.

Q7 For $W = \{w_1, w_2, w_3, w_4\}$ consider the set of probability distributions $\mathcal{P} = \{P_1, P_2, P_3, P_4\}$ where

$$P_1 : P_1(w_1) = 0.3, P_1(w_2) = 0.2, P_1(w_3) = 0.4, P_1(w_4) = 0.1$$
$$P_2 : P_2(w_1) = 0.1, P_2(w_2) = 0.3, P_2(w_3) = 0.5, P_2(w_4) = 0.1$$
$$P_3 : P_3(w_1) = 0.5, P_3(w_2) = 0.1, P_3(w_3) = 0.1, P_3(w_4) = 0.3$$
$$P_4 : P_4(w_1) = 0.2, P_4(w_2) = 0.1, P_4(w_3) = 0.3, P_4(w_4) = 0.4$$

- Determine $\underline{P}(\{w_1, w_2\})$ and $\overline{P}(\{w_3, w_4\})$ and indicate the distributions that realise these lower and upper bounds.
- Determine $\underline{P}(\{w_2, w_3\})$ and $\overline{P}(\{w_1, w_4\})$ and indicate the distributions that realise these lower and upper bounds.

Q8 For $\mathcal{P}$ a finite set of probability distributions on W, show that for all $A \subseteq W$, $\overline{P}(A) = 1 - \underline{P}(A)$.