



CMSC 5743

Efficient Computing of Deep Neural Networks

Mo04: Binary/Ternary Network

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2024 Fall

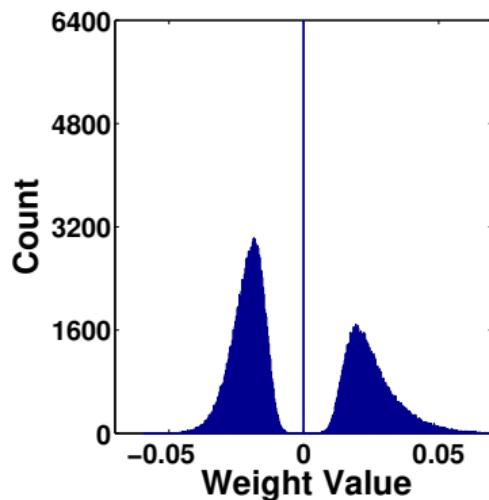


These slides contain/adapt materials developed by

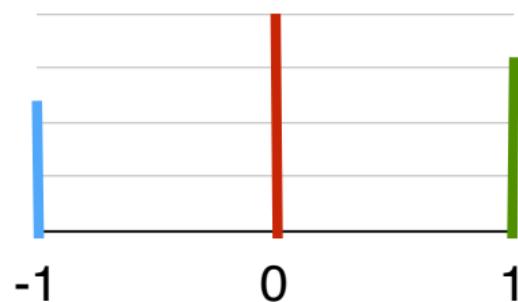
- Ritchie Zhao et al. (2017). “Accelerating binarized convolutional neural networks with software-programmable FPGAs”. In: *Proc. FPGA*, pp. 15–24
- Mohammad Rastegari et al. (2016). “XNOR-NET: Imagenet classification using binary convolutional neural networks”. In: *Proc. ECCV*, pp. 525–542



Binary / Ternary Net: Motivation



=>





Binarized Neural Networks (BNN)

CNN

$$\begin{array}{|c|c|c|} \hline & 2.4 & 6.2 \\ \hline & 3.3 & 1.8 \\ \hline \vdots & \ddots & \ddots \\ \hline \end{array} * \begin{array}{|c|c|} \hline & 0.8 & 0.1 \\ \hline & 0.3 & 0.8 \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline & 5.0 & 9.1 \\ \hline & 4.3 & 7.8 \\ \hline \vdots & \ddots & \ddots \\ \hline \end{array}$$

Input Map Weights Output Map

Key Differences

1. Inputs are binarized (-1 or +1)
2. Weights are binarized (-1 or +1)
3. Results are binarized after **batch normalization**

BNN

$$\begin{array}{|c|c|c|} \hline & 1 & -1 \\ \hline & 1 & 1 \\ \hline \vdots & \ddots & \ddots \\ \hline \end{array} * \begin{array}{|c|c|} \hline & 1 & -1 \\ \hline & 1 & -1 \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline & 1 & -3 \\ \hline & 3 & -7 \\ \hline \vdots & \ddots & \ddots \\ \hline \end{array}$$

Input Map (Binary) Weights (Binary) x_{ij} (Integer)

Batch Normalization

$$y_{ij} = \frac{x_{ij} - \mu}{\sqrt{\sigma^2 - \epsilon}} \gamma + \beta$$

\downarrow

$$z_{ij} = \begin{cases} +1 & \text{if } y_{ij} \geq 0 \\ -1 & \text{otherwise} \end{cases}$$

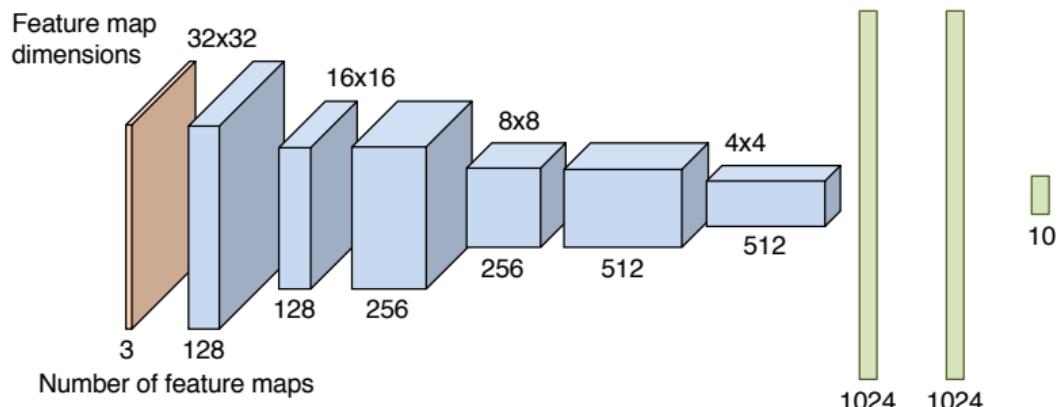
Binarization

→

$$\begin{array}{|c|c|c|} \hline & 1 & -1 \\ \hline & 1 & -1 \\ \hline \vdots & \ddots & \ddots \\ \hline \end{array}$$

Output Map (Binary)

BNN CIFAR-10 Architecture [2]



- ▶ 6 conv layers, 3 dense layers, 3 max pooling layers
- ▶ All conv filters are 3x3
- ▶ First conv layer takes in floating-point input
- ▶ **13.4 Mbits total model size** (after hardware optimizations)

[2] M. Courbariaux et al. **Binarized Neural Networks: Training Deep Neural Networks with Weights and Activations Constrained to +1 or -1**. arXiv:1602.02830, Feb 2016.

