Homework - Chapter 1

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1.1

1

다음과 같은 각 방정식들의 시스템을 풀기 위하여 역방향 대입을 사용하라.

(a)

$$\begin{aligned} x_1 + x_2 &= 7 \\ 2x_2 &= 6 \end{aligned}$$

$\begin{bmatrix} 1 & 1 & 7 \\ 0 & 2 & 6 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & 1 & 7 \\ 0 & 1 & 3 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 3 \end{bmatrix}$

$\therefore x_1 = 4, x_2 = 3$

(b)

$$\begin{aligned} x_1 + x_2 + x_3 &= 10 \\ 2x_2 + x_3 &= 11 \\ 2x_3 &= 14 \end{aligned}$$

$\begin{bmatrix} 1 & 1 & 1 & 10 \\ 0 & 2 & 1 & 11 \\ 0 & 0 & 2 & 14 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & 1 & 1 & 10 \\ 0 & 2 & 1 & 11 \\ 0 & 0 & 1 & 7 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & 1 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 7 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 7 \end{bmatrix}$

$\therefore x_1 = 1, x_2 = 2, x_3 = 7$

(c)

$$\begin{aligned} x_1 + 2x_2 + 3x_3 + 4x_4 &= 6 \\ 7x_2 - x_3 + 2x_4 &= 5 \\ x_3 - 4x_4 &= -9 \\ 4x_4 &= 8 \end{aligned}$$

$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 6 \\ 0 & 7 & -1 & 2 & 5 \\ 0 & 0 & 1 & -4 & -9 \\ 0 & 0 & 0 & 4 & 8 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 6 \\ 0 & 7 & -1 & 2 & 5 \\ 0 & 0 & 1 & -4 & -9 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 2 & 3 & 0 & -2 \\ 0 & 7 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 2 & 0 & 0 & 1 \\ 0 & 7 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right]$$

$\therefore x_1 = 1, x_2 = 0, x_3 = -1, x_4 = 2$

(d)

$$\begin{aligned} x_1 + x_2 + 16x_3 + 3x_4 + x_5 &= 5 \\ 4x_2 + 4x_3 + 6x_4 + 3x_5 &= 1 \\ -8x_3 + 27x_4 - 7x_5 &= 7 \\ 3x_4 + 11x_5 &= 1 \\ x_5 &= 0 \end{aligned}$$

$$\left[\begin{array}{ccccc|c} 1 & 1 & 16 & 3 & 1 & 5 \\ 0 & 4 & 4 & 6 & 3 & 1 \\ 0 & 0 & -8 & 27 & -7 & 7 \\ 0 & 0 & 0 & 3 & 11 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccccc|c} 1 & 1 & 16 & 3 & 0 & 5 \\ 0 & 4 & 4 & 6 & 0 & 1 \\ 0 & 0 & -8 & 27 & 0 & 7 \\ 0 & 0 & 0 & 3 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccccc|c} 1 & 1 & 16 & 0 & 0 & 4 \\ 0 & 4 & 4 & 0 & 0 & -1 \\ 0 & 0 & -8 & 0 & 0 & -2 \\ 0 & 0 & 0 & 3 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & -2 \\ 0 & 0 & -4 & 0 & 0 & -1 \\ 0 & 0 & 0 & 3 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 2 & 0 & 0 & 0 & -1 \\ 0 & 0 & 4 & 0 & 0 & 1 \\ 0 & 0 & 0 & 3 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$\therefore x_1 = \frac{1}{2}, x_2 = -\frac{1}{2}, x_3 = \frac{1}{4}, x_4 = \frac{1}{3}, x_5 = 0$

7

다음의 두 시스템은 계수 행렬은 같지만 우변이 다르다.

$$\begin{aligned} x_1 + 2x_2 &= 8 \qquad & x_1 + 2x_2 &= 7 \\ 4x_1 - 3x_2 &= -1 \qquad & 4x_1 - 3x_2 &= 6 \end{aligned}$$

다음의 증가된 행렬의 두 번째 행에 있는 첫 번째 원소를 제거하고 우변에 해당하는 각 열들에 대하여 역방향 대입을 수행하여 두 시스템의 해를 동시에 구하라.

