Homework - Chapter 1

과제 제출 기간 : ~03.29일 금요일 자정(30일 00시00분)

1.1

1

다음과 같은 각 방정식들의 시스템을 풀기 위하여 역바향 대입을 사용하라.

(a)

$$\begin{aligned} x_1 + x_2 = 7 \ 2x_2 = 6 \end{aligned}$$

$\begin{bmatrix} 1 & 1 & 7 \ 0 & 2 & 6 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & 1 & 7 \ 0 & 1 & 3 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & 0 & 4 \ 0 & 1 & 3 \end{bmatrix}$

$\therefore x_1 = 4, x_2 = 3$

(b)

$$\begin{aligned} x_1 + x_2 + x_3 = 10\ 2x_2 + x_3 = 11\ 2x_3 = 14 \end{aligned}$$

$\begin{bmatrix} 1 & 1 & 1 & 10 \ 0 & 2 & 1 & 11 \ 0 & 0 & 2 & 14 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & 1 & 1 & 10 \ 0 & 2 & 1 & 11 \ 0 & 0 & 1 & 7 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & 1 & 0 & 3 \ 0 & 1 & 0 & 2 \ 0 & 0 & 1 & 7 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & 0 & 0 & 1 \ 0 & 1 & 0 & 2 \ 0 & 0 & 1 & 7 \end{bmatrix}$

$\therefore x_1 = 1, x_2 = 2, x_3 = 7$

(c)

$$\begin{aligned} x_1 + 2x_2 + 3x_3 + 4x_4 &= 6\ 7x_2 - x_3 + 2x_4 &= 5\ x_3 - 4x_4 &= -9\ 4x_4 &= 8 \end{aligned}$$

$\left[\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 6 \ 0 & 7 & -1 & 2 & 5 \ 0 & 0 & 1 & -4 & -9 \ 0 & 0 & 0 & 4 & 8 \end{array}\right]$

$\Rightarrow \left[\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 6 \ 0 & 7 & -1 & 2 & 5 \ 0 & 0 & 1 & -4 & -9 \ 0 & 0 & 0 & 1 & 2 \end{array}\right]$

$\Rightarrow \left[\begin{array}{cccc|c} 1 & 2 & 3 & 0 & -2 \ 0 & 7 & -1 & 0 & 1 \ 0 & 0 & 1 & 0 & -1 \ 0 & 0 & 0 & 1 & 2 \end{array}\right]$

$\Rightarrow \left[\begin{array}{cccc|c} 1 & 2 & 0 & 0 & 1 \ 0 & 7 & 0 & 0 & 0 \ 0 & 0 & 1 & 0 & -1 \ 0 & 0 & 0 & 1 & 2 \end{array}\right]$

$\therefore x_1 = 1, x_2 = 0, x_3 = -1, x_4 = 2$

(d)

$$\begin{aligned} x_1 + x_2 + 16x_3 + 3x_4 + x_5 &= 5\ 4x_2 + 4x_3 + 6x_4 + 3x_5 &= 1\ -8x_3 + 27x_4 - 7x_5 &= 7\ 3x_4 + 11x_5 &= 1\ x_5 &= 0 \end{aligned}$$

$\left[\begin{array}{ccccc|c} 1 & 1 & 16 & 3 & 1 & 5 \ 0 & 4 & 4 & 6 & 3 & 1 \ 0 & 0 & -8 & 27 & -7 & 7 \ 0 & 0 & 0 & 3 & 11 & 1 \ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$

$\Rightarrow \left[\begin{array}{ccccc|c} 1 & 1 & 16 & 3 & 0 & 5 \ 0 & 4 & 4 & 6 & 0 & 1 \ 0 & 0 & -8 & 27 & 0 & 7 \ 0 & 0 & 0 & 3 & 0 & 1 \ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$

$\Rightarrow \left[\begin{array}{ccccc|c} 1 & 1 & 16 & 0 & 0 & 4 \ 0 & 4 & 4 & 0 & 0 & -1 \ 0 & 0 & -8 & 0 & 0 & -2 \ 0 & 0 & 0 & 3 & 0 & 1 \ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$

$\Rightarrow \left[\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 0 & 0 \ 0 & 4 & 0 & 0 & 0 & -2 \ 0 & 0 & -4 & 0 & 0 & -1 \ 0 & 0 & 0 & 3 & 0 & 1 \ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$

$\Rightarrow \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 0 & \dfrac12 \ 0 & 2 & 0 & 0 & 0 & -1 \ 0 & 0 & 4 & 0 & 0 & 1 \ 0 & 0 & 0 & 3 & 0 & 1 \ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$

$\therefore x_1 = \dfrac12, x_2 = -\dfrac12, x_3 = \dfrac14, x_4 = \dfrac13, x_5 = 0$

7

다음의 두 시스템은 계수 행렬은 같지만 우변이 다르다.

$$\begin{aligned} x_1 + 2x_2 &= 8 \ 4x_1 - 3x_2 &= -1 \end{aligned}\qquad \begin{aligned} x_1 + 2x_2 &= 7 \ 4x_1 - 3x_2 &= 6 \end{aligned}$$

다음의 증가된 행렬의 두 번째 행에 있는 첫 번째 원소를 제거하고 우변에 해당하는 각 열들에 대하여 역방향 대입을 수행하여 두 시스템의 해를 동시에 구하라.

