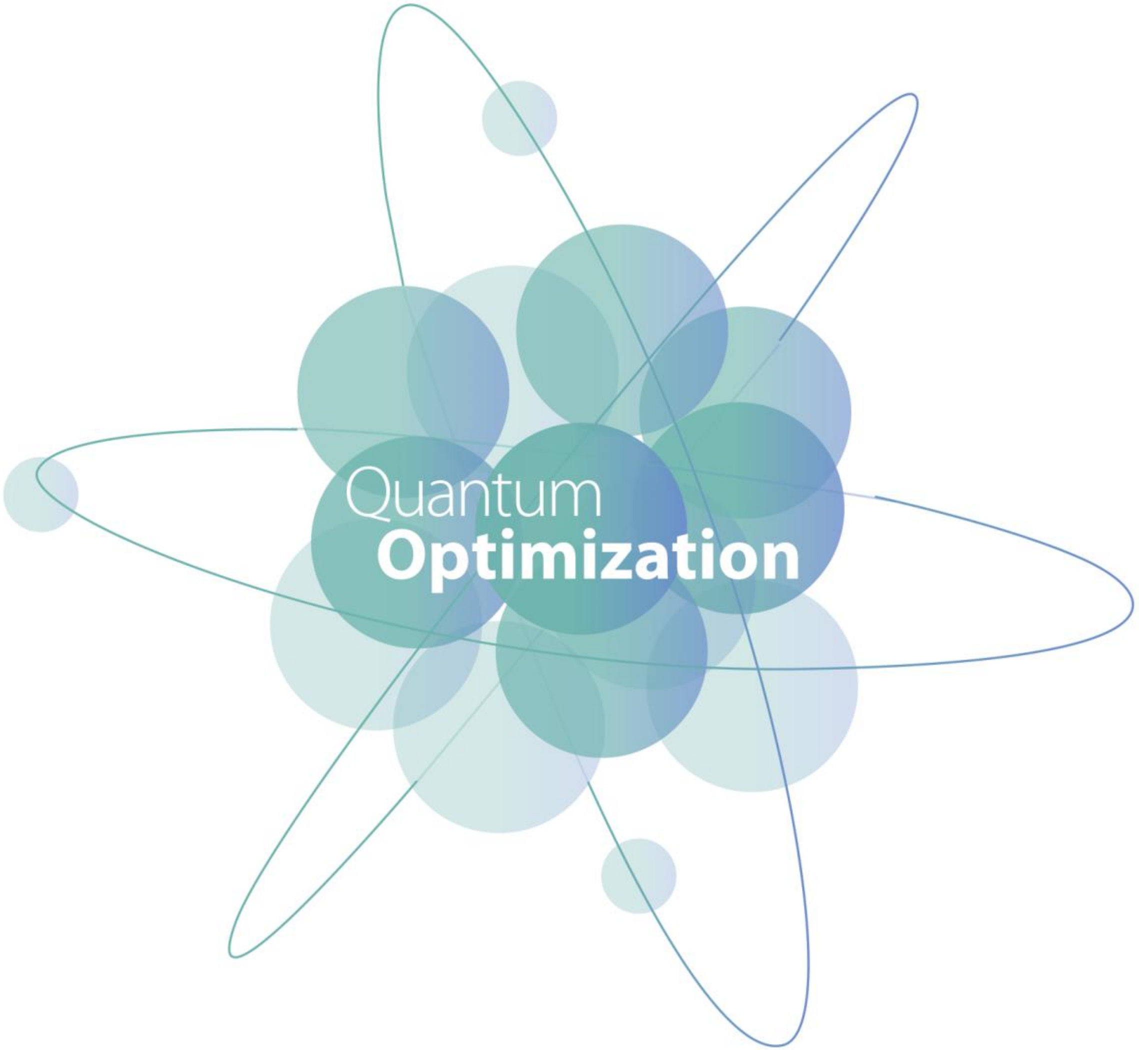

Alejandro Giraldo Quintero
Computer Science and Engineering
PhD Candidate

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Quantum Computing

Quantum Optimization

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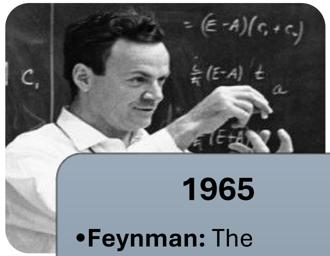
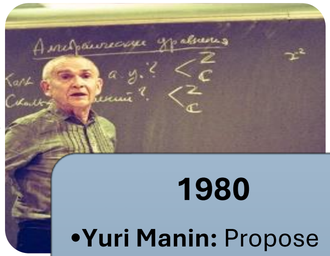
Milestones



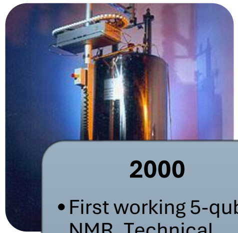
1965
- **Feynman:** The Development of the Space-Time View of Quantum Electrodynamics



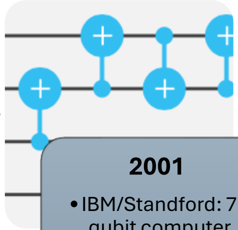
1980
- **Yuri Manin:** Propose the first idea of Quantum Computing

1985
- **David Deutsch:** Quantum theory, the Church-Turing principle and the universal quantum computer

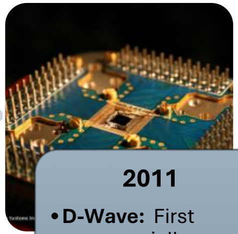
1994
- **Peter Shor:** Algorithm to find prime factors of a number



2000
- First working 5-qubit NMR. Technical University of Munich

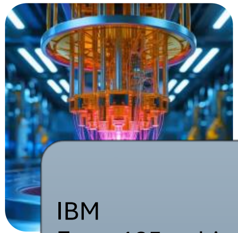


2001
- IBM/Standford: 7-qubit computer, factor number 15



2011
- **D-Wave:** First commercially available Quantum Computer

2016
- **IBM:** 5-qubit quantum processor available in the cloud IBM Q Experience



IBM
Free: 125 qubit
Pay: 1125 qubit
Dwave: 4400

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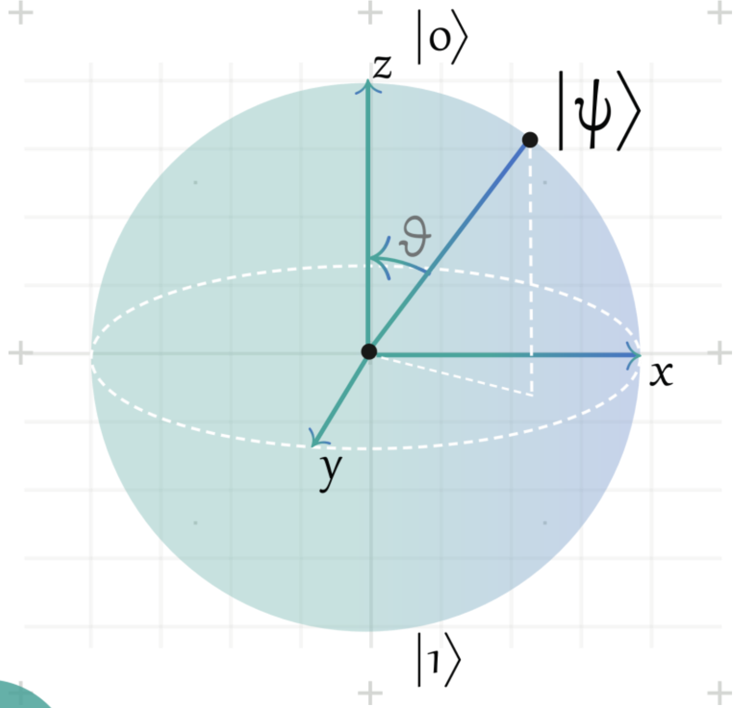
Qubit



Superposition

$$|\psi\rangle = a|0\rangle + b|1\rangle$$

$$a, b \in \mathbb{C}$$

Interference

$$|a|^2 + |b|^2 = 1$$

Entaglement

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

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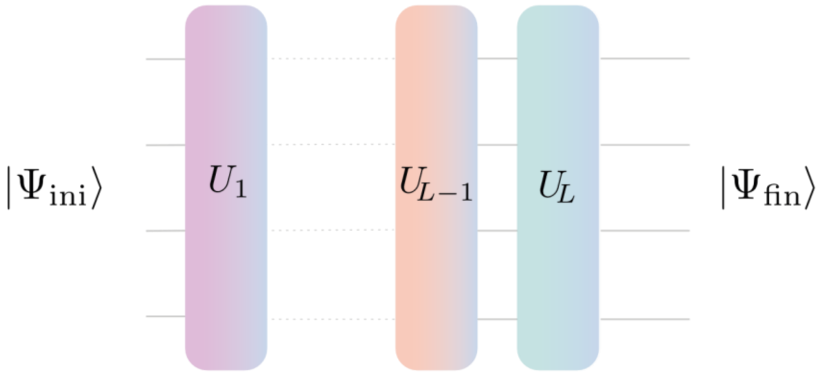
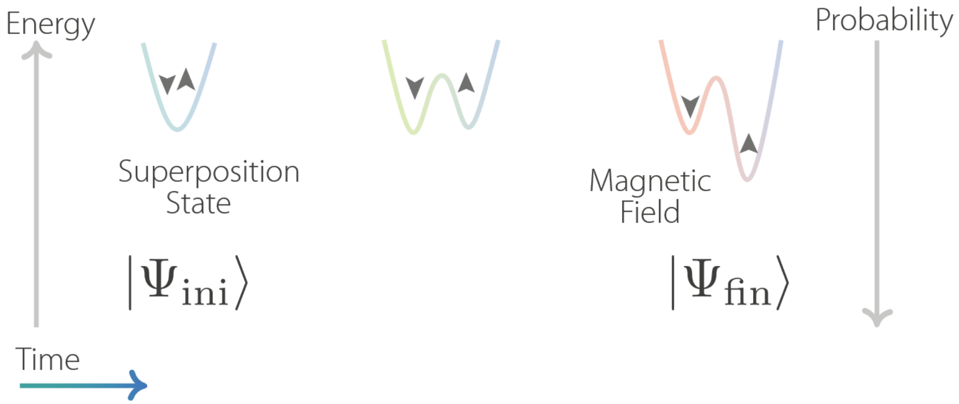
Approaches