$$\left[\begin{array}{cc|cc} 1 & 2 & 8 & 7 \\ 4 & -3 & -1 & 6 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{cc|cc} 1 & 2 & 8 & 7 \\ 0 & -11 & -33 & -22 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{cc|cc} 1 & 0 & 2 & 3 \\ 0 & 1 & 3 & 2 \end{array}\right]$$

$$\therefore (x_1 = 2, x_2 = 3), (x_1 = 3, x_2 = 2)$$

8

다음 두 시스템의 해를 3 × 5 증가된 행렬에 대해 소거를 하고 역방향 대입을 두 번 시행하여 구하라.

$$\begin{aligned} x_1 + 2x_2 - x_3 &= 6 \qquad & x_1 + 2x_2 - x_3 &= 9 \\ 2x_1 - x_2 + 3x_3 &= -3 \qquad & 2x_1 - x_2 + 3x_3 &= -2 \\ x_1 + x_2 - 4x_3 &= 7 \qquad & x_1 + x_2 - 4x_3 &= 9 \end{aligned}$$

$$\left[\begin{array}{ccc|cc} 1 & 2 & -1 & 6 & 9 \\ 2 & -1 & 3 & -3 & -2 \\ 1 & 1 & -4 & 7 & 9 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|cc} 1 & 2 & -1 & 6 & 9 \\ 0 & -5 & 5 & -15 & -20 \\ 0 & -1 & -3 & 1 & 0 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|cc} 1 & 2 & -1 & 6 & 9 \\ 0 & -1 & 1 & -3 & -4 \\ 0 & 0 & -4 & 4 & 4 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|cc} 1 & 2 & 0 & 5 & 8 \\ 0 & 1 & 0 & 2 & 3 \\ 0 & 0 & 1 & -1 & -1 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|cc} 1 & 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 2 & 3 \\ 0 & 0 & 1 & -1 & -1 \end{array}\right]$$

$$\therefore (x_1 = 1, x_2 = 2, x_3 = -1), (x_1 = 2, x_2 = 3, x_3 = -1)$$

1.2

2

다음의 각각에서, 증가된 행렬이 행 사다리꼴 형태이다. 각 경우에 대하여, 대응하는 선형 시스템이 모순이 아닌지 표시하라. 만일 그 시스템이 유일한 해를 갖는다면 그 해를 구하라.

(a)

$$\left[\begin{array}{cc|c} 1 & -1 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{cc|c} 1 & 0 & 10 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right]$$

$\therefore x_1 = 10, x_2 = 3$

(b)

$$\left[\begin{array}{cc|c} 1 & 2 & 4 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{array}\right]$$

마지막 행에서 모순 발생

(c)

$$\left[\begin{array}{ccc|c} 1 & 7 & -3 & 9 \\ 0 & 1 & 2 & -4 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 7 & 0 & 4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$\therefore x_1 = 3, x_2 = 0, x_3 = -2$

(d)

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

모순은 아니지만, 유일해를 갖지 않는다.

(e)

$$\left[\begin{array}{ccc|c} 1 & -5 & 3 & 5 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

마지막 행에서 모순을 갖는다.

(f)

$$\left[\begin{array}{ccc|c} 1 & 7 & 1 & 2 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 7 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\therefore x_1 = 0, x_2 = 0, x_3 = 2$

3

다음의 각각에서, 증가된 행렬이 감소된 행 사다리꼴 형태이다. 각 경우에 대하여, 대응하는 선형 시스템의 해 집합을 구하여라.

(a)

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

$\{(3, -2, 5)\}$

(b)

$$\left[\begin{array}{ccc|c} 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 15 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\{(\alpha, -3, 15) \mid \alpha \in \mathbb{R}\}$

(c)

$$\left[\begin{array}{ccc|c} 1 & 0 & 4 & 2 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & -3 \end{array}\right]$$

$\emptyset$

(d)

$$\left[\begin{array}{ccc|c} 1 & -2 & 0 & 5 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\{(5 + 2\alpha, \alpha, -1) \mid \alpha \in \mathbb{R}\}$

(e)

$$\left[\begin{array}{cccc|c} 1 & -6 & 0 & -5 & 0 \\ 0 & 0 & 1 & 3 & -6 \end{array}\right]$$

let $x_2 = \alpha, x_4 = \beta$

$\{(6\alpha + 5\beta, \alpha, -6 - 3\beta, \beta) \mid \alpha, \beta \in \mathbb{R}\}$

(f)

$$\left[\begin{array}{cccc|c} 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

let $x_2 = \alpha, x_3 = \beta$

$\{(-2\alpha - \beta, \alpha, \beta, 3) \mid \alpha, \beta \in \mathbb{R}\}$

6

다음의 시스템들을 풀기 위하여 가우스-조던 소거법을 사용하라.