$\left[\begin{array}{cc|cc} 1 & 2 & 8 & 7 \ 4 & -3 & -1 & 6 \end{array}\right]$

$\Rightarrow \left[\begin{array}{cc|cc} 1 & 2 & 8 & 7 \ 0 & -11 & -33 & -22 \end{array}\right]$

$\Rightarrow \left[\begin{array}{cc|cc} 1 & 0 & 2 & 3 \ 0 & 1 & 3 & 2 \end{array}\right]$

$\therefore \left(x_1 = 2, x_2 = 3\right), \left(x_1 = 3, x_2 = 2\right)$

8

다음 두 시스템의 해를 3 × 5 증가된 행렬에 대해 소거를 하고 역방향 대입을 두 번 시행하여 구하라.

$$\begin{aligned} x_1 + 2x_2 - x_3 &= 6 \ 2x_1 - x_2 + 3x_3 &= -3 \ x_1 + x_2 - 4x_3 &= 7 \end{aligned}\qquad \begin{aligned} x_1 + 2x_2 - x_3 &= 9 \ 2x_1 - x_2 + 3x_3 &= -2 \ x_1 + x_2 - 4x_3 &= 9 \end{aligned}$$

$\left[\begin{array}{ccc|cc} 1 & 2 & -1 & 6 & 9\ 2 & -1 & 3 & -3 & -2 \ 1 & 1 & -4 & 7 & 9 \end{array}\right]$

$\Rightarrow \left[\begin{array}{ccc|cc} 1 & 2 & -1 & 6 & 9\ 0 & -5 & 5 & -15 & -20 \ 0 & -1 & -3 & 1 & 0 \end{array}\right]$

$\Rightarrow \left[\begin{array}{ccc|cc} 1 & 2 & -1 & 6 & 9\ 0 & -1 & 1 & -3 & -4 \ 0 & 0 & -4 & 4 & 4 \end{array}\right]$

$\Rightarrow \left[\begin{array}{ccc|cc} 1 & 2 & 0 & 5 & 8\ 0 & 1 & 0 & 2 & 3 \ 0 & 0 & 1 & -1 & -1 \end{array}\right]$

$\Rightarrow \left[\begin{array}{ccc|cc} 1 & 0 & 0 & 1 & 2\ 0 & 1 & 0 & 2 & 3 \ 0 & 0 & 1 & -1 & -1 \end{array}\right]$

$\therefore \left(x_1 = 1, x_2 = 2, x_3 = -1\right), \left(x_1 = 2, x_2 = 3, x_3 = -1\right)$

1.2

2

다음의 각각에서, 증가된 행렬이 행 사다리꼴 형태이다. 각 경우에 대하여, 대응하는 선형 시스템이 모순이 아닌지 표시하라. 만일 그 시스템이 유일한 해를 갖는다면 그 해를 구하라.

(a)

$$\left[\begin{array}{cc|c} 1 & -1 & 7 \ 0 & 1 & 3 \ 0 & 0 & 0 \end{array}\right]$$

$\Rightarrow \left[\begin{array}{cc|c} 1 & 0 & 10 \ 0 & 1 & 3 \ 0 & 0 & 0 \end{array}\right]$

$\therefore x_1 = 10, x_2 = 3$

(b)

$$\left[\begin{array}{cc|c} 1 & 2 & 4 \ 0 & 1 & 2 \ 0 & 0 & 1 \end{array}\right]$$

마지막 행에서 모순 발생

(c)

$$\left[\begin{array}{ccc|c} 1 & 7 & -3 & 9 \ 0 & 1 & 2 & -4 \ 0 & 0 & 1 & -2 \end{array}\right]$$

$\Rightarrow \left[\begin{array}{ccc|c} 1 & 7 & 0 & 4 \ 0 & 1 & 0 & 0 \ 0 & 0 & 1 & -2 \end{array}\right]$

$\Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \ 0 & 1 & 0 & 0 \ 0 & 0 & 1 & -2 \end{array}\right]$

$\therefore x_1 = 3, x_2 = 0, x_3 = -2$

(d)

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \ 0 & 0 & 1 & 3 \ 0 & 0 & 0 & 0 \end{array}\right]$$

모순은 아니지만, 유일해를 갖지 않는다.