Advantages of BNN

1. Floating point ops replaced with binary logic ops

b_1	b_2	$b_1 \times b_2$
+1	+1	+1
+1	-1	-1
-1	+1	-1
-1	-1	+1

b_1	b_2	$b_1 \text{XOR } b_2$
0	0	0
0	1	1
1	0	1
1	1	0

- Encode $\{+1, -1\}$ as $\{0, 1\}$ → multiplies become XORs
- Conv/dense layers do dot products → XOR and popcount
- Operations can map to LUT fabric as opposed to DSPs

2. Binarized weights may reduce total model size

- Fewer bits per weight may be offset by having more weights



BNN vs CNN Parameter Efficiency

Architecture	Depth	Param Bits (Float)	Param Bits (Fixed-Point)	Error Rate (%)
ResNet [3] (CIFAR-10)	164	51.9M	13.0M*	11.26
BNN [2]	9	-	13.4M	11.40

* Assuming each float param can be quantized to 8-bit fixed-point

- ▶ **Comparison:**
 - Conservative assumption: ResNet can use 8-bit weights
 - BNN is based on VGG (less advanced architecture)
 - BNN seems to hold promise!

[2] M. Courbariaux et al. **Binarized Neural Networks: Training Deep Neural Networks with Weights and Activations Constrained to +1 or -1.** *arXiv:1602.02830*, Feb 2016.

[3] K. He, X. Zhang, S. Ren, and J. Sun. **Identity Mappings in Deep Residual Networks.** *ECCV 2016*.



- ① Minimize the Quantization Error
- ② Improve Network Loss Function
- ③ Reduce the Gradient Error



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- ② Improve Network Loss Function
- ③ Reduce the Gradient Error



*	Operations	Memory	Computation
\mathbb{R} * \mathbb{R}	+ - \times	1x	1x

Binary Weight Networks

XNOR-Networks

¹Mohammad Rastegari et al. (2016). "XNOR-NET: Imagenet classification using binary convolutional neural networks". In: *Proc. ECCV*, pp. 525–542.



*	Operations	Memory	Computation
$\mathbb{R} \text{ } * \text{ } \mathbb{R}$	+ - \times	1x	1x
$\mathbb{R} \text{ } * \text{ } \mathbb{B}$	+ -	$\sim 32x$	$\sim 2x$
$\mathbb{B} \text{ } * \text{ } \mathbb{B}$	XNOR Bit-count	$\sim 32x$	$\sim 58x$

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$$\begin{matrix} \text{R} \\ \text{X} \end{matrix} \odot \begin{matrix} \text{R} \\ \text{W} \end{matrix} \approx \begin{matrix} \text{R} \\ \text{X} \end{matrix} \odot \begin{matrix} \text{B} \\ \text{W}^B \end{matrix}$$

$$\begin{matrix} \text{R} \\ \text{W} \end{matrix} \approx \begin{matrix} \text{B} \\ \text{W}^B \end{matrix}$$

$$\mathbf{W}^B = \text{sign}(\mathbf{W})$$

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Quantization Error

$$W^B = \text{sign}(W)$$

$$\left\| \begin{matrix} W \\ \mathbb{R} \end{matrix} - \begin{matrix} W^B \\ \mathbb{B} \end{matrix} \right\| \approx 0.75$$

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Optimal Scaling Factor

$$\frac{\mathbb{R}}{\mathbf{W}} \approx \alpha \frac{\mathbb{B}}{\mathbf{W}^B}$$

$$\alpha^*, \mathbf{W}^{B^*} = \arg \min_{\mathbf{W}^B, \alpha} \{ \|\mathbf{W} - \alpha \mathbf{W}^B\|^2 \}$$

$$\boxed{\begin{aligned}\mathbf{W}^{B^*} &= \text{sign}(\mathbf{W}) \\ \alpha^* &= \frac{1}{n} \|\mathbf{W}\|_{\ell_1}\end{aligned}}$$

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How to train a CNN with binary filters?

$$\mathbb{R} \quad * \quad \mathbb{R} \quad \approx \quad (\quad \mathbb{R} \quad * \quad \mathbb{B} \quad) \alpha$$

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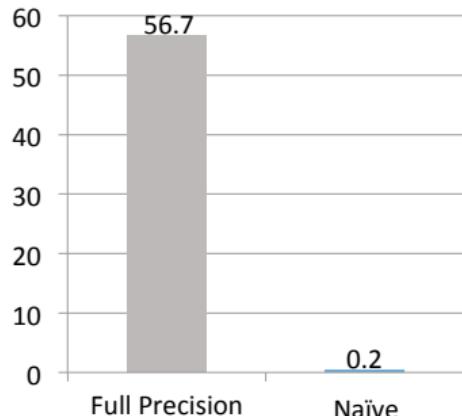
Training Binary Weight Networks

Naive Solution:

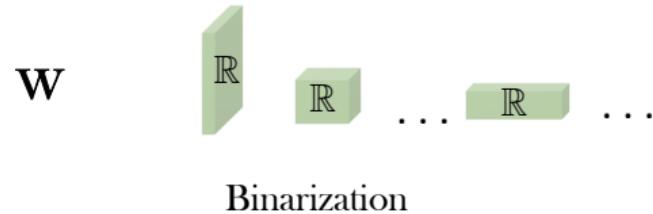
1. Train a network with real value parameters
2. Binarize the weight filters