(a)

$$\begin{aligned} 2x + y &= 1 \\ 7x + 6y &= 1 \end{aligned}$$

$$\left[\begin{array}{cc|c} 2 & 1 & 1 \\ 7 & 6 & 1 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{cc|c} 14 & 7 & 7 \\ 14 & 12 & 2 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{cc|c} 2 & 1 & 1 \\ 0 & 5 & -5 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{cc|c} 2 & 0 & 2 \\ 0 & 1 & -1 \end{array}\right]$$

$\therefore \{(1, -1)\}$

(b)

$$\begin{aligned} x_1 + x_2 - x_3 + x_4 &= 6 \\ 2x_1 - x_2 + x_3 - x_4 &= -3 \\ 3x_1 + x_2 - 2x_3 + x_4 &= 9 \end{aligned}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & -1 & 1 & 6 \\ 2 & -1 & 1 & -1 & -3 \\ 3 & 1 & -2 & 1 & 9 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 1 & -1 & 1 & 6 \\ 0 & -3 & 3 & -3 & -15 \\ 0 & -2 & 1 & -2 & -9 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 1 & -1 & 1 & 6 \\ 0 & 1 & -1 & 1 & 5 \\ 0 & 0 & -1 & 0 & 1 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 1 & 0 & 1 & 5 \\ 0 & 1 & 0 & 1 & 4 \\ 0 & 0 & 1 & 0 & -1 \end{array}\right]$$

$\therefore \{(1, \alpha, -1, 4 - \alpha) \,|\, \alpha \in \mathbb{R}\}$

(c)

$$\begin{aligned} x_1 - 10x_2 + 5x_3 &= -4 \\ x_1 + x_2 + x_3 &= 1 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & -10 & 5 & -4 \\ 1 & 1 & 1 & 1 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -11 & 4 & -3 \end{array}\right]$$

let $x_2 = \alpha$

$\therefore \{\left(1 - \dfrac{15\alpha - 3}{4},\ \alpha,\ \dfrac{11\alpha - 3}{4}\right) \,|\, \alpha \in \mathbb{R}\}$

(d)

$$\begin{aligned} x_1 - 2x_2 + 3x_3 + x_4 &= 4 \\ 2x_1 + x_2 - x_3 + x_4 &= 1 \\ x_1 + 3x_2 + x_3 + x_4 &= 3 \end{aligned}$$

$$\left[\begin{array}{cccc|c} 1 & -2 & 3 & 1 & 4 \\ 2 & 1 & -1 & 1 & 1 \\ 1 & 3 & 1 & 1 & 3 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & -2 & 3 & 1 & 4 \\ 0 & 5 & -7 & -1 & -7 \\ 0 & 5 & -2 & 0 & -1 \end{array}\right]$$

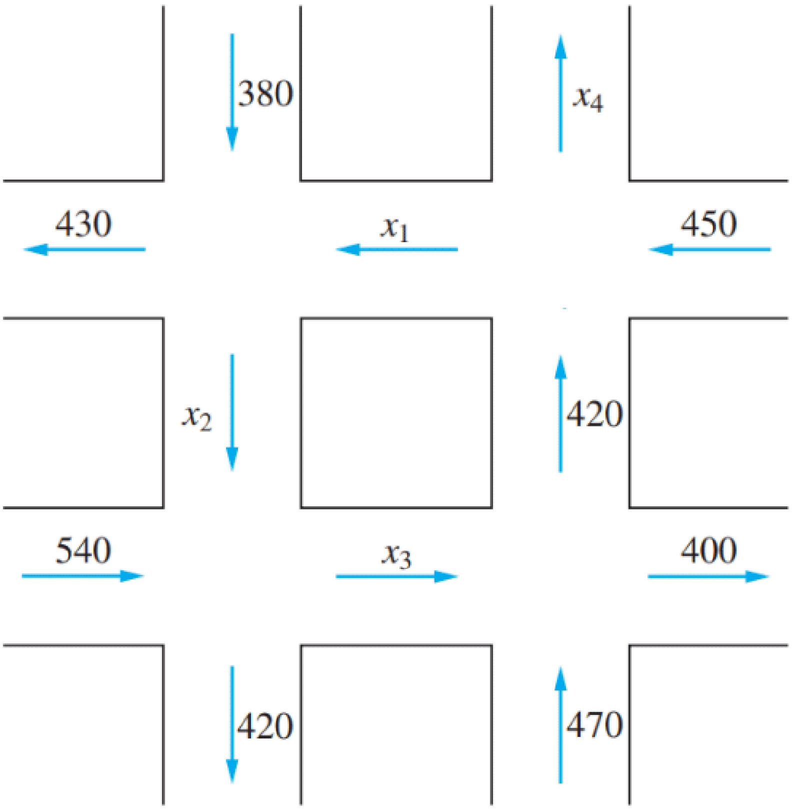
$$\Rightarrow \left[\begin{array}{cccc|c} 1 & -2 & 3 & 1 & 4 \\ 0 & 5 & -7 & -1 & -7 \\ 0 & 0 & 5 & 1 & 6 \end{array}\right]$$

