Assignment on computing Solution for solid mechanics problems

C programming code for calculating centroid for all regular shapes

Subject code:20CE101T

Subject name: Elements of civil engineering and solid mechanics

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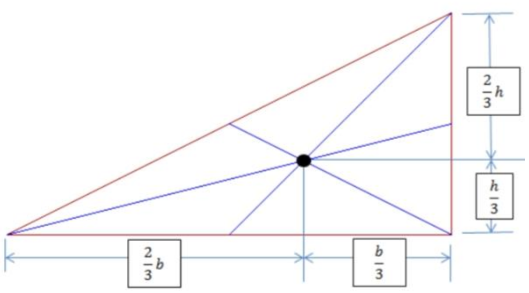
CENTROID:

A centroid is the central point of a figure and is also called the geometric centre. It is the point that matches the centre of gravity of a particular shape. It is the point which corresponds to the mean position of all the points in a figure. The centroid is the term for 2-dimensional shapes. The centre of mass is the term for 3-dimensional shapes.

Centroid Of Some Regular Shapes:

Centroid of a Right Angle Triangle:

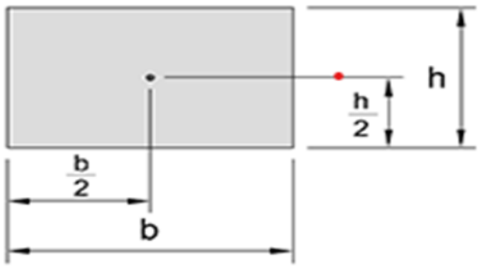
Centroid of triangle is a point where medians of geometric figures intersect each other. In case of right triangle this point is located at 2b/3 horizontally from reference y-axis or from extreme left vertical line. And h/3 vertically from reference x-axis or from extreme bottom horizontal line.



Centroid of Triangle

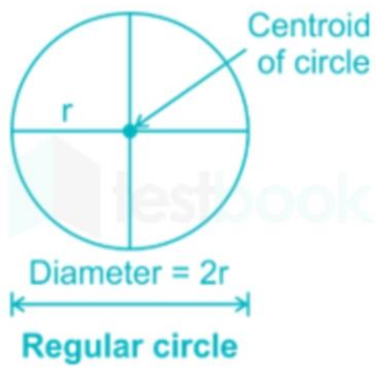
Centroid of a Rectangle:

Centroid of rectangle lies at intersection of two diagonals. Diagonals intersect at width (b/2) from reference x-axis and at height (h/2) from reference y-axis. As shown below.

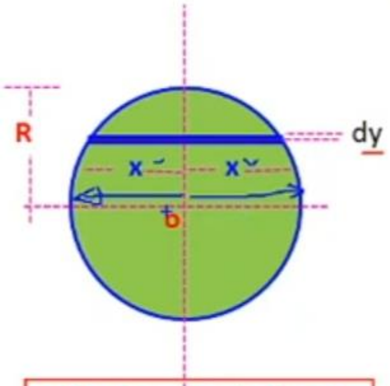


Centroid of Circle:

Centroid of a circle is very easy to determine. Centroid of circle lies at the centre of a circle that is also called as the radius of circle from edges of a circle. As shown below.



Centroid of a Circle



reference axis is at center of circle i.e. O

Solution:
Let us assume a strip of height 'dy' having an elemental area '2x dy' is exists in the Circular object.

One of the properties of a circle is $R^2 = Y^2 + X^2$

$$\therefore X = \sqrt{R^2 - Y^2} \qquad \text{---(1)}$$

dA= 2xdy

We know that first moment of area

$Q_x = y\int dA = \int y dA$

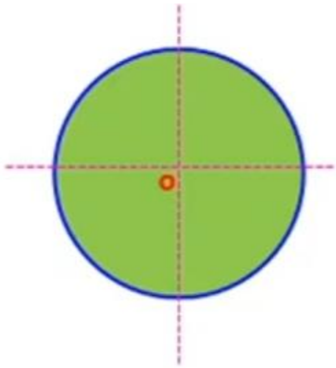
$Q_x = \int y\, 2xdy$

The limits are considered from center of circle.

from (1) $\boldsymbol{Qx} = \int_{-R}^{+R} 2y\sqrt{R^2 - Y^2}\ dy$

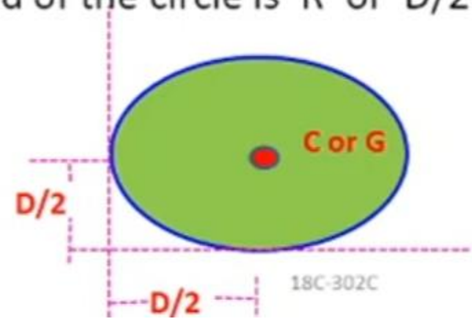
$\therefore Q_x = 0$ i.e. $\boxed{\bar{Y} = 0}$

Similarly $Q_y = 0$ i.e. $\boxed{\bar{X} = 0}$



That implies, the Centroid of the circle is its 'Center' only.

