Matrix Arithmetic Laws

- Commutative Law (교환 법칙)

$$AB \neq BA$$

Note:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

- Associative Law (결합 법칙)

$$A(BC)=(AB)C$$

- Distributive Law (분배 법칙)

$$A(B+C)=AB+AC$$

$$(A+B)C=AC+BC$$

Inverse Matrix

- If a matrix $\boldsymbol{B}$ satisfies the following condition, it is called an inverse matrix of $\boldsymbol{A}$
  - A is called nonsingular matrix, invertible matrix, or regular matrix

$$AB=BA=I_n$$

- The inverse matrix of $\boldsymbol{A}$ is denoted by $\boldsymbol{A}^{-1}$

$$AA^{-1}=A^{-1}A=I_n$$

- Ex) The inverse matrix of 2x2 matrix

$$A=\begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad |A|=ad-bc \qquad A^{-1}=\frac{1}{det(A)}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Inverse Matrix

- Inverse of matrix multiplication

$$(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$$

- The inverse of the inverse matrix is the matrix itself

$$A^{-1} = B,\ B^{-1} = A$$

- Matrix transpose

$$(A^T)^{-1} = (A^{-1})^T$$

Solving Linear Equations

- A linear equation is given as follows:

$$x' = ax + by \qquad 2x + 3y = 1$$

$$y' = cx + dy \qquad x - 2y = 4$$

- The equation can be represented by matrix-vector multiplication

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix} \qquad \underbrace{\begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 1 \\ 4 \end{bmatrix}}_{b}$$

Solving Linear Equations

- Calculate the inverse matrix

$$det(A) = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$(2 \times -2) - (3 \times 1) = -4 + (-3) = -7$$

$$A^{-1} = \frac{1}{-7}\begin{bmatrix} -2 & -3 \\ -1 & 2 \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix}$$

- Multiply the inverse matrix on both sides

$$A^{-1}Ax = A^{-1}b$$

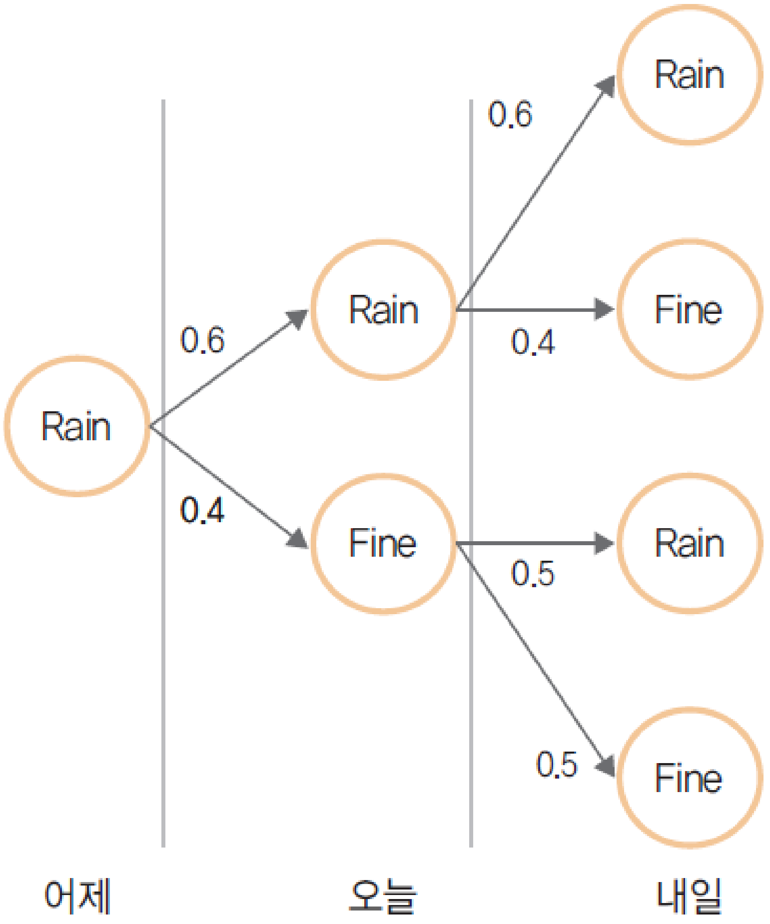
$$x = \frac{1}{7}\begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ 4 \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 14 \\ -7 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

Markov Chain

- A stochastic model describing a sequence of possible events

- The probability of each event depends only on the previous state

- Ex) Predicting weather condition of tomorrow based on today's weather
  - What will be the probability of rain tomorrow?
  - What will be the probability of sunny 3 days later?