let $x_3 = \alpha$

$$\therefore \{\left(-\frac{6\alpha+12}{5}, \frac{2\alpha-1}{5}, \alpha, 6-5\alpha\right) \mid \alpha \in \mathbb{R}\}$$

15

다음과 같은 교통 흐름도에서 x_1, x_2, x_3, x_4의 값을 구하여라.



각 교차로를 기준으로 식을 세운다.

$$\begin{aligned} x_1 + 380 &= x_2 + 430 \\ x_2 + 540 &= x_3 + 420 \\ x_3 + 470 &= 400 + 420 \\ x_1 + x_4 &= 420 + 450 \end{aligned}$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & -1 & 0 & 0 & 50 \\ 0 & 1 & -1 & 0 & -120 \\ 0 & 0 & 1 & 0 & 350 \\ 1 & 0 & 0 & 1 & 870 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 280 \\ 0 & 1 & 0 & 0 & 230 \\ 0 & 0 & 1 & 0 & 350 \\ 0 & 0 & 0 & 1 & 590 \end{array}\right]$$

$$\therefore x_1 = 280,\ x_2 = 230,\ x_3 = 350,\ x_4 = 590$$

1.3

2

다음의 각 행렬들의 쌍에 대하여, 첫 번째 행렬과 두 번째 행렬을 곱하는 것이 가능한가? 만일 가능하다면 곱셈을 수행하라.

(a)

$$\begin{bmatrix} 3 & 5 & 1 \\ -2 & 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 3 \\ 4 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 6+5+4 & 3+15+1 \\ -4+0+8 & -2+0+2 \end{bmatrix} = \begin{bmatrix} 15 & 19 \\ 4 & 0 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 4 & -2 \\ 6 & -4 \\ 8 & -6 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$$

불가능

(c)

$$\begin{bmatrix} 1 & 4 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 1 & 1 \\ 4 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 3+4+12 & 2+4+15 \\ 0+1+16 & 0+1+20 \\ 0+0+8 & 0+0+10 \end{bmatrix} = \begin{bmatrix} 19 & 21 \\ 17 & 21 \\ 8 & 10 \end{bmatrix}$$

(d)

$$\begin{bmatrix} 4 & 6 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 & 5 \\ 4 & 1 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 12+24 & 4+6 \\ 6+4 & 2+1 \end{bmatrix} = \begin{bmatrix} 36 & 10 \\ 10 & 3 \end{bmatrix}$$

(e)

$$\begin{bmatrix} 4 & 6 & 1 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 & 5 \\ 4 & 1 & 6 \end{bmatrix}$$

불가능

(f)

$$\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} \begin{bmatrix} 3 & 2 & 4 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 4 & 8 & 10 \\ -3 & -2 & -4 & -5 \\ 9 & 6 & 12 & 15 \end{bmatrix}$$

1.4

8

만일 $A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ 일 때, A^2, A^3을 계산하라. A^{2n}, A^{2n+1}은 어떻게 될까?

$$A^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$A^{2n} = I$

$A^{2n+1} = A$

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$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ 일 때, R은 불특이 행렬이고 $R^{-1} = R^T$임을 보여라.