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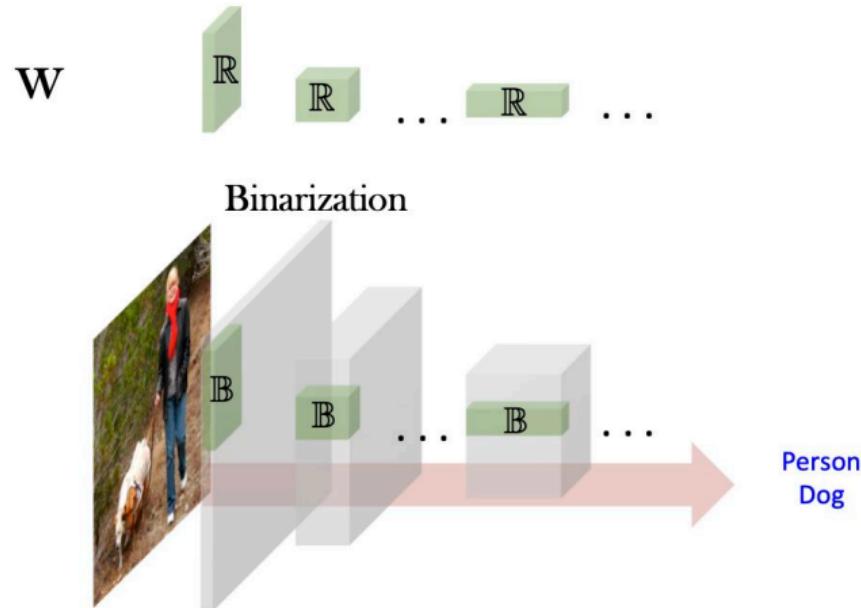
AlexNet Top-1 (%) ILSVRC2012



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Binary Weight Network

Train for binary weights:

1. Randomly initialize \mathbf{W}
2. For $iter = 1$ to N
3. Load a random input image \mathbf{X}
4. $\mathbf{W}^B = \text{sign}(\mathbf{W})$
5. $\alpha = \frac{\|\mathbf{W}\|_{\ell_1}}{n}$
6. Forward pass with α, \mathbf{W}^B
7. Compute loss function \mathbf{C}
8. $\frac{\partial \mathbf{C}}{\partial \mathbf{W}} = \text{Backward pass with } \alpha, \mathbf{W}^B$
9. Update \mathbf{W} ($\mathbf{W} = \mathbf{W} - \frac{\partial \mathbf{C}}{\partial \mathbf{W}}$)

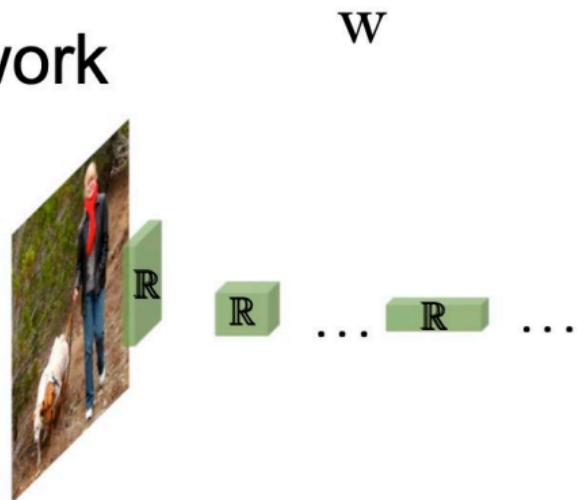


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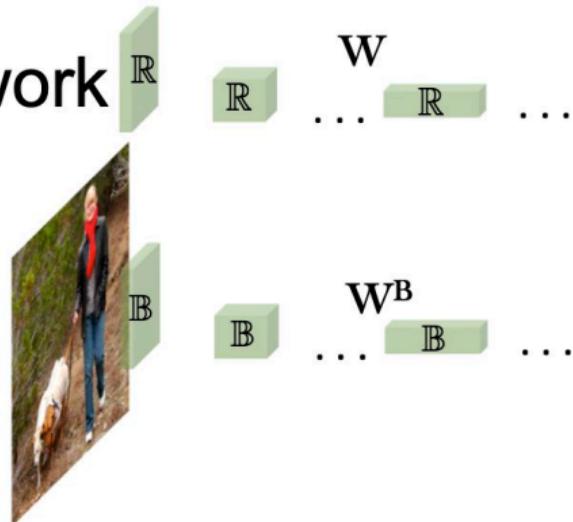


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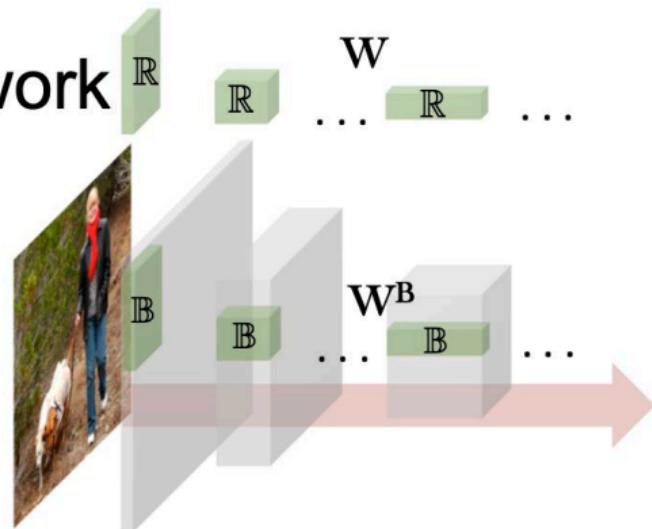


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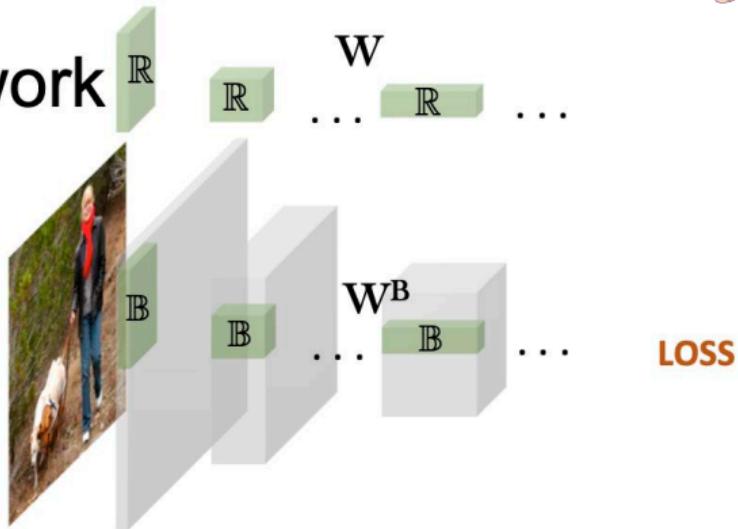


