

15. 某系统如图 3.29 所示, 试求单位阶跃响应的最大超调量 σ_p 、上升时间 t_r 和调整时间 t_s 。

解: 如图:

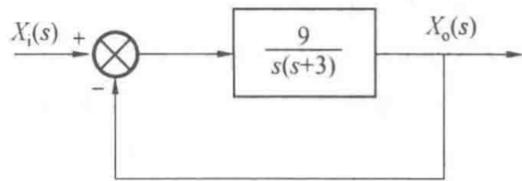


图 3.29 15 题图

$$(X_i - X_o) \frac{9}{s(s+3)} = X_o$$

∴ 应有传递函数 $G(s)$:

$$G(s) = \frac{X_o(s)}{X_i(s)} = \frac{9}{9 + s(s+3)} = \frac{9}{s^2 + 3s + 9}$$

⇒ 对应到二阶系统的标准形式传递函数有

$$\begin{cases} 9 = w_n^2 \\ 2\zeta w_n = 3 \end{cases} \quad \text{解得 } \begin{cases} w_n = 3 \text{ rad/s} \\ \zeta = 0.5 \end{cases} \quad 0 < \zeta < 1 \text{ 反阻尼情形}$$

∴ σ_p 对应 t_p : $[Y(s) = \frac{1}{2} G(s) \cdot \mathcal{L}[sY(s)]|_{t_p=0}] \frac{\pi}{\omega_d} \star$

$$\text{有 } \sigma_p = e^{-\frac{3}{\sqrt{1-3^2}} \cdot \pi} \times 100\% \approx 16.303\%$$

上升时间 t_r 有: $\int_0^{t_r} \sin(\omega_d t + \varphi) dt = 0 \Rightarrow \varphi = \arccos 3 = \frac{\pi}{3}$

$$\text{有 } t_r = \frac{\pi - \varphi}{\omega_d} = \frac{\pi - \frac{\pi}{3}}{\sqrt{1-3^2} w_n} = \frac{\frac{2}{3}\pi}{\frac{\sqrt{10}}{2} \times 3} = \frac{4\sqrt{3}}{27}\pi s (\approx 0.81s)$$

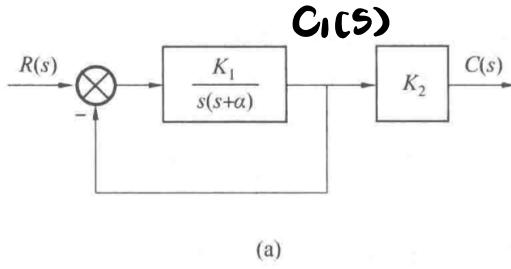
调整时间 t_s 有:

$$t_s(5\%) = \frac{3}{3w_n} = 2s \quad t_s(2\%) = \frac{4}{3w_n} = \frac{8}{3} \approx 2.67s$$

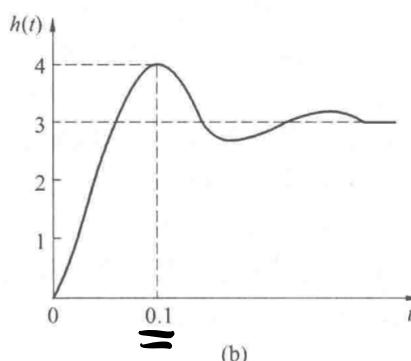
⇒ σ_p 约 16.303%, t_r 约 0.81s ($\frac{4\sqrt{3}}{27}\pi s$)

t_s 对应误差容限取 5% 为 2s 取 2% 为 $\frac{8}{3}s$ ($\approx 2.67s$)

24. 图 3.34(a) 所示系统的单位阶跃响应如图 3.34(b) 所示。试确定系统参数 K_1 、 K_2 和 α 。



(a)



(b)

图 3.34 24 题图

∴ 如图所示: $\lim_{t \rightarrow \infty} h(t) = 3$, $h(t)_{\max} = 4$ 对应 $t_p = 0.1s$

$$\therefore \text{最大超调量 } OP = \frac{\lim_{t \rightarrow \infty} h(t) - \lim_{t \rightarrow 0^+} h(t)}{\lim_{t \rightarrow \infty} h(t)} = \frac{1}{3} \approx 33.3\%$$

$$t_p = 0.15$$

\Rightarrow 针对单位阶跃响应，有正弦振荡衰减分量，为双阻尼系统

$$OP = e^{-\frac{3}{\sqrt{1-\zeta^2}}\pi} \times 100\% = \frac{1}{3} \times 100\%$$

$$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}} = 0.15$$

解得：对应二阶系统标准形式 3 与 ω_n

$$\begin{cases} 3 \approx 0.33 \\ \omega_n \approx 33.28 \text{ rad/s} \end{cases}$$