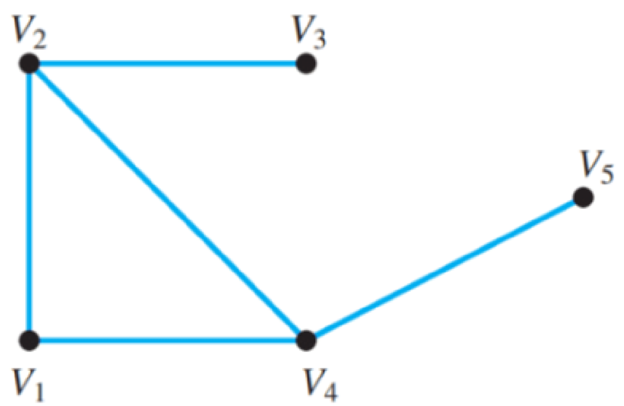
$R^T = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ 에서

$$R \cdot R^T = \begin{bmatrix} \cos^2\theta + \sin^2\theta & 0 \\ 0 & \cos^2\theta + \sin^2\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\therefore R^{-1} = R^T$, R is nonsingular

33

다음 그래프에 대해



(a)

그래프의 인접 행렬 A를 구하여라.

$$A = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

(b)

A^2를 계산하라. A^2의 첫 번째 행에 있는 원소들이 여러분에게 V_1에서 출발하여 길이 2인 걸음에 관하여 말하는 것은 무엇인가?

$$A^2 = \begin{bmatrix} 2 & 1 & 1 & 1 & 1 \\ 1 & 3 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 3 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

V_1에서 출발하여 V_n에 도착하는 데 A^2_{1n}가지의 경로가 있는 의미이다.

(c)

A^3을 계산하라. V_2부터 V_4까지의 길이가 3인 걸음은 얼마나 있는가? V_2부터 V_4까지의 길이가 3 이하인 걸음은 얼마나 있는가?

$$A^3 = \begin{bmatrix} 2 & 4 & 1 & 4 & 1 \\ 4 & 2 & 3 & 5 & 1 \\ 1 & 3 & 0 & 1 & 1 \\ 4 & 5 & 1 & 2 & 3 \\ 1 & 1 & 1 & 3 & 0 \end{bmatrix}$$

V_2부터 V_4까지의 길이가 3인 걸음은 $A^3_{24} = 5$이다.
V_2부터 V_4까지의 길이가 3 이하인 걸음은 $A^3_{24} + A^2_{24} + A_{24} = 5 + 1 + 1 = 7$이다.

1.5

4

다음의 각 행렬의 쌍에 대하여, $AE = B$가 되는 기본 행렬 E를 구하라.

(a)

$$A = \begin{bmatrix} 4 & -1 & 0 \\ 3 & 4 & 1 \\ 2 & 5 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 4 & -2 & 0 \\ 3 & 8 & 1 \\ 2 & 10 & 4 \end{bmatrix}$$

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(b)

$$A = \begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & -3 \\ -5 & -2 \end{bmatrix}$$

$$E = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

(c)

$$A = \begin{bmatrix} 4 & -1 & 1 \\ 0 & 2 & 3 \\ 5 & 1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} -1 & 4 & 1 \\ 2 & 0 & 3 \\ 1 & 5 & 0 \end{bmatrix}$$

$$E = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

6

$A = \begin{bmatrix} 2 & 0 & 4 \\ -6 & 3 & -9 \\ -4 & 3 & 2 \end{bmatrix}$
에 대해

(a)

다음 식을 만족하는 기본 행렬 E_1, E_2, E_3을 구하라.

$$E_1 E_2 E_3 A = U$$

여기서 U는 위쪽 삼각 행렬이다.

$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$

$E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$

(b)

E_1, E_2, E_3의 역을 구하고 $L = E_1^{-1} E_2^{-1} E_3^{-1}$이라 정한다. L은 어떤 형태의 행렬인가? $A = LU$임을 증명하라.

$E_1^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $E_2^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$, $E_3^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

L은 단위 아래쪽 삼각 행렬이다.

주어진 식의 양변에 $L = E_1^{-1} E_2^{-1} E_3^{-1}$을 곱하면 (교환이 가능한 경우이므로) 좌변의 E들이 모두 소거되어 $A = LU$임을 알 수 있다.

8

다음의 각 행렬에 대하여 LU 분해를 하라.

(a)

$$\begin{bmatrix} 3 & 1 \\ 9 & 5 \end{bmatrix}$$

$\begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$

(b)

$$\begin{bmatrix} 2 & 4 \\ -2 & 1 \end{bmatrix}$$

$\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 0 & 5 \end{bmatrix}$

(c)

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & 5 & 6 \\ -2 & 2 & 7 \end{bmatrix}$$

$\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & 3 \end{bmatrix}$

(d)

$$\begin{bmatrix} -2 & 1 & 2 \\ 4 & 1 & -2 \\ -6 & -3 & 4 \end{bmatrix}$$

$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -3 & 2 & 1 \end{bmatrix} \begin{bmatrix} -2 & 1 & 2 \\ 0 & 3 & 2 \\ 0 & 0 & 2 \end{bmatrix}$

1.6

1

A를 특이하지 않은 $n \times n$ 행렬이라 하자. 다음의 곱셈들을 수행하라.

