



CMSC 5743

Efficient Computing of Deep Neural Networks

Implementation 04: Sparse Conv

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2024 Fall



- ① Kernel Sparse Convolution
- ② Submanifold Sparse Convolution
- ③ Sparse Hardware Architecture



① Kernel Sparse Convolution

② Submanifold Sparse Convolution

③ Sparse Hardware Architecture

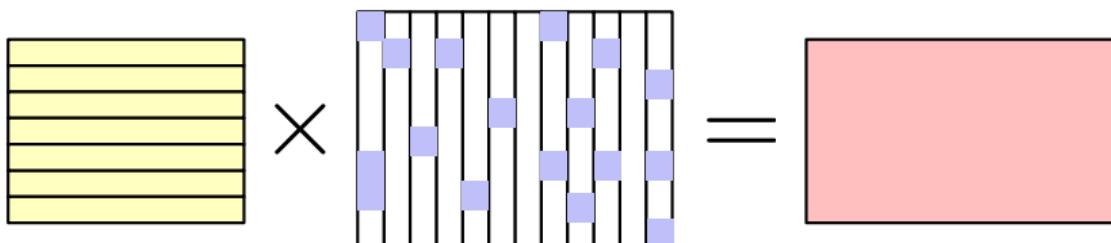


Kernel Sparse Convolution

Sparse Convolution



- Our DNN may be **redundant**, and sometimes the filters may be **sparse**
- Sparsity can be helpful to **overcome over-fitting**



Sparse Convolution: Naive Implementation 1



$$\begin{matrix} X & \quad \\ \begin{array}{|c|c|c|c|}\hline 0 & 0 & 3 & 0 \\ \hline 7 & 0 & 0 & 0 \\ \hline 0 & 0 & 4 & 8 \\ \hline 6 & 5 & 3 & 0 \\ \hline 2 & 0 & 0 & 1 \\ \hline 0 & 0 & 0 & 8 \\ \hline \end{array} & \begin{array}{l} * \\ \begin{array}{|c|}\hline w \\ \hline 0 \\ 0 \\ 4 \\ 8 \\ \hline \end{array} \end{array} \end{matrix}$$

Algorithm Sparse Convolution Naive 1

```
1: for all  $w[i]$  do
2:   if  $w[i] = 0$  then
3:     Continue;
4:   end if
5:   output feature map  $Y \leftarrow X \times w[i];$ 
6: end for
```



$$\begin{array}{c} X \\ \hline
 \begin{array}{|c|c|c|c|} \hline 0 & 0 & 3 & 0 \\ \hline 7 & 0 & 0 & 0 \\ \hline 0 & 0 & 4 & 8 \\ \hline 6 & 5 & 3 & 0 \\ \hline 2 & 0 & 0 & 1 \\ \hline 0 & 0 & 0 & 8 \\ \hline \end{array}
 \end{array}
 \begin{array}{c} * \\ \hline
 \begin{array}{|c|} \hline w \\ \hline \begin{array}{|c|} \hline 0 \\ \hline 0 \\ \hline 4 \\ \hline 8 \\ \hline \end{array} \end{array}
 \end{array}$$

Algorithm Sparse Convolution Naive 1

```

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```

BAD implementation for Pipeline!

Instr. No.	Pipeline Stage							
	IF	ID	EX	MEM	WB			
1								
2			EX	MEM	WB			
3			IF	ID	EX	MEM	WB	
4				IF	ID	EX	MEM	
5					IF	ID	EX	
Clock Cycle	1	2	3	4	5	6	7	

Sparse Matrix Representation



A

0	0	3	0
7	0	0	0
0	0	4	8
6	5	3	0
2	0	0	1
0	0	0	8

A matrix example

rowptr

- row0 (3,2)
- row1 (7,0)
- row2 (4,2), (8,3)
- row3 (6,0), (5,1), (3,2)
- row4 (2,0), (1,3)
- row5 (8,3)

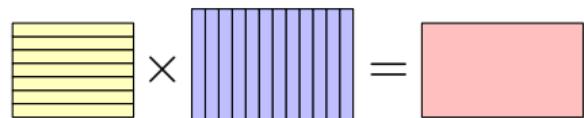
Compressed
Sparse Row
(CSR)

colptr

- col0 (7,1), (6,3), (2,4)
- col1 (5,3)
- col2 (3,0), (4,2), (3,3)
- col3 (8,2), (1,4), (8,5)

Compressed
Sparse Column
(CSC)

- CSR: Good for operation on **feature maps**
- CSC: Good for operation on **filters**
- We have **better control on filters**, thus usually CSC.



Sparse Convolution: Naive Implementation 2



matrix * sparse vector

$$\begin{array}{c} \text{X} \\ \boxed{0 \ 0 \ 3 \ 0} \\ \boxed{7 \ 0 \ 0 \ 0} \\ \boxed{0 \ 0 \ 4 \ 8} \\ \boxed{6 \ 5 \ 3 \ 0} \\ \boxed{2 \ 0 \ 0 \ 1} \\ \boxed{0 \ 0 \ 0 \ 8} \end{array} * \begin{array}{c} \text{W} \\ \boxed{0} \\ \boxed{0} \\ \boxed{4} \\ \boxed{8} \end{array} = \begin{array}{c} \text{Y} \\ 12 \\ 0 \\ 16 \\ 12 \\ 0 \\ 0 \end{array}$$

- **BAD** implementation for Spatial Locality!
- **Poor** memory access patterns

$$\begin{array}{c} 0 \ 0 \ 3 \ 0 \\ 7 \ 0 \ 0 \ 0 \\ 0 \ 0 \ 4 \ 8 \\ 6 \ 5 \ 3 \ 0 \\ 2 \ 0 \ 0 \ 1 \\ 0 \ 0 \ 0 \ 8 \end{array} * \begin{array}{c} 0 \\ 0 \\ 4 \\ 8 \end{array} = \begin{array}{c} 12 \\ 0 \\ 80 \\ 12 \\ 8 \\ 64 \end{array}$$

SOTA 2: Sparse Convolution

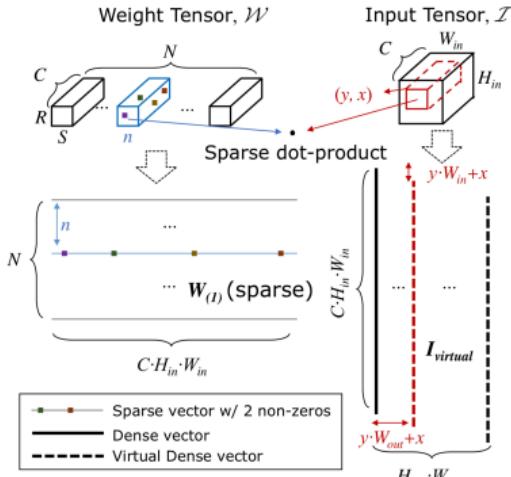


Figure 1: Conceptual view of the direct sparse convolution algorithm. Computation of output value at (y, x) th position of n th output channel is highlighted.