Gates



$$|\Psi_{\text{fin}}\rangle = U|\Psi_{\text{ini}}\rangle = U_L U_{L-1} \cdots U_2 U_1 |\Psi_{\text{ini}}\rangle$$

Annealing



$$\hat{H}(t) = A(t)\hat{H}_P + B(t)\hat{H}_D$$

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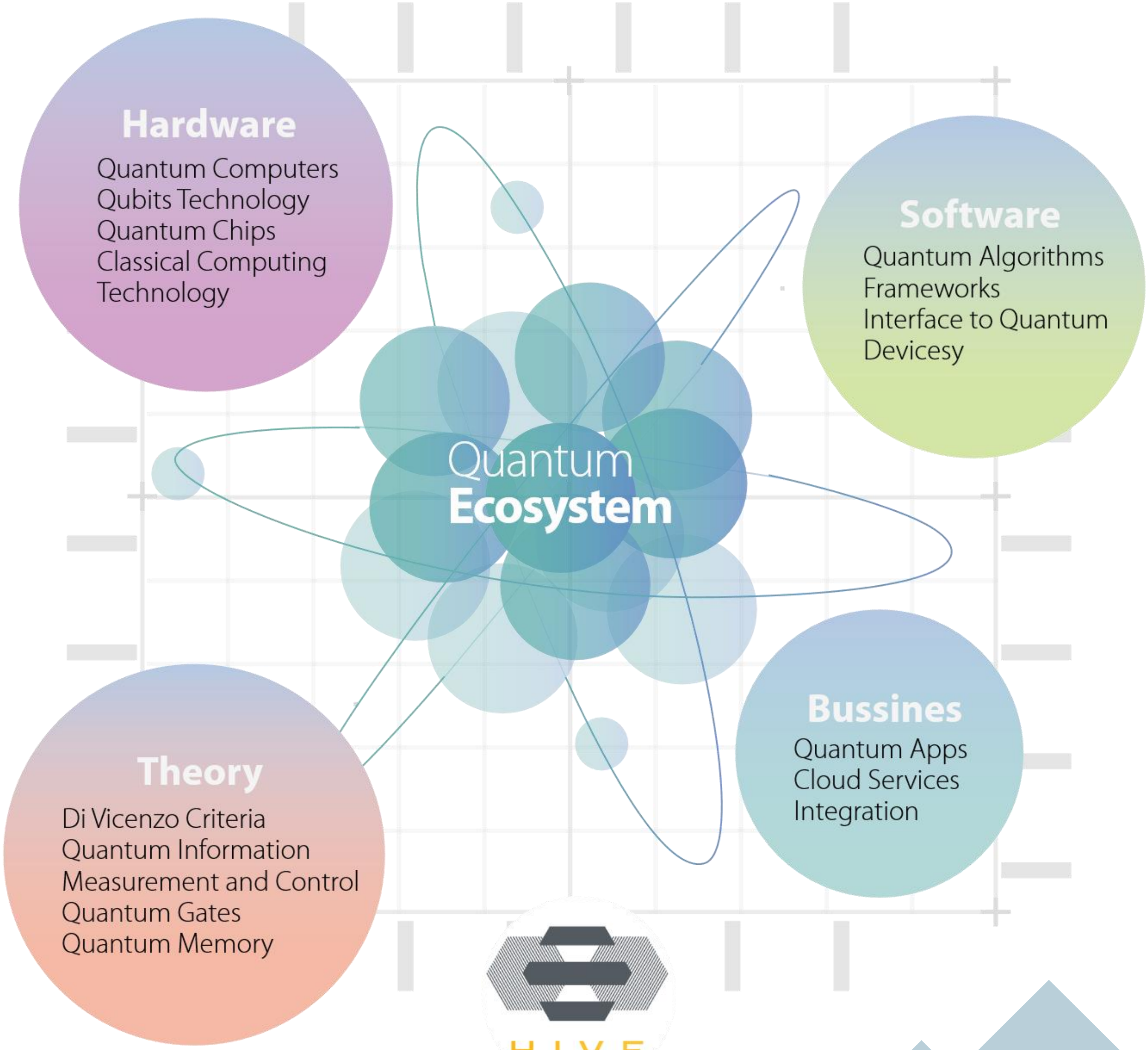

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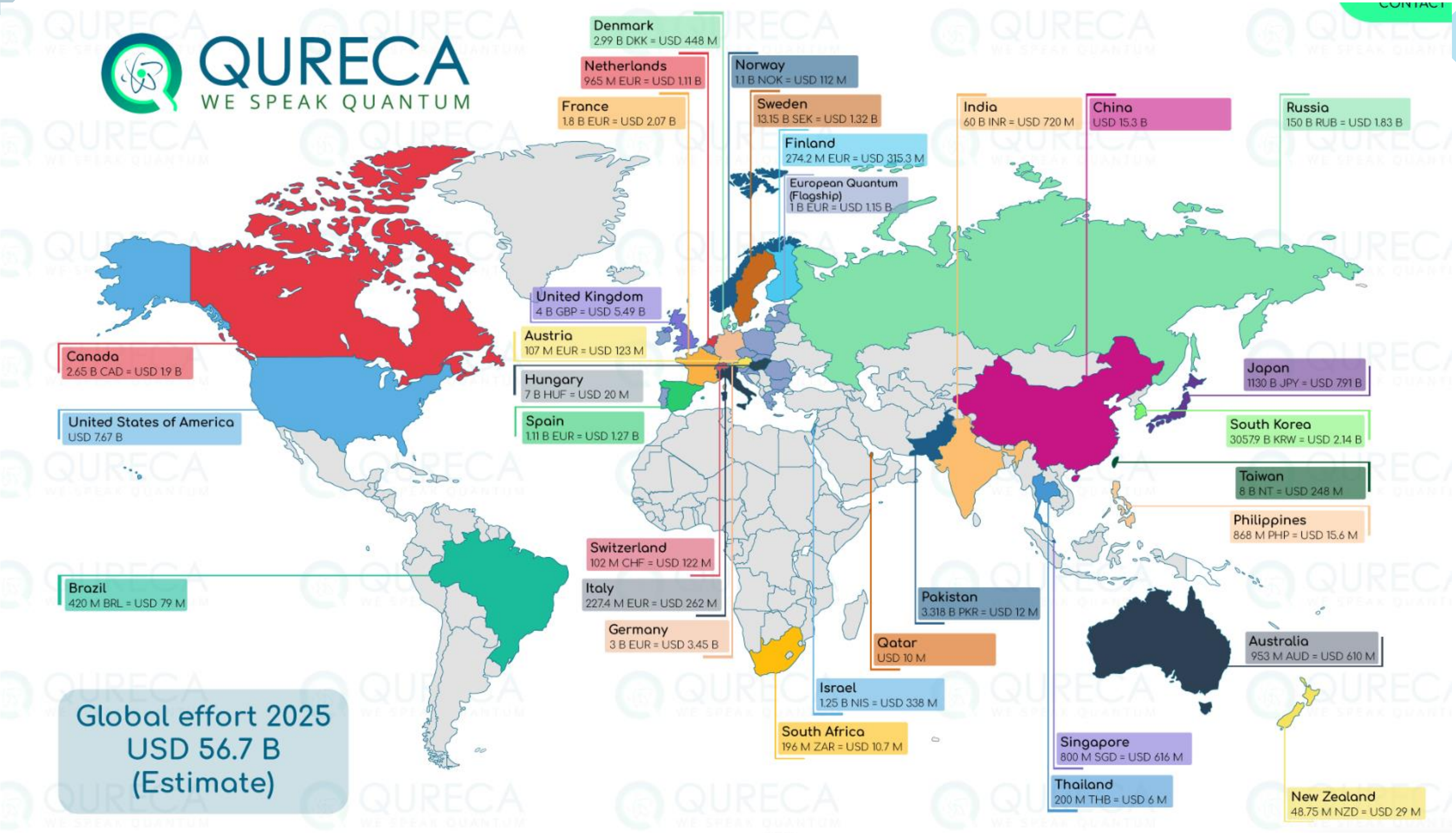

