例 3.9: 某反馈控制系统的开环传递函数为

22-PSP

$$G_0(s) = \frac{K}{s(\frac{1}{6}s+1)(\frac{1}{2}s+1)}$$

要求满足下列性能指标：

(1) 系统为 I 型，开环增益为 $K_v = 180\ \mathrm{s}^{-1}$；

(2) 剪切频率 $\omega_c \geq 3.5$ rad/s;

(3) 相角裕度 $\gamma \geq 40°$。

解： $G_0(s) = \dfrac{180}{s(\frac{1}{6}s+1)(\frac{1}{2}s+1)}$ 转折频率：2, 6

$$L(\omega) = \begin{cases} 20(\lg\frac{180}{\omega}) & 0<\omega<2 \\ 20(\lg\frac{180}{\omega} - \lg\frac{\omega}{2}) & 2<\omega<6 \\ 20(\lg\frac{180}{\omega} - \lg\frac{\omega}{2} - \lg\frac{\omega}{6}) & 6<\omega \end{cases}$$

$\omega_{c0} = 12.927$ rad/s

$\angle G_0(j\omega) = -90° - \mathrm{atan}(\frac{\omega}{6}) - \mathrm{atan}(\frac{\omega}{2})$

$\Rightarrow \gamma_0(j\omega_c) = 180° + \angle G_0(j\omega_c) = -56.31°$

$\gamma_0(j\omega_{c1}) = 180° + \angle G_0(j3.5\,\mathrm{rad/s}) = -0.5°$，相位储备不足

故迟后-超前校正，设 $\omega_c = 3.6$ rad/s

① 超前校正，$G_{c1}(s) = \dfrac{\alpha Ts+1}{Ts+1}$，$\alpha>1$，$\varphi_m \geq \gamma - \gamma_0(j\omega_c) + \Delta_2 = 40° + 6° = 46°$，选 $\varphi_m = 55°$

$\alpha = \dfrac{1+\sin\varphi_m}{1-\sin\varphi_m} = 10.06$，$T = \dfrac{1}{\omega_m\sqrt{\alpha}} = 0.0876$，$\alpha T = 0.881$

$G_{c1}(s) = \dfrac{0.881s+1}{0.0876s+1}$

$$G_1(s) = \frac{180(\overset{1.13}{0.881s+1})}{s(\underset{6}{\tfrac{1}{6}s+1})(\underset{2}{\tfrac{1}{2}s+1})(\underset{11.4}{0.0876s+1})}$$

$L(\omega) = 20(\lg\frac{180}{\omega} + \lg 0.881\omega - \lg\frac{\omega}{2})$ $2<\omega<6$

② 迟后校正，$G_{c2}(s) = \dfrac{\tau s+1}{\beta\tau s+1}$，$\beta>1$

$\beta = |G_1(j\omega_c)| = \dfrac{180 \cdot 0.881 \cdot 2}{\omega_c} = 88.1$

$\frac{1}{\tau} = \frac{1}{10}\omega_c \Rightarrow \tau = \frac{10}{\omega_c} = 2.78$，$\beta\tau = 244.7$

$G_{c2}(s) = \dfrac{2.78s+1}{244.7s+1}$

$$G(s) = \frac{180(\overset{1.13}{0.881s+1})(2.78s+1)}{s(\underset{6}{\tfrac{1}{6}s+1})(\underset{2}{\tfrac{1}{2}s+1})(\underset{11.4}{0.0876s+1})(244.7s+1)}$$

$L(\omega) = 20(\lg\frac{180}{\omega} + \lg 0.881\omega + \lg 2.78\omega - \cdots)$ $2<\omega<6$

$\Rightarrow \omega_c = 3.60$ rad/s

$\gamma = 180° + \angle G(j\omega_c) = 47.45°$ 满足要求

例 3.10: 某单位反馈系统的不可变部分为传递函数为

22-PSP

$$G_0(s) = \frac{K}{s(0.1s+1)(0.2s+1)}$$

若要求校正后系统满足
(1) 速度误差系数 $K_v = 30$;
(2) 相角裕度 $\gamma \geqslant 40°$;
(3) 增益裕度 $20\lg K_g \geqslant 10\text{dB}$;
(t) 剪切频率 $\omega_c \geqslant 5\text{rad/s}$
试设计串联校正装置。

$$G_0(s) = \frac{30}{s(0.2s+1)(0.1s+1)} \quad 5 \quad 10$$

$$L(w) = \begin{cases} 20\lg\frac{30}{w} & 0<w<5 \\ 20(\lg\frac{30}{w} - \lg 0.2w) & 5<w<10 \\ 20(\lg\frac{30}{w} - \lg 0.2w - \lg 0.1w) & 10<w \end{cases}$$