(a)

$$A^{-1} \begin{bmatrix} A & I \end{bmatrix}$$

$\begin{bmatrix} I & A^{-1} \end{bmatrix}$

(b)

$$\begin{bmatrix} A \\ I \end{bmatrix} A^{-1}$$

$\begin{bmatrix} I \\ A^{-1} \end{bmatrix}$

(c)

$$\begin{bmatrix} A & I \end{bmatrix}^T \begin{bmatrix} A & I \end{bmatrix}$$

$\begin{bmatrix} A^T A & A^T \\ A & I \end{bmatrix}$

(d)

$$\begin{bmatrix} A & I \end{bmatrix} \begin{bmatrix} A & I \end{bmatrix}^T$$

$AA^T + I$

(e)

$$\begin{bmatrix} A^{-1} \\ I \end{bmatrix} \begin{bmatrix} A & I \end{bmatrix}$$

$\begin{bmatrix} I & A^{-1} \\ A & I \end{bmatrix}$

5

다음의 각 블록 곱셈들을 수행하라.

(a)

$$\left[\begin{array}{ccc|c} 2 & -1 & 3 & 1 \\ 4 & -1 & 0 & 2 \end{array}\right] \left[\begin{array}{ccc} 1 & 2 & 4 \\ 2 & 1 & 1 \\ 4 & 0 & 1 \\ \hline 1 & 0 & 2 \end{array}\right]$$

$$\begin{bmatrix} 2 & -1 & 3 \\ 4 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & 4 \\ 2 & 1 & 1 \\ 4 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 12 & 3 & 10 \\ 2 & 7 & 15 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & 4 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 12 & 3 & 10 \\ 2 & 7 & 15 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 13 & 3 & 12 \\ 4 & 7 & 19 \end{bmatrix}$$

(b)

$$\left[\begin{array}{cc} 1 & 2 \\ 2 & 1 \\ 4 & 0 \\ \hline 1 & 0 \end{array}\right] \left[\begin{array}{ccc|c} 2 & -1 & 3 & 1 \\ 4 & -1 & 0 & 2 \end{array}\right]$$

$$\begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} 2 & -1 & 3 \\ 4 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 10 & -3 & 3 \\ 8 & -3 & 6 \\ 8 & -4 & 12 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 2 & -1 & 3 \\ 4 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 1$$

$$\therefore \left[\begin{array}{ccc|c} 10 & -3 & 3 & 5 \\ 8 & -3 & 6 & 4 \\ 8 & -4 & 12 & 4 \\ \hline 2 & -1 & 3 & 1 \end{array}\right]$$

(c)

$$\left[\begin{array}{cc|cc} \frac{1}{4} & \frac{3}{4} & \multicolumn{2}{c}{\multirow{2}{*}{O}} \\ -\frac{3}{4} & \frac{1}{4} & & \\ \hline \multicolumn{2}{c|}{O} & 1 & 1 \end{array}\right] \left[\begin{array}{cc|c} \frac{1}{4} & -\frac{3}{4} & \multirow{2}{*}{O} \\ \frac{3}{4} & \frac{1}{4} & \\ \hline \multicolumn{2}{c|}{\multirow{2}{*}{O}} & 1 \\ & & 1 \end{array}\right]$$

서로 전치인 행렬의 곱이다.

$$\left[\begin{array}{cc|c} \frac{5}{8} & 0 & 0 \\ 0 & \frac{5}{8} & 0 \\ \hline 0 & 0 & 2 \end{array}\right]$$

(d)

$$\left[\begin{array}{ccc|cc} 0 & 0 & 1 & & \\ 0 & 1 & 0 & & O \\ 1 & 0 & 0 & & \\ \hline & O & & 0 & 1 \\ & & & 1 & 0 \end{array}\right] \left[\begin{array}{cc} 1 & 0 \\ 2 & -9 \\ 3 & -8 \\ \hline 4 & -7 \\ 5 & -6 \end{array}\right]$$

$$\left[\begin{array}{cc} 3 & -8 \\ 2 & -9 \\ 1 & 0 \\ \hline 5 & -6 \\ 4 & -7 \end{array}\right]$$