In other words if we considered the axes at the bottom and left of the circle i.e. entire object in the First Quadrant then the Centroid of the circle is 'R' or 'D/2' about the both the axes.



To solve the integral let us assume

$$R^2 - Y^2 = t^2$$

Differentiate on both sides:
('R' is constant)
-2ydy=2tdt ------(2)

Substitute the above values in the Integral

$$\boldsymbol{Qx} = \int_{-R}^{+R} 2y\sqrt{R^2 - Y^2}\ dy$$

$$= \int -2tdt * t \quad = -2\int t^2 dt$$

$$= -2\left[\frac{t^3}{3}\right] \quad = \frac{-2}{3}\left[(R^2 - Y^2)^{\frac{3}{2}}\right]_{-R}^{+R} \quad = 0$$

4.6.1 Rectangular Area

Consider a rectangle of size $b \times d$ as shown in the Fig. 4.2. Though we know directly that area = b.d, the area also can be obtained by considering a vertical strip of thickness dx.

$\therefore$ Area of strip $dA = (dx)\, d$

$\therefore$ For complete rectangle, $A = \int_0^b d(dx)$

$= d[x]_0^b$ ... d is a constant

$= b.d$

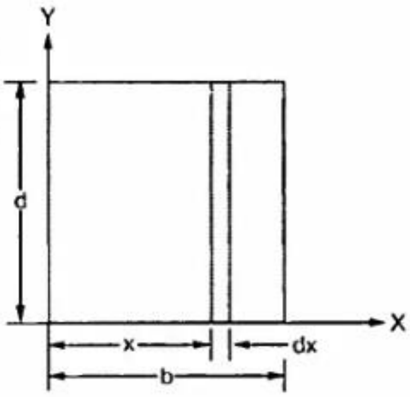


Fig. 4.2

Now first moment of dA about Y axis = x.dA

$= d\,(x.dx)$

Apply V.T.M. about Y axis.

$$\therefore \quad A\,x = d\int_0^b x\,dx$$

$$\therefore \quad b \cdot d\,\bar{x} = d\left(\frac{b^2}{2}\right)$$

$$\therefore \quad \boxed{\bar{x} = \frac{b}{2}}$$

Now refer Fig. 4.3 showing horizontal strip of thickness dy so that area of strip = $b \cdot dy$

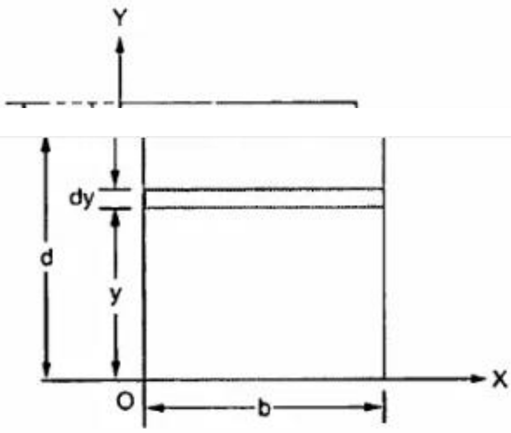


Fig. 4.3

$$\therefore \quad A\,\bar{y} = b\int_0^d y \cdot dy$$

(by applying V.T.M. about X axis.)

$$\therefore \quad b \cdot d \cdot \bar{y} = b\left(\frac{d^2}{2}\right)$$

$$\therefore \quad \boxed{\bar{y} = \frac{d}{2}}$$

Note : Same vertical strip (used for $\bar{x}$) can also be used for $\bar{y}$.

4.6.2 Right Angled Triangular Area

For the triangle of base 'b' and height 'h' as shown in Fig. 4.4, consider vertical strip of thickness dx and height 'h_1' at distance x from origin O.

