The proof of the Theorem 1

Proof. Introduce the following Lyapunov function

$$V(t)=\sum_{p=1}^{n}\sum_{k=1}^{m}\lambda_p\delta_k V_p^{(k)}\left(e_p^{(k)}(t),t\right),$$

with

$$V_p^{(k)}\left(e_p^{(k)}(t),t\right)=\left(e_p^{(k)}(t)\right)^{\mathrm{T}}e_p^{(k)}(t)+\int_{t-\Upsilon(t)}^{t}\left(e_p^{(k)}(s)\right)^{\mathrm{T}}e_p^{(k)}(s)\mathrm{d}s+\left(\xi_{1p}^{(k)}(t)-\bar{\xi}_{1p}^{(k)}\right)^2,$$

where λ_p and δ_k are respectively the cofactors of the pth and kth diagonal elements of the Laplacian matrices in directed graphs $(\mathcal{H},A)$ and $(\mathcal{G},B)$. It can be found from Assumption 3 that $\lambda_p>0$ and $\delta_k>0$. Then, one gets

$$\begin{aligned}\dot{V}(t)&=\sum_{p=1}^{n}\sum_{k=1}^{m}\lambda_p\delta_k\dot{V}_p^{(k)}\left(e_p^{(k)}(t),t\right)\\&=\sum_{p=1}^{n}\sum_{k=1}^{m}\lambda_p\delta_k\left[2\left(e_p^{(k)}(t)\right)^{\mathrm{T}}\dot{e}_p^{(k)}(t)+\left(e_p^{(k)}(t)\right)^{\mathrm{T}}e_p^{(k)}(t)-(1-\dot{\Upsilon}(t))\left(e_p^{(k)}(t-\Upsilon(t))\right)^{\mathrm{T}}e_p^{(k)}(t-\Upsilon(t))\right.\\&\quad\left.+2\left(\xi_{1p}^{(k)}(t)-\bar{\xi}_{1p}^{(k)}\right)\dot{\xi}_{1p}^{(k)}(t)\right].\end{aligned}$$

We get for any $t\neq t_i$,

$$\begin{aligned}\dot{V}(t)=\sum_{p=1}^{n}\sum_{k=1}^{m}\lambda_p\delta_k&\left[2(e_p^{(k)}(t))^{\mathrm{T}}\Bigg(F\left(x_p^{(k)}(t)\right)-F\left(x_0(t)\right)+\sum_{q=1}^{n}a_{pq}^{(k)}H_{pq}^{(k)}\left(e_q^{(k)}(t-\Upsilon(t))-e_p^{(k)}(t-\Upsilon(t))\right)\right.\\&+\sum_{l=1}^{m}b_p^{(kl)}G_p^{(kl)}\left(e_p^{(l)}(t-\Upsilon(t))-e_p^{(k)}(t-\Upsilon(t))\right)-\xi_p^{(k)}(t)e_p^{(k)}(t)\\&-\left(\xi_{2p}^{(k)}\int_{t-\Upsilon(t)}^{t}\left(e_p^{(k)}(s)\right)^{\mathrm{T}}e_p^{(k)}(s)\mathrm{d}s\right)\frac{e_p^{(k)}(t)}{\left\|e_p^{(k)}(t)\right\|^2}\\&-\left(\xi_{3p}^{(k)}\int_{t-\Upsilon(t)}^{t}\left(e_p^{(k)}(s)\right)^{\mathrm{T}}e^{(k)}(s)\mathrm{d}s\right)^{\frac{c+d}{2c}}\frac{e_p^{(k)}(t)}{\left\|e_p^{(k)}(t)\right\|^2}\\&-\left(\xi_{4p}^{(k)}\int_{t-\Upsilon(t)}^{t}\left(e_p^{(k)}(s)\right)^{\mathrm{T}}e_p^{(k)}(s)\mathrm{d}s\right)^{\frac{\alpha+\beta}{2d}}\frac{e_p^{(k)}(t)}{\left\|e_p^{(k)}(t)\right\|^2}\\&-\xi_{5p}^{(k)}\left(e_p^{(k)}(t)\right)^{\frac{d}{c}}-\xi_{6p}^{(k)}\left(e_p^{(k)}(t)\right)^{\frac{\beta}{\alpha}}\Bigg)\end{aligned}\tag{1}$$