$W_{c0} = \sqrt[3]{1500} = 11.45\,rad/s$

$\angle G_0(jw) = -90° - atan0.1w - atan0.2w$

$\gamma_0 = 180° + \angle G_0(jw_{c0}) = -25.28°$

$w_{c1} = 5rad/s,\ \gamma_0(w_{c1}) = 180° + \angle G_0(jw_{c1}) = 18.43°$

需超前-迟后校正,

①超前: $G_{c1}(s) = \frac{\alpha Ts+1}{Ts+1}$, $\alpha > 1$, 设 $w_c = 5rad/s$

$\varphi_m \geqslant \gamma - \gamma_0(w_c) + \Delta_2 = 40° - 18.43° + 6° = 27.57°$

选 $\varphi_m = 30°$, $\alpha = \frac{1+\sin\varphi_m}{1-\sin\varphi_m} = 3$, $T = \frac{1}{w_m\sqrt{\alpha}} = \frac{1}{5\sqrt{3}} = 0.115$

$\alpha T = 0.346$

$G_{c1}(s) = \frac{0.346s+1}{0.115s+1}$,

$G_1(s) = G_{c1}(s)G_0(s) = \frac{30}{s(0.2s+1)(0.1s+1)} \cdot \frac{0.346s+1}{0.115s+1}$ (5, 10; 2.89, 8.70)

②迟后: $G_{c2}(s) = \frac{\tau s+1}{\beta\tau s+1}$, $\beta > 1$, $w_c = 5rad/s$

$\beta = |G_1(jw_c)| = \frac{30}{w_c} \cdot \frac{0.346w_c}{0.2w_c} = 10.38$

$\tau = \frac{10}{w_c} = 2$, $\beta\tau = 20.76 \Rightarrow G_{c2}(s) = \frac{2s+1}{20.76s+1}$

$G(s) = \frac{30}{s(0.2s+1)(0.1s+1)} \cdot \frac{0.346s+1}{0.115s+1} \cdot \frac{2s+1}{20.76s+1}$ (5, 10; 2.89, 8.70; 0.5, 0.048)

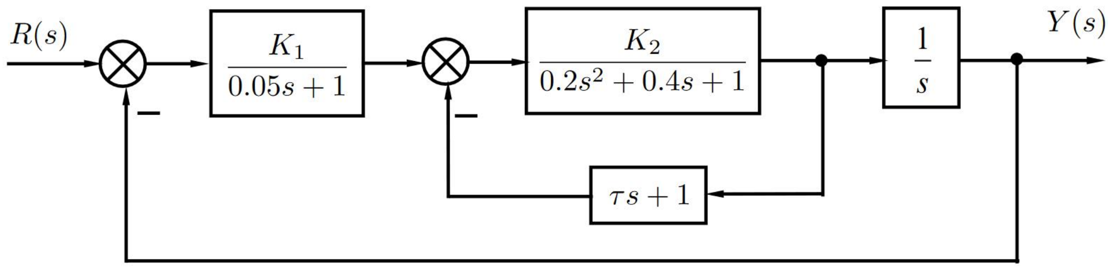
$L(w) = 20(\lg\frac{30}{w} + \lg 0.346w + \lg 2w - \lg 0.2w - \lg 20.76w) \quad 5<w<10$

$\Rightarrow w_c = 5.00\,rad/s$

$\gamma = 180° + \angle G(jw_c) = 43.34°$

令 $-180° = \angle G(jw_g) \Rightarrow w_g = 10.6\,rad/s$

$20\lg K_g = -20\lg|G(jw_g)| = 14.38dB$



例 4.7: 控制系统的结构图如4.16所示，欲采用局部反馈来改善系统的性能，要求大闭环系统的闭环主导极点为 $s_{1,2} = -3 \pm \mathrm{j}\sqrt{3}$，需要确定 K_1，K_2 和 τ 的值。

例 6.4: 将如下连续校正装置

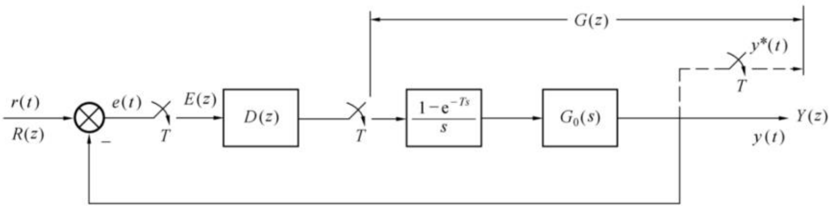
$$D(s) = \frac{1}{s^2 + 0.2s + 1}$$

在采样周期为 $T = 1\text{sec}$ 的情形下，通过双线性变换获得其离散化形式。

例 6.9: 设结构如图 6.10.1 所示单位反馈线性离散系统被控对象及零阶保持的传递函数分别为

$$G_0(s) = \frac{10}{s(s+1)}, \quad H_0(s) = \frac{1 - \mathrm{e}^{-Ts}}{s}$$

采样周期 $T = 1\text{s}$，试设计在控制输入为 $r(t) = t$ 时的最少拍无差系统。



例 8.19: 给定线性定常系统 22-PSP

$$\dot{x} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} x + \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} u \qquad y = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} x$$

