Teacher: **Yanjie Li**

Course: **Linear Algebra in Control Theory**

Assignment Number: **1**

Disclosure date: May 13, 2022

Problem 1

Show that the function that takes $((x_1, x_2), (y_1, y_2)) \in R^2 \times R^2$ to $|x_1 y_1| + |x_2 y_2|$ is not an inner product on R^2.

Problem 2

Suppose V is a real inner product space, show that:

a) the inner product $\langle u+v, u-v \rangle = \|u\|^2 - \|v\|^2$ for every $u, v \in V$.

b) if $u, v \in V$ have the same norm, then $u+v$ is orthogonal to $u-v$.

c) use part(b) to show that the diagonals of a rhombus are perpendicular to each other.

Problem 3

Suppose $u, v \in V$, prove that the inner product $\langle u, v \rangle = 0$ if and only if $\|u\| \leqslant \|u + av\|$ for all $a \in F$.

Problem 4

Suppose $u, v \in V$, prove that $\|au + bv\| = \|bu + av\|$ for all $a, b \in R$ if and only if $\|u\| = \|v\|$.

Problem 5

Suppose $u, v \in V$, $\|u\| = \|v\| = 1$ and $\langle u, v \rangle = 1$, prove that $u = v$.

Problem 6

Find vectors $u, v \in R^2$ such that u is a scalar multiple of $(1,3)$, v is orthogonal to $(1,3)$, and $(1,2) = u + v$.

Problem 7

Prove that $(x_1 + \cdots + x_n)^2 \leqslant n\left(x_1^2 + \cdots + x_n^2\right)$ for all positive integers n and all real numbers $x_1, ..., x_n$.

Problem 8

Suppose V is a real inner product space, prove that

$$\langle u, v\rangle = \frac{\|u+v\|^2 - \|u-v\|^2}{4}$$

for all $u, v \in V$.

Pay Attention

a) Mark your class number, name and student number on the homework.
b) Please hand in your homework to your TA before class next Friday (May 20).

References

[1] Axler, S. (1997). Linear algebra done right. Springer Science & Business Media.

[2] Lay, D. C. . Linear algebra and its applications. Academic Press.