```

for each output channel n {
    for j in [W.rowptr[n], W.rowptr[n+1]) {
        off = W.colidx[j]; coeff = W.value[j]
        for (int y = 0; y < H_OUT; ++y) {
            for (int x = 0; x < W_OUT; ++x) {
                out[n][y][x] += coeff*in[off+f(0,y,x)]
            }
        }
    }
}
}

```

Figure 2: Sparse convolution pseudo code. Matrix \mathbf{W} has *compressed sparse row* (CSR) format, where $\text{rowptr}[n]$ points to the first non-zero weight of n th output channel. For the j th non-zero weight at (n, c, r, s) , $\text{W.colidx}[j]$ contains the offset to (c, r, s) th element of tensor in , which is pre-computed by layout function as $f(c, r, s)$. If in has CHW format, $f(c, r, s) = (cH_{in} + r)W_{in} + s$. The “virtual” dense matrix is formed on-the-fly by shifting in by $(0, y, x)$.

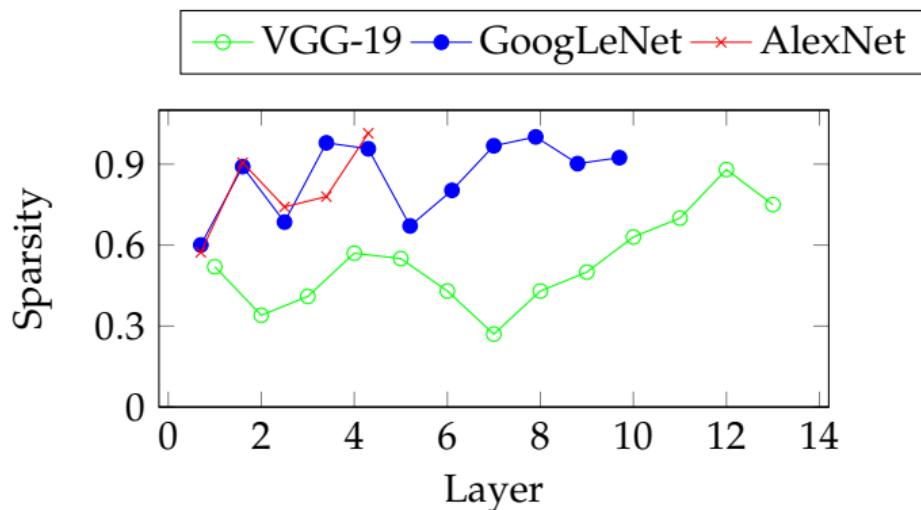
1

¹Jongsoo Park et al. (2017). “Faster CNNs with direct sparse convolutions and guided pruning”. In: *Proc. ICLR*.

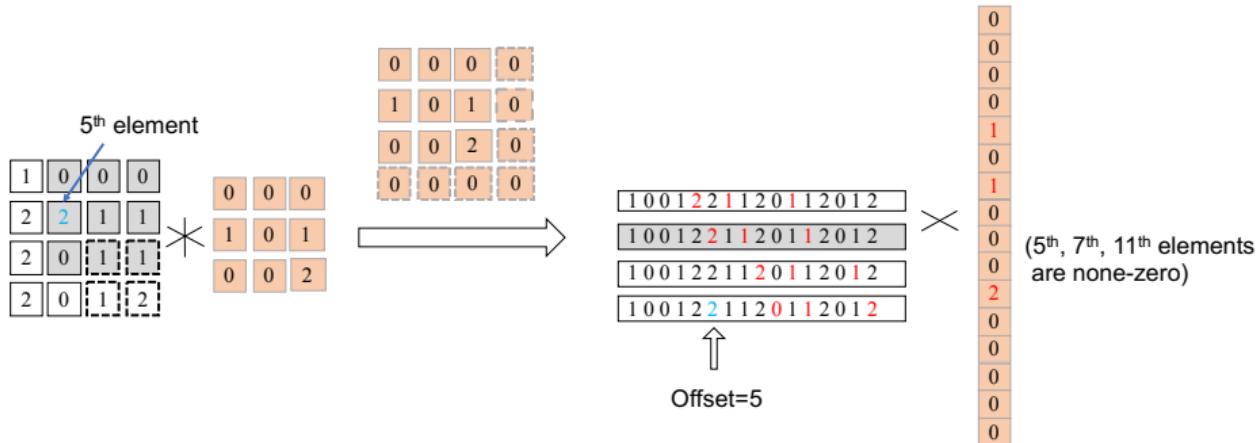
Discussion: Sparse-Sparse Convolution



- Sparsity is a desired property for computation acceleration. (cuSPARSE library, direct sparse convolution, etc.)
- Sometimes not only the filters but also the **input feature maps** are sparse.



Discussion: Sparse-Sparse Convolution



- Efficient programming implementation required; (**Improve pipeline efficiency**)
 - When sparsity(*input*) = 0.9, sparsity(*weight*) = 0.8, more than **10×** speedup;
 - Some other issues:
 - How to be compatible with pooling layer?
 - Transform between dense & sparse formats



① Kernel Sparse Convolution

② Submanifold Sparse Convolution

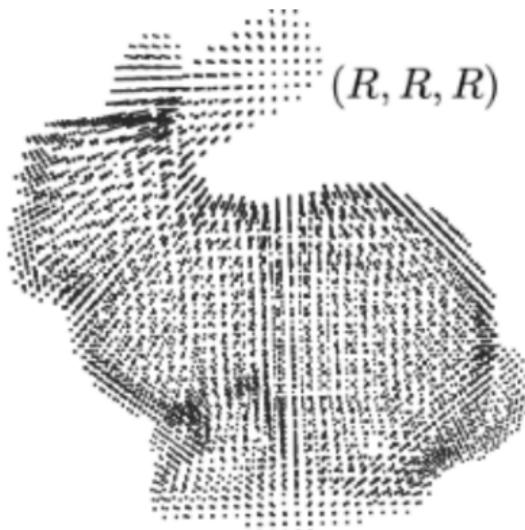
③ Sparse Hardware Architecture



Submanifold Sparse Convolution

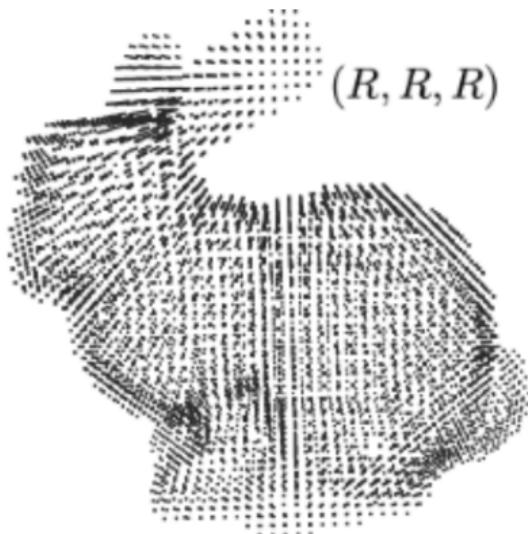


In real world, we have to handle voxel data sometimes. For example, in point cloud analysis, 3D voxel data is widely used. A simple example is shown here and it can be viewed as $V \in (1, R, R, R)$.





Here is a rabbit with shape $V \in (1, 64, 64, 64)$. If using traditional convolution to extract its feature, the GPU will run out of memory very soon because the input $V \in (1, 64, 64, 64)$ can be viewed as an image $I \in (1, 4096, 64)$.



Submanifold Sparse Convolution



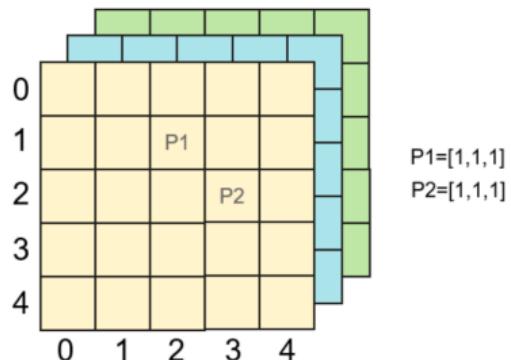
To overcome this issue, we use 3D sparse convolution for voxel data analysis. Sparse convolution only calculate the data points where voxel data exists.



Submanifold Sparse Convolution



In this Lab, we are going to build a sparse convolution from scratch. Here we use the example input:



where P1 and P2 has pixel value of 1 in 3 channels.



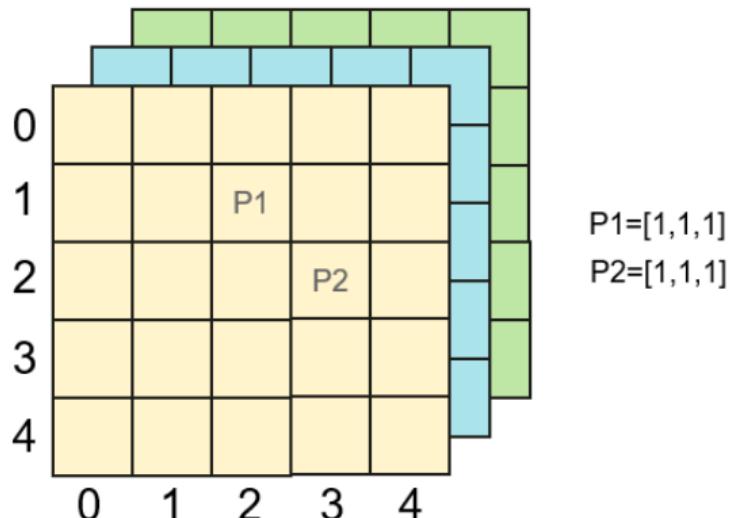
Firstly, we build a hash table to store the input data. Considering the following case:

```
conv2D(kernel_size=3, out_channels=2, stride=1, padding=0)
```

Submanifold Sparse Convolution



We can build an input table H_{in} like this:



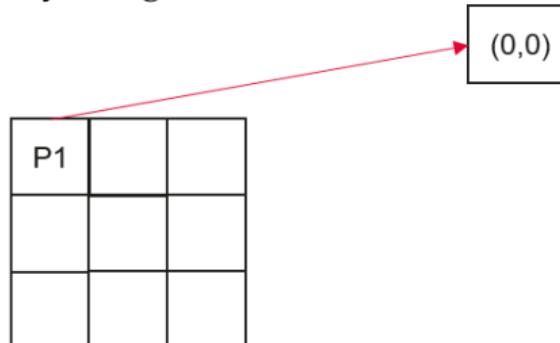
H_{in}	
0	(2,1)
1	(3,2)

Submanifold Sparse Convolution



Then we build an output hash table. Firstly, we generate a P_{out} table as follow:

		P1		



Submanifold Sparse Convolution



Then we build an output hash table. Firstly, we generate a P_{out} table as follow:

		P1		

P1	P1	

(0,0)
(1,0)

Submanifold Sparse Convolution



Then we build an output hash table. Firstly, we generate a P_{out} table as follow:

		P1		

P1	P1	P1

(0,0)
(1,0)
(2,0)

Submanifold Sparse Convolution



Then we build an output hash table. Firstly, we generate a P_{out} table as follow:

		P1		

P1	P1	P1
P1		

(0,0)
(1,0)
(2,0)
(0,1)

Submanifold Sparse Convolution



Then we build an output hash table. Firstly, we generate a P_{out} table as follow:

		P1		

P1	P1	P1
P1	P1	

(0,0)
(1,0)
(2,0)
(0,1)
(1,1)

Submanifold Sparse Convolution



Then we build an output hash table. Firstly, we generate a P_{out} table as follow:

		P1		

P1	P1	P1
P1	P1	P1

(0,0)
(1,0)
(2,0)
(0,1)
(1,1)
(2,1)

Submanifold Sparse Convolution



After applying the same process to P_2 , we get an output hash table H_{out} via P_{out} merging

	P1		
			P2

P1	P1	P1
P1	P1	P1

	P2	P2
	P2	P2
	P2	P2

P_{out}

(0,0)
(1,0)
(2,0)
(0,1)
(1,1)
(2,1)

H_{out}

0	(0,0)
1	(1,0)
2	(2,0)
3	(0,1)
4	(1,1)
5	(2,1)
6	(1,2)
7	(2,2)

