Mobile Robotics
Assignment 1
Team:20

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Question 1:

First, we load the points from load velodyne_points () function which is already provided.

Now as we can see that LiDAR gives distance of points in according to the XYZ direction given but the camera XYZ axis seems different.

So now we now find the rotation matrix by taking the components LiDAR-XYZ to camera's X-axis than LiDAR-XYZ to camera's Y-axis and finally camera's Z-axis.

For Camera's X = -1*Y OF LiDAR Frame

Camera's Y = -1*Z of LiDAR Frame

Camera's Z = X of LiDAR Frame

Now we add the translational points to the matrix such that to get our Projection matrix. We also need to append one's column to the XYZ data of LiDAR points so as to make all calculations in vector form. Now points in camera frame is equal to CalibrationMatrix*ProjectionMatrix*LiDARpoints

Projection matrix has combination of Rotational and translation components.

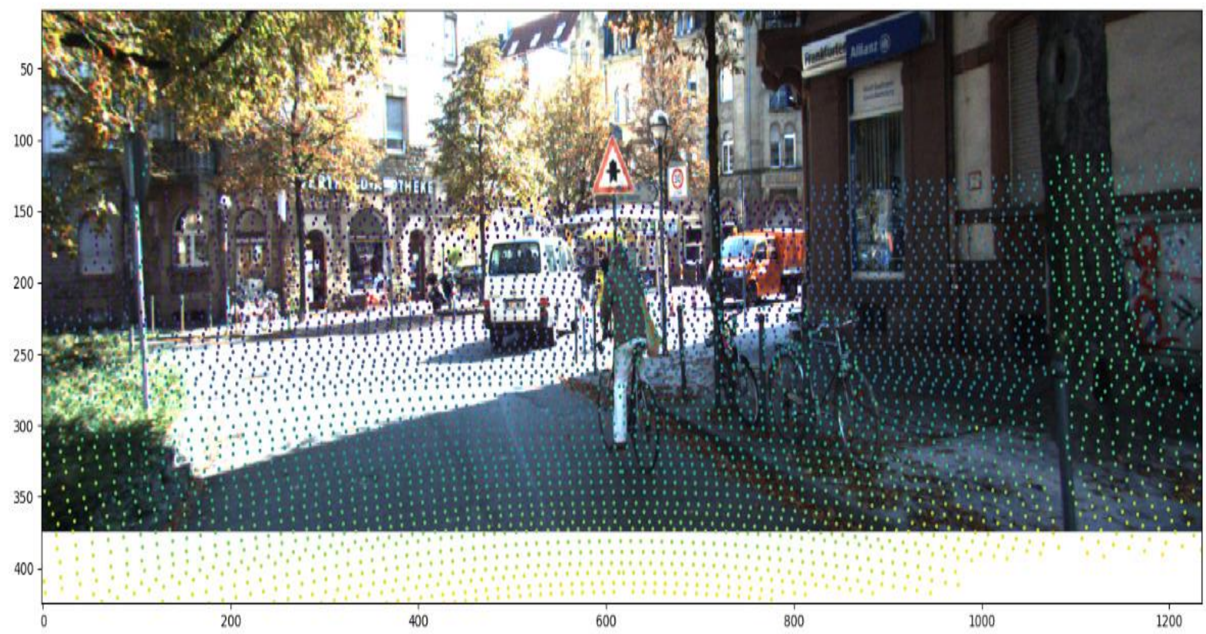
Matrix size verification: [3*x] = [3*3] * [3*4] * [4*x]