$$
\begin{aligned}
&+\left(e_p^{(k)}(t)\right)^{\mathrm{T}} e_p^{(k)}(t)-\left((1-\dot{\Upsilon}(t)\right)\left(e_p^{(k)}(t-\Upsilon(t))\right)^{\mathrm{T}} e_p^{(k)}(t-\Upsilon(t)) \\
&+2\left(\xi_{1p}^{(k)}(t)-\bar{\xi}_{1p}^{(k)}\right)\left(\left(ep^{(k)}(t)\right)^{\mathrm{T}} e_p^{(k)}(t)-\left(\xi_{1p}^{(k)}(t)-\bar{\xi}_{1p}^{(k)}\right)^{\frac{d}{c}}-\left(\xi_{1p}^{(k)}(t)-\bar{\xi}_{1p}^{(k)}\right)^{\frac{\beta}{\alpha}}-\left(\xi_{1p}^{(k)}(t)-\bar{\xi}_{1p}^{(k)}\right)\right)\Bigg]. \qquad (2)
\end{aligned}
$$

According to Assumptions 1 and 2, one gets that

$$
2\left(e_p^{(k)}(t)\right)^{\mathrm{T}}\left(F\left(x_p^{(k)}(t)\right)-F(x_0(t))\right) \leq 2L\left\|e_p^{(k)}(t)\right\|^2, \qquad (3)
$$

$$
\begin{aligned}
&2\left(e_p^{(k)}(t)\right)^{\mathrm{T}} \sum_{q=1}^{n} a_{pq}^{(k)} H_{pq}^{(k)}\left(e_q^{(k)}(t-\Upsilon(t))-e_p^{(k)}(t-\Upsilon(t))\right) \\
\leq& 2\sum_{q=1}^{n} a_{pq}^{(k)} h_{pq}^{(k)}\left\|e_p^{(k)}(t)\right\| \| e_q^{(k)}(t-\Upsilon(t))-e_p^{(k)}(t-\Upsilon(t)) \| \\
\leq& \sum_{q=1}^{n} a_{pq}^{(k)} h_{pq}^{(k)}\left\|e_p^{(k)}(t)\right\|^2+2\sum_{q=1}^{n} a_{pq}^{(k)} h_{pq}^{(k)}\left\|e_p^{(k)}(t-\Upsilon(t))\right\|^2+2\sum_{q=1}^{n} a_{pq}^{(k)} h_{pq}^{(k)}\left\|e_q^{(k)}(t-\Upsilon(t))\right\|^2 \qquad (4) \\
\leq& \sum_{q=1}^{n} a_{pq}\left\|e_p^{(k)}(t)\right\|^2+4\sum_{q=1}^{n} a_{pq}\left\|e_p^{(k)}(t-\Upsilon(t))\right\|^2+2\sum_{q=1}^{n} a_{pq}\left(\left\|e_q^{(k)}(t-\Upsilon(t))\right\|^2-\left\|e_p^{(k)}(t-\Upsilon(t))\right\|^2\right),
\end{aligned}
$$

and

$$
\begin{aligned}
&2\left(e_p^{(k)}(t)\right)^{\mathrm{T}} \sum_{l=1}^{m} b_p^{(kl)} G_p^{(kl)}\left(e_p^{(l)}(t-\Upsilon(t))-e_p^{(k)}(t-\Upsilon(t))\right) \\
\leq& 2\sum_{t=1}^{M} b_p^{(kl)} g_p^{(kl)}\left\|e_p^{(k)}(t)\right\|\left\|e_p^{(l)}(t-\Upsilon(t))-e_p^{(k)}(t-\Upsilon(t))\right\| \\
\leq& \sum_{l=1}^{m} b_p^{(kl)} g_p^{(kl)}\left\|e_p^{(k)}(t)\right\|^2+2\sum_{l=1}^{m} b_p^{(kl)} g_p^{(kl)}\left\|e_p^{(k)}(t-\Upsilon(t))\right\|^2+2\sum_{l=1}^{m} b_p^{(kl)} g_p^{(kl)}\left\|e_p^{(l)}(t-\Upsilon(t))\right\|^2 \qquad (5) \\
\leq& \sum_{l=1}^{m} b^{(kl)}\left\|e_p^{(k)}(t)\right\|^2+4\sum_{l=1}^{m} b^{(kl)}\left\|e_p^{(k)}(t-\Upsilon(t))\right\|^2+2\sum_{l=1}^{m} b^{(kl)}\left(\left\|e_p^{(l)}(t-\Upsilon(t))\right\|^2-\left\|e_p^{(k)}(t-\Upsilon(t))\right\|^2\right).
\end{aligned}
$$