(e)

$$\left[\begin{array}{ccc|c} 1 & -5 & 3 & 5 \ 0 & 0 & 1 & 2 \ 0 & 0 & 0 & 1 \end{array}\right]$$

마지막 행에서 모순을 갖는다.

(f)

$$\left[\begin{array}{ccc|c} 1 & 7 & 1 & 2 \ 0 & 1 & 1 & 2 \ 0 & 0 & 1 & 2 \ 0 & 0 & 0 & 0 \end{array}\right]$$

$\Rightarrow \left[\begin{array}{ccc|c} 1 & 7 & 0 & 0 \ 0 & 1 & 0 & 0 \ 0 & 0 & 1 & 2 \ 0 & 0 & 0 & 0 \end{array}\right]$

$\Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \ 0 & 1 & 0 & 0 \ 0 & 0 & 1 & 2 \ 0 & 0 & 0 & 0 \end{array}\right]$

$\therefore x_1 = 0, x_2 = 0, x_3 = 2$

3

다음의 각각에서, 증가된 행렬이 감소된 행 사다리꼴 형태이다. 각 경우에 대하여, 대응하는 선형 시스템의 해 집합을 구하여라.

(a)

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \ 0 & 1 & 0 & -2 \ 0 & 0 & 1 & 5 \end{array}\right]$$

${ \left(3, -2, 5 \right) }$

(b)

$$\left[\begin{array}{ccc|c} 0 & 1 & 0 & -3 \ 0 & 0 & 1 & 15 \ 0 & 0 & 0 & 0 \end{array}\right]$$

${ \left(\alpha, -3, 15 \right) ,\rvert, \alpha \in \mathbb{R} }$

(c)

$$\left[\begin{array}{ccc|c} 1 & 0 & 4 & 2 \ 0 & 1 & 2 & 1 \ 0 & 0 & 0 & -3 \end{array}\right]$$

$\emptyset$

(d)

$$\left[\begin{array}{ccc|c} 1 & -2 & 0 & 5 \ 0 & 0 & 1 & -1 \ 0 & 0 & 0 & 0 \end{array}\right]$$

${ \left(5 + 2\alpha, \alpha, -1 \right) ,\rvert, \alpha \in \mathbb{R} }$

(e)

$$\left[\begin{array}{cccc|c} 1 & -6 & 0 & -5 & 0 \ 0 & 0 & 1 & 3 & -6 \end{array}\right]$$

let $x_2 = \alpha, x_4 = \beta$

${ \left(6\alpha + 5\beta, \alpha, -6-3\beta, \beta \right) ,\rvert, \alpha, \beta \in \mathbb{R} }$

(f)

$$\left[\begin{array}{cccc|c} 1 & 2 & 1 & 0 & 0 \ 0 & 0 & 0 & 1 & 3 \ 0 & 0 & 0 & 0 & 0 \ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

let $x_2 = \alpha, x_3 = \beta$

${ \left(-2\alpha - \beta, \alpha, \beta, 3 \right) ,\rvert, \alpha, \beta \in \mathbb{R} }$

6

다음의 시스템들을 풀기 위하여 가우스-조던 소거법을 사용하라.

(a)

$$\begin{aligned} 2x + y &= 1 \ 7x + 6y &= 1 \end{aligned}$$

$\left[\begin{array}{cc|c} 2 & 1 & 1 \ 7 & 6 & 1 \end{array}\right]$

$\Rightarrow \left[\begin{array}{cc|c} 14 & 7 & 7 \ 14 & 12 & 2 \end{array}\right]$

$\Rightarrow \left[\begin{array}{cc|c} 2 & 1 & 1 \ 0 & 5 & -5 \end{array}\right]$

$\Rightarrow \left[\begin{array}{cc|c} 2 & 0 & 2 \ 0 & 1 & -1 \end{array}\right]$

$\therefore { \left(1, -1 \right) }$

(b)

$$\begin{aligned} x_1 + x_2 - x_3 + x_4 &= 6 \ 2x_1 - x_2 + x_3 - x_4 &= -3 \ 3x_1 + x_2 - 2x_3 + x_4 &= 9 \end{aligned}$$

$\left[\begin{array}{cccc|c} 1 & 1 & -1 & 1 & 6 \ 2 & -1 & 1 & -1 & -3 \ 3 & 1 & -2 & 1 & 9 \end{array}\right]$

$\Rightarrow \left[\begin{array}{cccc|c} 1 & 1 & -1 & 1 & 6 \ 0 & -3 & 3 & -3 & -15 \ 0 & -2 & 1 & -2 & -9 \end{array}\right]$