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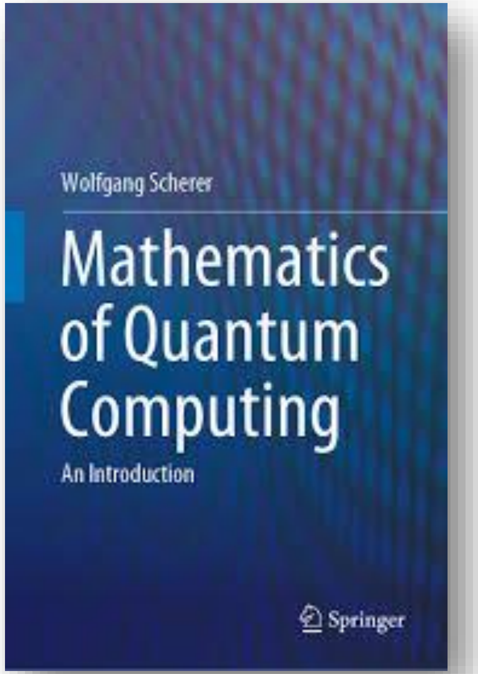
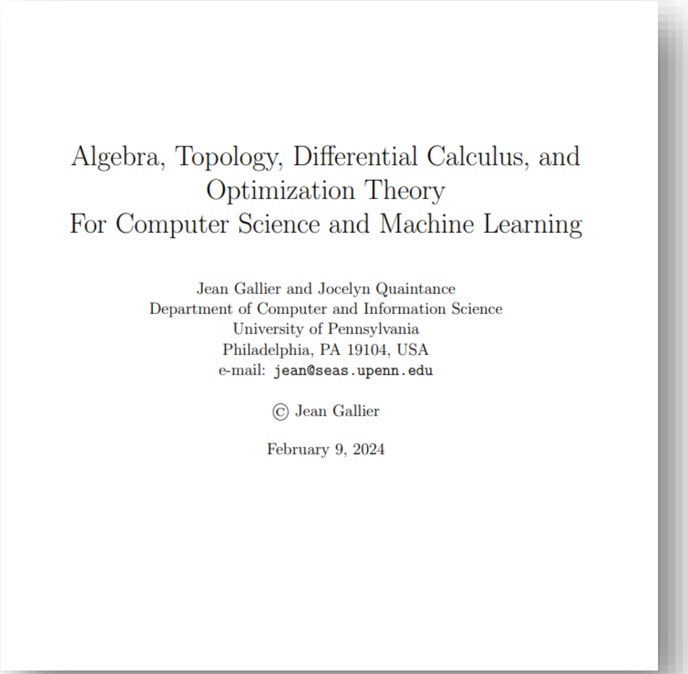
Main Research Areas:

- Chemistry and materials
- Optimization
- Logistics
- Machine Learning

Tools




Algebra, Topology, Differential Calculus, and Optimization Theory
For Computer Science and Machine Learning

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February 9, 2024

https://www.cis.upenn.edu/~jean/math-deep.pdf

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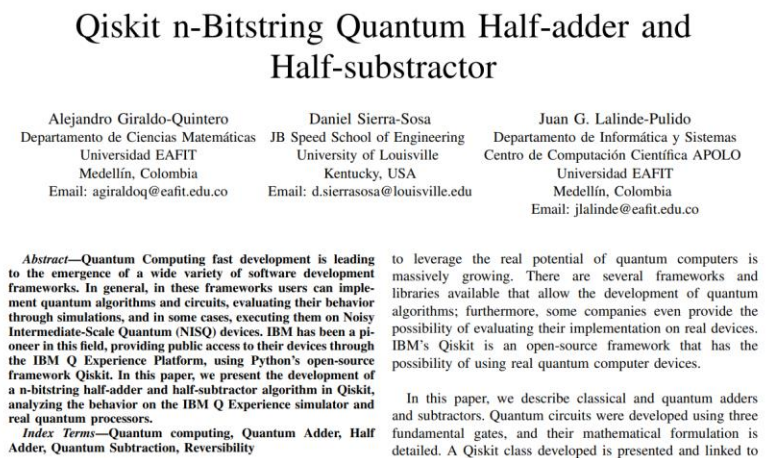
Basic Operations



Qiskit n-Bitstring Quantum Half-adder and Half-substractor

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***Abstract*—Quantum Computing fast development is leading to the emergence of a wide variety of software development frameworks. In general, in these frameworks users can implement quantum algorithms and circuits, evaluating their behavior through simulations, and in some cases, executing them on Noisy Intermediate-Scale Quantum (NISQ) devices. IBM has been a pioneer in this field, providing public access to their devices through the IBM Q Experience Platform, using Python's open-source framework Qiskit. In this paper, we present the development of a n-bitstring half-adder and half-subtractor algorithm in Qiskit, analyzing the behavior on the IBM Q Experience simulator and real quantum processors.**

***Index Terms*—Quantum computing, Quantum Adder, Half Adder, Quantum Subtraction, Reversibility**

to leverage the real potential of quantum computers is massively growing. There are several frameworks and libraries available that allow the development of quantum algorithms; furthermore, some companies even provide the possibility of evaluating their implementation on real devices. IBM's Qiskit is an open-source framework that has the possibility of using real quantum computer devices.

In this paper, we describe classical and quantum adders and subtractors. Quantum circuits were developed using three fundamental gates, and their mathematical formulation is detailed. A Qiskit class developed is presented and linked to

https://ieeexplore.ieee.org/document/9408852

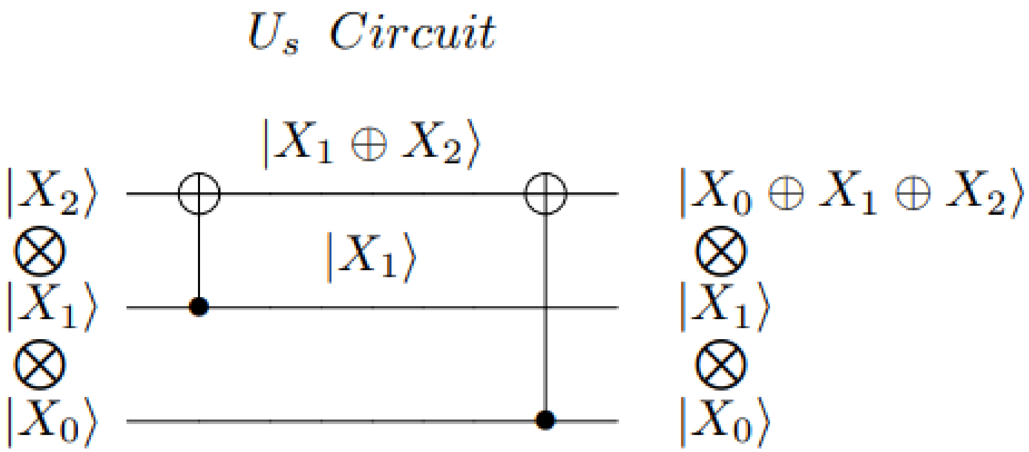

Fig. 8: Quantum Circuit U_s

Optimization

Optimization

Euclides

Lagrange
Newton
Leibniz
Gauss
Euler

Metaheuristics
Lineal Prog
A.I.

Objective Function

Optimization Variables

Constraints

Quantum **Optimization**

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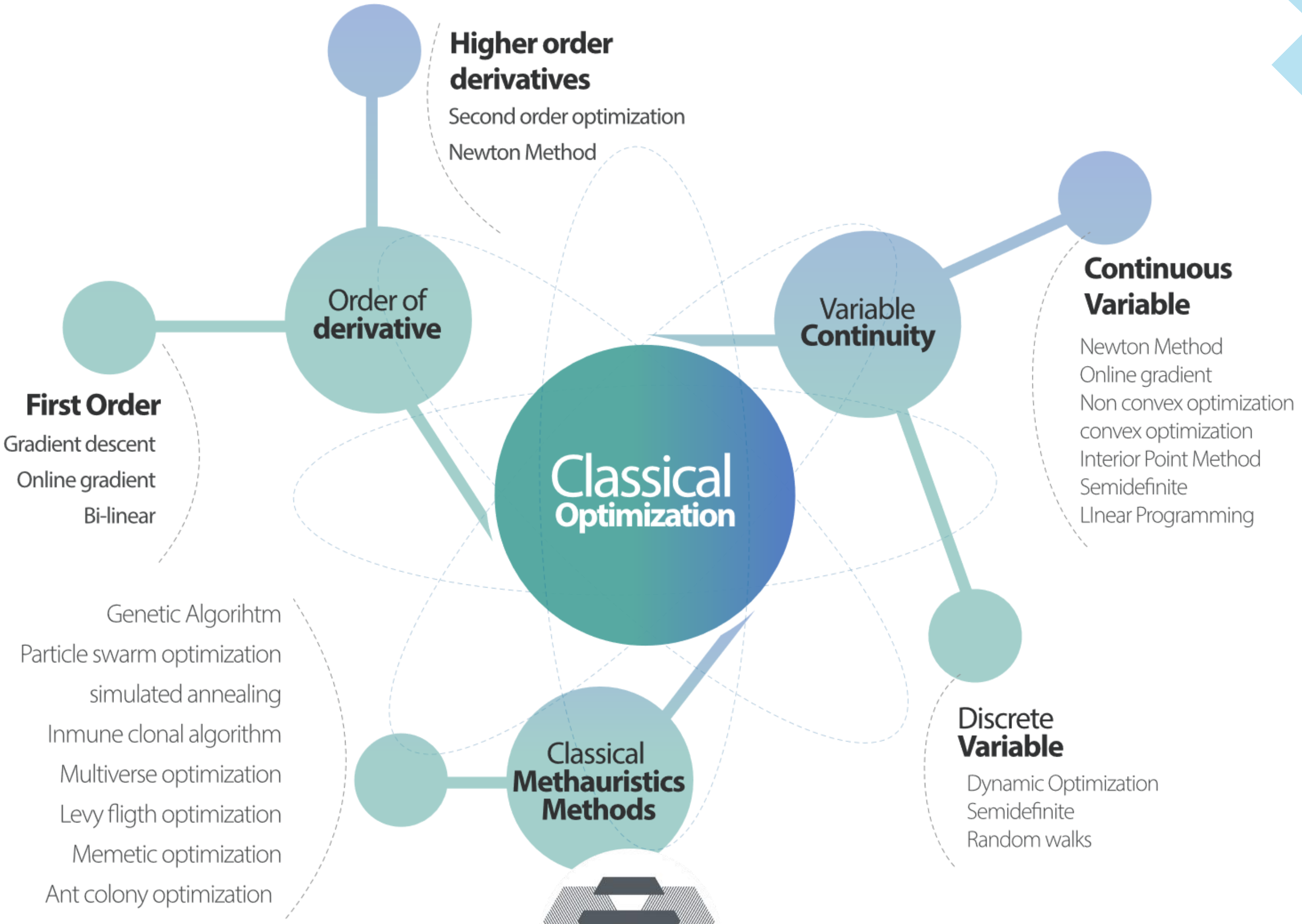