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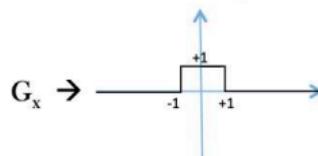
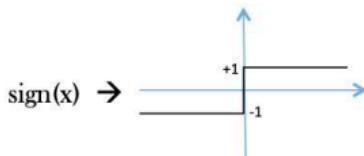
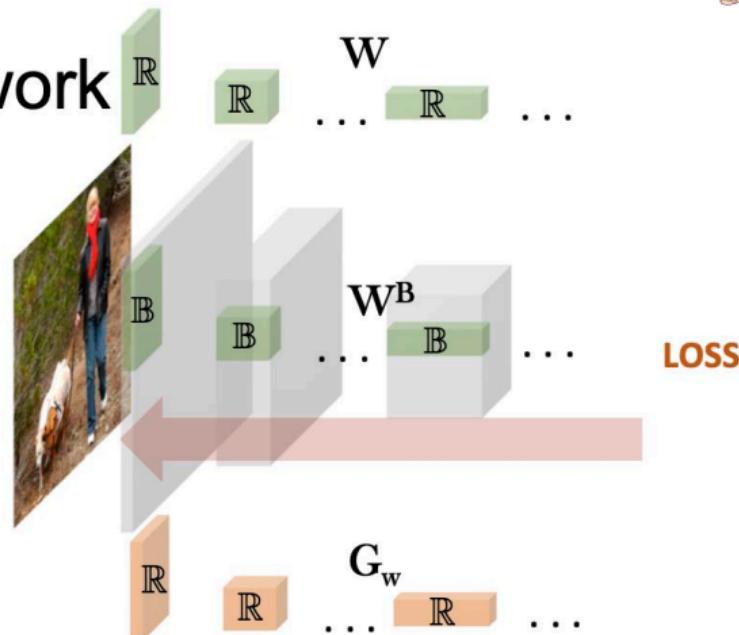


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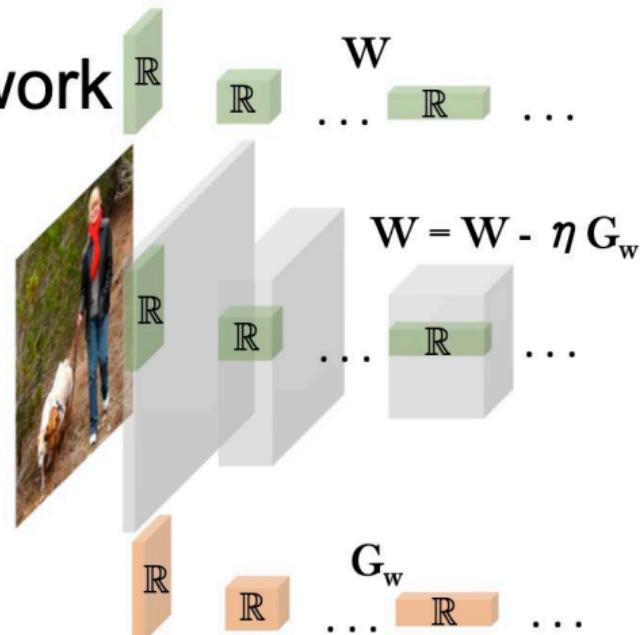
[Hinton et al. 2012]

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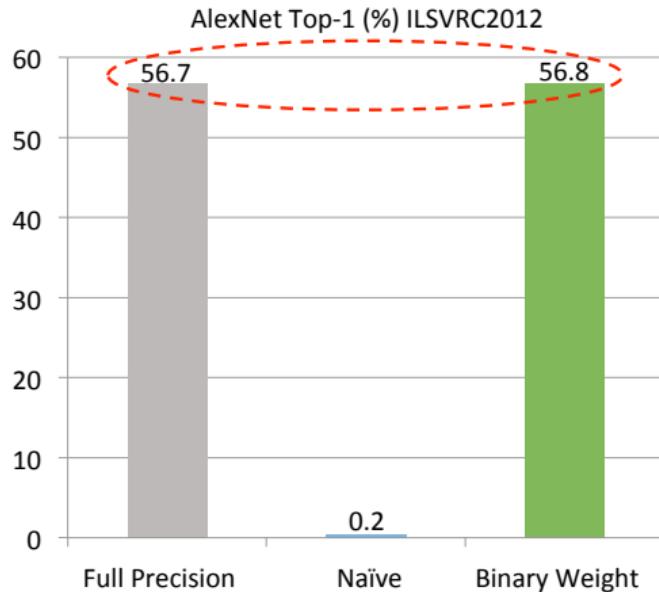
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$\mathbb{B} * \mathbb{B}$ XNOR-Networks	XNOR Bit-count	$\sim 32x$	$\sim 58x$