$\Rightarrow \left[\begin{array}{cccc|c} 1 & 1 & -1 & 1 & 6 \ 0 & 1 & -1 & 1 & 5 \ 0 & 0 & -1 & 0 & 1 \end{array}\right]$

$\Rightarrow \left[\begin{array}{cccc|c} 1 & 1 & 0 & 1 & 5 \ 0 & 1 & 0 & 1 & 4 \ 0 & 0 & 1 & 0 & -1 \end{array}\right]$

$\therefore { \left(1, \alpha, -1, 4 - \alpha \right) ,\rvert, \alpha \in \mathbb{R} }$

(c)

$$\begin{aligned} x_1 - 10x_2 + 5x_3 = -4 \ x_1 + x_2 + x_3 = 1 \end{aligned}$$

$\left[\begin{array}{ccc|c} 1 & -10 & 5 & -4 \ 1 & 1 & 1 & 1 \end{array}\right]$

$\Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \ 0 & -11 & 4 & -3 \end{array}\right]$

let $x_2 = \alpha$

$\therefore { \left(1 - \dfrac{15\alpha - 3}{4},, \alpha,, \dfrac{11\alpha - 3}{4} \right) ,\rvert, \alpha \in \mathbb{R} }$

(d)

$$\begin{aligned} x_1 - 2x_2 + 3x_3 + x_4 &= 4 \ 2x_1 + x_2 - x_3 + x_4 &= 1 \ x_1 + 3x_2 + x_3 + x_4 &= 3 \end{aligned}$$

$\left[\begin{array}{cccc|c} 1 & -2 & 3 & 1 & 4 \ 2 & 1 & -1 & 1 & 1 \ 1 & 3 & 1 & 1 & 3 \end{array}\right]$

$\Rightarrow \left[\begin{array}{cccc|c} 1 & -2 & 3 & 1 & 4 \ 0 & 5 & -7 & -1 & -7 \ 0 & 5 & -2 & 0 & -1 \end{array}\right]$

$\Rightarrow \left[\begin{array}{cccc|c} 1 & -2 & 3 & 1 & 4 \ 0 & 5 & -7 & -1 & -7 \ 0 & 0 & 5 & 1 & 6 \end{array}\right]$

let $x_3 = \alpha$

$\therefore { \left(-\dfrac{6\alpha + 12}{5},, \dfrac{2\alpha - 1}{5},, \alpha,, 6 - 5\alpha \right) ,\rvert, \alpha \in \mathbb{R} }$

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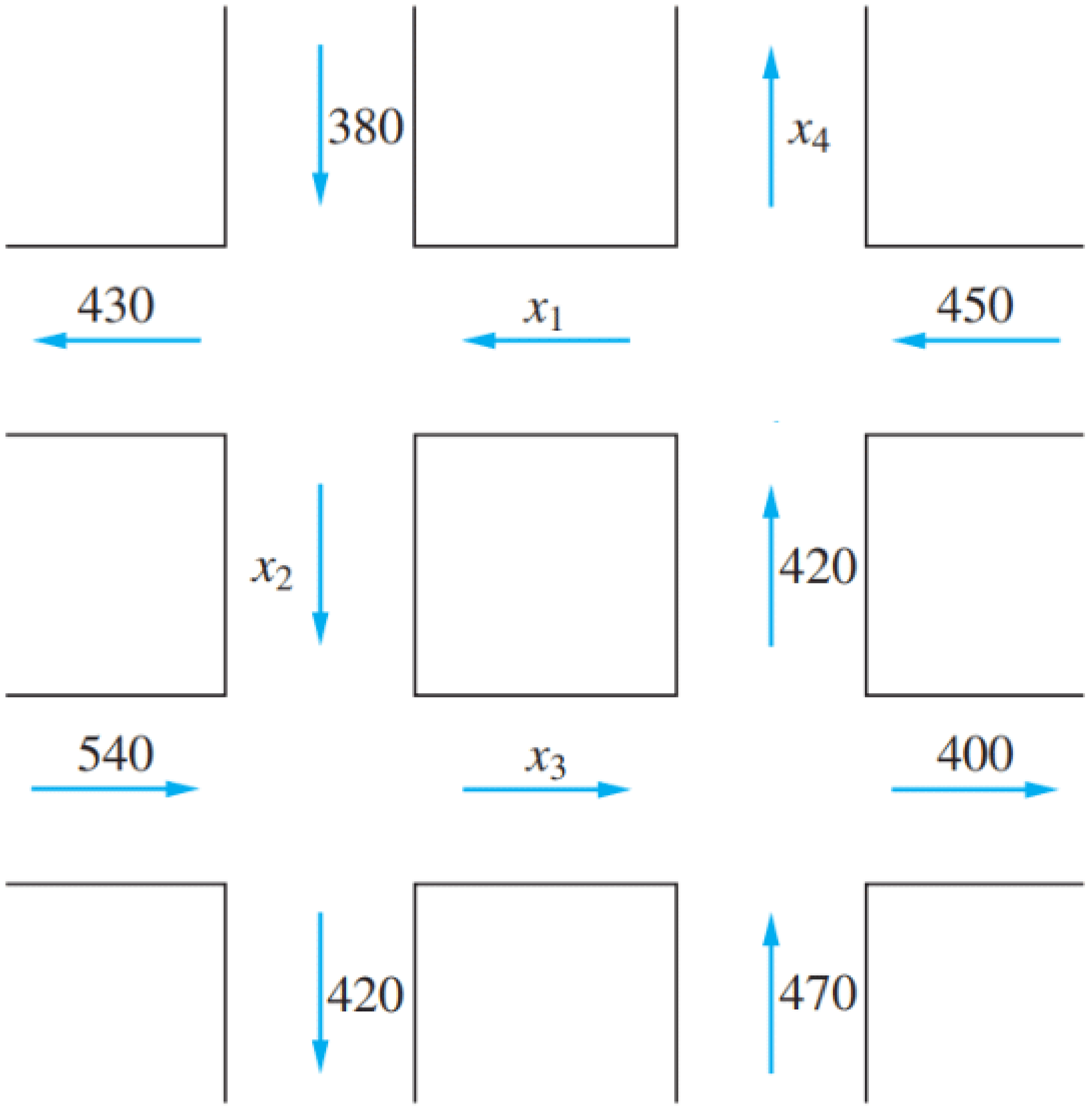
다음과 같은 교통 흐름도에서 x_1, x_2, x_3, x_4의 값을 구하여라.