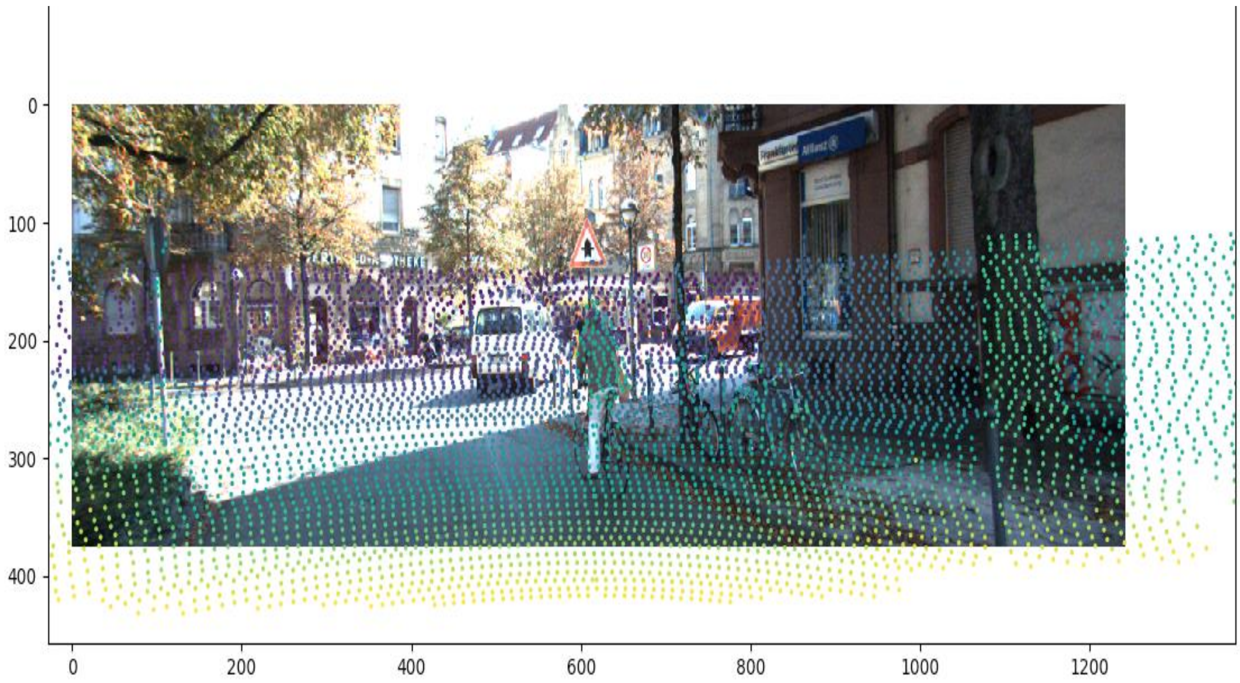
Finally taking transpose of our result we get x*3 matrix which gives us x point in camera frame.

Now we plot the image and points over the image with gradients set as 100/z such that all the points with same z will have same colour and thus if all distant point will have different colour as from near points.

So, we are finally able to plot the point with different colours so as to differentiate closer and far away objects/points in the image.

Output Images:





As we can see that all the equi-distance points are having same color.

Question 2:

We can do this question by using many methods and can implement also using many methods, in other words we have many ways to implement this question so in this report we have discuss two methods, one is where we have already given the K and one of the three coordinates of the camera from ground in world frame and other in which we don’t have a general situation.

1st Method:

let $\vec{X_c}$ be the vector consists of the coordinates of the object in camera frame.

let $\vec{X_w}$ be the vector consists of the coordinates of the object in world frame.

let P be the Projection matrix,

then $\vec{X_c}_{3\times1} = P\vec{X_w}_{4\times1}$,

where $P = K_{3\times3}[R\ t]_{3\times4}$.

$$\Rightarrow \vec{X_c}_{3\times1} = K_{3\times3}[R\ t]_{3\times4}\vec{X_w}_{4\times1},$$

Now, as $\theta = 5^\circ$, ~~So we can~~

$$\Rightarrow R = \begin{bmatrix} \cos\theta & \sin 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

Now, we can assume $\sin\theta \simeq 0$ and $\cos\theta \simeq 1$ as "θ" is very small. So, R can be approximately written as,

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

$$\Rightarrow \vec{X_c} = K_{3\times3}[I_{3\times3}\ t]\vec{X_w}$$

Now, assuming one corners of the object as origin in world frame. and then we find rest of the 7 co-ordinates using dimension of the object relative to the origin.

