Linear Algebra and applications - Chapter 1 homework

1.1

1. Use back substitution to solve each of the following systems of equations:

(a) $$\begin{aligned} x_1 + x_2 &= 7 \\ 2x_2 &= 6 \end{aligned}$$

(b) $$\begin{aligned} x_1 + x_2 + x_3 &= 10 \\ 2x_2 + x_3 &= 11 \\ 2x_3 &= 14 \end{aligned}$$

(c) $$\begin{aligned} x_1 + 2x_2 + 3x_3 + 4x_4 &= 6 \\ 7x_2 - x_3 + 2x_4 &= 5 \\ x_3 - 4x_4 &= -9 \\ 4x_4 &= 8 \end{aligned}$$

(d) $$\begin{aligned} x_1 + x_2 + 16x_3 + 3x_4 + x_5 &= 5 \\ 4x_2 + 4x_3 + 6x_4 + 3x_5 &= 1 \\ -8x_3 + 27x_4 - 7x_5 &= 7 \\ 3x_4 + 11x_5 &= 1 \\ x_5 &= 0 \end{aligned}$$

7. The two systems

$$\begin{aligned} x_1 + 2x_2 &= 8 \\ 4x_1 - 3x_2 &= -1 \end{aligned} \quad \text{and} \quad \begin{aligned} x_1 + 2x_2 &= 7 \\ 4x_1 - 3x_2 &= 6 \end{aligned}$$

have the same coefficient matrix but different right-hand sides. Solve both systems simultaneously by eliminating the first entry in the second row of the augmented matrix

$$\left[\begin{array}{cc|cc} 1 & 2 & 8 & 7 \\ 4 & -3 & -1 & 6 \end{array}\right]$$

and then performing back substitutions for each of the columns corresponding to the right-hand sides.

8. Solve the two systems

$$\begin{aligned} x_1 + 2x_2 - x_3 &= 6 \\ 2x_1 - x_2 + 3x_3 &= -3 \\ x_1 + x_2 - 4x_3 &= 7 \end{aligned} \qquad \begin{aligned} x_1 + 2x_2 - x_3 &= 9 \\ 2x_1 - x_2 + 3x_3 &= -2 \\ x_1 + x_2 - 4x_3 &= 9 \end{aligned}$$

by doing elimination on a 3×5 augmented matrix and then performing two back substitutions.

1.2

2. The augmented matrices that follow are in row echelon form. For each case, indicate whether the corresponding linear system is consistent. If the system has a unique solution, find it.

(a) $$\left[\begin{array}{cc|c} 1 & -1 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right]$$

(b) $$\left[\begin{array}{cc|c} 1 & 2 & 4 \\ 0 & 1 & 2 \\ 0 & 0 & -1 \end{array}\right]$$

(c) $$\left[\begin{array}{ccc|c} 1 & 7 & -3 & 9 \\ 0 & 1 & 2 & -4 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

(d) $$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

(e) $$\left[\begin{array}{ccc|c} 1 & -5 & 3 & 5 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

(f) $$\left[\begin{array}{ccc|c} 1 & 7 & 1 & 2 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

3. The augmented matrices that follow are in reduced row echelon form. In each case, find the solution set of the corresponding linear system.

(a) $$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

(b) $$\left[\begin{array}{ccc|c} 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 15 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

(c) $$\left[\begin{array}{ccc|c} 1 & 0 & 4 & 2 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & -3 \end{array}\right]$$

(d) $$\left[\begin{array}{ccc|c} 1 & -2 & 0 & 5 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

(e) $$\left[\begin{array}{cccc|c} 1 & -6 & 0 & -5 & 0 \\ 0 & 0 & 1 & 3 & -6 \end{array}\right]$$

(f) $$\left[\begin{array}{cccc|c} 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

6. Use Gauss–Jordan reduction to solve each of the following systems:

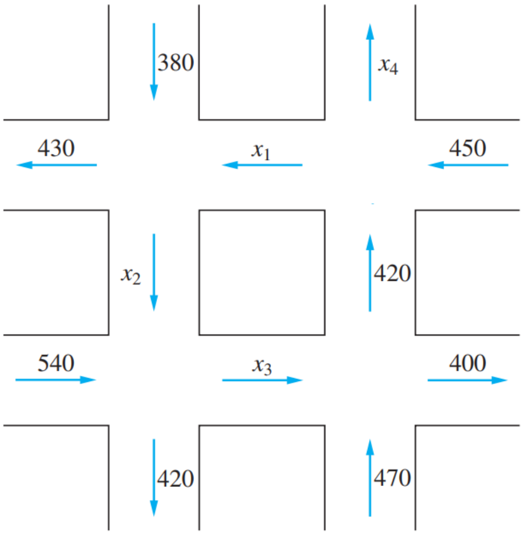
(a) $$\begin{aligned} 2x + y &= 1 \\ 7x + 6y &= 1 \end{aligned}$$

(b) $$\begin{aligned} x_1 + x_2 - x_3 + x_4 &= 6 \\ 2x_1 - x_2 + x_3 - x_4 &= -3 \\ 3x_1 + x_2 - 2x_3 + x_4 &= 9 \end{aligned}$$

(c) $$\begin{aligned} x_1 - 10x_2 + 5x_3 &= -4 \\ x_1 + x_2 + x_3 &= 1 \end{aligned}$$

(d) $$\begin{aligned} x_1 - 2x_2 + 3x_3 + x_4 &= 4 \\ 2x_1 + x_2 - x_3 + x_4 &= 1 \\ x_1 + 3x_2 + x_3 + x_4 &= 3 \end{aligned}$$

15. Determine the values of x_1, x_2, x_3, and x_4 for the following traffic flow diagram:



1.3

2. For each of the pairs of matrices that follow, determine whether it is possible to multiply the first matrix times the second. If it is possible, perform the multiplication.