각 교차로를 기준으로 식을 세운다.

$$\begin{aligned} x_1 + 380 &= x_2 + 430 \ x_2 + 540 &= x_3 + 420 \ x_3 + 470 &= 400 + 420 \ x_1 + x_4 &= 420 + 450 \end{aligned}$$

$\Rightarrow \left[\begin{array}{cccc|c} 1 & -1 & 0 & 0 & 50 \ 0 & 1 & -1 & 0 & -120 \ 0 & 0 & 1 & 0 & 350 \ 1 & 0 & 0 & 1 & 870 \end{array}\right]$

$\Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 280 \ 0 & 1 & 0 & 0 & 230 \ 0 & 0 & 1 & 0 & 350 \ 0 & 0 & 0 & 1 & 590 \end{array}\right]$

$\therefore x_1 = 280,, x_2 = 230,, x_3 = 350,, x_4 = 590$

1.3

2

다음의 각 행렬들의 쌍에 대하여, 첫 번째 행렬과 두 번째 행렬을 곱하는 것이 가능한가? 만일 가능하다면 곱셈을 수행하라.

(a)

$$\begin{bmatrix} 3 & 5 & 1 \ -2 & 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 \ 1 & 3 \ 4 & 1 \end{bmatrix}$$

$\begin{bmatrix} 6 + 5 + 4 & 3 + 15 + 1 \ -4 + 0 + 8 & -2 + 0 + 2 \end{bmatrix} = \begin{bmatrix} 15 & 19 \ 4 & 0 \end{bmatrix}$

(b)

$$\begin{bmatrix} 4 & -2 \ 6 & -4 \ 8 & -6 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$$

불가능

(c)

$$\begin{bmatrix} 1 & 4 & 3 \ 0 & 1 & 4 \ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 3 & 2 \ 1 & 1 \ 4 & 5 \end{bmatrix}$$

$\begin{bmatrix} 3 + 4 + 12 & 2 + 4 + 15 \ 0 + 1 + 16 & 0 + 1 + 20 \ 0 + 0 + 8 & 0 + 0 + 10 \end{bmatrix} = \begin{bmatrix} 19 & 21 \ 17 & 21 \ 8 & 10 \end{bmatrix}$

(d)

$$\begin{bmatrix} 4 & 6 \ 2 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 & 5 \ 4 & 1 & 6 \end{bmatrix}$$

$\begin{bmatrix} 12 + 24 & 4 + 6 \ 6 + 4 & 2 + 1 \end{bmatrix} = \begin{bmatrix} 36 & 10 \ 10 & 3 \end{bmatrix}$

(e)

$$\begin{bmatrix} 4 & 6 & 1 \ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 & 5 \ 4 & 1 & 6 \end{bmatrix}$$

불가능

(f)

$\begin{bmatrix} 2 \ -1 \ 3 \end{bmatrix} \begin{bmatrix} 3 & 2 & 4 & 5 \end{bmatrix}$

$\begin{bmatrix} 6 & 4 & 8 & 10 \ -3 & -2 & -4 & -5 \ 9 & 6 & 12 & 15 \end{bmatrix}$

1.4

8

만일 $A = \begin{bmatrix} 0 & 1 & 0 & 0 \ 0 & 0 & 1 & 0 \ 0 & 0 & 0 & 1 \ 0 & 0 & 0 & 0 \end{bmatrix}$ 일 때, A^2, A^3을 계산하라. A^{2n}, A^{2n+1}은 어떻게 될까?

