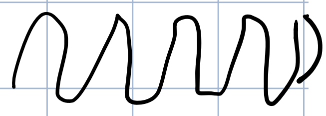
1、How to represent arbitary function

Fourier base function (Non-compact support )

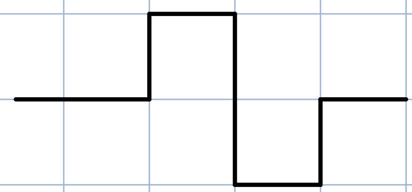
$h_p(x) = e^{j\frac{2\pi p x}{T}}$

$$\langle h_p(x), h_q(x)\rangle = \int_0^T e^{j\frac{2\pi p x}{T}} e^{-j\frac{2\pi q x}{T}} dx$$

$$= \begin{cases} 0 & p \neq q \\ T & p = q \end{cases}$$

Attenuation term

↳ Windowed STFT $h_p(x) = g(x)\, e^{j\frac{2\pi p(x)}{T}}$

Wavelet base function (Compact Support: )

$\psi_{n,k}(x) = 2^{n/2}\psi(2^n x - k)$

$$\langle \psi_{n_1 k_1}(x), \psi_{n_2 k_2}(x)\rangle = \begin{cases} 0 & \text{otherwise} \\ 1 & n_1 = k_1, n_2 = k_2 \end{cases}$$

Subspace V_0 has sampling interval T_0

↳ Subspace V_j has sampling interval $T_j = 2^{-j} T_0$ $\begin{cases} j<0 \text{ coarser scale } T_j \uparrow \\ j>0 \text{ finer scale } T_j \downarrow \end{cases}$

↳ V_j is the approximation of original signal at scale j

V_j is the linear span of the scaled and translated scaling function

vector space formed by linear combination of base vectors.

$\phi_{jn}(t) = 2^{\frac{j}{2}} \phi(2^j t - n)$ (Scale, translate)

$\phi_{jn}(t)$ are orthonormal for each j : $\langle \phi_{jn}(t), \phi_{jm}(t) \rangle = \delta_{mn} = \begin{cases} 0 & m \neq n \\ 1 & m = n \end{cases}$