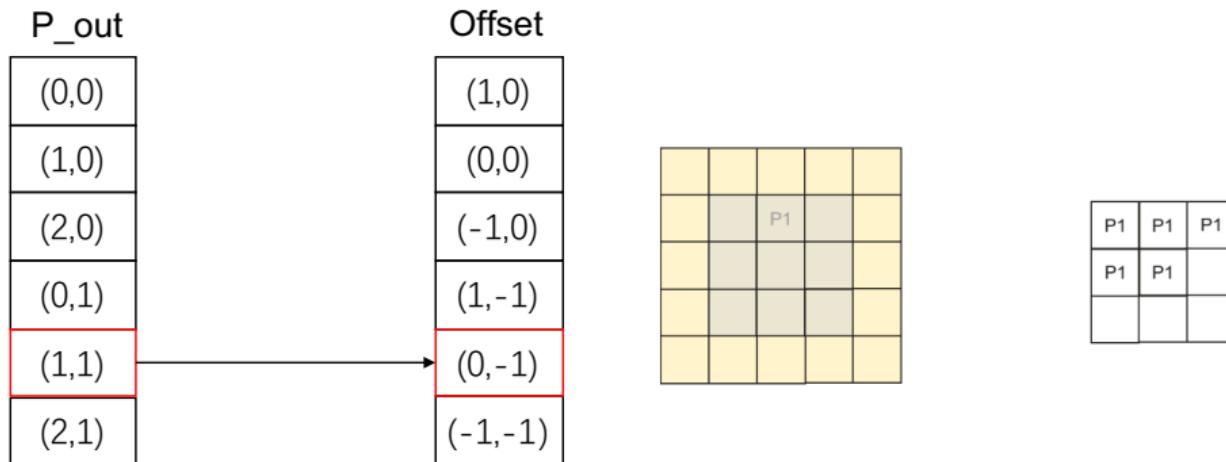
Merge P_{out}

(1,0)
(2,0)
(1,1)
(2,1)
(1,2)
(2,2)

Submanifold Sparse Convolution



- Next we build up a Rulebook to realize H_{in} to H_{out} .
- To build the rule book, we have to build an offset map like this:





Quick Question:

Please write the offset map of $P2$ by yourself.

Submanifold Sparse Convolution



After obtaining the offset map, we can finally build up the rule book as follow:

P_out

(0,0)
(1,0)
(2,0)
(0,1)
(1,1)
(2,1)

Offset

(1,0)
(0,0)
(-1,0)
(1,-1)
(0,-1)
(-1,-1)

Offset count in out

(-1,-1)	0	0	5
(0,-1)	0	0	4
	1	1	7
(1,-1)	0	0	3
	1	1	6
(-1,0)	0	0	2
(0,0)	0	0	1
	1	1	5
(1,0)	0	0	0
	1	1	4
(0,1)	0	1	2
(1,1)	0	1	1

P_out

(1,0)
(2,0)
(1,1)
(2,1)
(1,2)
(2,2)

Offset

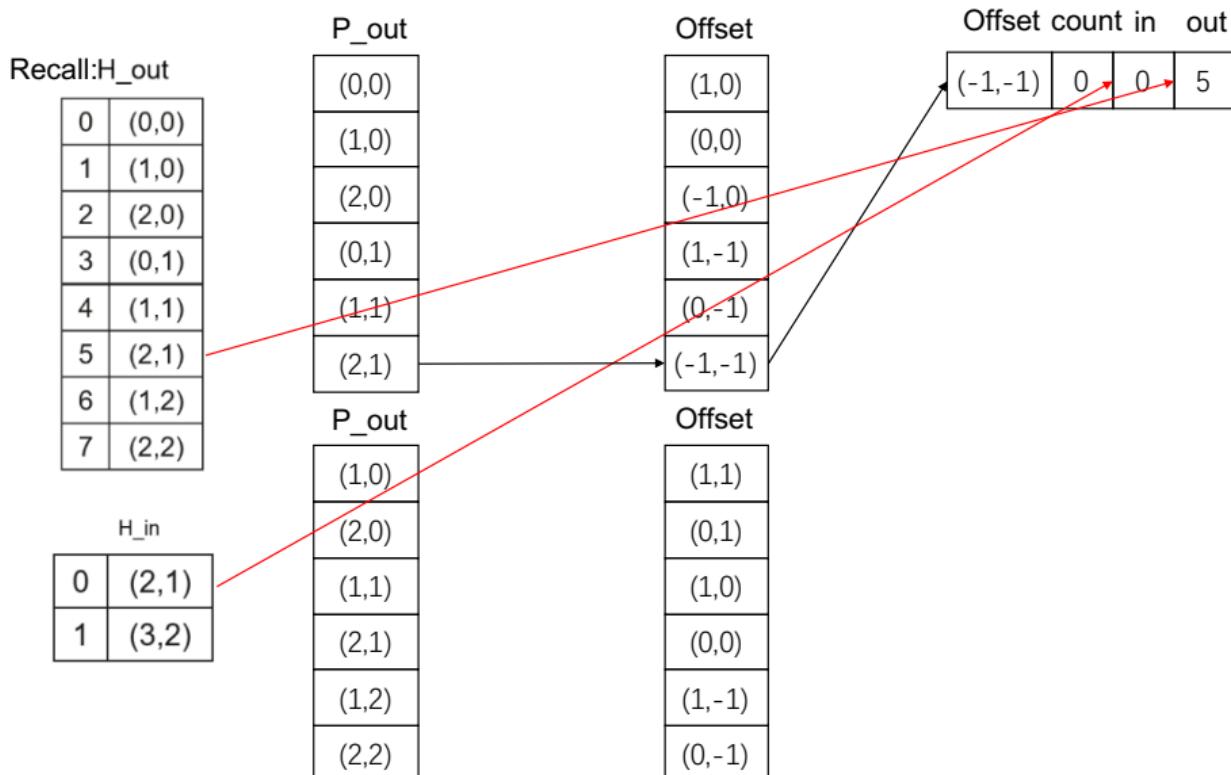
(1,1)
(0,1)
(1,0)
(0,0)
(1,-1)
(0,-1)

RuleBook

Submanifold Sparse Convolution



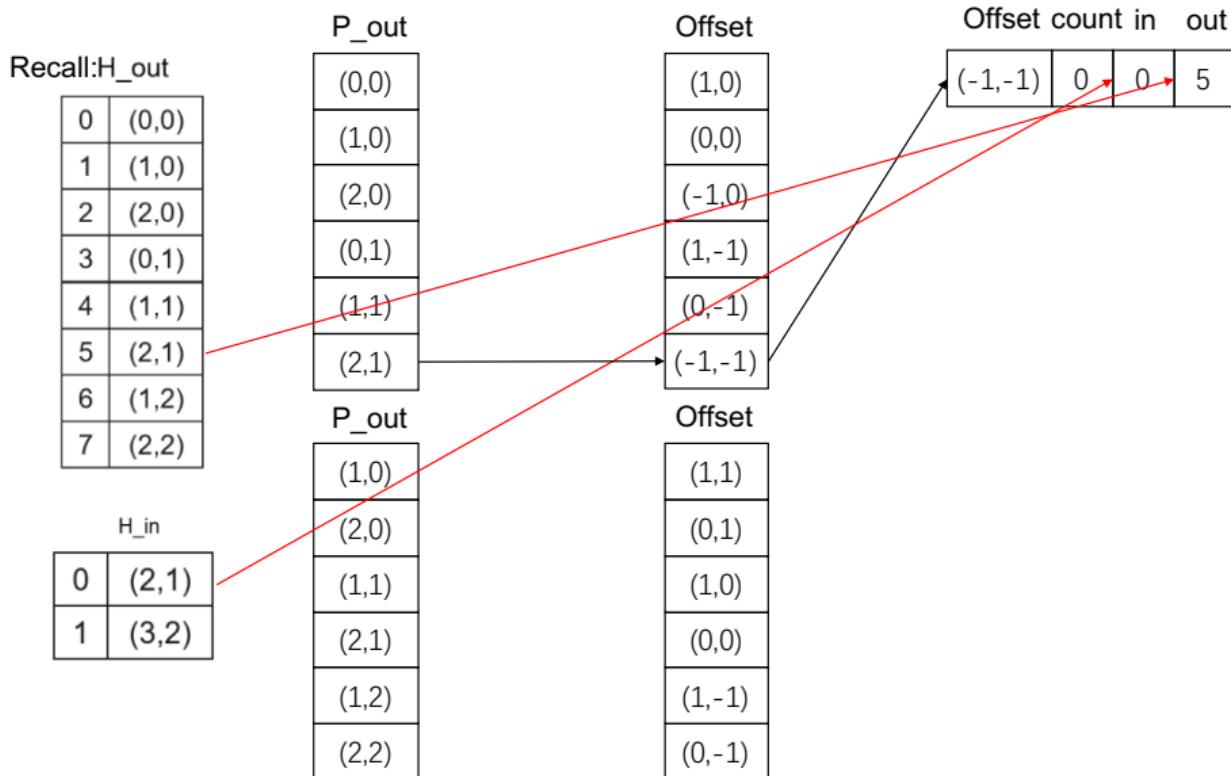
Recalling the H_{in} and H_{out} , the rulebook is generated as follow:



Submanifold Sparse Convolution



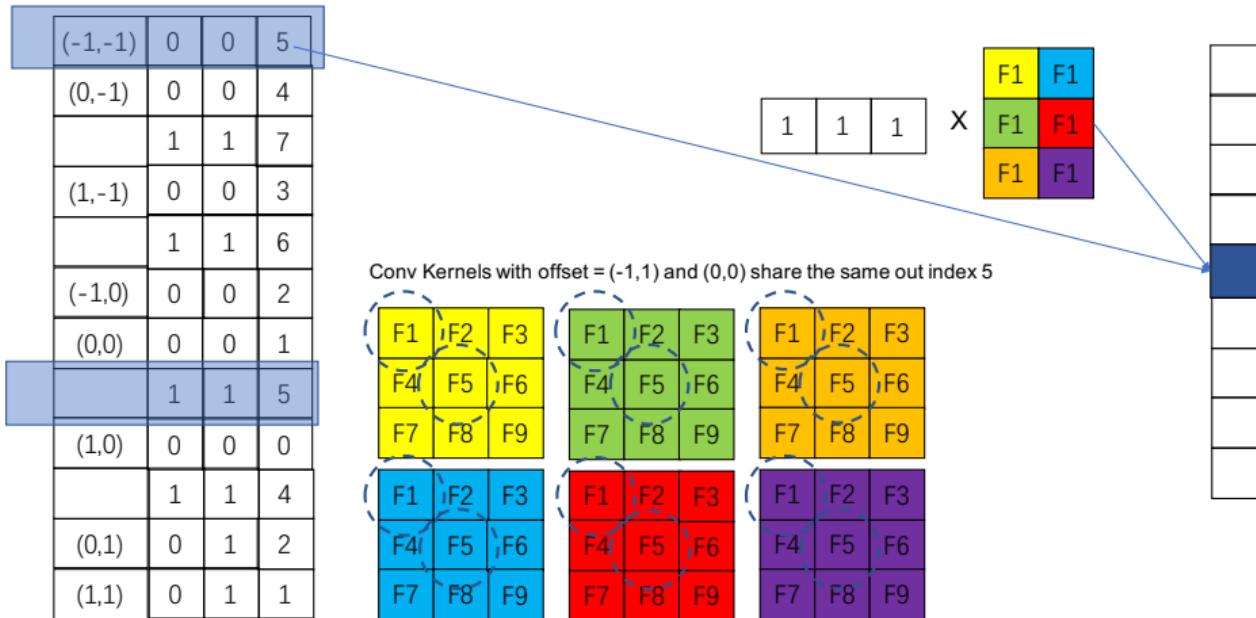
If the offset already exists, we simply add 1 in count:



Submanifold Sparse Convolution



After getting rulebook, we can apply sparse convolution:



For P_1 , the results is shown above, which is the blue points in 5-th row. Please practice P_2 by yourself



- ① Kernel Sparse Convolution
- ② Submanifold Sparse Convolution
- ③ Sparse Hardware Architecture



Sparse Hardware Architecture