So. Now consider $\vec{Xw_1} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$, and $\vec{Xc_1} = \begin{bmatrix} x_1 \\ y_1 \\ f \end{bmatrix}$, so,

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By Putting that in eq^n above, we get.

$$\begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} = K_{3\times3} \cdot [I_{3\times3} \; t_{3\times1}]_{3\times4} \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\Rightarrow K^{-1}_{3\times3} \vec{X_c}_{3\times1} = [I_{3\times3} \; t_{3\times1}]_{3\times4} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$K^{-1}_{3\times3} \vec{X}_{c_{3V}} = t_{3\times1} = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$$; so, we will get t_x, t_y, t_z from eq^n.

Now, as h = camera height from ground, so :

~~t_x~~

$t_x = (t_x/t_y) \times h_y$

$t_z = (t_z/t_y) \times h_y$

$t_y = h_y$.

As ~~t~~ y-coordinate is given to us., ~~which is the~~ i.e., we have given height of the camera centre from ground.

$\Rightarrow$ ~~$P = K_{3\times3} [R_{3\times3} \; t_{3\times1}]$~~.

$\Rightarrow$ ~~From here~~ Now, we have R, t and K.

So we can find Project matrix, ~~by equ~~ as,

$$P = K_{3\times3} [R_{3\times3} \; t_{3\times1}]$$

Now, we find all the 8-coordinates of the object in camera frame using $\boxed{\vec{X_c} = P \vec{X_w}}$,

and then we can plot our desired result

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2nd Method:

To do this we need the coordinates of the object in the camera frame as well as in the world frame. So, we start by taking 6 points in the camera frame, and then in world frame, we assume one left-down corner as origin and then we find the rest of the coordinates in world frame relative to the origin. Our main goal is to find the rest of the 2 coordinates in the camera frame, so to do that, firstly we need to find the projection matrix.

NOTE: We have implemented the 2nd method because it is more general and can be use in any situation like in this we don't require K matrix or the height of the matrix

Process to find the projection matrix:

Derivation to find Projection matrix:-

let $\vec{X_c}$ be the coordinates vector in camera frame.
let $\vec{X_w}$ be the coordinates vector in world frame.

Also, let $\vec{P}$ be the projection matrix.

$$\Rightarrow \vec{X_c} = P\vec{X_w}$$

$$\Rightarrow \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix} = P_{3\times4} \begin{bmatrix} X_{wi} \\ Y_{wi} \\ Z_{wi} \\ 1 \end{bmatrix}, \text{ where } P_{3\times4} = \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \end{bmatrix}$$

Each point gives two observation equations, one for each coordinate.

$$X_{ci} = \frac{P_{11}X_{wi} + P_{12}Y_{wi} + P_{13}Z_{wi} + P_{14}}{P_{31}X_{wi} + P_{32}Y_{wi} + P_{33}Z_{wi} + P_{34}}$$

$$Y_{ci} = \frac{P_{21}X_{wi} + P_{22}Y_{wi} + P_{23}Z_{wi} + P_{24}}{P_{31}X_{wi} + P_{32}Y_{wi} + P_{33}Z_{wi} + P_{34}}$$

For each coordinate we get 2 such equations, so for 6 coordinates we will get 12 equations.

So, now we will solved this using Direct linear transform.