对该系统进行能控性分解。

解: $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$ $AB = \begin{bmatrix} 1 & 2 \\ 1 & 0 \\ 1 & 2 \end{bmatrix}$ $A^2B = \begin{bmatrix} 3 & 4 \\ 1 & 0 \\ 3 & 4 \end{bmatrix}$

$Q_c = [B \quad AB \quad A^2B] = \begin{bmatrix} 0 & 1 & 1 & 2 & 3 & 4 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 2 & 3 & 4 \end{bmatrix} \Rightarrow r = 2 < 3$, 不能控

行满秩要求

取 Q_c 前两列加上 $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ 有 $P = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

$\Rightarrow A_c = P^{-1}AP = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 2 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ $\qquad B_c = P^{-1}B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$

$C_c = CP = (0 \quad 2 \quad 1)$

3. 两个子系统 Σ_1 和 Σ_2 串联，如图8.7 所示。Σ_1 和 Σ_2 的系统矩阵、输入矩阵和输出矩阵分别为:

$$\Sigma_1 : A_1 = -2, B_1 = 1, C_1 = 1$$

$$\Sigma_2 : A_2 = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C_2 = \begin{bmatrix} 2 & 1 \end{bmatrix}$$

(1) 求串联后的状态空间描述；(5 分)

(2) 判断 Σ_1 和 Σ_2 串联后的状态能控性和能观性；(5 分)

(3) 求串联后的传递函数。(5 分)

1. 某系统的状态空间表达式为

$$\begin{cases}\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}0 & 1\\ -2 & -4\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}u\\ y=\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}\end{cases}$$

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试设计一个带全维状态观测器的状态反馈控制系统，使观测器的极点均为 −3，闭环系统的极点为 $-5\pm j5$，要求写出观测器方程、状态反馈控制律之表达式，并画出带观测器闭环系统的系统结构图。

解：$A=\begin{pmatrix}0 & 1\\ -2 & -4\end{pmatrix}$ $B=\begin{pmatrix}0\\ 1\end{pmatrix}$ $C=(1\ \ 0)$

设状态反馈 $K=(k_1\ \ k_2)$ $A+BK=\begin{pmatrix}0 & 1\\ k_1-2 & k_2-4\end{pmatrix}$

$|\lambda I-(A+BK)|=\begin{vmatrix}\lambda & -1\\ 2-k_1 & \lambda-k_2+4\end{vmatrix}=\lambda^2+(4-k_2)\lambda+(2-k_1)$

$\alpha^*(s)=(s+5-j5)(s+5+j5)=s^2+10s+50\Rightarrow k_1=-48,\ k_2=-6$

$K=(-48\ \ -6)$

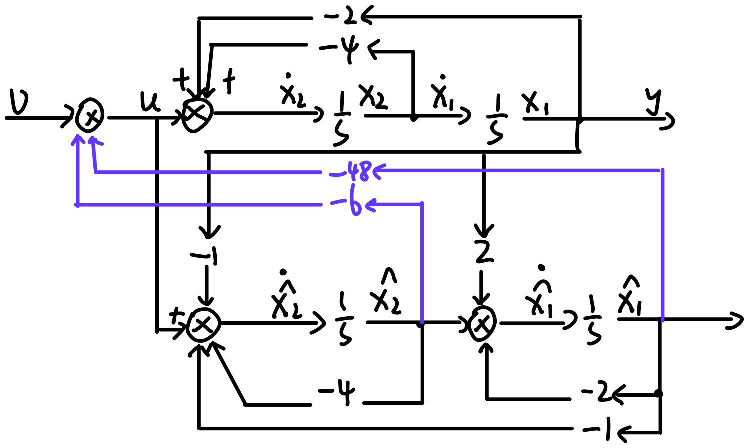
设 $L=\begin{pmatrix}l_1\\ l_2\end{pmatrix}$ $A+LC=\begin{pmatrix}l_1 & 1\\ l_2-2 & -4\end{pmatrix}$