(a) $\begin{bmatrix} 3 & 5 & 1 \\ -2 & 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 3 \\ 4 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 4 & -2 \\ 6 & -4 \\ 8 & -6 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 4 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 1 & 1 \\ 4 & 5 \end{bmatrix}$

(d) $\begin{bmatrix} 4 & 6 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 & 5 \\ 4 & 1 & 6 \end{bmatrix}$

(e) $\begin{bmatrix} 4 & 6 & 1 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 & 5 \\ 4 & 1 & 6 \end{bmatrix}$

(f) $\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} \begin{bmatrix} 3 & 2 & 4 & 5 \end{bmatrix}$

1.4

8. Let

$$A = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

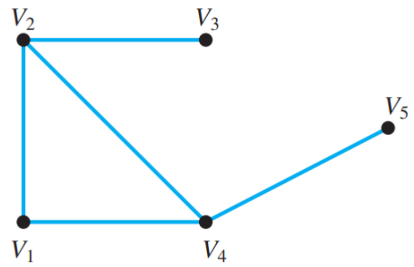
Compute A^2 and A^3. What will A^{2n} and A^{2n+1} turn out to be?

21. Given

$$R = \begin{Bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{Bmatrix}$$

show that R is nonsingular and $R^{-1} = R^T$.

33. Consider the graph



(a) Determine the adjacency matrix A of the graph.
(b) Compute A^2. What do the entries in the first row of A^2 tell you about walks of length 2 that start from V_1?
(c) Compute A^3. How many walks of length 3 are there from V_2 to V_4? How many walks of length less than or equal to 3 are there from V_2 to V_4?

1.5

4. For each of the following pairs of matrices, find an elementary matrix E such that $AE = B$:

(a) $A = \begin{bmatrix} 4 & -1 & 0 \\ 3 & 4 & 1 \\ 2 & 5 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 4 & -2 & 0 \\ 3 & 8 & 1 \\ 2 & 10 & 4 \end{bmatrix}$

(b) $A = \begin{bmatrix} 3 & -1 \\ 2 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -3 \\ -5 & -2 \end{bmatrix}$

(c) $A = \begin{bmatrix} 4 & -1 & 1 \\ 0 & 2 & 3 \\ 5 & 1 & 0 \end{bmatrix}$,

$B = \begin{bmatrix} -1 & 4 & 1 \\ 2 & 0 & 3 \\ 1 & 5 & 0 \end{bmatrix}$

6. Let

$$A = \begin{bmatrix} 2 & 0 & 4 \\ -6 & 3 & -9 \\ -4 & 3 & 2 \end{bmatrix}$$

(a) Find elementary matrices E_1, E_2, E_3 such that

$$E_3 E_2 E_1 A = U$$

where U is an upper triangular matrix.

(b) Determine the inverses of E_1, E_2, E_3 and set $L = E_1^{-1} E_2^{-1} E_3^{-1}$. What type of matrix is L? Verify that $A = LU$.

8. Compute the LU factorization of each of the following matrices:

(a) $\begin{bmatrix} 3 & 1 \\ 9 & 5 \end{bmatrix}$ **(b)** $\begin{bmatrix} 2 & 4 \\ -2 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 & 1 \\ 3 & 5 & 6 \\ -2 & 2 & 7 \end{bmatrix}$ **(d)** $\begin{bmatrix} -2 & 1 & 2 \\ 4 & 1 & -2 \\ -6 & -3 & 4 \end{bmatrix}$

1.6

1. Let A be a nonsingular $n \times n$ matrix. Perform the following multiplications:

(a) $A^{-1}\begin{pmatrix} A & I \end{pmatrix}$ **(b)** $\begin{pmatrix} A \\ I \end{pmatrix} A^{-1}$

(c) $\begin{pmatrix} A & I \end{pmatrix}^T \begin{pmatrix} A & I \end{pmatrix}$

(d) $\begin{pmatrix} A & I \end{pmatrix} \begin{pmatrix} A & I \end{pmatrix}^T$

(e) $\begin{pmatrix} A^{-1} \\ I \end{pmatrix} \begin{pmatrix} A & I \end{pmatrix}$

5. Perform each of the following block multiplications:

(a) $\left(\begin{array}{ccc|c} 2 & -1 & 3 & 1 \\ 4 & -1 & 0 & 2 \end{array}\right) \left(\begin{array}{ccc} 1 & 2 & 4 \\ 2 & 1 & 1 \\ 4 & 0 & 1 \\ \hline 1 & 0 & 2 \end{array}\right)$

(b) $\left(\begin{array}{cc} 1 & 2 \\ 2 & 1 \\ 4 & 0 \\ \hline 1 & 0 \end{array}\right) \left(\begin{array}{ccc|c} 2 & -1 & 3 & 1 \\ 4 & -1 & 0 & 2 \end{array}\right)$

(c) $\left(\begin{array}{cc|cc} \frac{1}{4} & \frac{3}{4} & 0 & 0 \\ -\frac{3}{4} & \frac{1}{4} & 0 & 0 \\ \hline 0 & 0 & 1 & 1 \end{array}\right) \left(\begin{array}{cc|c} \frac{1}{4} & -\frac{3}{4} & 0 \\ \frac{3}{4} & \frac{1}{4} & 0 \\ \hline 0 & 0 & 1 \\ 0 & 0 & 0 \end{array}\right)$

(d) $\left(\begin{array}{ccc|cc} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right) \left(\begin{array}{cc} 1 & 0 \\ 2 & -9 \\ 3 & -8 \\ \hline 4 & -7 \\ 5 & -6 \end{array}\right)$