So,

$$X_{ci} = P_{3\times4}X_{wi} = \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \end{bmatrix} X_{wi}$$

$i = 1, \cdots 6.$

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let the ~~Fors~~ First Row of Projection matrix ~~represent~~ written as A^T, Second Row be B^T and third row be C^T, i.e..,

$$x_{ci} = \begin{bmatrix} A^T \\ B^T \\ C^T \end{bmatrix} X_{wi} \quad , i = 1, \cdots, 6.$$

$$x_{ci} = \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix} , \quad X_{wi} = \begin{bmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{bmatrix}$$

So, $x_i = \dfrac{A^T_{1\times 4} X_i}{C^T_{1\times 4} X_i}$ and. $y_i = \dfrac{B^T_{1\times 4} X_i}{C^T_{1\times 4} X_i}$

$\Rightarrow \quad A_{1\times 4}^T X_i - C^T_{1\times 4} X_i \cdot x_i = 0$ and $B^T_{1\times 4} X_i - y_i C^T X_i = 0$

Now taking transpose on both equation we get.

$$X_i^T A - x_i X_i^T C = 0 \quad \text{and} \quad X_i^T B - y_i X_i^T C = 0.$$

This two equation leads to an system of equation, which is linear in the Parameters A, B and C., as

$$-X_i^T A + 0 + x_i X_i^T C = 0$$
$$0 + (-X_i^T B) + y_i X_i^T C = 0$$

$\Rightarrow$ The Solution is of the form $MP = 0$, where $P_{12\times 1} = \begin{bmatrix} A \\ B \\ C \end{bmatrix}$

and M is 12×12 matrix.

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So, to solve. we need to take care of all the cases possible, because in some ~~cases,~~ cases, there are redundant observation. for which MP is non-zero matrix.

~~So,~~ i.e., in ~~that~~ such cases, we have.

$MP = \omega$, so, we need to find.

P, such that $\omega^T\omega$ minimizes., So for that we use SVD, as

$M = USV^T$. and to minimize w^Tw, we choose. P as the last column of V,

and. then reshape it to 3×4 matrix. to get Projection matrix.

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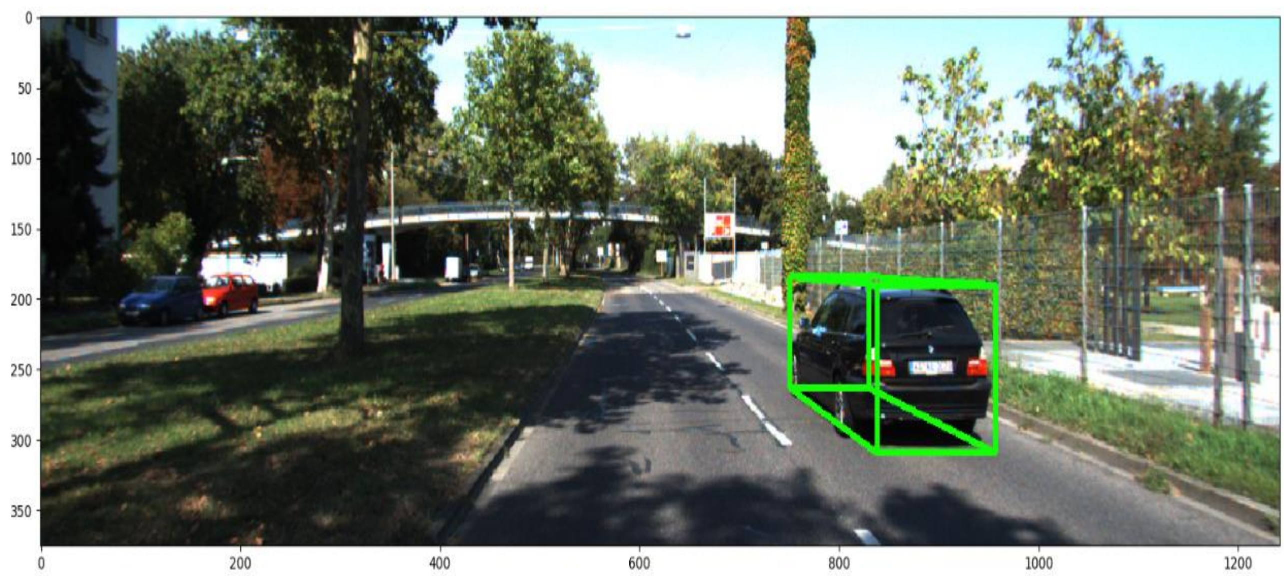
Now, using P we find the rest of the 2 coordinates in camera frame and then we join the points according to the requirements.