$A^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \ 0 & 1 & 0 & 0 \ 0 & 0 & 1 & 0 \ 0 & 0 & 0 & 1 \end{bmatrix}$

$A^3 = \begin{bmatrix} 0 & 1 & 0 & 0 \ 0 & 0 & 1 & 0 \ 0 & 0 & 0 & 1 \ 0 & 0 & 0 & 0 \end{bmatrix}$

$A^{2n} = I$

$A^{2n+1} = A$

21

$R = \begin{bmatrix} \cos \theta & -\sin \theta \ \sin \theta & \cos \theta \end{bmatrix}$일 때, R은 불특이 행렬이고 $R^{-1} = R^T$임을 보여라.

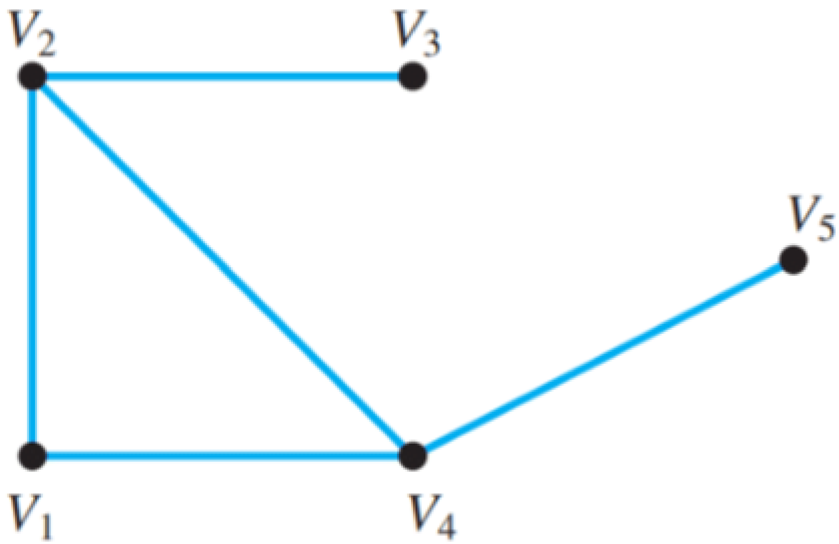
$R^T = \begin{bmatrix} \cos \theta & \sin \theta \ -\sin \theta & \cos \theta \end{bmatrix}$에서

$R \cdot R^T = \begin{bmatrix} \cos^2 \theta + \sin^2 \theta & 0 \ 0 & \cos^2 \theta + \sin^2 \theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \ 0 & 1 \end{bmatrix}$

$\therefore R^{-1} = R^T$, R is nonsingular

33

다음 그래프에 대해



(a)

그래프의 인접 행렬 A를 구하여라.

$A = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \ 1 & 0 & 1 & 1 & 0 \ 0 & 1 & 0 & 0 & 0 \ 1 & 1 & 0 & 0 & 1 \ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

(b)

A^2를 계산하라. A^2의 첫 번째 행에 있는 원소들이 여러분에게 V_1에서 출발하여 길이 2인 걸음에 관하여 말하는 것은 무엇인가?

$A^2 = \begin{bmatrix} 2 & 1 & 1 & 1 & 1 \ 1 & 3 & 0 & 1 & 1 \ 1 & 0 & 1 & 1 & 0 \ 1 & 1 & 1 & 3 & 0 \ 1 & 1 & 0 & 0 & 1 \end{bmatrix}$

V_1에서 출발하여 V_n에 도착하는 데 A^2_{1n}가지의 경로가 있는 의미이다.

(c)

A^3을 계산하라. V_2부터 V_4까지의 길이가 3인 걸음은 얼마나 있는가? V_2부터 V_4까지의 길이가 3 이하인 걸음은 얼마나 있는가?

$A^3 = \begin{bmatrix} 2 & 4 & 1 & 4 & 1 \ 4 & 2 & 3 & 5 & 1 \ 1 & 3 & 0 & 1 & 1 \ 4 & 5 & 1 & 2 & 3 \ 1 & 1 & 1 & 3 & 0 \end{bmatrix}$

V_2부터 V_4까지의 길이가 3인 걸음은 $A^3_{24} = 5$이다. V_2부터 V_4까지의 길이가 3 이하인 걸음은 $A^3_{24} + A^2_{24} + A_{24} = 5 + 1 + 1 = 7$이다.