내일의 상태

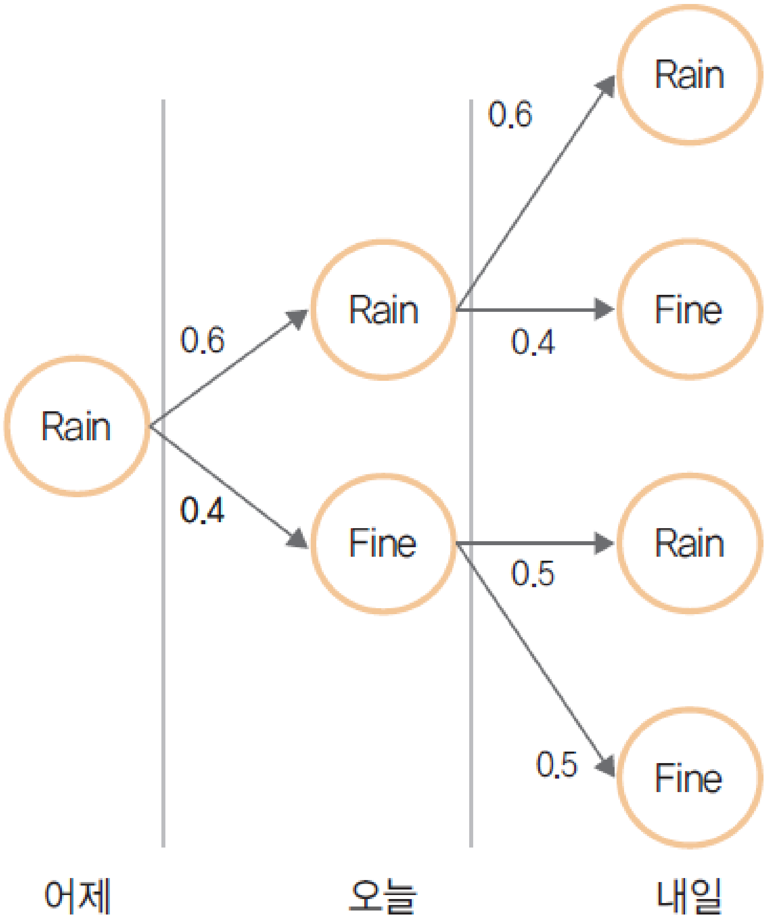
오늘의 상태		Rain	Fine
	Rain	0.6	0.4
	Fine	0.5	0.5

Markov Chain

- Using transition matrix, we can predict probabilities of next states

		내일의 상태	
		Rain	Fine
오늘의 상태	Rain	0.6	0.4
	Fine	0.5	0.5

$$\text{전이행렬} \times \text{전이행렬} = \begin{bmatrix} 0.6 & 0.4 \\ 0.5 & 0.5 \end{bmatrix} \begin{bmatrix} 0.6 & 0.4 \\ 0.5 & 0.5 \end{bmatrix}$$



- By multiplying transition matrix multiple times, it does not change anymore
  - Steady state transition matrix

Example Python Codes

01_matrix_example.ipynb

파일 수정 보기 삽입 런타임 도구 도움말 변경사항이 저장되지 않음

+ 코드 + 텍스트 Drive로 복사

```
# Numpy 라이브러리 import
import numpy as np
```

```
A = np.random.random((2, 2))
B = np.random.random((2, 2))
```

```
print(A)
```

```
[[0.51944086 0.92782027]
 [0.54917301 0.81377701]]
```

```
print(B)
```

```
[[0.59374053 0.61851152]
 [0.43431437 0.07217217]]
```

```
type(A)
```

```
numpy.ndarray
```

```
# Not recommended anymore
Amat = np.matrix(A)
```

```
type(Amat)
```

```
numpy.matrix
```

https://colab.research.google.com/drive/1BtIPjZ5DdSs9gLbXQtNujAQ6_SfgWvvM?usp=sharing

Thank You