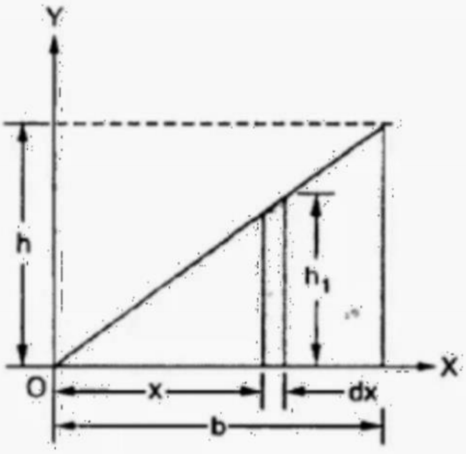


Fig. 4.4

$$\therefore \quad dA = h_1 \cdot dx$$

$$\therefore \quad A = \int_0^b h_1 \cdot dx$$

From similar triangles, we have $\frac{h_1}{x} = \frac{h}{b}$.

Hence $dA = \frac{hx}{b} dx$

$$\therefore \quad A = \int_0^b \left(\frac{h}{b}\right) x \, dx$$

$$= \frac{h}{b}\left(\frac{b^2}{2}\right)$$

$$\therefore \quad A = \frac{1}{2} b h$$

Now consider first moment of the strip about Y axis and apply V.T.M.

$$\therefore \quad A\bar{x} = \int_0^b x \, dA$$

$$\therefore \quad \frac{1}{2} bh\bar{x} = \int_0^b \left(\frac{h}{b}\right) x^2 \, dx$$

$$\therefore \quad \frac{1}{2} bh\bar{x} = \frac{h}{b}\left(\frac{b^3}{3}\right)$$

$$\therefore \quad \boxed{\bar{x} = \frac{2}{3} b}$$

Now consider first moment of the same strip about X axis. (Alternatively, a new horizontal strip also can be considered). Apply V.T.M. also.

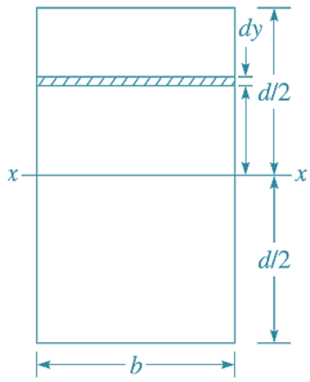
$$\therefore \quad \frac{1}{2} b h \bar{y} = \int_0^b \left(\frac{h_1}{2}\right) dA$$

$$= \int_0^b \left(\frac{hx}{2b}\right)\left(\frac{hx}{b}\right) dx$$

MOMENT OF INERTIA OF RECTANGLE

***Moment of Inertia of a Rectangle about the Centroidal Axis*:**

- Consider a rectangle of width $\boldsymbol{b}$ and depth $\boldsymbol{d}$.
- Moment of inertia about the centroial axis $\boldsymbol{x}$–$\boldsymbol{x}$ parallel to the short side is required.
- Consider an elemental strip of width $\boldsymbol{dy}$ at a distance $\boldsymbol{y}$ from the axis.
- Moment of inertia of the elemental strip about the centroidal axis xx is: $= y^2 dA$



$$= y^2 b\,dy$$

$$I_{xx} = \int_{-d/2}^{d/2} y^2 b\,dy$$

$$= b\left[\frac{y^3}{3}\right]_{-d/2}^{d/2}$$

$$= b\left[\frac{d^3}{24} + \frac{d^3}{24}\right]$$

$$\boxed{I_{xx} = \frac{bd^3}{12}}$$

MOMENT OF INERTIA OF A TRIANGLE

Moment of Inertia of a Triangle About its Base:

- Moment of inertia of a triangle with base width $\boldsymbol{b}$ and height $\boldsymbol{h}$ is to be determined about the base **AB**.
- Consider an elemental strip at a distance $\boldsymbol{y}$ from the base $\boldsymbol{AB}$.
- Let $\boldsymbol{dy}$ be the thickness of the strip and $\boldsymbol{dA}$ its area.
- Width of this strip is given by: $b_1 = \frac{(h-y)}{h} \times b$

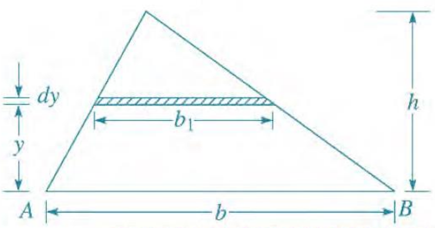
(By property of similar triangle)