Because $c > d$ and $\alpha < \beta$, we get $0 < \dfrac{c+d}{2c} < 1$ and $\dfrac{\alpha+\beta}{2\alpha} > 1$. Moreover, one has

$$
\begin{aligned}
&-2\left(e_p^{(k)}(t)\right)^{\mathrm{T}}\left(\xi_{3p}^{(k)} \int_{t-\Upsilon(t)}^{t}\left(e_p^{(k)}(s)\right)^{\mathrm{T}} e_p^{(k)}(s)\mathrm{d}s\right)^{\frac{c+d}{2c}} \frac{e_p^{(k)}(t)}{\left\|e_p^{(k)}(t)\right\|^2}-2\left(e_p^{(k)}(t)\right)^{\mathrm{T}} \xi_{5p}^{(k)}\left(e_p^{(k)}(t)\right)^{\frac{d}{c}} \\
\leq& -2\left(\xi_{3p}^{(k)} \int_{t-\Upsilon(t)}^{t}\left(e_p^{(k)}(s)\right)^{\mathrm{T}} e_p^{(k)}(s)\mathrm{d}s\right)^{\frac{c+d}{2c}}-2\xi_{5p}^{(k)}\left(\left\|e_p^{(k)}(t)\right\|^2\right)^{\frac{c+d}{2c}}, \qquad (6)
\end{aligned}
$$

and

$$
-2\left(e_p^{(k)}(t)\right)^{\mathrm{T}}\left(\xi_{4p}^{(k)} \int_{t-\Upsilon(t)}^{t}\left(e^{(k)}(s)\right)^{\mathrm{T}} e_p^{(k)}(s)\mathrm{d}s\right)^{\frac{\alpha+\beta}{2\alpha}} \frac{e_p^{(k)}(t)}{\left\|e_p^{(k)}(t)\right\|^2}-2\left(e_p^{(k)}(t)\right)^{\mathrm{T}} \xi_{6p}^{(k)}\left(e_p^{(k)}(t)\right)^{\frac{\beta}{\alpha}}
$$

$$\leq -2\left(\xi_{4p}^{(k)}\int_{t-\Upsilon(t)}^{t}\left(e_p^{(k)}(s)\right)^{\mathrm{T}}e_p^{(k)}(s)\mathrm{d}s\right)^{\frac{\alpha+\beta}{2\alpha}}-2\sigma^{\frac{\alpha-\beta}{2\alpha}}\xi_{6p}^{(k)}\left(\left\|e_p^{(k)}(t)\right\|\right)^{\frac{\alpha+\beta}{2\alpha}}. \tag{7}$$

