Homework Chapter 5

1.

Find the point on the line $y = 2x + 1$ that is closest to the point $(5, 2)$.

2.

Find the distance from the point $(1, 1, 1)$ to the plane $2x + 2y + z = 0$.

3.

Find the distance from the point $(2, -3, 4)$ to the plane

$$8(x - 2) + 6(y + 2) - (z - 4) = 0$$

4.

For each of the following matrices, determine a basis for each of the subspaces $R(A^T)$, $N(A)$, $R(A)$, and $N(A^T)$.

(a) $A = \begin{bmatrix} 2 & 4 \\ -4 & -8 \end{bmatrix}$ **(b)** $A = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 6 \end{bmatrix}$

(c) $A = \begin{bmatrix} 4 & 2 \\ -2 & 3 \\ 1 & 4 \\ 5 & 1 \end{bmatrix}$ **(d)** $A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 4 & 4 \\ 2 & 2 & 0 & 1 \\ 2 & 1 & 1 & 1 \end{bmatrix}$

5.

For each of the following systems $A\mathbf{x} = \mathbf{b}$, find all least squares solutions:

(a) $A = \begin{bmatrix} 3 & -6 \\ 2 & -4 \\ -3 & 6 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 & 3 & -1 \\ 2 & 1 & 1 \\ 2 & 6 & -2 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

6.

(a) Find the best least squares fit by a linear function to the data

x	-1	0	1	2
y	4	2	1	0

(b) Plot your linear function from part (a) along with the data on a coordinate system.

7.

Let $\mathbf{x} = (-1, -1, 1, 1)^T$ and $\mathbf{y} = (1, 1, 5, -3)^T$. Show that $\mathbf{x} \perp \mathbf{y}$. Calculate $\|\mathbf{x}\|_2$, $\|\mathbf{y}\|_2$, $\|\mathbf{x} + \mathbf{y}\|_2$ and verify that the Pythagorean law holds.

8.

Let $\mathbf{x} = (2, 3, 1)^T$ and $\mathbf{y} = (5, -6, 2)^T$. Compute $\|\mathbf{x} - \mathbf{y}\|_1$, $\|\mathbf{x} - \mathbf{y}\|_2$, and $\|\mathbf{x} - \mathbf{y}\|_\infty$. Under which norm are the two vectors closest together? Under which norm are they farthest apart?

9.

Let $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ be an orthonormal basis for an inner product space V and let

$$\mathbf{u} = \mathbf{u}_1 + 2\mathbf{u}_2 + 2\mathbf{u}_3 \quad \text{and} \quad \mathbf{v} = \mathbf{u}_1 + 7\mathbf{u}_3$$

Determine the value of each of the following:

(a) $\langle \mathbf{u}, \mathbf{v} \rangle$

(b) $\|\mathbf{u}\|$ and $\|\mathbf{v}\|$

(c) The angle θ between $\mathbf{u}$ and $\mathbf{v}$

10.

Given the basis $\{(1, 2, -2)^T, (4, 3, 2)^T, (1, 2, 1)^T\}$ for $\mathbb{R}^3$, use the Gram–Schmidt process to obtain an orthonormal basis.