Moment of inertia of this strip about AB

$$= y^2 dA$$

$$= y^2\, b_1\, dy$$

$$= y^2 \frac{(h-y)}{h} \times b \times dy$$



$\therefore$ Moment of inertia of the triangle about AB,

$$I_{AB} = \int_0^h \frac{y^2 (h-y) b\,dy}{h}$$

$$= \int_0^h \left(y^2 - \frac{y^3}{h}\right) b\,dy$$

$$= b\left[\frac{y^3}{3} - \frac{y^4}{4h}\right]_0^h$$

$$= b\left[\frac{h^3}{3} - \frac{h^4}{4h}\right]$$

$$\boxed{I_{AB} = \frac{bh^3}{12}}$$

MOMENT OF INERTIA OF A CIRCLE

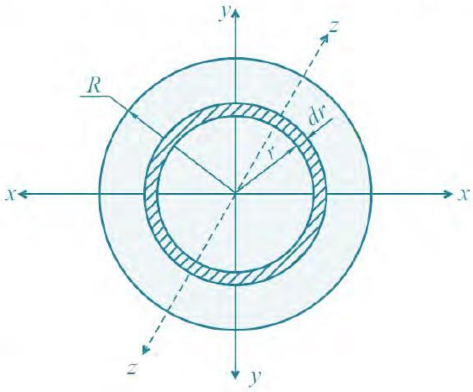
Moment of Inertia of a Circle About Diametrical Axis:

- Consider an elemental thin circular ring of width ***dr*** and radius ***r***, as shown in Fig.
- So, area of ring ***dA = 2πr dr.***
- By polar moment of inertia, we have:

$$I_{zz} = \int r^2\, dA$$

$$= \int_0^R r^2\, 2\pi r\, dr = 2\pi \int_0^R r^3\, dr = 2\pi \left[\frac{r^4}{4}\right]_0^R$$

$$\boxed{I_{zz} = \frac{\pi R^4}{2}}$$



Since circle is symmetric about x-x and y-y axis, we have

$$I_{xx} = I_{yy}$$

By perpendicular axis theorem, we have

$$I_{zz} = I_{xx} + I_{yy}$$

$$I_{zz} = 2I_{xx} = 2I_{yy} \quad \therefore \quad I_{xx} = I_{yy} = \frac{I_{zz}}{2}$$

$$\therefore \boxed{I_{xx} = \frac{\pi R^4}{4}} \text{ and } \boxed{I_{yy} = \frac{\pi R^4}{4}}$$

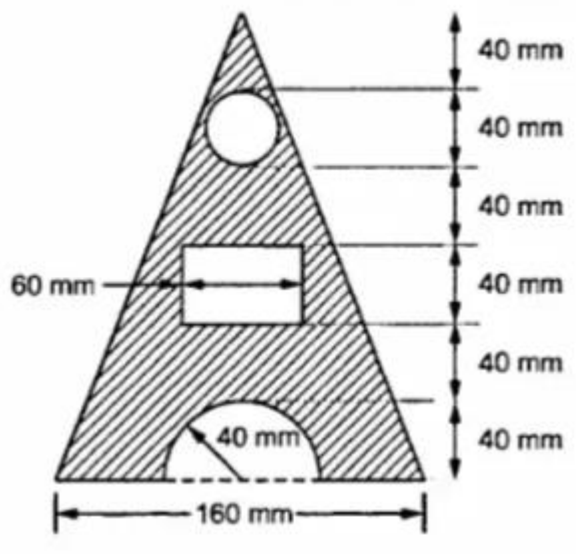
COMPOSITE FIGURES:

For any non-standard or composite figure, either mathematical method (integration) can be used or given shape can be divided into basic/standard shapes or formulas given in the tables can be applied. (This is incidently, the 'principle of superposition')

Formulas,

$$\bar{x} = \frac{\sum (a \cdot x)}{\sum (a)} \quad \text{and} \quad \bar{y} = \frac{\sum (a \cdot y)}{\sum (a)}$$

Example: Determine centroid of the shaded area with reference apex.