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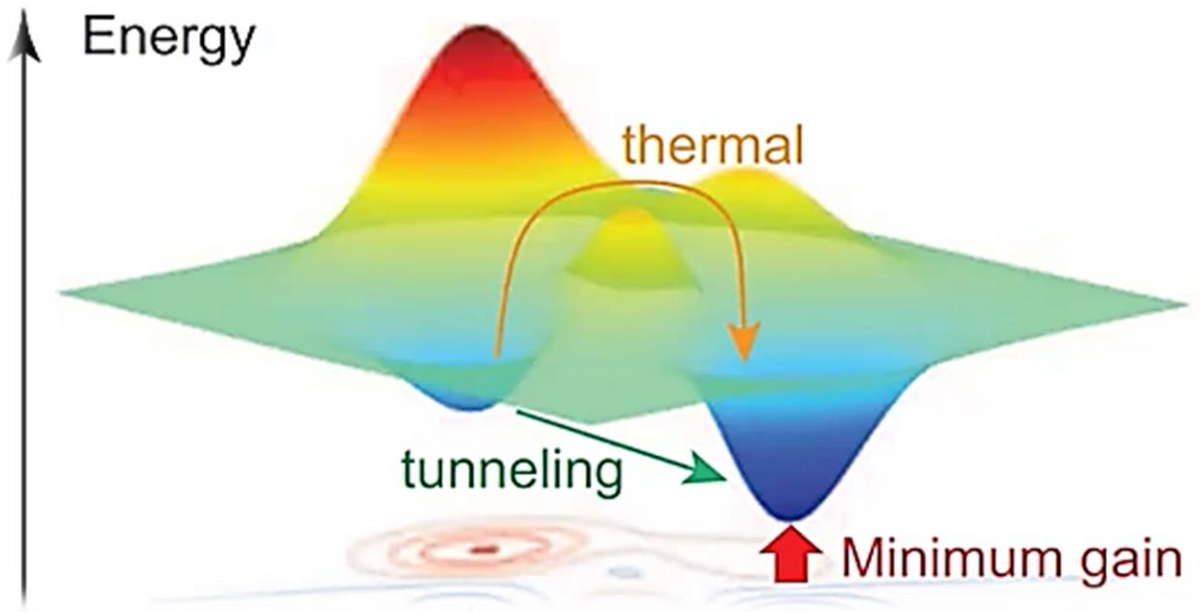

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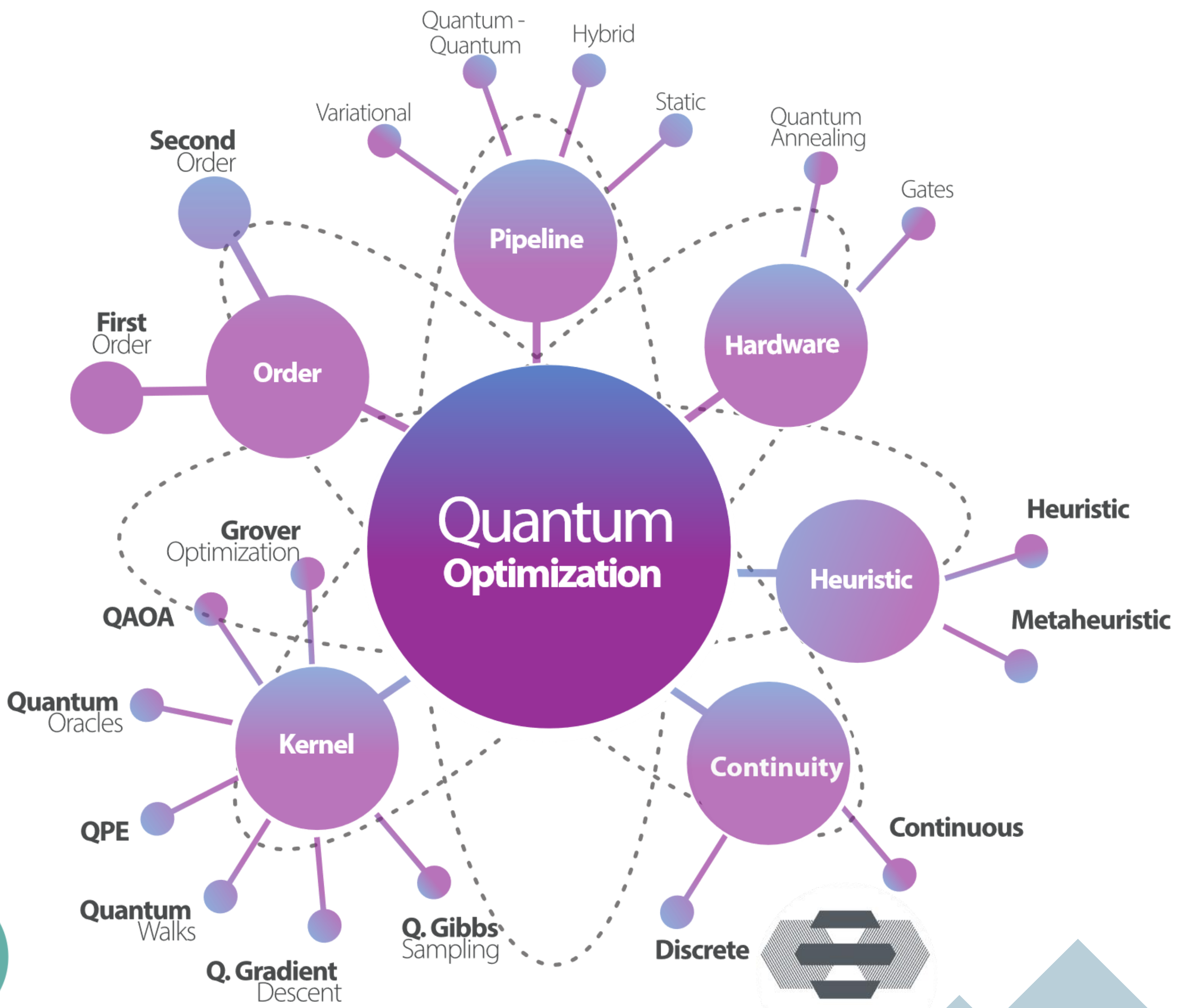

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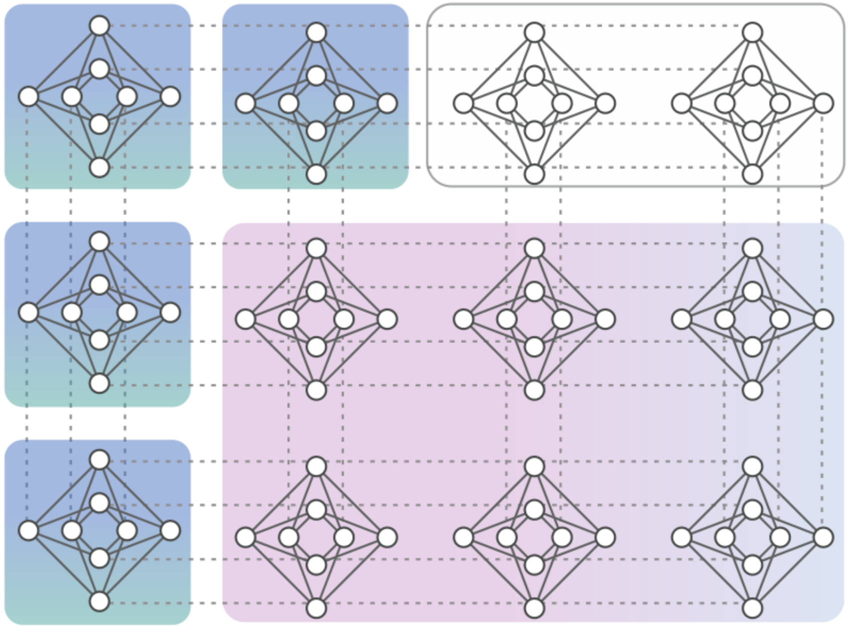
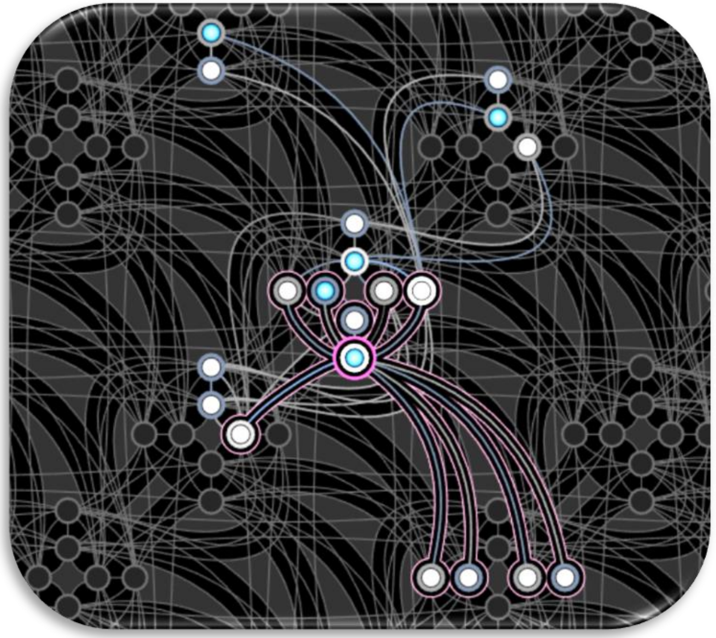
Clasification by Hardware