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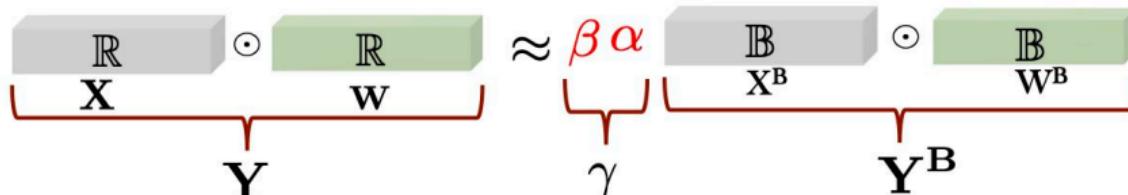
Binary Input and Binary Weight (XNOR-Net)

$$\begin{matrix} \mathbb{R} \\ \mathbf{x} \end{matrix} \odot \begin{matrix} \mathbb{R} \\ \mathbf{w} \end{matrix} \approx \beta \begin{matrix} \mathbb{B} \\ \mathbf{x}^B \end{matrix} \odot \alpha \begin{matrix} \mathbb{B} \\ \mathbf{w}^B \end{matrix}$$

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Binary Input and Binary Weight (XNOR-Net)



$$\mathbf{Y} \approx \gamma \mathbf{Y}^B$$

$$\mathbf{Y}^{B*}, \gamma^* = \arg \min_{\mathbf{Y}^B, \gamma} \|\mathbf{Y} - \gamma \mathbf{Y}^B\|^2$$

$$\boxed{\mathbf{Y}^{B*} = \text{sign}(\mathbf{Y}) \quad \gamma^* = \frac{1}{n} \|\mathbf{Y}\|_{\ell_1}}$$

$$\boxed{\mathbf{X}^{B*} = \text{sign}(\mathbf{X}) \quad \mathbf{W}^{B*} = \text{sign}(\mathbf{W})} \quad \boxed{\alpha^* = \frac{1}{n} \|\mathbf{W}\|_{\ell_1} \quad \beta^* = \frac{1}{n} \|\mathbf{X}\|_{\ell_1}}$$

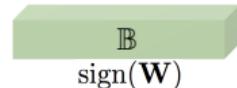
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(1) Binarizing Weights

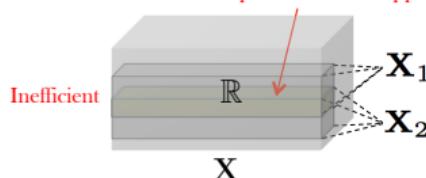


$$\frac{1}{n} \|\mathbf{W}\|_{\ell_1} = \alpha$$



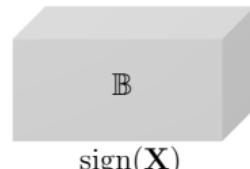
(2) Binarizing Input

Redundant computation in overlapping areas



$$\begin{aligned} \frac{1}{n} \|\mathbf{X}_1\|_{\ell_1} &= \beta_1 \\ \frac{1}{n} \|\mathbf{X}_2\|_{\ell_1} &= \beta_2 \end{aligned}$$

\mathbb{B}
 \mathbf{K}

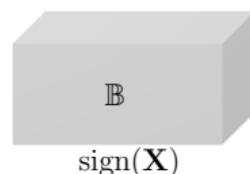


(2) Binarizing Input

$$\text{Efficient} \quad \frac{\sum |\mathbf{X}_{:,i}|}{c} \rightarrow \quad \mathbf{X} = \mathbf{K}$$

c

$$\text{Average Filter} \quad * = \mathbf{K} = \beta_1 \quad \beta_2$$



(3) Convolution with XNOR-Bitcount

$$\mathbb{R} * \mathbb{R} \approx \left[\mathbb{B} \otimes \mathbb{B} \right] \odot \alpha$$

\mathbf{W}

\mathbb{B}
 $\text{sign}(\mathbf{X})$

\mathbb{B}
 $\text{sign}(\mathbf{W})$

\mathbf{K}

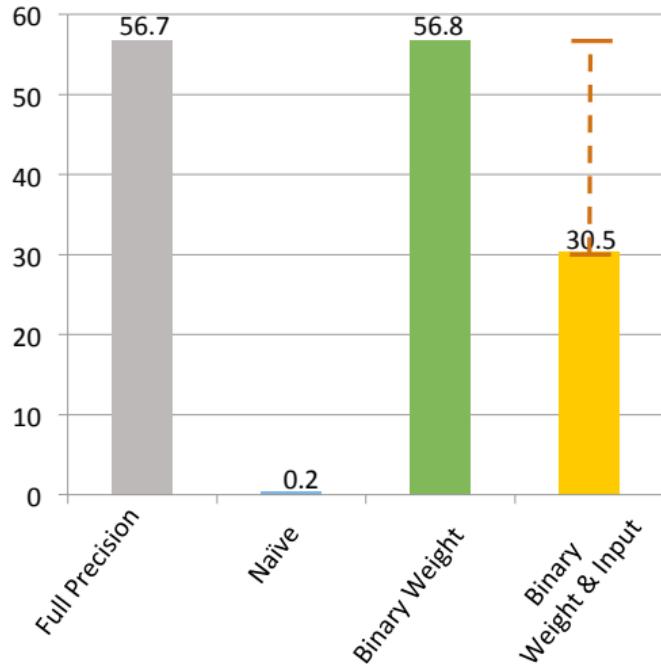
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$$\mathbb{R} * \mathbb{R} \approx \left[\begin{matrix} \mathbb{B} \\ \text{sign}(\mathbf{X}) \end{matrix} * \begin{matrix} \mathbb{B} \\ \text{sign}(\mathbf{W}) \end{matrix} \right] \odot \beta \odot \alpha$$