EIE: Efficient Inference Engine on Compressed Deep Neural Network

Han et al.
ISCA 2016



Deep Learning Accelerators

- First Wave: Compute (Neu Flow)
- Second Wave: Memory (Diannao family)
- Third Wave: Algorithm / Hardware Co-Design (EIE)

Google TPU: “This unit is designed for dense matrices. Sparse architectural support was omitted for time-to-deploy reasons. Sparsity will have high priority in future designs”



EIE: the First DNN Accelerator for Sparse, Compressed Model

$$0 * A = 0$$

$$W * 0 = 0$$

~~2.09, 1.92=> 2~~

Sparse Weight

90% *static* sparsity

Sparse Activation

70% *dynamic* sparsity

Weight Sharing

4-bit weights



10x less computation



3x less computation



5x less memory footprint



8x less memory footprint

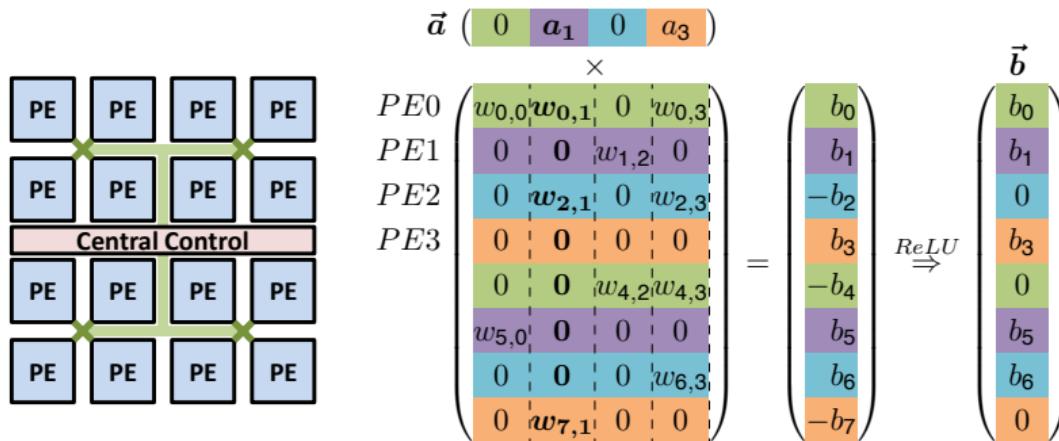


EIE: Parallelization on Sparsity

$$\begin{array}{c}
 \vec{a} \left(\begin{array}{cccc} 0 & \mathbf{a}_1 & 0 & a_3 \end{array} \right) \\
 \times \\
 \left(\begin{array}{cc|cc} w_{0,0} & \mathbf{w}_{0,1} & 0 & w_{0,3} \\ 0 & \mathbf{0} & w_{1,2} & 0 \\ 0 & \mathbf{w}_{2,1} & 0 & w_{2,3} \\ 0 & \mathbf{0} & 0 & 0 \\ 0 & \mathbf{0} & w_{4,2} & w_{4,3} \\ w_{5,0} & \mathbf{0} & 0 & 0 \\ 0 & \mathbf{0} & 0 & w_{6,3} \\ 0 & \mathbf{w}_{7,1} & 0 & 0 \end{array} \right) = \left(\begin{array}{c} b_0 \\ b_1 \\ -b_2 \\ b_3 \\ -b_4 \\ b_5 \\ b_6 \\ -b_7 \end{array} \right) \xrightarrow{\text{ReLU}} \vec{b} \left(\begin{array}{c} b_0 \\ b_1 \\ 0 \\ b_3 \\ 0 \\ b_5 \\ b_6 \\ 0 \end{array} \right)
 \end{array}$$