$|\lambda I-(A+LC)|=\begin{vmatrix}\lambda-l_1 & -1\\ 2-l_2 & \lambda+4\end{vmatrix}=\lambda^2+(4-l_1)\lambda+2-4l_1-l_2$

$\beta^*(s)=(s+3)^2=s^2+6s+9\Rightarrow l_1=-2,\ l_2=1\Rightarrow L=\begin{pmatrix}-2\\ 1\end{pmatrix}$

$\dot{\hat{x}}=A\hat{x}+Bu+L(C\hat{x}-y)=(A+LC)\hat{x}+Bu-Ly$

$=\begin{pmatrix}-2 & 1\\ -1 & -4\end{pmatrix}\hat{x}+\begin{pmatrix}0\\ 1\end{pmatrix}u+\begin{pmatrix}2\\ -1\end{pmatrix}y$



22-PSP

8.设非线性系统如图10.49所示。试求：

(1) 两个非线性环节串联后的等效非线性特性；

(2) 用描述函数法求此系统的自振角频率 ω 和振幅 A。

已知：$N_1=\frac{2K}{\pi}\left[\arcsin\frac{a}{A}+\frac{a}{A}\sqrt{1-(\frac{a}{A})^2}\right], A\geqslant a$

$N_2=\frac{4b}{\pi A}\sqrt{1-(\frac{\Delta}{A})^2}, A\geqslant\Delta$

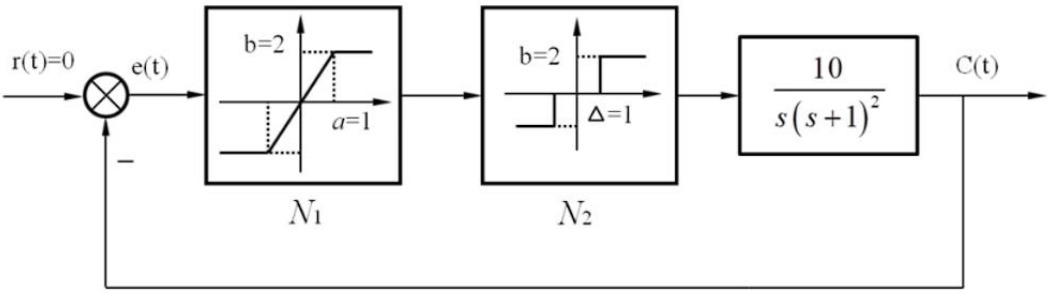


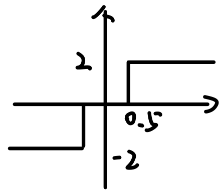
图 10.49

解：① $-0.5<e<0.5$ 时，$x=2e\in(-1,1)$，$y=0$

② $e<-0.5$ 时，$x<-1$，$y=-2$，

③ $e>0.5$ 时，$x>1$，$y=2$

⇒



相当于 $\Delta=0.5$，$b=2$ 的带死区继电，

$N(A)=\frac{8}{\pi A}\sqrt{1-(\frac{1}{2A})^2}=\frac{16}{\pi}\cdot\frac{1}{2A}\sqrt{1-(\frac{1}{2A})^2}\leqslant\frac{8}{\pi}$，当且仅当 $A=\frac{\sqrt{2}}{2}$ 等号成立

故 $A\to0.5$ 时，$-\frac{1}{N(A)}\to-\infty$，$A\to\frac{\sqrt{2}}{2}$ 时，$-\frac{1}{N(A)}\to-\frac{\pi}{8}$，$A\to+\infty$ 时，$-\frac{1}{N(A)}\to-\infty$

线性部分：

$G(s)=\frac{10}{s(s+1)^2}$，$G(j\omega)=\frac{10}{-2\omega^2+j(\omega-\omega^3)}=\frac{10[-2\omega^2-j(\omega-\omega^3)]}{4\omega^4+\omega^6-2\omega^4+\omega^2}=\frac{10[-2\omega^2-j(\omega-\omega^3)]}{\omega^6+2\omega^4+\omega^2}$

$Re=\frac{-20}{\omega^4+2\omega^2+1}$，$Im=\frac{-10(1-\omega^2)}{\omega^5+2\omega^3+\omega}$

令 $Im=0\Rightarrow\omega=1rad/s$，$Re=-5<-\frac{\pi}{8}$，即 $G(j\omega)$ 与 $-\frac{1}{N(A)}$ 有交点，

$-5=-\frac{1}{\frac{8}{\pi A}\sqrt{1-(\frac{1}{2A})^2}}\Rightarrow A=12.72$，由于 $A_1>\frac{\sqrt{2}}{2}$，是上面的交点，是稳定的

会产生自激振荡，振幅12.72，频率 1rad/s