Hardware

- Quantum Gates
- Quantum Annealing

Hardware





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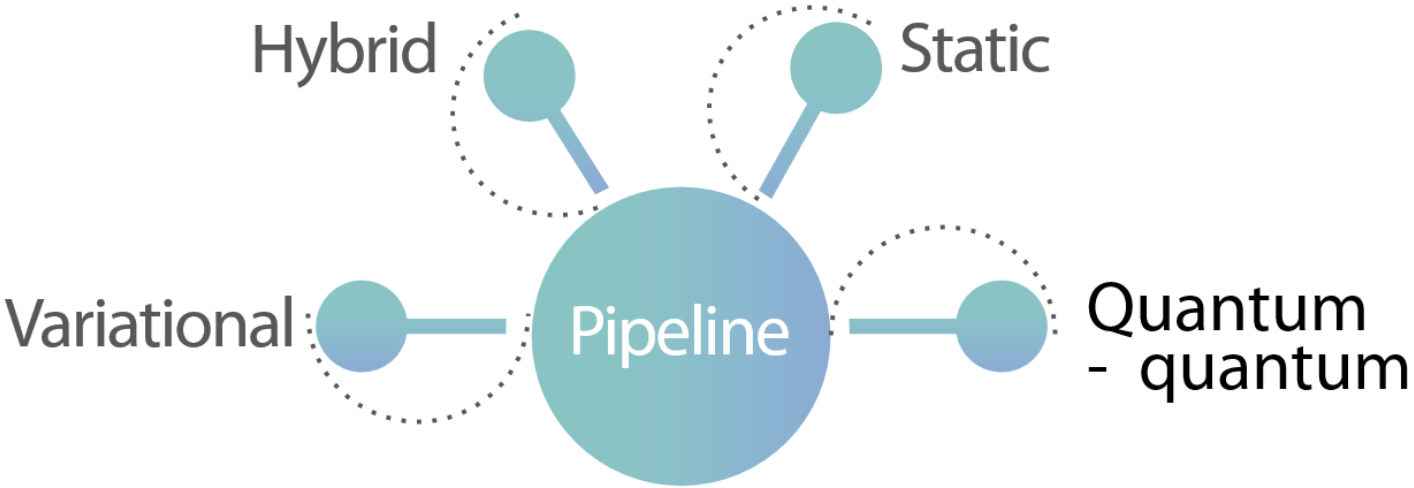
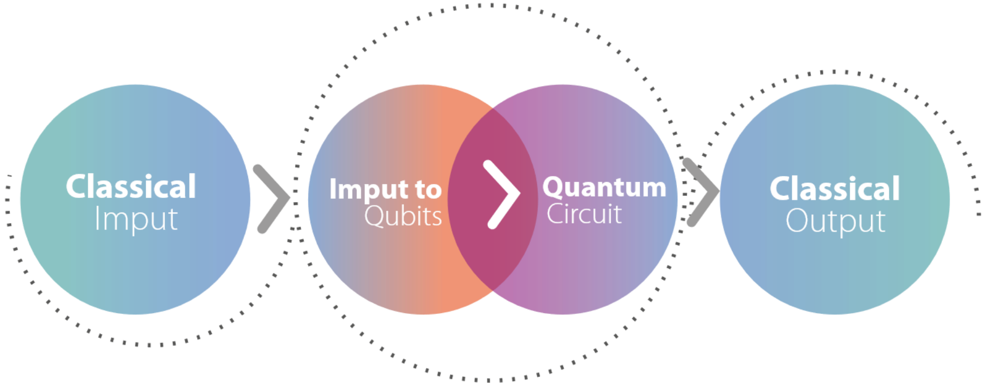
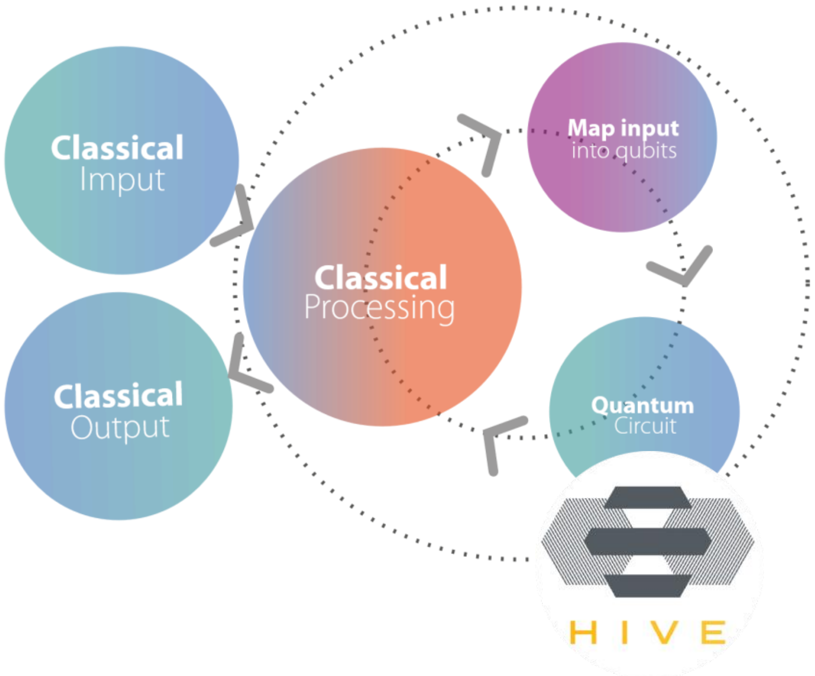
Clasification by Pipeline



Static



Hybrid



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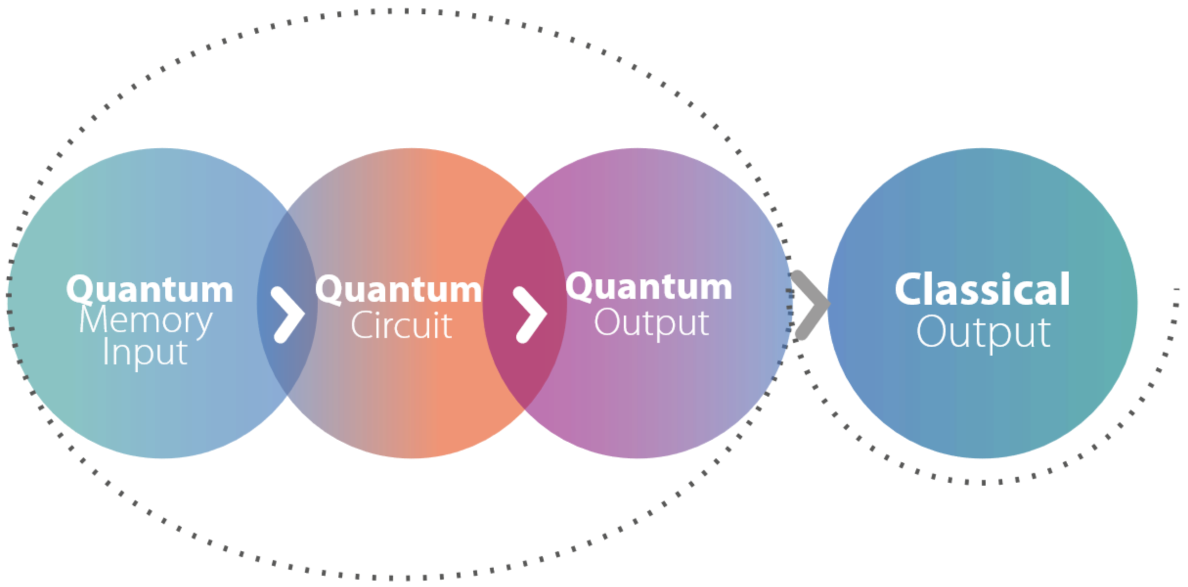
Quantum-Quantum