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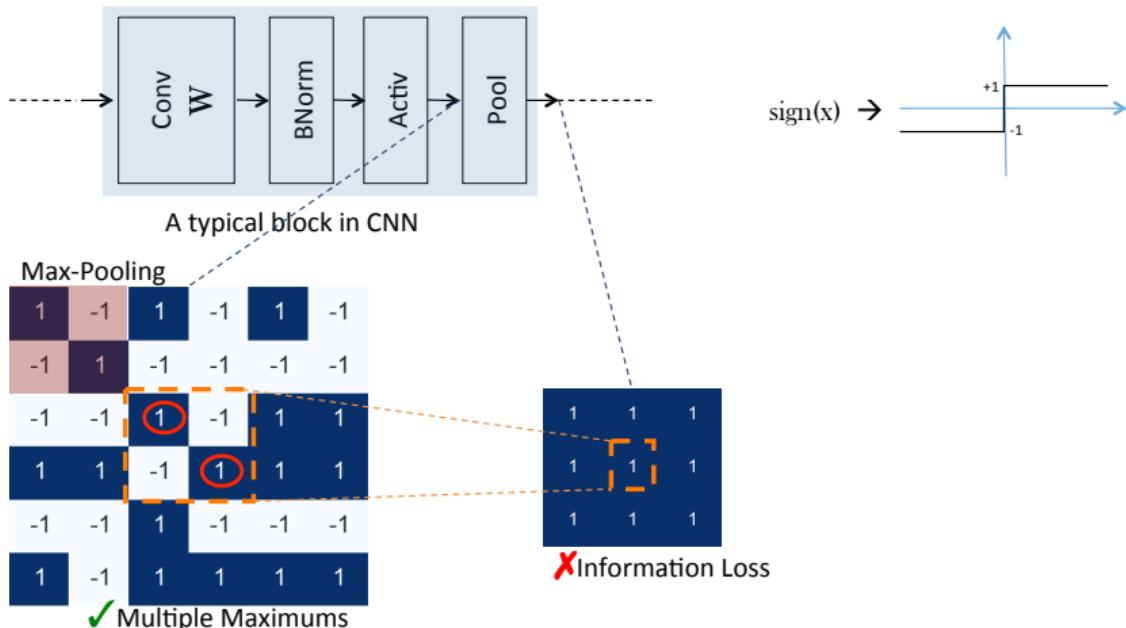
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AlexNet Top-1 (%) ILSVRC2012



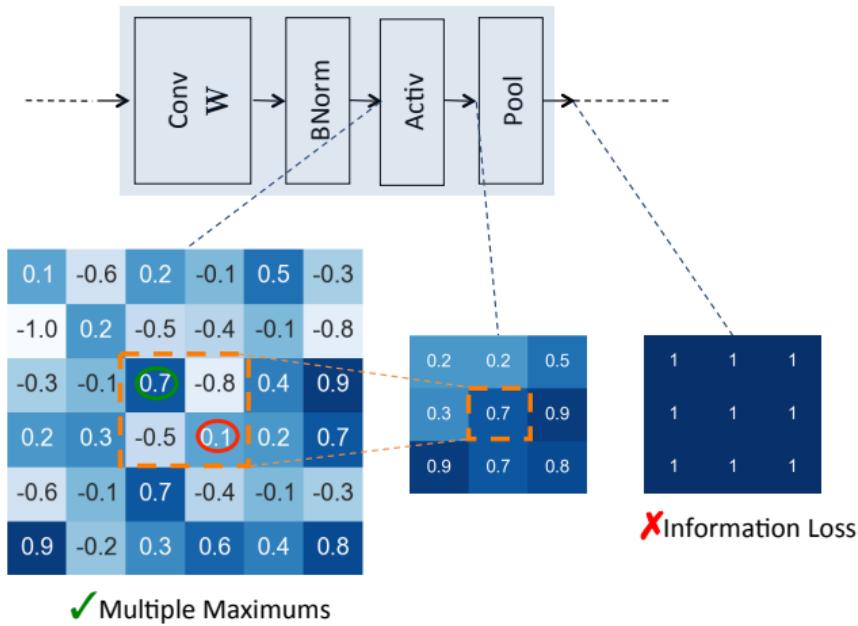
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Network Structure in XNOR-Networks