For the given fig, there is a vertical axis of symmetry. Hence we have to find $\bar{y}$ only. Given shaded area=Triangle- Circle- Rectangle- Semicircle. Assuming center of semicircle as origin, measure the centroidal y distances as shown in the table below:

Component area a (mm^2)	Vertical (y) centroidal distances (mm)	Product a.y (mm^3)
$\frac{1}{2}(160)\ 240 = 19200$	$\frac{1}{3}(240) = 80$	+1536000
$-\frac{\pi}{4}(40)^2 = -1256.64$	$160+\frac{40}{2} = 180$	− 226195.20
$-(40)(60) = -2400$	$80+\frac{40}{2} = 100$	− 240000
$-\frac{\pi}{2}(40)^2 = -2513\ 27$	$\frac{4(40)}{3\pi} = 16\ 98$	− 42675.325
$\sum A = 13030.09$	–	1027129.5

Apply $(\sum A)\ \bar{y} = \sum (a\ .y)$

$$\boxed{\bar{y} = 78.83mm}$$

SOURCE CODE:

```
#include<stdio.h>

#include<math.h> int

main()

{

   float b,h, d, x,y,i;

int ch;     do{

   printf ("\n CHOOSE A SHAPE FROM MENU");

printf("\n 1.Rectangle");     printf("\n 2.Circle");

printf("\n 3.Right Triangle");     printf("\n 4.Exit

Menu");

   printf("\n SHAPE SELECTED FROM MENU IS:  ");

scanf("%d", &ch);

   printf("-----------------------------------------------------------------");

switch(ch)

   {

case 1:

          printf("\n Enter your  depth d(mm):");

scanf("%f", &d);
```

```
        printf("\n Enter your breath b(mm):");
scanf("%f", &b);        x=b/2;
        y=d/2;
        printf("\n Centroid of rectangle (x,y) =%f,%f\n", x,y);
i=(b/12)*pow(d,3);

        printf(" Moment of inertia of rectangle is,I=%f(m^4)\n",i);
        printf("--------------------------------------------------------------");
break;      case 2:

        printf("\n Enter the diameter of circle:");
scanf("%f", &d);        x=d/2;        y=d/2;

        i=(3.14/4)*pow(x,4);
        printf("\n Centroid of circle (x,y) =%f, %f\n", x,y);
printf("\n Moment of inertia of circle,I=%f(m^4)\n",i);

        printf("--------------------------------------------------------------");
break;      case 3:

        printf("\n Enter the base of right triangle:");
scanf("%f", &b);

        printf("\n Enter the height of right triangle:");
scanf("%f", &h);        x=b/3;        y=h/3;
```

```
        i=(b/12)*pow(h,3);
        printf("\n Centroid of right triangle (x,y)= %f,%f\n", x,y);
printf("\n Moment of inertia of triangle,I=%f(m^4)\n",i);

        printf("------------------------------------------------------------------");
break;


     default:

        printf(" Invalid Choice.Exit Menu\n");
ch=4;          break;

   }
   if (ch==4)break;




   }while(ch>0 && ch<4);
return 0;

}
```

Link for Code:

https://github.com/DURGSINGH15/CIVIL.git

OUTPUT:

```
CHOOSE A SHAPE FROM MENU
1.Rectangle
2.Circle
3.Right Triangle
4.Exit Menu
SHAPE SELECTED FROM MENU IS:  1
------------------------------------------------------------------
Enter your  depth d(mm):21

Enter your breath b(mm):42

Centroid of rectangle (x,y) =21.000000,10.500000
Moment of inertia of rectangle is,I=32413.500000(m^4)
------------------------------------------------------------------
CHOOSE A SHAPE FROM MENU
1.Rectangle
2.Circle
3.Right Triangle
4.Exit Menu
SHAPE SELECTED FROM MENU IS:  2
------------------------------------------------------------------
Enter the diameter of circle:43

Centroid of circle (x,y) =21.500000, 21.500000

Moment of inertia of circle,I=167734.921875(m^4)
------------------------------------------------------------------
CHOOSE A SHAPE FROM MENU
1.Rectangle
2.Circle
3.Right Triangle
4.Exit Menu
SHAPE SELECTED FROM MENU IS:  3
------------------------------------------------------------------
Enter the base of right triangle:55

Enter the height of right triangle:99

Centroid of right triangle (x,y)= 18.333334,33.000000

Moment of inertia of triangle,I=4447204.000000(m^4)
------------------------------------------------------------------
CHOOSE A SHAPE FROM MENU
1.Rectangle
2.Circle
3.Right Triangle
4.Exit Menu
SHAPE SELECTED FROM MENU IS:  4
------------------------------------------------------------------
Invalid Choice.Exit Menu

[Program finished]
```