Substituting (4)-(7) into (1), one has

$$\begin{aligned}
\dot{V}(t)\leq&\sum_{p=1}^{n}\sum_{k=1}^{m}\lambda_p\delta_k\left(2L+\sum_{q=1}^{n}a_{pq}+\sum_{l=1}^{m}b^{(kl)}-2\bar{\xi}_{1p}^{(k)}\right)\left\|e_p^{(k)}(t)\right\|^2-2\sum_{p=1}^{n}\sum_{k=1}^{m}\lambda_p\delta_k\left(\xi_{1p}^{(k)}(t)-\bar{\xi}_{1p}^{(k)}\right)^2\\
&+\sum_{p=1}^{n}\sum_{k=1}^{m}\lambda_p\delta_k\left(4\sum_{q=1}^{n}a_{pq}+4\sum_{l=1}^{m}b^{(kl)}-(1-\Upsilon)\right)\left\|e_p^{(k)}(t-\Upsilon(t))\right\|^2\\
&+2\sum_{p=1}^{n}\sum_{k=1}^{m}\sum_{q=1}^{n}\lambda_p\delta_k a_{pq}\left(\left\|e_q^{(k)}(t-\Upsilon(t))\right\|^2-\left\|e_p^{(k)}(t-\Upsilon(t))\right\|^2\right)\\
&+2\sum_{p=1}^{n}\sum_{k=1}^{m}\sum_{l=1}^{n}\lambda_p\delta_k b^{(kl)}\left(\left\|e_q^{(l)}(t-\Upsilon(t))\right\|^2-\left\|e_p^{(k)}(t-\Upsilon(t))\right\|^2\right)\\
&-2\sum_{p=1}^{n}\sum_{k=1}^{m}\lambda_p\delta_k\xi_{2p}^{(k)}\int_{t-\Upsilon(t)}^{t}\left(e_p^{(k)}(s)\right)^{\mathrm{T}}e_p^{(k)}(s)\mathrm{d}s\\
&-2\sum_{p=1}^{n}\sum_{k=1}^{m}\lambda_p\delta_k\left[\left(\xi_{3p}^{(k)}\int_{t-\Upsilon(t)}^{t}\left(e_p^{(k)}(s)\right)^{\mathrm{T}}e_p^{(k)}(s)\mathrm{d}s\right)^{\frac{c+d}{2c}}+\xi_{5p}^{(k)}\left(\left\|e_p^{(k)}(t)\right\|^2\right)^{\frac{c+d}{2c}}+\left(\left(\xi_{1p}^{(k)}(t)-\bar{\xi}_{1p}^{(k)}\right)^2\right)^{\frac{c+d}{2c}}\right]\\
&-2\sum_{p=1}^{n}\sum_{k=1}^{m}\lambda_p\delta_k\left[\left(\xi_{4p}^{(k)}\int_{t-\Upsilon(t)}^{t}\left(e_p^{(k)}(s)\right)^{\mathrm{T}}e_p^{(k)}(s)\mathrm{d}s\right)^{\frac{\alpha+\beta}{2\alpha}}+2\sigma^{\frac{\alpha-\beta}{2\alpha}}\xi_{6p}^{(k)}\left(\left\|e_p^{(k)}(t)\right\|^2\right)^{\frac{\alpha+\beta}{2\alpha}}+\left(\left(\xi_{1p}^{(k)}(t)-\bar{\xi}_{1p}^{(k)}\right)^2\right)^{\frac{\alpha+\beta}{2\alpha}}\right].
\end{aligned}\tag{8}$$

Based on Assumption 3, when digraphs $(\mathcal{G}, B)$ and $(\mathcal{H}, A)$ are strongly connected, it yields

$$\sum_{p=1}^{n}\sum_{k=1}^{m}\sum_{q=1}^{n}\lambda_p\delta_k a_{pq}\left(\left\|e_q^{(k)}(t-\Upsilon(t))\right\|^2-\left\|e_p^{(k)}(t-\Upsilon(t))\right\|^2\right)=0, \tag{9}$$

and

$$\sum_{p=1}^{n}\sum_{k=1}^{m}\sum_{l=1}^{m}\lambda_p\delta_k b^{(kl)}\left(\left\|e_p^{(l)}(t-\Upsilon(t))\right\|^2-\left\|e_p^{(k)}(t-\Upsilon(t))\right\|^2\right)=0. \tag{10}$$