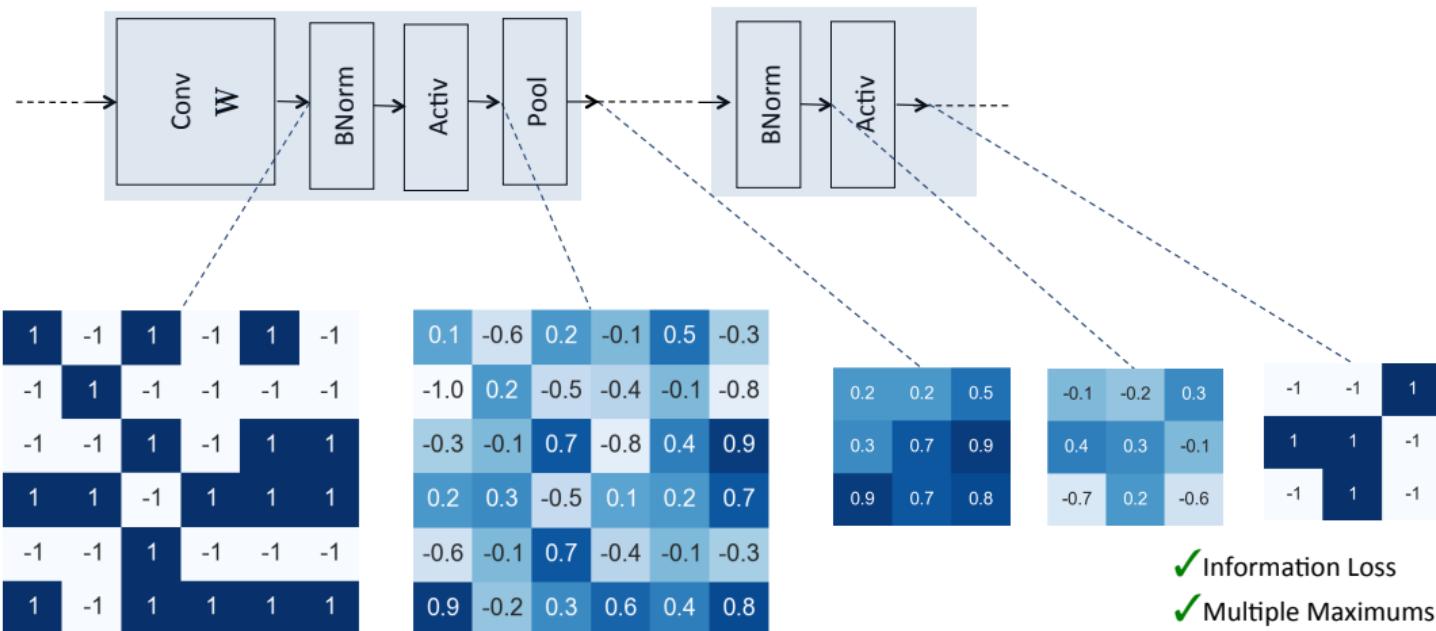
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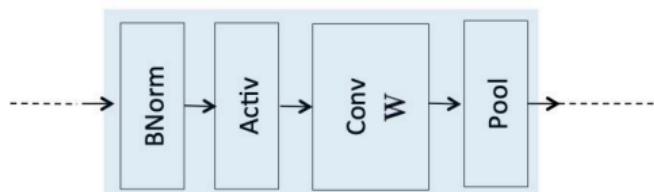
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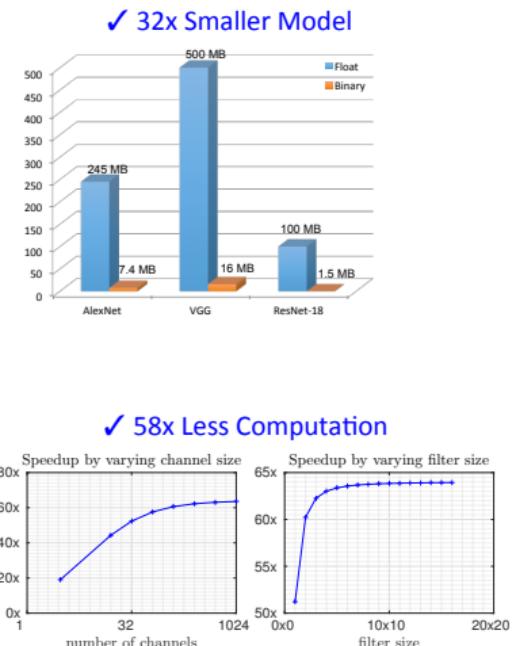
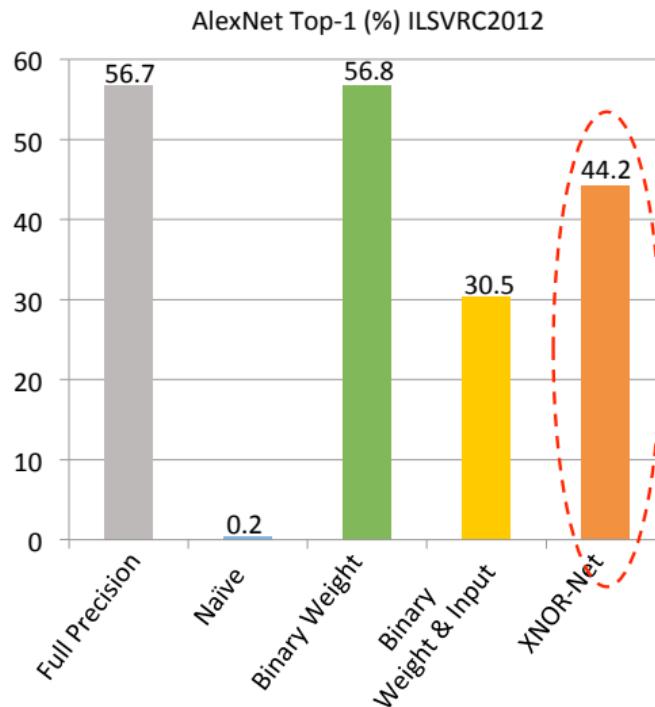
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8. $\frac{\partial C}{\partial W} = \text{Backward pass with } \alpha, \mathbf{W}^B$
9. Update \mathbf{W} ($\mathbf{W} = \mathbf{W} - \frac{\partial C}{\partial W}$)

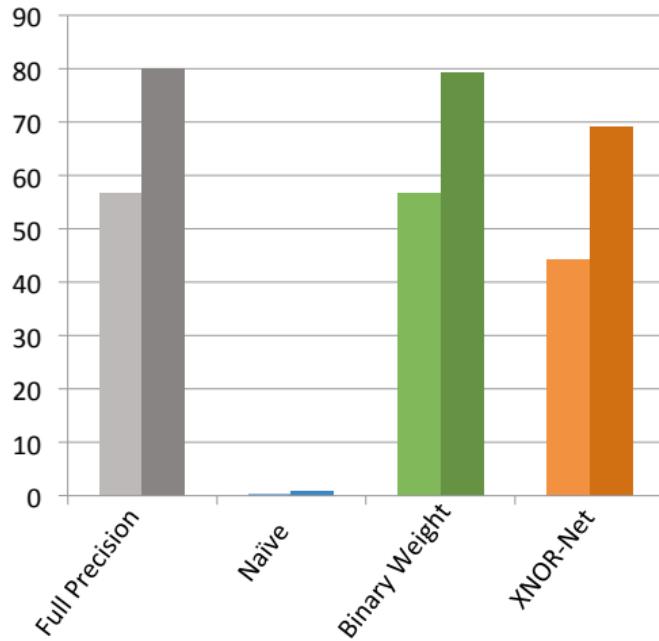


¹Mohammad Rastegari et al. (2016). "XNOR-NET: Imagenet classification using binary convolutional neural networks". In: *Proc. ECCV*, pp. 525–542.