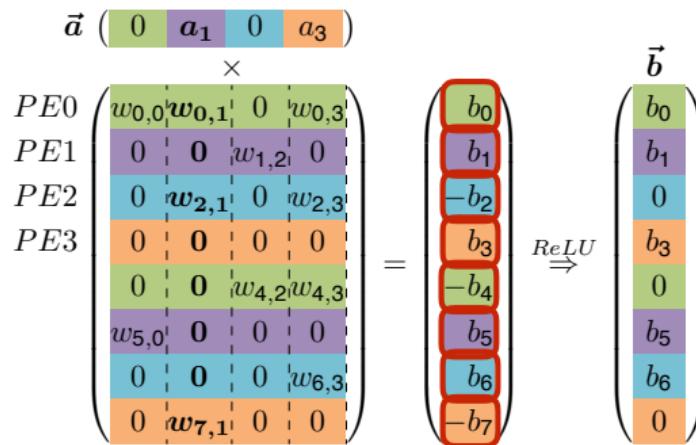


EIE: Parallelization on Sparsity





Dataflow

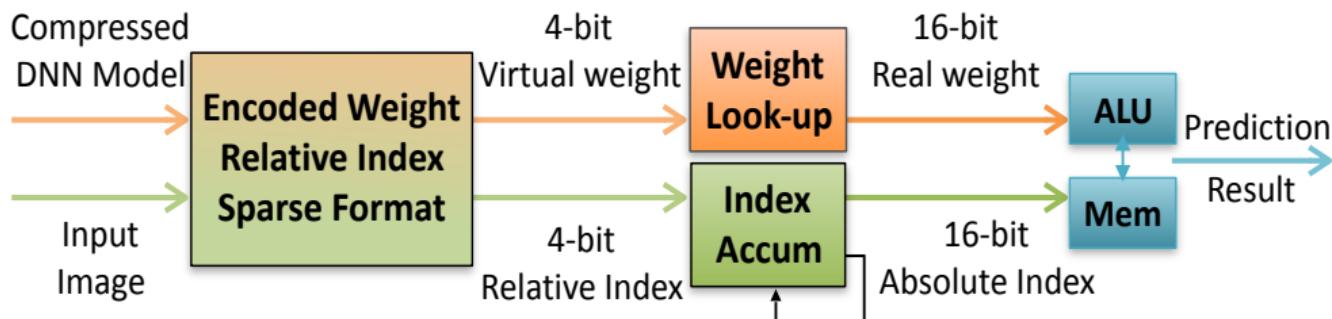


rule of thumb:
 $0 * A = 0$ $W * 0 = 0$



EIE Architecture

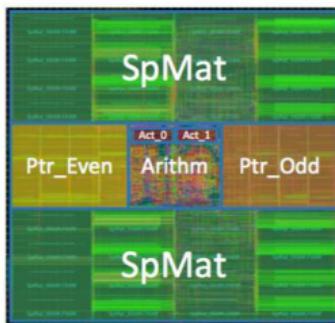
Weight decode



rule of thumb: $0 * A = 0$ $W * 0 = 0$ ~~$2.09, 1.92 \Rightarrow 2$~~



Post Layout Result of EIE

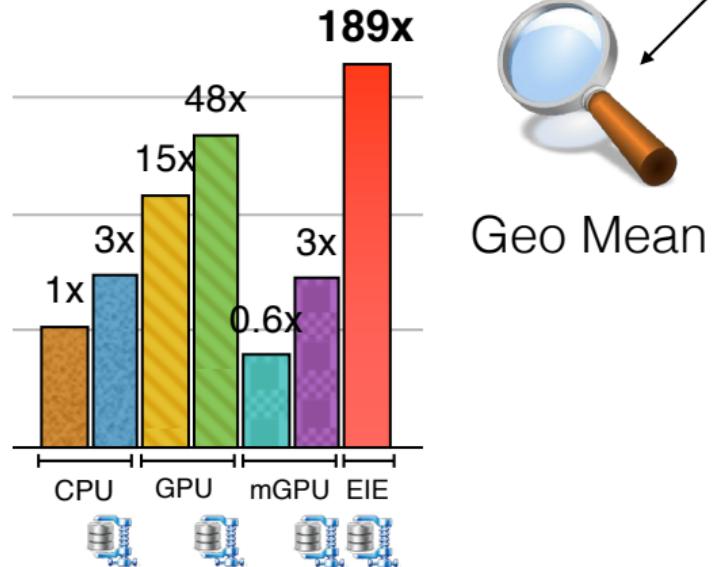
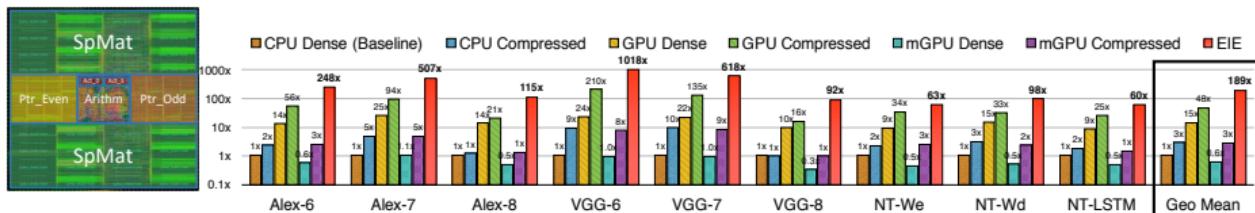


Technology	40 nm
# PEs	64
on-chip SRAM	8 MB
Max Model Size	84 Million
Static Sparsity	10x
Dynamic Sparsity	3x
Quantization	4-bit
ALU Width	16-bit
Area	40.8 mm²
MxV Throughput	81,967 layers/s
Power	586 mW

- Post layout result**
- Throughput measured on AlexNet FC-7**



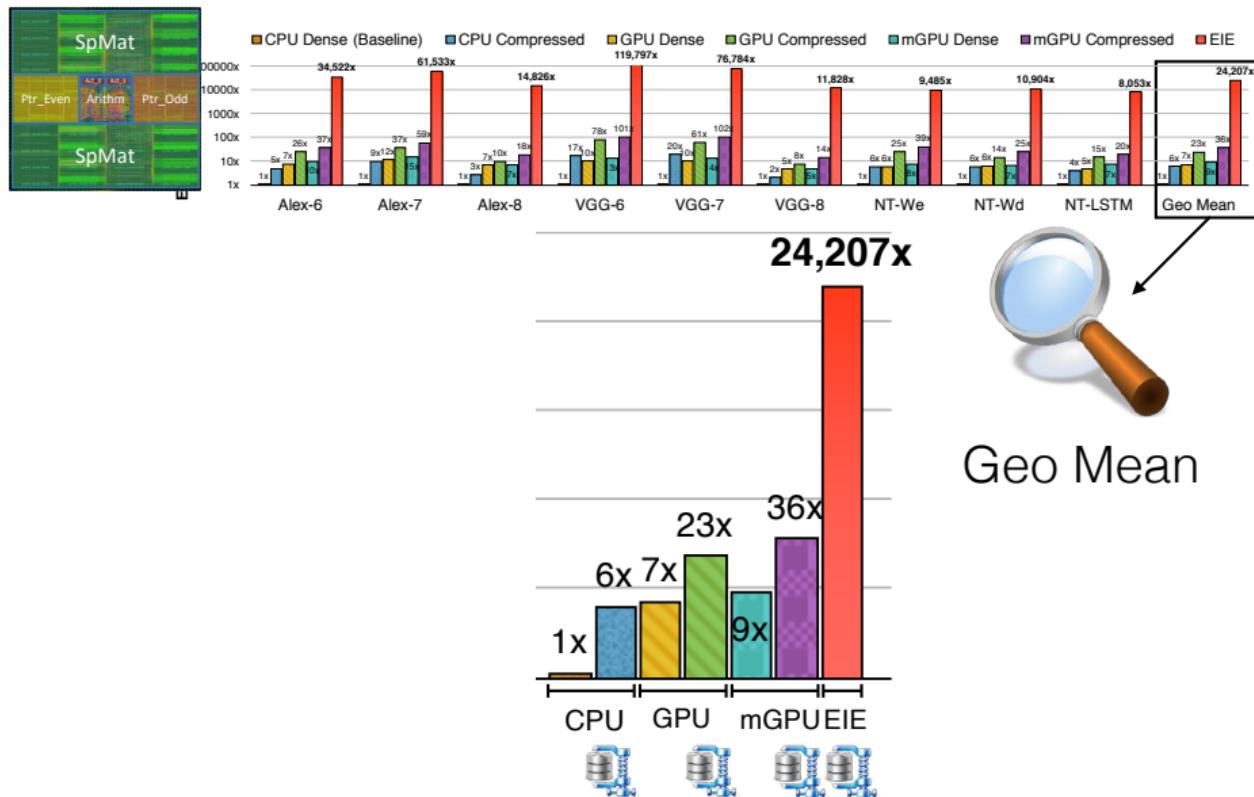
Speedup on EIE



Geo Mean

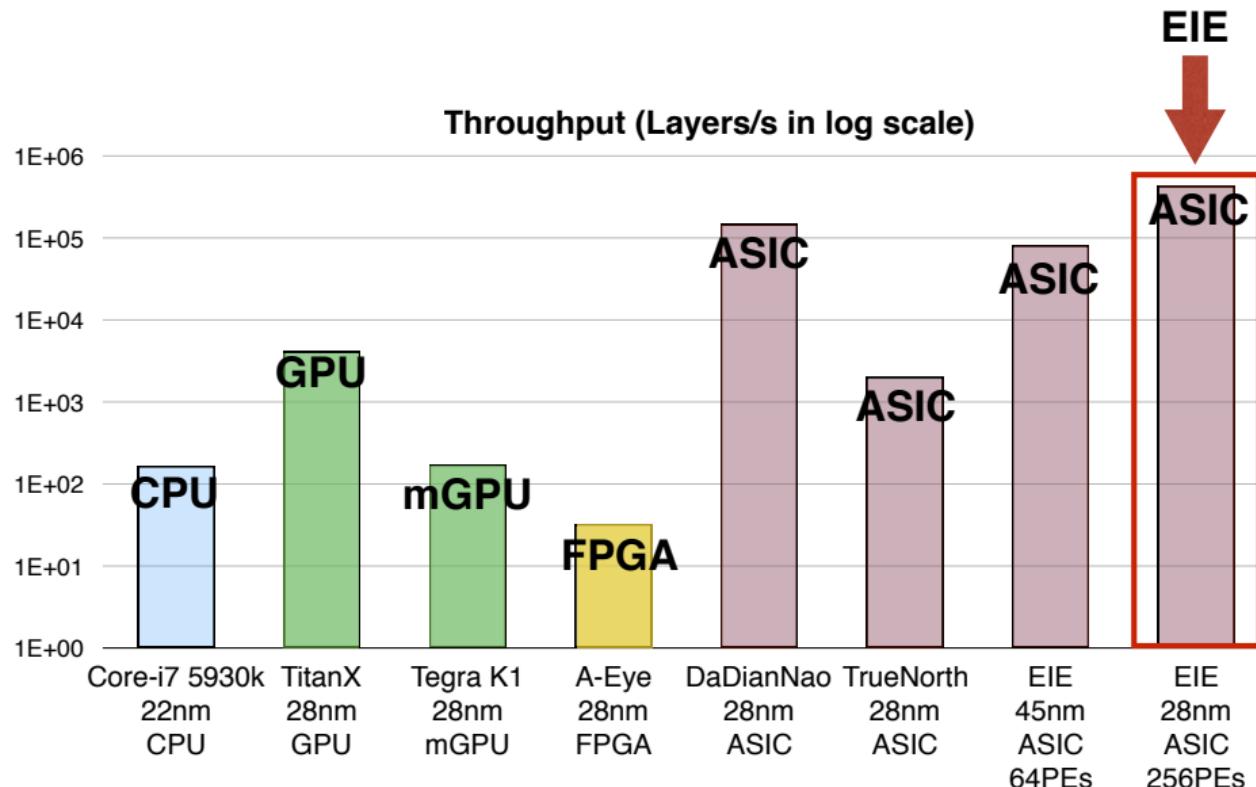


Energy Efficiency on EIE



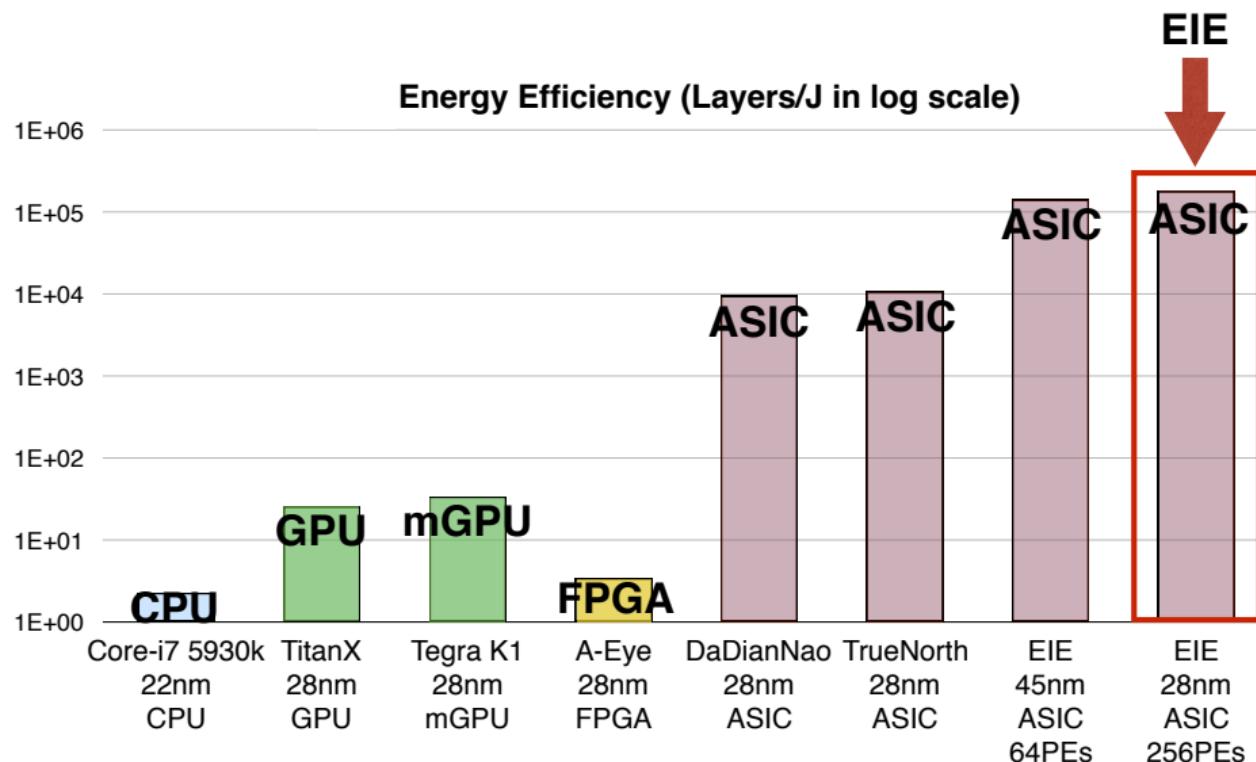


Comparison: Throughput



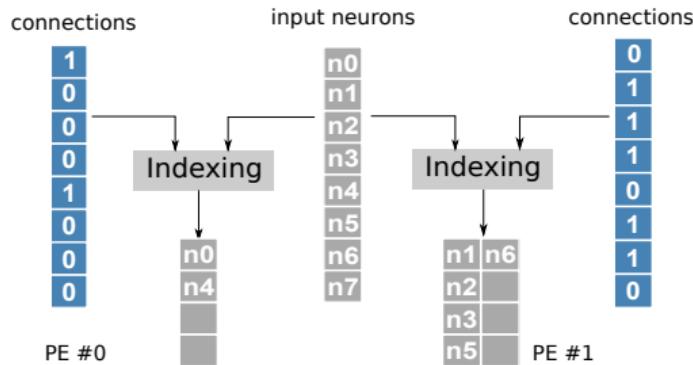


Comparison: Energy Efficiency





Indexing Module (IM) for sparse data

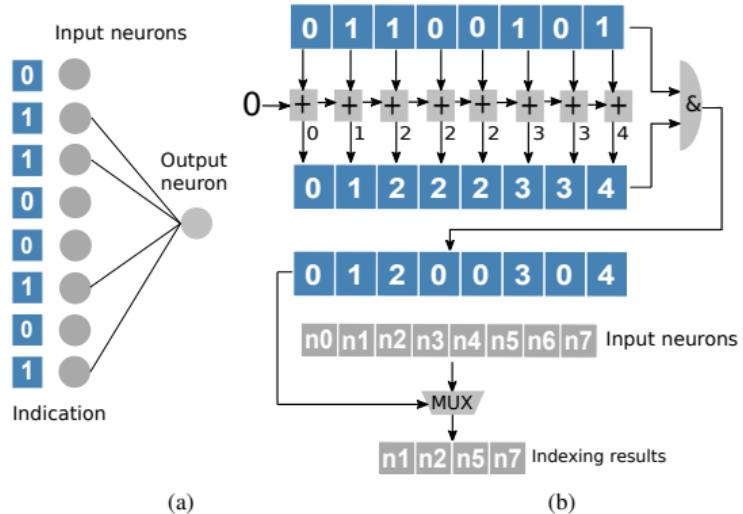


- IM is used for indexing needed neurons of sparse networks with different levels of sparsities.
- A centralized IM is designed in the buffer controller and only transfer the indexed neurons to processing engines.

²Shijin Zhang et al. (2016). "Cambricon-x: An accelerator for sparse neural networks". In: Proc. MICRO. IEEE, pp. 1-12.



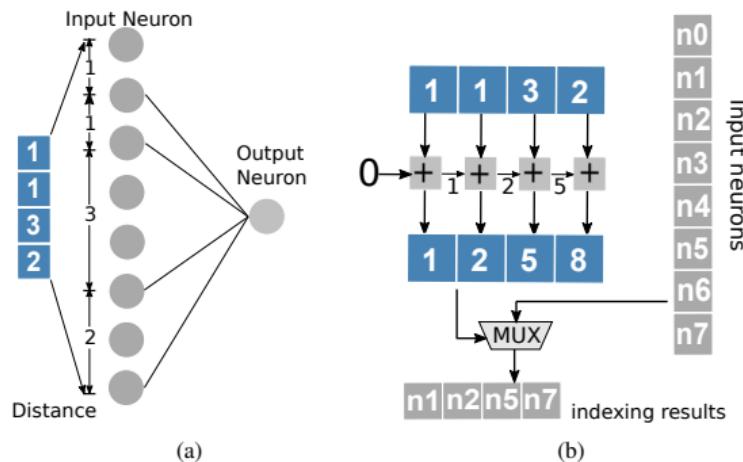
Direct indexing and hardware implementation