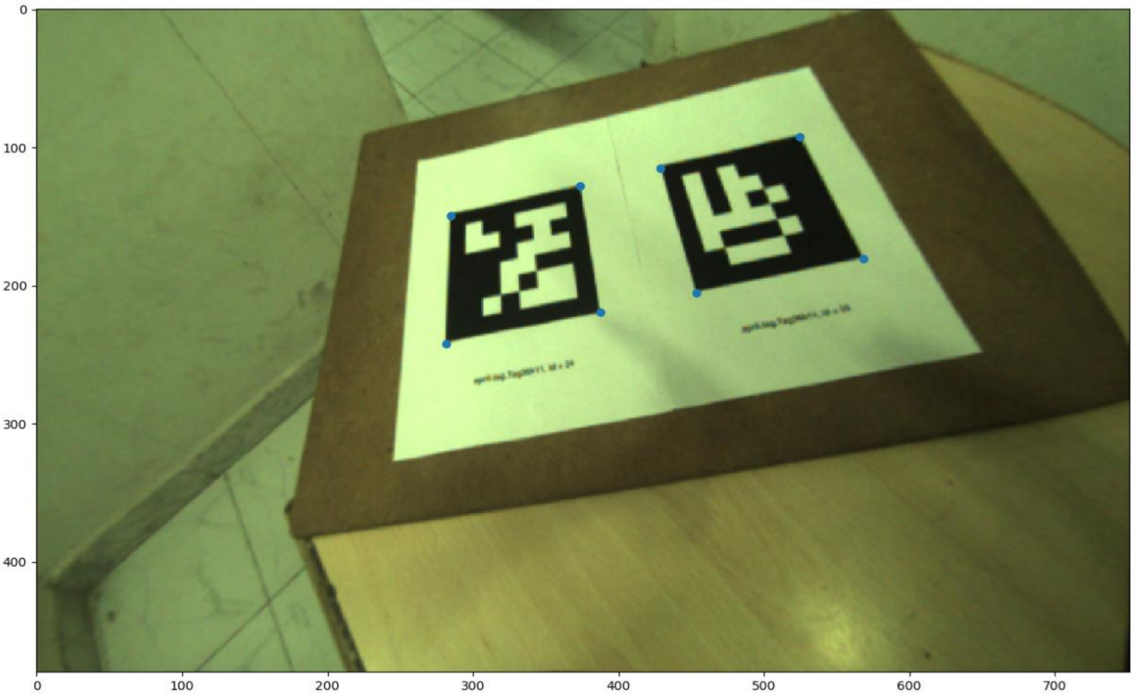
Output Images:



Question 3:

In this question, as we have given the coordinates of the object in camera frame and the dimension in world frame, so by assuming one of the coordinates as origin and then using this dimension to find rest of the 7 coordinates relative to the origin in world frame. Now our first aim is to find the homography matrix, so to find to find the homography matrix we use the same process as we follow in Q2 i.e., by using SVD. Now we have find homography matrix, so now we multiply the homography matrix with the matrix that contains the coordinates of the object in world frame to verify the accuracy of the homography matrix i.e., as we all know, x = HX, where x is the vector consists of the coordinates in image frame and H is homography matrix and X is the vector that consists of world coordinates.

Output Image:



For bonus part, as we know,

H = K[R t], where K is given to us. So to find [R t] we firstly find the inverse of K and then pre multiply it with H.

Now we have [R t] so now we divide our result into two parts, where one part is R and other is t. As [R t] is a 3x3 matrix so its last column will give us t or translation matrix and 1st 2 column will give us the 1st two columns of Rotational matrix and to find the 3rd column of R we take the cross product of its 1st two columns and we assign the resulting column as the 3rd column of Rotational matrix. And then we finally display this result on terminal.