X :

$$C(s) = K_2 C_1(s)$$

$$(R(s) - C_1(s)) \frac{K_1}{s(s+a)} = C_1(s) \quad \frac{R(s)}{C_1(s)} = \frac{K_1 + s(s+a)}{K_1}$$

$$\therefore \text{解得 } C(s) = \frac{K_1 K_2}{s^2 + as + K_1} R(s)$$

$$\Rightarrow G(s) = \frac{C(s)}{R(s)} = \frac{K_1 K_2}{s^2 + as + K_1} = K_2 \cdot \frac{K_1}{s^2 + as + K_1}$$

$$\therefore K_2 = \lim_{t \rightarrow \infty} h(t) = 3 = \lim_{s \rightarrow 0} s \cdot G(s) \cdot \frac{1}{s} \quad (\text{稳态增益})$$

$$\begin{cases} K_1 = \omega_n^2 \approx 1107.56 \\ a = 2\zeta\omega_n \approx 21.96 \end{cases}$$

\therefore 参数 K_1 为 1107.56 a 取 21.96 K_2 取 3
 $(\approx 1108) \quad (\approx 22)$

11. 某系统的方框图如图 6.27 所示。

(2) 当输入 $x_i(t) = (4+6t+3t^2) \cdot 1(t)$ 时, 试求其稳态误差。

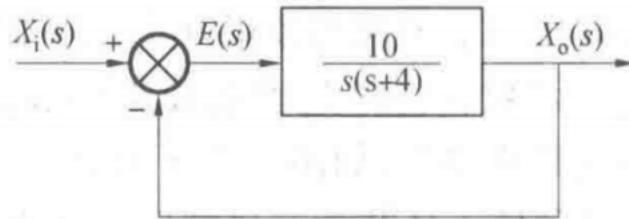


图 6.27 11 题图

11(2):

$$\because \text{先求 } \Phi_e(s) = \frac{E(s)}{X_i(s)}$$

$$= \frac{X_i(s) - X_o(s)}{X_i(s)}$$

$$= 1 - \Phi_e(s)$$

$$\text{而 } \Phi_e(s) = \frac{G(s)}{1 + G(s)H(s)}$$

$$\therefore \Phi_e(s) = \frac{1}{1 + \frac{10}{s(s+4)}} \\ = \frac{s^2 + 4s}{s^2 + 4s + 10}$$

$$\because x_i(t) = (4+6t+3t^2)1(t)$$

$$\therefore X_i(s) = \mathcal{L}\{x_i(t)\} = 6\left(\frac{1}{s} + \frac{1}{s^2} + \frac{3}{s^3}\right)$$

$$= \frac{4}{s} + \frac{6}{s^2} + \frac{6}{s^3}$$

$$\Rightarrow E(s) = X_i(s) \cdot \Phi_e(s)$$

$$= \frac{4(s+4)}{s^2 + 4s + 10} + \frac{6(1 + \frac{4}{s})}{s^2 + 4s + 10} + \frac{\frac{6(\frac{1}{s} + \frac{4}{s^2})}{s^2 + 4s + 10}}{s^2 + 4s + 10} = E_1(s) + E_2(s) + E_3(s)$$

\therefore 由终值定理知, 稳态误差

$$ess_1 = \lim_{s \rightarrow 0} s E_1(s)$$

$$= \lim_{s \rightarrow 0} \frac{4(s+4)}{s^2 + 4s + 10} = 0$$

$$ess_2 = \lim_{s \rightarrow 0} s E_2(s)$$

$$= \lim_{s \rightarrow 0} \frac{6(1 + \frac{4}{s})}{s^2 + 4s + 10} = 2.4$$

$$ess_3 = \lim_{s \rightarrow 0} s E_3(s)$$

$$= \lim_{s \rightarrow 0} \frac{6(\frac{1}{s} + \frac{4}{s^2})}{s^2 + 4s + 10} = \infty$$

$$\therefore ess = ess_1 + ess_2 + ess_3$$

$$= \infty$$

∴ 稳态误差取 ∞