One gets

$$\begin{aligned}
&-2\sum_{p=1}^{n}\sum_{k=1}^{m}\lambda_p\delta_k\left[\left(\xi_{3p}^{(k)}\int_{t-\Upsilon(t)}^{t}\left(e_p^{(k)}(s)\right)^{\mathrm{T}}e_p^{(k)}(s)\mathrm{d}s\right)^{\frac{c+d}{2c}}+\xi_{5p}^{(k)}\left(\left\|e_p^{(k)}(t)\right\|^2\right)^{\frac{c+d}{2c}}+\left(\left(\xi_{1p}^{(k)}(t)-\bar{\xi}_{1p}^{(k)}\right)^2\right)^{\frac{c+d}{2c}}\right]\\
&\leq -2\xi\sum_{p=1}^{n}\sum_{k=1}^{m}\left(\lambda_p\delta_k\right)^{\frac{c+d}{2c}}\left[\left(\int_{t-\Upsilon(t)}^{t}\left(e_p^{(k)}(s)\right)^{\mathrm{T}}e_p^{(k)}(s)\mathrm{d}s\right)^{\frac{c+d}{2c}}+\left(\|e_p^{(k)}(t)\|^2\right)^{\frac{c+d}{2c}}+\left(\left(\xi_{1p}^{(k)}(t)-\bar{\xi}_{1p}^{(k)}\right)^2\right)^{\frac{c+d}{2c}}\right]\\
&\leq -\xi_2\sum_{p=1}^{n}\sum_{k=1}^{m}\left[\lambda_p\delta_k\int_{t-\Upsilon(t)}^{t}\left(e_p^{(k)}(s)\right)^{\mathrm{T}}e_p^{(k)}(s)\mathrm{d}s+\left\|e_p^{(k)}(t)\right\|^2+\left(\xi_{1p}^{(k)}(t)-\bar{\xi}_{1p}^{(k)}\right)^2\right]^{\frac{c+d}{2c}}
\end{aligned}\tag{11}$$

$$= -\xi_2 V^{\frac{c+d}{2c}}(t),$$

where $\xi = \min\limits_{p,k}\left\{\left(\lambda_p\delta_k\right)^{\frac{c-d}{2c}}\left(\xi_{3p}^{(k)}\right)^{\frac{c+d}{2c}}, \left(\lambda_p\delta_k\right)^{\frac{c-d}{2c}}\xi_{5p}^{(k)}\right\}$, $\xi_1 = 2\xi$. Similarly, we obtain

$$\begin{aligned}
&-2\sum_{p=1}^{n}\sum_{k=1}^{m}\lambda_p\delta_k\left[\left(\xi_{4p}^{(k)}\int_{t-\Upsilon(t)}^{t}\left(e_p^{(k)}(s)\right)^{\mathrm{T}}e_p^{(k)}(s)\mathrm{d}s\right)^{\frac{\alpha+\beta}{2\alpha}} + 2\sigma^{\frac{\alpha-\beta}{2\alpha}}\xi_{6p}^{(k)}\left(\left\|e_p^{(k)}(t)\right\|^2\right)^{\frac{\alpha+\beta}{2\alpha}} + \left(\left(\xi_{1p}^{(k)}(t)-\bar{\xi}_{1p}^{(k)}(t)\right)^2\right)^{\frac{\alpha+\beta}{2\alpha}}\right]\\
&\le -2(nm)^{\frac{\alpha-\beta}{2\alpha}}\tilde{\xi}\sum_{p=1}^{n}\sum_{k=1}^{m}\left[\lambda_p\delta_k\left(\int_{t-\Upsilon(t)}^{t}\left(e_p^{(k)}(s)\right)^{\mathrm{T}}e_p^{(k)}(s)\mathrm{d}s + \left\|e_p^{(k)}(t)\right\|^2 + \left(\xi_{1p}^{(k)}(t)-\bar{\xi}_{1p}^{(k)}\right)^2\right]^{\frac{\alpha+\beta}{2\alpha}} \qquad (12)\\
&= -\xi_2 V^{\frac{c+d}{2c}}(t),
\end{aligned}$$

where $\tilde{\xi} = \min\limits_{p,k}\left\{\left(\lambda_p\delta_k\right)^{\frac{\alpha-\beta}{2\alpha}}\left(\xi_{4p}^{(k)}\right)^{\frac{\alpha+\beta}{2\alpha}}, 2\left(\lambda_p\delta_k\right)^{\frac{\alpha-\beta}{2\alpha}}\sigma^{\frac{\alpha-\beta}{2\alpha}}\xi_{6p}^{(k)}\right\}$, $\xi_1 = 2(nm)^{\alpha-\beta}2\alpha\tilde{\xi}$. Combining (A_1) and (A_2), one obtains