- Neurons are selected from all input neurons directly based on existed connections in the binary string.



Step indexing and hardware implementation

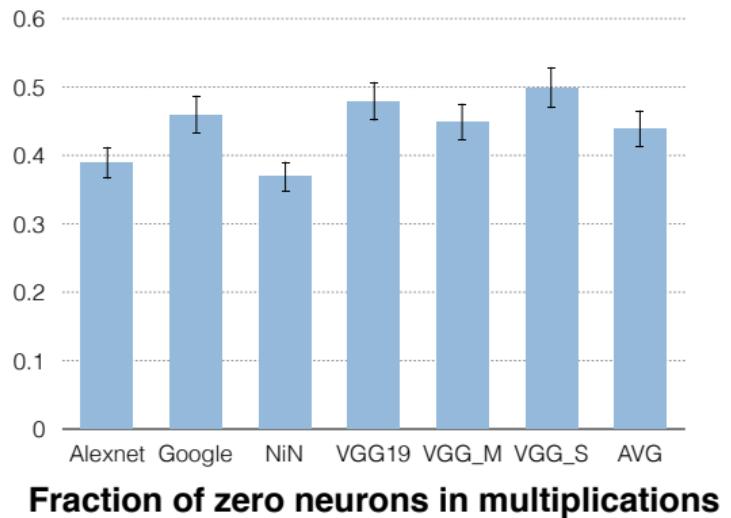


- Neurons are selected based on the distances between input neurons with existed synapses.



Lots of Runtime Zeroes

Ineffectual zero computations.



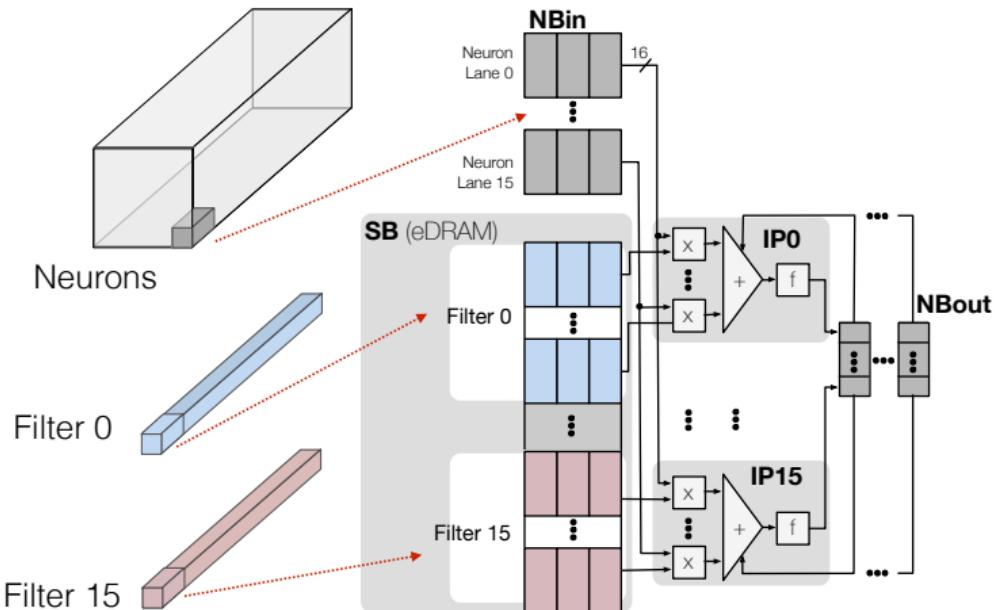
Fraction of zero neurons in multiplications

³Jorge Albericio et al. (2016). "Cnvlutin: Ineffectual-neuron-free deep neural network computing". In: *ACM SIGARCH Computer Architecture News* 44.3, pp. 1-13.

Feature Sparsity



DaDianNao⁴



⁴Yunji Chen et al. (2014). "Dadiannao: A machine-learning supercomputer". In: 2014 47th Annual IEEE/ACM International Symposium on Microarchitecture. IEEE, pp. 609–622.



Processing in DaDianNao

Neuron	0	1	1	2	0
Lanes	1	2	1	0	3
	⋮	⋮	⋮	⋮	⋮
	15	0	1	1	1

Synapse	0	1	2	3	4
Lanes	1	2	1	0	3
	⋮	⋮	⋮	⋮	⋮
	15	0	1	1	1

⋮

Synapse	0	1	2	3	4
Lanes	1	2	1	0	3
	⋮	⋮	⋮	⋮	⋮
	15	0	1	1	1

Feature Sparsity



Processing in DaDianNao

Neuron	0	1	1	2	0
Lanes	1	2	1	0	3
	:	:	:	:	
	15	0	1	1	1

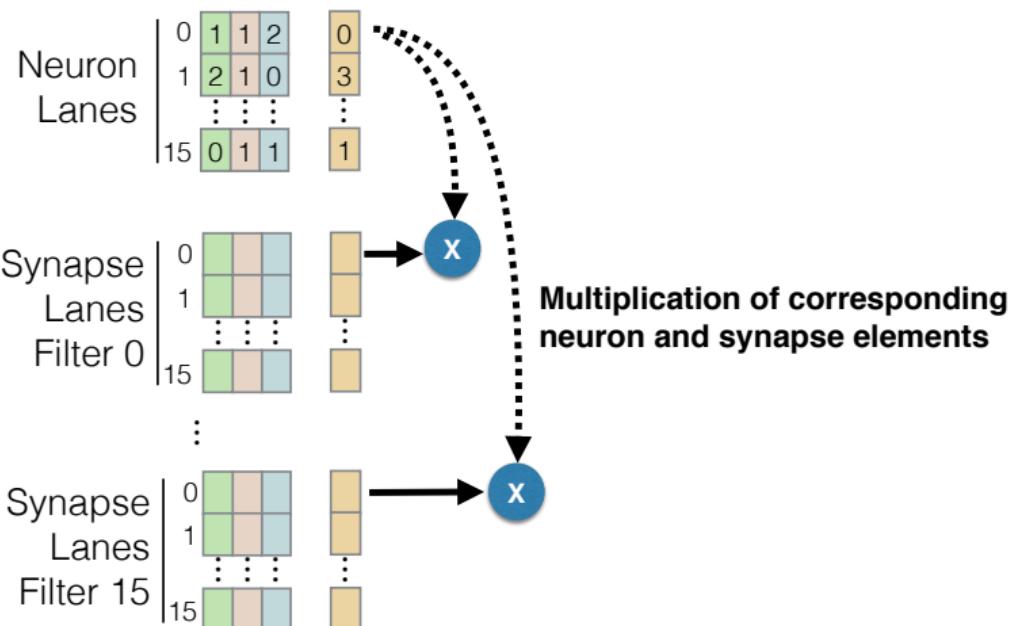
Synapse	0	1	2	3
Lanes	1	2	1	0
	:	:	:	
	15	0	1	1

⋮

Synapse	0	1	2	3
Lanes	1	2	1	0
	:	:	:	
	15	0	1	1



Processing in DaDianNao

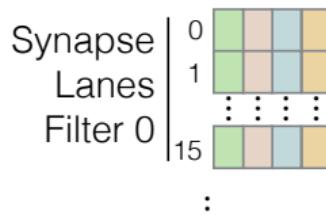
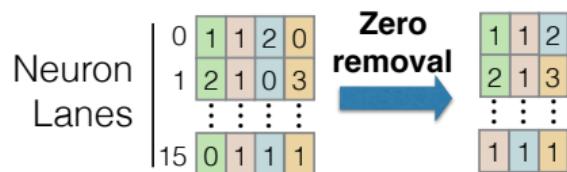


Feature Sparsity

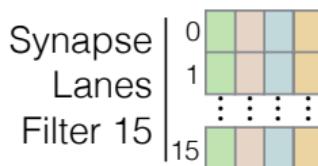


Processing in DaDianNao

Zero removal.



⋮

