1.

Determine whether the following vectors are linearly independent in P_3:

(a) $x^2, 1, x^2 - 1$ **(b)** $3, x, x^2, x - 2$

(c) $x + 1, x^2, x - 1$ **(d)** $x^2 + 2x, x + 1$

2.

Let $\{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$ be a spanning set for the vector space V, and let $\mathbf{v}$ be any other vector in V. Show that $\mathbf{v}, \mathbf{v}_1, \ldots, \mathbf{v}_n$ are linearly dependent.

3.

Let

$$\mathbf{x}_1 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, \quad \mathbf{x}_2 = \begin{bmatrix} 3 \\ -1 \\ 4 \end{bmatrix}, \quad \mathbf{x}_3 = \begin{bmatrix} 2 \\ 6 \\ 4 \end{bmatrix}$$

(a) Show that $\mathbf{x}_1, \mathbf{x}_2$, and $\mathbf{x}_3$ are linearly dependent.

(b) Show that $\mathbf{x}_1$ and $\mathbf{x}_2$ are linearly independent.

(c) What is the dimension of Span$(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3)$?

(d) Give a geometric description of Span $(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3)$.

4.

The vectors

$$\mathbf{x}_1 = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}, \quad \mathbf{x}_2 = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix},$$

$$\mathbf{x}_3 = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}, \quad \mathbf{x}_4 = \begin{bmatrix} 2 \\ 0 \\ 6 \end{bmatrix}, \quad \mathbf{x}_5 = \begin{bmatrix} 0 \\ 4 \\ 5 \end{bmatrix}$$

span $\mathbb{R}^3$. Pare down the set $\{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4, \mathbf{x}_5\}$ to form a basis for $\mathbb{R}^3$.

5.

For each of the following, find the transition matrix corresponding to the change of basis from $\{\mathbf{u}_1, \mathbf{u}_2\}$ to $\{\mathbf{e}_1, \mathbf{e}_2\}$:

(a) $\mathbf{u}_1 = (1, -1)^T, \quad \mathbf{u}_2 = (1, 2)^T$

(b) $\mathbf{u}_1 = (2, 3)^T, \quad \mathbf{u}_2 = (4, 7)^T$

(c) $\mathbf{u}_1 = (1, 0)^T, \quad \mathbf{u}_2 = (0, 1)^T$

6.

Let $\mathbf{u}_1 = (1, 1, 1)^T$, $\mathbf{u}_2 = (1, 2, 2)^T$, and $\mathbf{u}_3 = (2, 3, 4)^T$.

(a) Find the transition matrix corresponding to the change of basis from $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ to $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$.

(b) Find the coordinates of each of the following vectors with respect to the ordered basis $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$:

(i) $(3, 2, 5)^T$ **(ii)** $(1, 1, 2)^T$ **(iii)** $(2, 3, 2)^T$

7.

Given

$$\mathbf{v}_1 = \begin{pmatrix} 3 \\ 1 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 4 \\ 2 \end{pmatrix}, \quad S = \begin{pmatrix} 2 & -5 \\ -1 & 3 \end{pmatrix}$$

find vectors $\mathbf{w}_1$ and $\mathbf{w}_2$ so that S will be the transition matrix from $\{\mathbf{w}_1, \mathbf{w}_2\}$ to $\{\mathbf{v}_1, \mathbf{v}_2\}$.

8.

Let $[x, 1]$ and $[2x - 1, 2x + 1]$ be ordered bases for P_2.

(a) Find the transition matrix representing the change in coordinates from $[2x - 1, 2x + 1]$ to $[x, 1]$.

(b) Find the transition matrix representing the change in coordinates from $[x, 1]$ to $[2x - 1, 2x + 1]$.

9.

For each of the following matrices, find a basis for the row space, a basis for the column space, and a basis for the null space:

(a) $\begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & 4 \\ 4 & 7 & 8 \end{bmatrix}$

(b) $\begin{bmatrix} -3 & 1 & 3 & 4 \\ 1 & 2 & -1 & -2 \\ -3 & 8 & 4 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 3 & -2 & 1 \\ 2 & 1 & 3 & 2 \\ 3 & 4 & 5 & 6 \end{bmatrix}$

10.

Let A be a 4×4 matrix with reduced row echelon form given by

$$U = \begin{bmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

If

$$\mathbf{a}_1 = \begin{bmatrix} 3 \\ 2 \\ -1 \\ 2 \end{bmatrix} \quad \text{and} \quad \mathbf{a}_2 = \begin{bmatrix} 1 \\ 1 \\ -2 \\ 3 \end{bmatrix}$$

find $\mathbf{a}_3$ and $\mathbf{a}_4$.