$$\begin{aligned}
\dot{V}(t) \le& \sum_{p=1}^{n}\sum_{k=1}^{m}\lambda_p\delta_k\left(2L+\sum_{q=1}^{n}a_{pq}+\sum_{l=1}^{m}b^{(kl)}-2\bar{\xi}_{1p}^{(k)}\right)\left\|e_p^{(k)}(t)\right\|^2 - 2\sum_{p=1}^{n}\sum_{k=1}^{m}\lambda_p\delta_k\left(\xi_{1p}^{(k)}(t)-\bar{\xi}_{1p}^{(k)}\right)^2\\
&-2\sum_{p=1}^{n}\sum_{k=1}^{M}\lambda_p\delta_k\xi_{2p}^{(k)}\int_{t-\Upsilon(t)}^{t}\left(e_p^{(k)}(s)\right)^{\mathrm{T}}e_p^{(k)}(s)\mathrm{d}s - \xi_1 V^{\frac{\alpha+\beta}{2\alpha}}(t) - \xi_2 V^{\frac{c+d}{2c}}(t)\\
\le& -\xi_1 V^{\frac{\alpha+\beta}{2\alpha}}(t) - \xi_2 V^{\frac{c+d}{2c}}(t) - \xi_3 V(t), \qquad (13)
\end{aligned}$$

where $\xi_3 = \min\limits_{p,k}\left\{2\bar{\xi}_{1p}^{(k)} - 2L - \sum\limits_{q=1}^{n}a_{pq} - \sum\limits_{l=1}^{m}b^{(kl)}, 2, 2\delta_{2p}^{(k)}\right\}$.

When $t = t_i$, it can be calculated that

$$\begin{aligned}
V\left(t_i^{+}\right) =& \sum_{p=1}^{n}\sum_{k=1}^{m}\lambda p\delta_k V_p^{(k)}\left(e_p^{(k)}\left(t^{+}\right), t\right)\\
=& \sum_{p=1}^{n}\sum_{k=1}^{m}\lambda_p\delta_k\left[\left(e_p^{(k)}\left(t_i^{+}\right)\right)^{\mathrm{T}}e_p^{(k)}\left(t_i^{+}\right) + \int_{t_i^{+}-\Upsilon(t_i^{+})}^{t_i^{+}}\left(e_p^{(k)}(s)\right)^{\mathrm{T}}e_p^{(k)}(s)\mathrm{d}s + \left(\xi_{1p}^{(k)}\left(t_i^{+}\right)-\bar{\xi}_{1p}^{(k)}\right)^2\right]\\
\le& \sum_{p=1}^{n}\sum_{k=1}^{m}\lambda_p\delta_k\left(1+\rho_i\right)^2\left\|e_p^{(k)}\left(t_i\right)\right\|^2 + \sum_{p=1}^{n}\sum_{k=1}^{m}\lambda_p\delta_k\int_{t_i-\Upsilon(t_i)}^{t_i}\left(e_p^{(k)}(s)\right)^{\mathrm{T}}e_p^{(k)}(s)\mathrm{d}s\\
&+\sum_{p=1}^{m}\sum_{k=1}^{m}\lambda_p\delta_k\left(\xi_{1p}^{(k)}\left(t_i^{+}\right)-\bar{\xi}_{1p}^{(k)}\right)^2\\
\le& \rho V(t_i), \qquad (14)
\end{aligned}$$

where $\rho = \sup\limits_{p,k}\left\{\left(1+\rho_p^{(k)}\right)^2\right\}$. According to (13) and (14), it can be found that system (1) realizes Fd-tS, where the settling time is predicted:

$$T = \frac{\ln\left[1+\frac{\left(\xi_3-\frac{\ln\rho}{I_a}\right)\rho^{\frac{(\beta-\alpha)N_0}{2\alpha}}}{\xi_1}\right]}{\left(\xi_3-\frac{\ln\rho}{I_a}\right)\frac{\beta-\alpha}{2\alpha}} + \frac{\ln\left[\frac{\xi_2\rho^{\frac{(c-d)N_0}{2c}}}{\rho^{\frac{(d-c)N_0}{2c}}\left(\xi_3-\frac{\ln\rho}{I_a}\right)+\xi_2\rho^{\frac{(c-d)N_0}{2c}}}\right]}{\left(\frac{\ln\rho}{I_a}-\xi_3\right)\frac{c-d}{2c}}.$$

The proof is completed. □