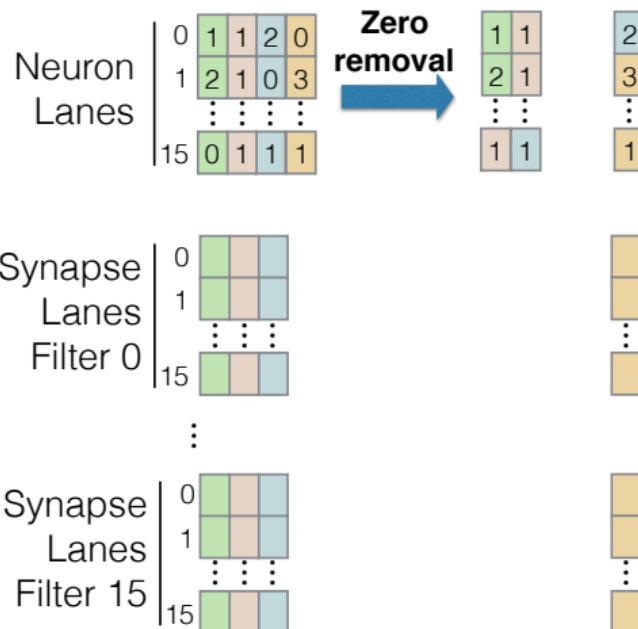


Feature Sparsity



Processing in DaDianNao

Zero removal.

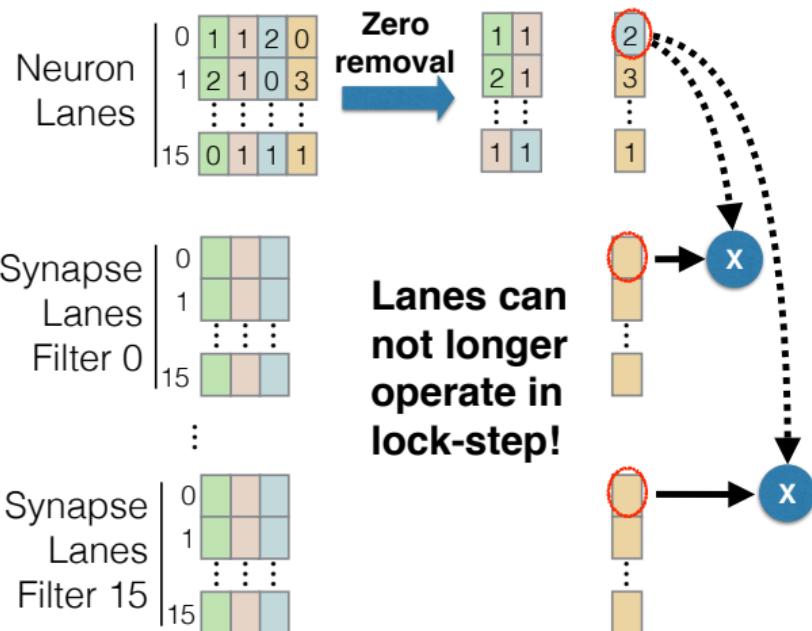


Feature Sparsity



Processing in DaDianNao

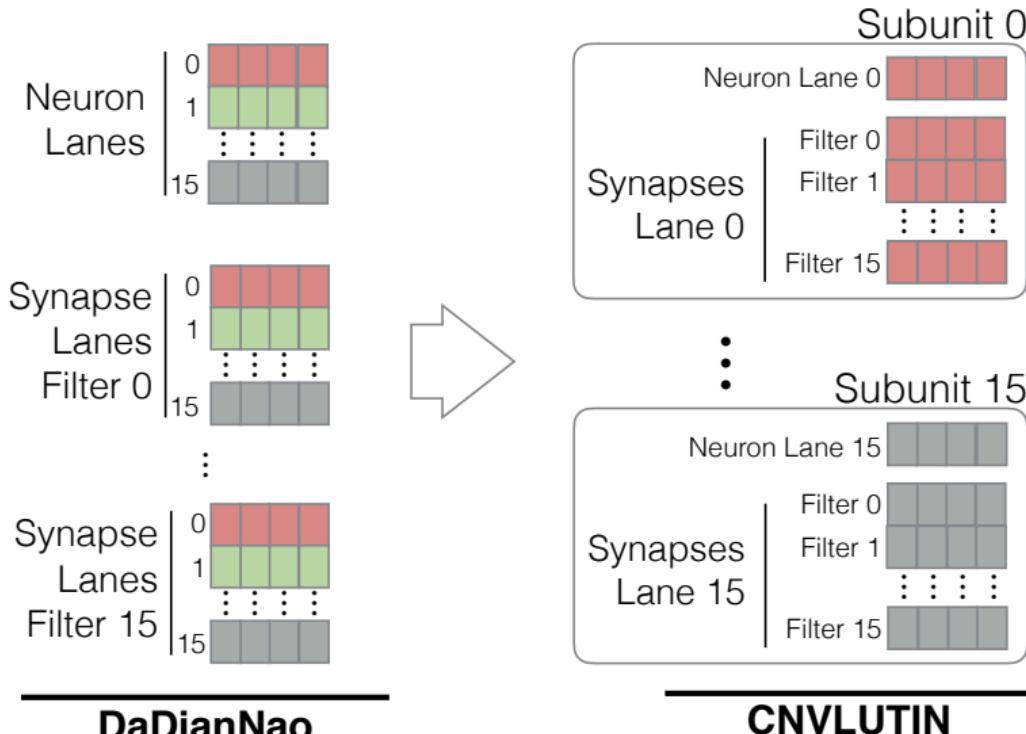
Lanes can no longer operate in lock-step.



Feature Sparsity



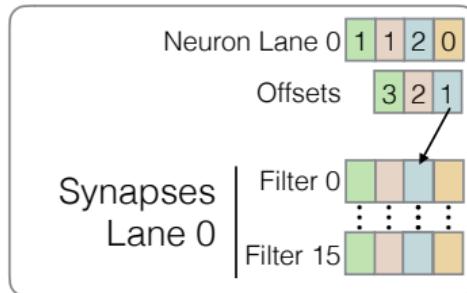
CNVLUTIN: Decoupling Lanes





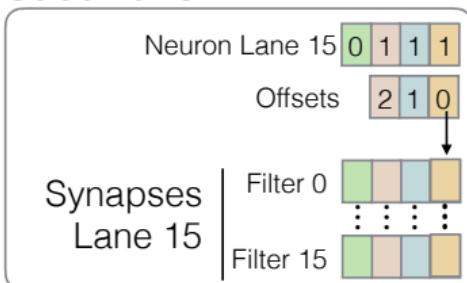
CNVLUTIN: Decoupling Lanes

Subunit 0



Subunit 15

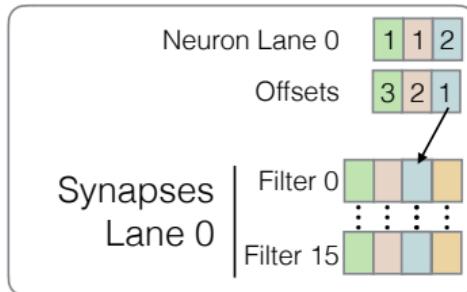
⋮





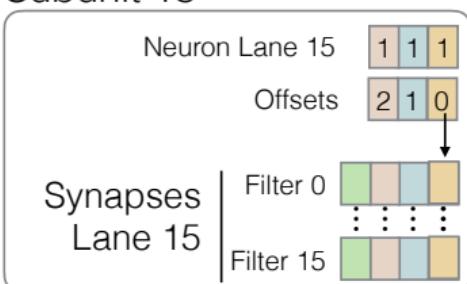
CNVLUTIN: Decoupling Lanes

Subunit 0



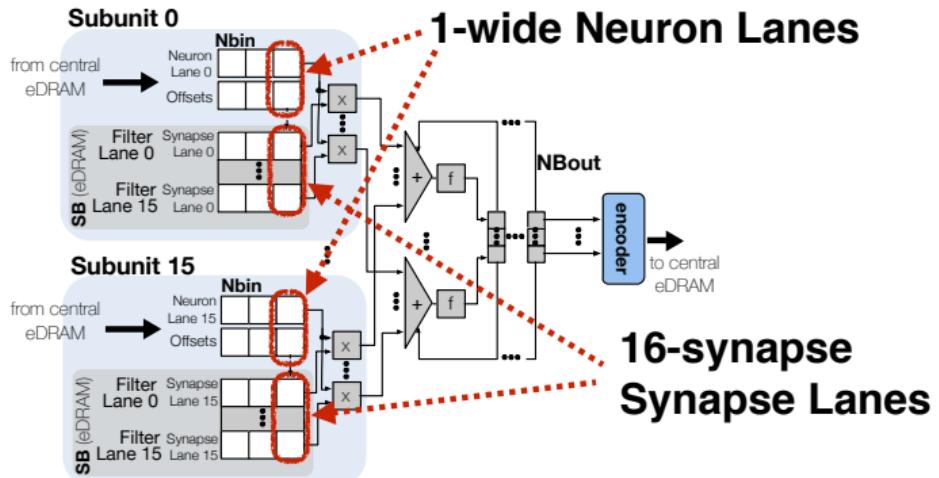
Subunit 15

⋮





CNVLUTIN: Decoupling Lanes



Decoupled Neuron Lanes:

Neuron + coordinate
Proceed independently

Partitioned SB:

16-wide accesses
1 synapse per filter

Further Discussion: Reading List



- Wenlin Chen et al. (2015). “Compressing neural networks with the hashing trick”. In: *Proc. ICML*, pp. 2285–2294
- Huizi Mao et al. (2017). “Exploring the granularity of sparsity in convolutional neural networks”. In: *CVPR Workshop*, pp. 13–20
- Zhuang Liu et al. (2017). “Learning efficient convolutional networks through network slimming”. In: *Proc. ICCV*, pp. 2736–2744
- Chenglong Zhao et al. (June 2019). “Variational convolutional neural network pruning”. In: *Proc. CVPR*
- Junru Wu et al. (2018). “Deep k -Means: Re-training and parameter sharing with harder cluster assignments for compressing deep convolutions”. In: *Proc. ICML*