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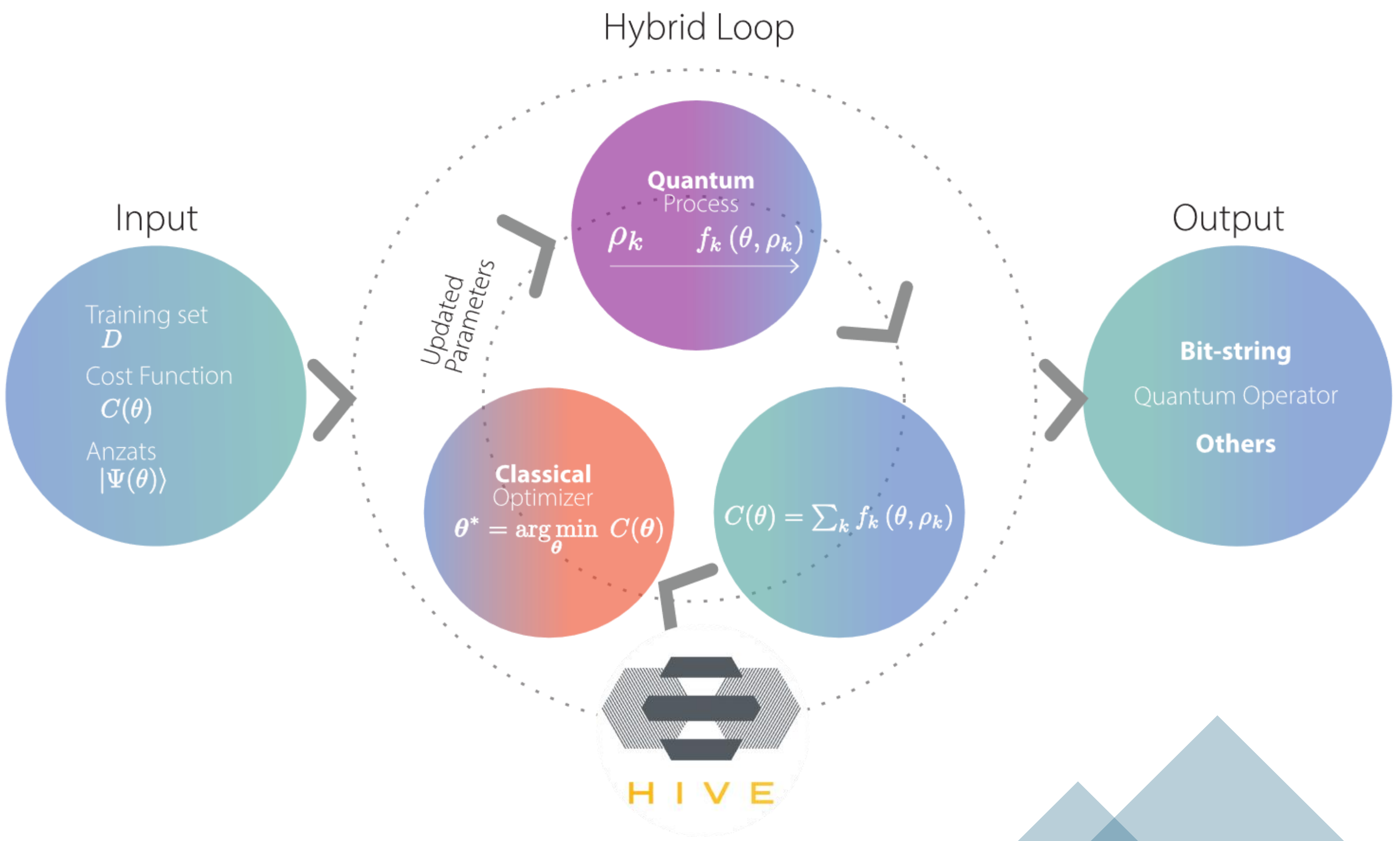
Variational



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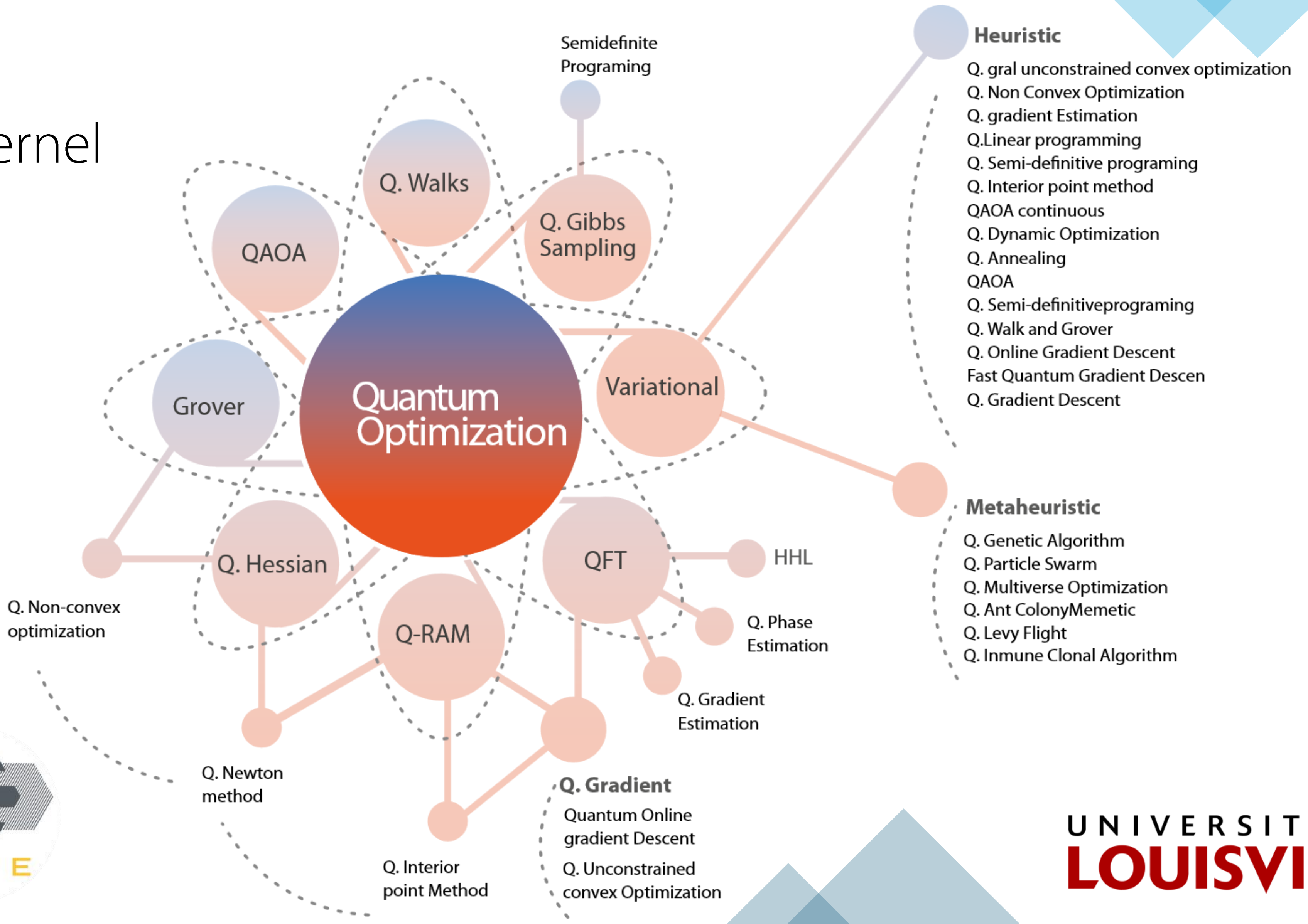
Clasification by Kernel



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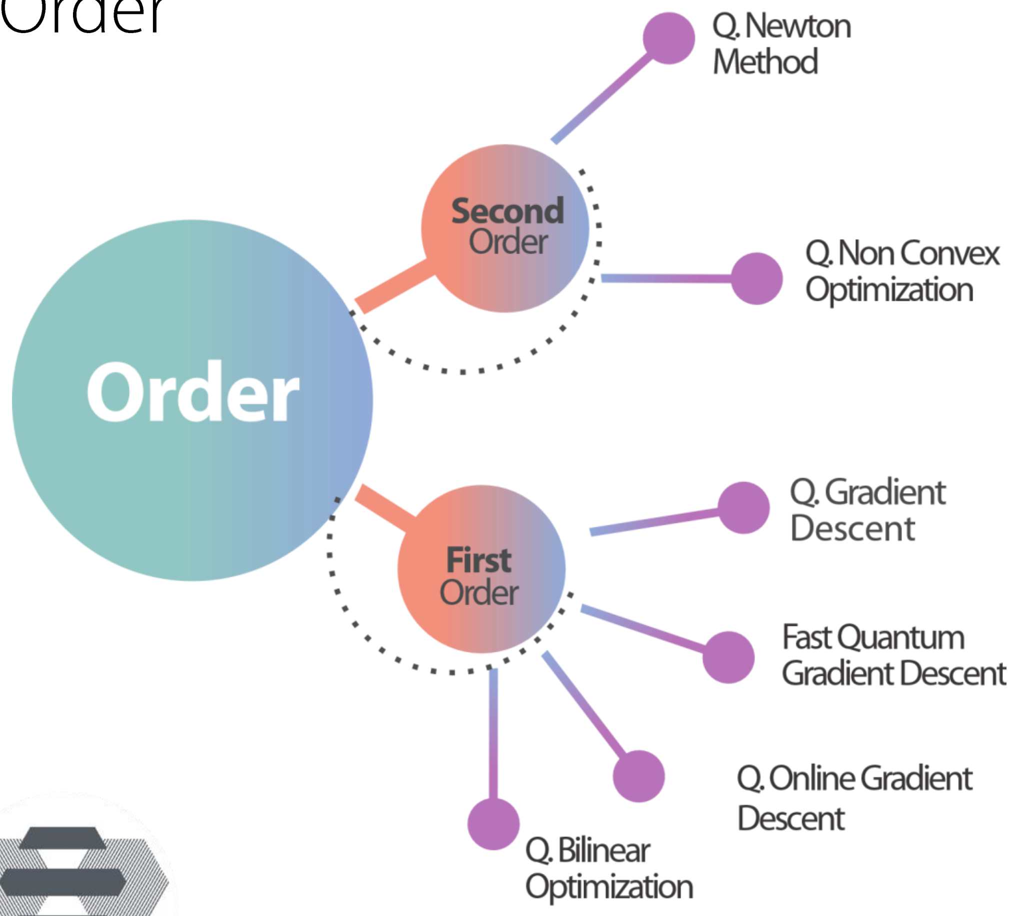
Clasification by Order