1.5

4

다음의 각 행렬의 쌍에 대하여, $AE=B$가 되는 기본 행렬 E를 구하라.

(a)

$$A = \begin{bmatrix} 4 & -1 & 0 \ 3 & 4 & 1 \ 2 & 5 & 4 \end{bmatrix}\qquad B = \begin{bmatrix} 4 & -2 & 0 \ 3 & 8 & 1 \ 2 & 10 & 4 \end{bmatrix}$$

$E = \begin{bmatrix} 1 & 0 & 0 \ 0 & 2 & 0 \ 0 & 0 & 1 \end{bmatrix}$

(b)

$$A = \begin{bmatrix} 3 & -1 \ 2 & 4 \end{bmatrix}\qquad B = \begin{bmatrix} 1 & -3 \ -5 & -2 \end{bmatrix}$$

$E = \begin{bmatrix} 0 & -1 \ -1 & 0 \end{bmatrix}$

(c)

$$A = \begin{bmatrix} 4 & -1 & 1 \ 0 & 2 & 3 \ 5 & 1 & 0 \end{bmatrix}\qquad B = \begin{bmatrix} -1 & 4 & 1 \ 2 & 0 & 3 \ 1 & 5 & 0 \end{bmatrix}$$

$E = \begin{bmatrix} 0 & 1 & 0 \ 1 & 0 & 0 \ 0 & 0 & 1 \end{bmatrix}$

6

$A = \begin{bmatrix} 2 & 0 & 4 \ -6 & 3 & -9 \ -4 & 3 & 2 \end{bmatrix}$ 에 대해

(a)

다음 식을 만족하는 기본 행렬 E_1, E_2, E_3을 구하라.

$$E_1E_2E_3A = U$$

여기서 U는 위쪽 삼각 행렬이다.

$E_1 = \begin{bmatrix} 1 & 0 & 0 \ 3 & 1 & 0 \ 0 & 0 & 1 \end{bmatrix}$

$E_2 = \begin{bmatrix} 1 & 0 & 0 \ 0 & 1 & 0 \ 2 & 0 & 1 \end{bmatrix}$

$E_3 = \begin{bmatrix} 1 & 0 & 0 \ 0 & 1 & 0 \ 0 & -1 & 1 \end{bmatrix}$

(b)

E_1, E_2, E_3의 역을 구하고 $L=E_1^{-1}E_2^{-1}E_3^{-1}$이라 정한다. L은 어떤 형태의 행렬인가? $A=LU$임을 증명하라.

$E_1^{-1} = \begin{bmatrix} 1 & 0 & 0 \ -3 & 1 & 0 \ 0 & 0 & 1 \end{bmatrix},, E_2^{-1} = \begin{bmatrix} 1 & 0 & 0 \ 0 & 1 & 0 \ -2 & 0 & 1 \end{bmatrix},, E_3^{-1} = \begin{bmatrix} 1 & 0 & 0 \ 0 & 1 & 0 \ 0 & 1 & 1 \end{bmatrix}$

L은 단위 아래쪽 삼각 행렬이다.

주어진 식의 양변에 $L=E_1^{-1}E_2^{-1}E_3^{-1}$을 곱하면 (교환이 가능한 경우이므로) 좌변의 E들이 모두 소거되어 $A=LU$임을 알 수 있다.

8

다음의 각 행렬에 대하여 LU 분해를 하라.

(a)

$$\begin{bmatrix} 3 & 1 \ 9 & 5 \end{bmatrix}$$

$\begin{bmatrix} 1 & 0 \ -3 & 1 \end{bmatrix}\begin{bmatrix} 3 & 1 \ 0 & 2 \end{bmatrix}$

(b)

$$\begin{bmatrix} 2 & 4 \ -2 & 1 \end{bmatrix}$$

$\begin{bmatrix} 1 & 0 \ -1 & 1 \end{bmatrix}\begin{bmatrix} 2 & 4 \ 0 & 5 \end{bmatrix}$

(c)

$$\begin{bmatrix} 1 & 1 & 1 \ 3 & 5 & 6 \ -2 & 2 & 7 \end{bmatrix}$$

$\begin{bmatrix} 1 & 0 & 0 \ 3 & 1 & 0 \ 2 & -2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \ 0 & 2 & 3 \ 0 & 0 & 3 \end{bmatrix}$

(d)

$$\begin{bmatrix} -2 & 1 & 2 \ 4 & 1 & -2 \ -6 & -3 & 4 \end{bmatrix}$$

$\begin{bmatrix} 1 & 0 & 0 \ 2 & 1 & 0 \ -3 & 2 & 1 \end{bmatrix}\begin{bmatrix} -2 & 1 & 2 \ 0 & 3 & 2 \ 0 & 0 & 2 \end{bmatrix}$

1.6

1

A를 특이하지 않은 $n \times n$ 행렬이라 하자. 다음의 곱셈들을 수행하라.

