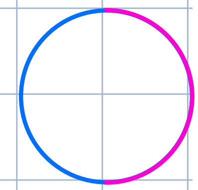
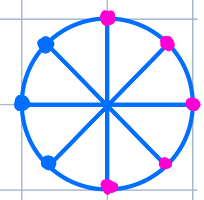
Filter vs zeroing DFT: Why Filter better?

DTFT − $X(\Omega)=\sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n}$ $\Omega=\frac{2\pi f}{f_s}$.



DFT − $X(\Omega_k)=\sum_{n=0}^{N-1} x[n]e^{-j\Omega_k n}$ $\Omega=\frac{2\pi k}{N}$, $k=0,1\dots N-1$



IDTFT − $x[n]=\frac{1}{2\pi}\int_0^{2\pi} X(\Omega)e^{j\Omega n}\,d\omega$, $n=(-\infty,+\infty)$

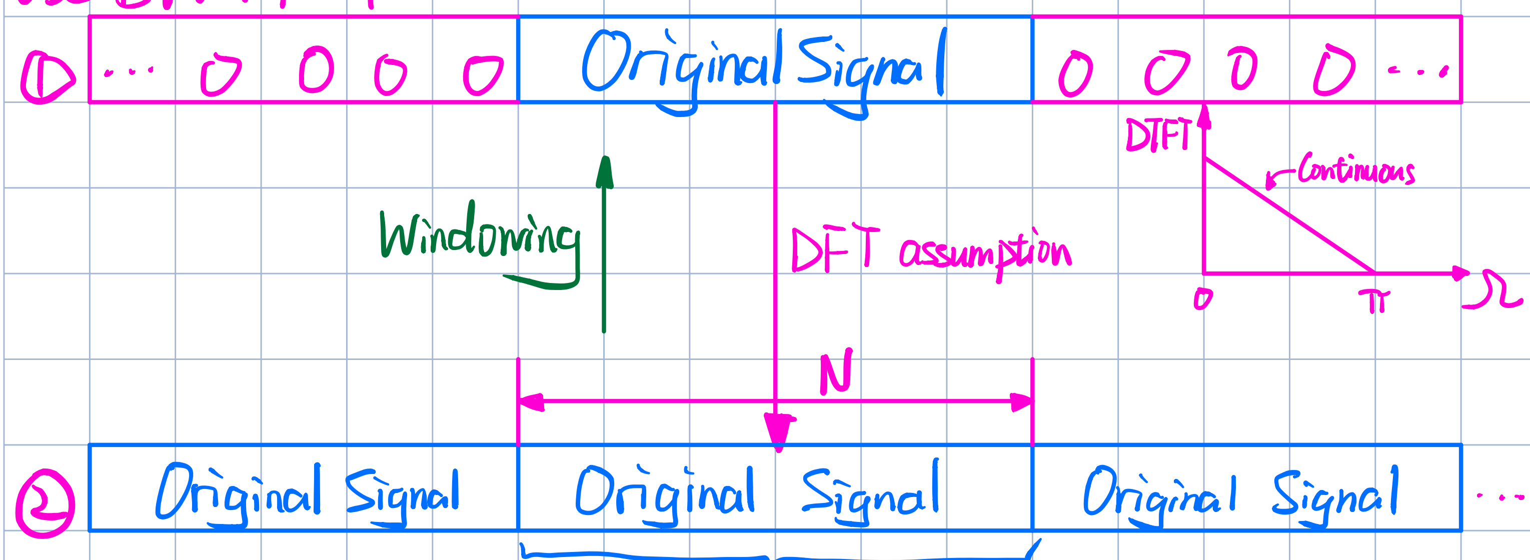
IDFT − $x[n]=\frac{1}{N}\sum_{k=0}^{N-1} X(\Omega_k)e^{j\Omega_k n}$ $n=[0,N-1]$

↳ IDFT has same result over the truncated region of the result of IDTFT

Zeroing DFT Coefficient

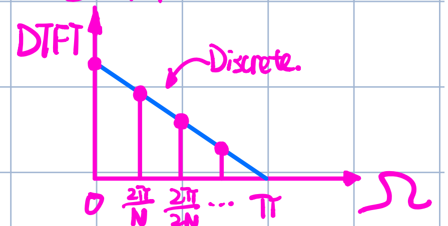
= Applying and designing a filter with frequency sampling method.
+ truncation

Use DTFT perspective

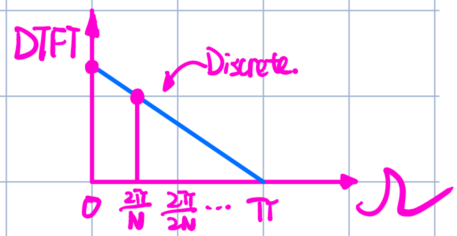


DFT Assume the original signal to be periodical

Under this assumption, DFT = DTFT



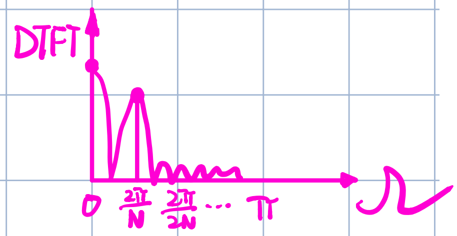
③ | Manipulated Signal | Manipulated Signal | Manipulated. Signal | . . .



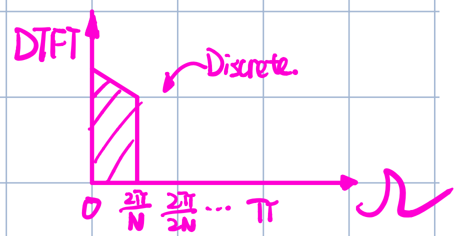
(output)

④ | 0 0 0 0 0 | Manipulated Signal | 0 0 0 0 0 | . . .

What we got for filtered signal :



What we expected for filtered signal :



Trade-off between frequency resolution and leakage in window selection

Selection of window shape

Good Frequency Resolution ⟷ Small leakage

The ability to precisely know the distribution of energy in frequency domain (When each frequency component has similar energy)

↑

The width of main lobe

The ability to detect frequency components with low energy (When there is difference in the energy of frequency component)

↑

The attenuation between mainlobe and sidelobe

1. This trade-off is introduced by the fact that Fourier analysis assume the signal to be periodic or with infinite length.

Narrow mainlobe → Good Frequency Resolution
High sidelobe

Wide mainlobes → Small leakage
Low sidelobes