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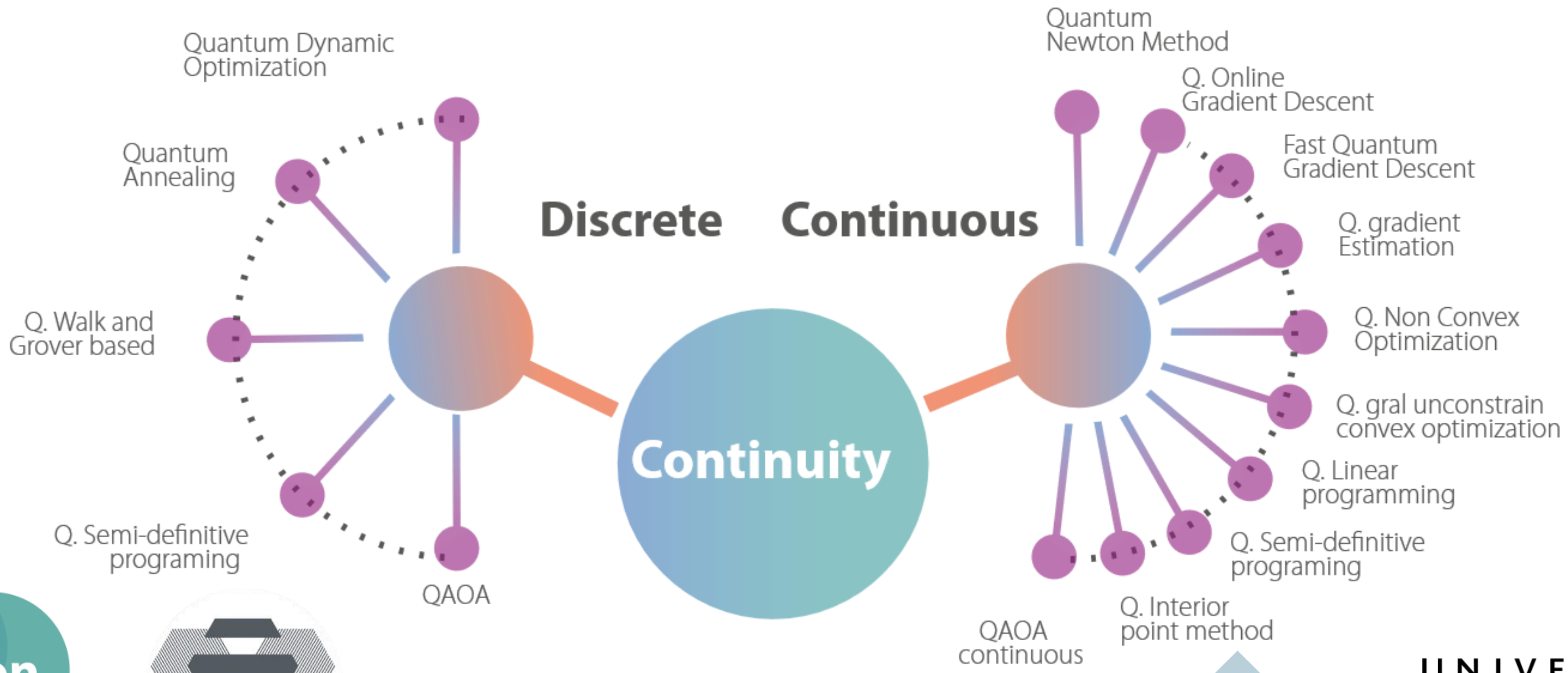
Clasification by Continuity



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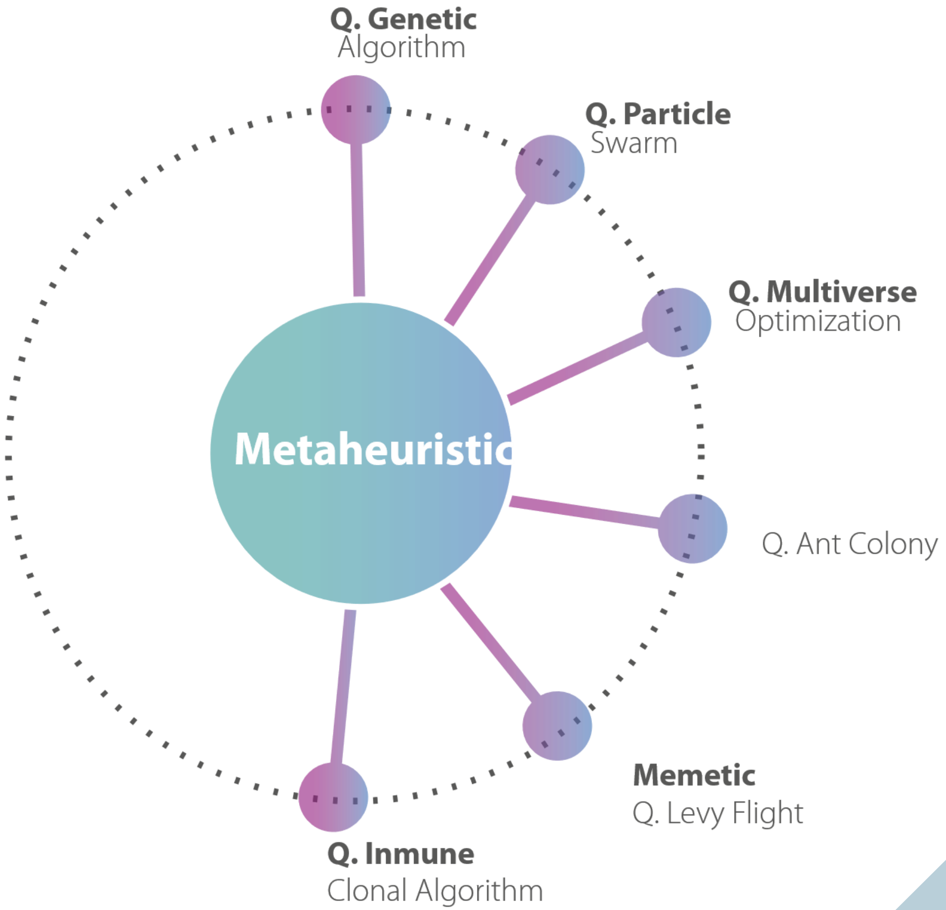
Clasification by Heuristic And Metaheuristic



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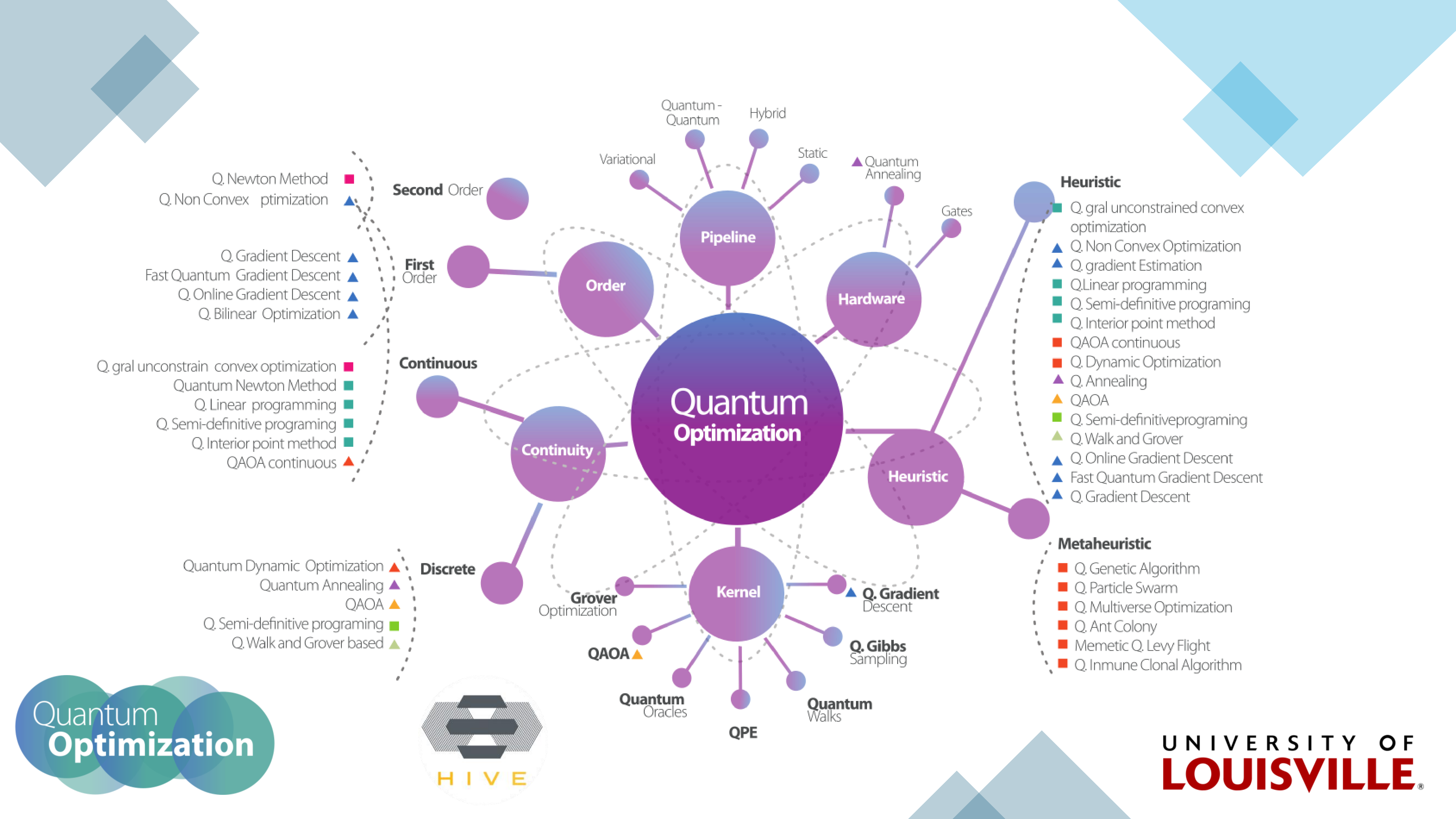
Quantum Optimization
Pipeline
Variational
Quantum - Quantum
Hybrid
Static
Hardware
Quantum Annealing
Gates
Order
Second Order
First Order
Q. Newton Method
Q. Non Convex ptimization
Q. Gradient Descent
Fast Quantum Gradient Descent
Q. Online Gradient Descent
Q. Bilinear Optimization
Continuity
Continuous
Q. gral unconstrain convex optimization
Quantum Newton Method
Q. Linear programming
Q. Semi-definitive programing
Q. Interior point method
QAOA continuous
Discrete
Quantum Dynamic Optimization
Quantum Annealing
QAOA
Q. Semi-definitive programing
Q. Walk and Grover based
Heuristic
Q. gral unconstrained convex optimization
Q. Non Convex Optimization
Q. gradient Estimation
Q.Linear programming
Q. Semi-definitive programing
Q. Interior point method
QAOA continuous
Q. Dynamic Optimization
Q. Annealing
QAOA
Q. Semi-definitiveprograming
Q. Walk and Grover
Q. Online Gradient Descent
Fast Quantum Gradient Descent
Q. Gradient Descent
Metaheuristic
Q. Genetic Algorithm
Q. Particle Swarm
Q. Multiverse Optimization
Q. Ant Colony
Memetic Q. Levy Flight
Q. Inmune Clonal Algorithm
Kernel
Grover Optimization
QAOA
Quantum Oracles
QPE
Quantum Walks
Q. Gibbs Sampling
Q. Gradient Descent
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An Example