(a)

$$A^{-1}\begin{bmatrix} A & I \end{bmatrix}$$

$\begin{bmatrix} I & A^{-1} \end{bmatrix}$

(b)

$$\begin{bmatrix} A \ I \end{bmatrix}A^{-1}$$

$\begin{bmatrix} I \ A^{-1} \end{bmatrix}$

(c)

$$\begin{bmatrix} A & I \end{bmatrix}^T\begin{bmatrix} A & I \end{bmatrix}$$

$\begin{bmatrix} A^TA & A^T \ A & I \end{bmatrix}$

(d)

$$\begin{bmatrix} A & I \end{bmatrix}\begin{bmatrix} A & I \end{bmatrix}^T$$

$AA^T + I$

(e)

$$\begin{bmatrix} A^{-1} \ I \end{bmatrix}\begin{bmatrix} A & I \end{bmatrix}$$

$\begin{bmatrix} I & A^{-1} \ A & I \end{bmatrix}$

5

다음의 각 블록 곱셈들을 수행하라.

(a)

$$\left[\begin{array}{ccc|c} 2 & -1 & 3 & 1 \ 4 & -1 & 0 & 2 \end{array}\right]\begin{bmatrix} 1 & 2 & 4 \ 2 & 1 & 1 \ 4 & 0 & 1 \ \hline 1 & 0 & 2 \end{bmatrix}$$

$\begin{bmatrix} 2 & -1 & 3 \ 4 & -1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 2 & 4 \ 2 & 1 & 1 \ 4 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 12 & 3 & 10 \ 2 & 7 & 15 \end{bmatrix}$

$\begin{bmatrix} 1 \ 2 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 2 \ 2 & 0 & 4 \end{bmatrix}$

$\therefore \begin{bmatrix} 12 & 3 & 10 \ 2 & 7 & 15 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 2 \ 2 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 13 & 3 & 12 \ 4 & 7 & 19 \end{bmatrix}$

(b)

$$\begin{bmatrix} 1 & 2 \ 2 & 1 \ 4 & 0 \ \hline 1 & 0 \end{bmatrix}\left[\begin{array}{ccc|c} 2 & -1 & 3 & 1\ 4 & -1 & 0 & 2 \end{array}\right]$$

$\begin{bmatrix} 1 & 2 \ 2 & 1 \ 4 & 0 \end{bmatrix}\begin{bmatrix} 2 & -1 & 3\ 4 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 10 & -3 & 3 \ 8 & -3 & 6 \ 8 & -4 & 12 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} 2 & -1 & 3\ 4 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 3 \end{bmatrix}$

$\begin{bmatrix} 1 & 2 \ 2 & 1 \ 4 & 0 \end{bmatrix}\begin{bmatrix} 1 \ 2 \end{bmatrix} = \begin{bmatrix} 5 \ 4 \ 4 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \ 2 \end{bmatrix} = 1$

$\therefore \left[\begin{array}{ccc|c} 10 & -3 & 3 & 5 \ 8 & -3 & 6 & 4 \ 8 & -4 & 12 & 4 \ \hline 2 & -1 & 3 & 1 \end{array}\right]$

(c)

$$\left[\begin{array}{c|c} \begin{matrix} \dfrac14 & \dfrac34 \[1em] -\dfrac34 & \dfrac14\[1em] \end{matrix} & O \ \hline O & \begin{matrix} 1 & 1 \end{matrix} \end{array}\right]\left[\begin{array}{c|c} \begin{matrix} \dfrac14 & -\dfrac34 \[1em] \dfrac34 & \dfrac14\[1em] \end{matrix} & O \ \hline O & \begin{matrix} 1 \ 1 \end{matrix} \end{array}\right]$$

서로 전치인 행렬의 곱이다.

$\left[\begin{array}{cc|c} \dfrac58 & 0 & 0 \ 0 & \dfrac58 & 0 \[1em] \hline 0 & 0 & 2 \end{array}\right]$

(d)

$$\left[\begin{array}{c|c} \begin{matrix} 0 & 0 & 1 \ 0 & 1 & 0 \ 1 & 0 & 0 \end{matrix} & O \ \hline O & \begin{matrix} 0 & 1 \ 1 & 0 \end{matrix} \end{array}\right]\begin{bmatrix} 1 & 0 \ 2 & -9 \ 3 & -8 \ \hline 4 & -7 \ 5 & -6 \end{bmatrix}$$

$\begin{bmatrix} 3 & -8 \ 2 & -9 \ 1 & 0 \ \hline 5 & -6 \ 4 & -7 \end{bmatrix}$