Example:
Quantum portfolio Optimization

Quantum portfolio optimization is the application of quantum computing techniques, especially quantum optimization algorithms, to the portfolio selection problem in finance.

It aims to find the best combination of assets that maximizes expected return while minimizing risk, under given constraints (like budget or diversification).

Example:
Optimization Problem

$$\min_{x \in \{0,1\}^n} \quad q x^T \Sigma x - \mu^T x$$

$$\textit{subject to:} \quad 1^T x = B$$

$x \in \{0,1\}^n$ denotes the vector of binary decision variables, which indicate which assets to pick ($x[i] = 1$) and which not to pick ($x[i] = 0$),

$\mu \in \mathbb{R}^n$ defines the expected returns for the assets,

$\Sigma \in \mathbb{R}^{n \times n}$ specifies the covariances between the assets,

$q > 0$ controls the risk appetite of the decision maker,

and B denotes the budget, i.e. the number of assets to be selected out of n.

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Example:

Optimization Problem

We assume the following simplifications:

-all assets have the same price (normalized to 1),

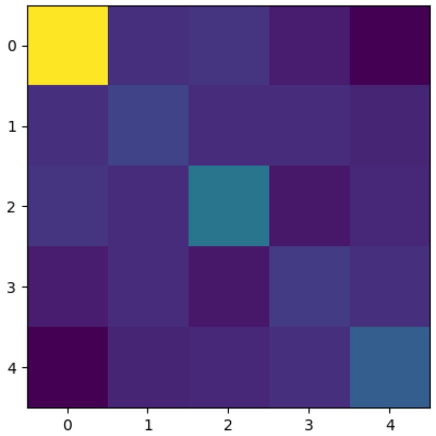
-the full budget B has to be spent, i.e. one has to select exactly B assets. parameters.

The equality constraint $1^T x = B$ is mapped to a penalty term $(1^T x - B)^2$ which is scaled by a parameter and subtracted from the objective function.

The resulting problem can be mapped to a Hamiltonian whose ground state corresponds to the optimal solution

Example:

Instances



Covariance Matrix

```
Optimal: selection [1. 0. 0. 0. 1.], value -0.0244

----------------- Full result ---------------------
selection        value            probability
---------------------------------------------------
[1 0 0 0 1]      -0.0244          1.0000
```

Output

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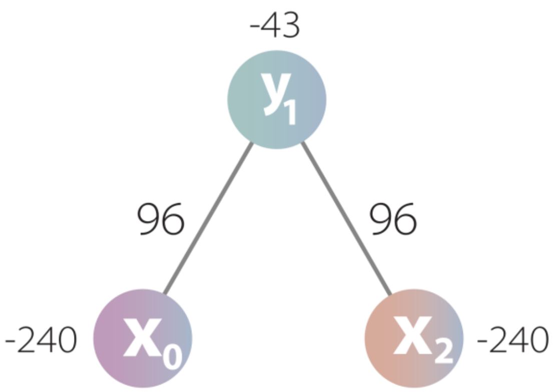
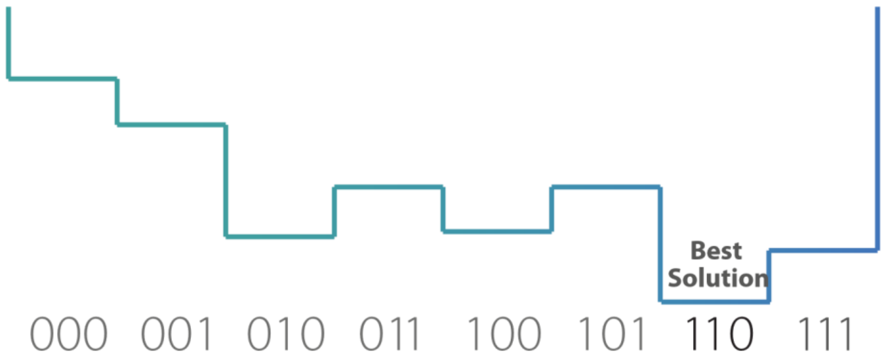
Example:
QUBO-Mapping in quantum annealers

$$-43y_1 - 240x_0 - 240x_2 + 96y_1x_0 + 96y_1x_2$$





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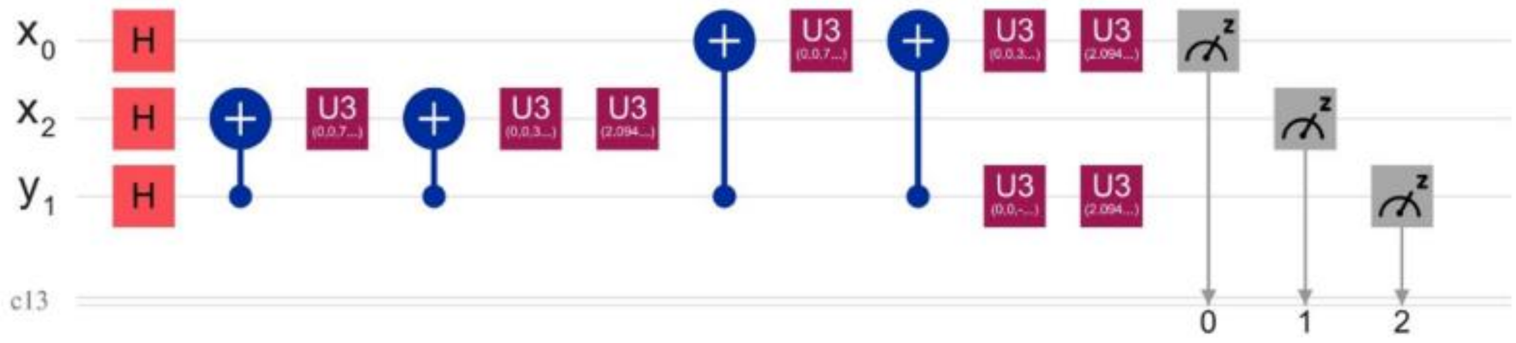
Example:

QUBO-Mapping in quantum gates

$$-43y_1 - 240x_0 - 240x_2 + 96y_1x_0 + 96y_1x_2$$



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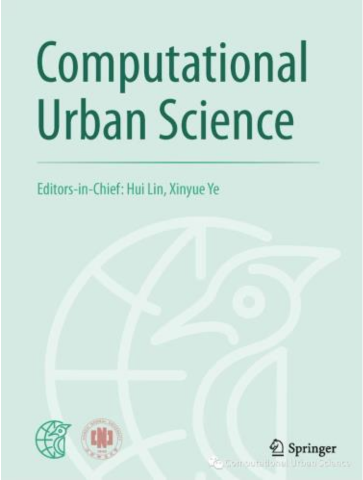
Example:
QUBO-Mapping in quantum gates

Hands-on practice
Jupyter Notebook

https://github.com/HIVE-AI-Studio/QuantumDay

Spatial Optimization Example:



Using quantum computing to solve the maximal covering location problem -2022

https://link.springer.com/article/10.1007/s43762-022-00070-x

GitHub

https://github.com/alejogq/QuantumR

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Conclusions

Conclusions:

- Quantum computing is showing clear advantages in speeding up many optimization methods. However, only a few have demonstrated true exponential improvements, leaving plenty of room for future exploration.

- Researchers agree that heuristics and metaheuristics will likely remain powerful tools for solving optimization problems, even in quantum environments.

- Promising techniques—such as **Quantum Gradient Descent** and the **Quantum Approximate Optimization Algorithm**—are opening new possibilities for tackling high-dimensional challenges.

- At the same time, creative ideas continue to emerge, like innovative attempts to solve the **Traveling Salesman Problem** using just a single qubit, showing how dynamic and evolving this field continues to be.

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Conclusions:

- There are many studies using **QUBO transformations**, and numerous ways to perform this mapping—from the use of **slack variables** to handling **decoherence** effects.
- However, we still need **comprehensive reviews** focused on specific optimization problems.
- Because **control theory** is closely related to optimization, this area offers **many opportunities for research and collaboration**.
- There is strong potential both in **theoretical exploration** and in **applied implementations**.
- Finally, we hope this review will **inspire more researchers** to join and contribute to this exciting field.

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Thanks !!

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