¹Mohammad Rastegari et al. (2016). "XNOR-NET: Imagenet classification using binary convolutional neural networks". In: *Proc. ECCV*, pp. 525–542.

AlexNet Top-1 & 5 (%) ILSVRC2012



¹Mohammad Rastegari et al. (2016). "XNOR-NET: Imagenet classification using binary convolutional neural networks". In: *Proc. ECCV*, pp. 525–542.



Motivation

- Naive methods (Matthieu Courbariaux, Yoshua Bengio, and Jean-Pierre David (2015). “Binaryconnect: Training deep neural networks with binary weights during propagations”. In: *Advances in neural information processing systems*, pp. 3123–3131, Matthieu Courbariaux, Itay Hubara, et al. (2016). “Binarized neural networks: Training deep neural networks with weights and activations constrained to +1 or -1”. In: *arXiv preprint arXiv:1602.02830*) suffer the accuracy loss

Intuition

- Quantized parameter should approximate the full precision parameter as closely as possible



DoReFa-net: Training low bitwidth convolutional neural networks with low bitwidth gradients



Contribution

- Succeeded in quantizing gradients to numbers with bitwidth less than 8 bits during the backward pass
- Creating DoReFa-Net which has arbitrary bitwidth in weights, activations and gradients
- Explore the the configuration space of bitwidth for weights, activations and gradients for DoReFa-Net



Weights Quantization

- Weights binarization



Activations Quantization

- Assume the output of the previous layer has passed through a bounded activation function h , which ensures $r \in [0, 1]$

$$f_\alpha^k(r) = \text{quantize}_k(r)$$

Gradient Quantization

- Gradients are unbounded and may have significantly larger value range than activations

$$f_\gamma^k(\mathbf{d}r) = 2 \max_0(|\mathbf{d}r|) [\text{quantize}_k \left[\frac{\mathbf{d}r}{2 \max_0(|\mathbf{d}r|)} + \frac{1}{2} \right] - \frac{1}{2}]$$

$$N(k) = \frac{\sigma}{2^k - 1} \text{ where } \sigma \sim \text{Uniform}(-0.5, 0.5)$$



Read the paper² if you want to learn the specific details of the algorithm

DOREFA-NET: TRAINING LOW BITWIDTH CONVOLUTIONAL NEURAL NETWORKS WITH LOW BITWIDTH GRADIENTS

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²Shuchang Zhou et al. (2016). "Dorefa-net: Training low bitwidth convolutional neural networks with low bitwidth gradients". In: *arXiv preprint arXiv:1606.06160*.



Towards Accurate Binary Convolutional Neural Network



Contribution

- Approximate full-precision weights with the linear combination of multiple binary weight bases
- Introduce multiple binary activations



Weights Binarization

- Weights tensors in one layer: $W \in \mathbb{R}^{w \times h \times c_{in} \times c_{out}}$

$$B_1, B_2, \dots, B_M \in \{-1, +1\}^{w \times h \times c_{in} \times c_{out}}$$

$$W \approx \alpha_1 B_1 + \alpha_2 B_2 + \dots + \alpha_M B_M$$

$$B_i = F_{u_i}(W) = \text{sign}(\bar{W} + u_i \text{std}(W)), i = 1, 2, \dots, M$$

where $\bar{W} = W - \text{mean}(W)$, u_i is a shift parameter (e.g. $u_i = -1 + (i-1)\frac{2}{M-1}$)
 α can be calculated via $\min_a J(\alpha) = \|W - B\alpha\|^2$



Forward and Backward

- Forward

$$B_1, B_2, \dots, B_M = F_{u_1}(W), F_{w_2}(W), \dots, F_{u,u}(W)$$

$$\text{solve } \min_a J(\alpha) = \|W - B\alpha\|^2 \text{ for } \alpha$$

$$O = \sum_{m=1}^M \alpha_m \text{Conv}(B_m, A)$$

- Backward

$$\frac{\partial c}{\partial W} = \frac{\partial c}{\partial O} \left(\sum_{m=1}^M \alpha_m \frac{\partial O}{\partial B_m} \frac{\partial B_m}{\partial W} \right) \stackrel{STE}{=} \frac{\partial c}{\partial O} \left(\sum_{m=1}^M \alpha_m \frac{\partial O}{\partial B_m} \right) = \sum_{m=1}^M \alpha_m \frac{\partial c}{\partial B_m}$$



Multiple Binary Activations

- Bounded Activation Function

$$h(x) \in [0, 1]$$

$$h_r(x) = \text{clip}(x + v, 0, 1)$$

where v is a shift parameter

- Binarization Function

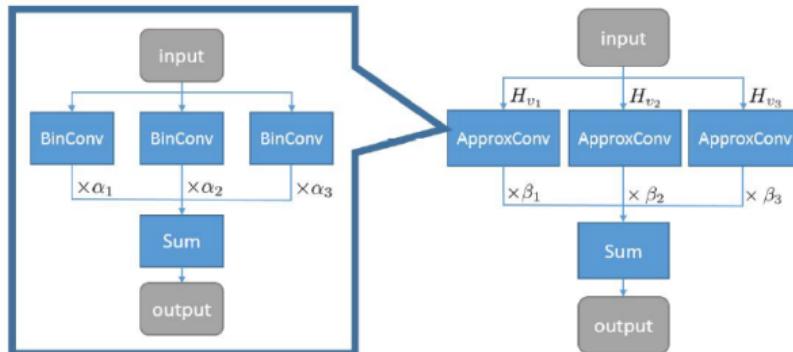
$$H_v(R) := 2\mathbb{I}_{h_v(R) \geq 0.5} - 1$$

$$A_1, A_2, \dots, A_N = H_{v_1}(R), H_{v_2}(R), \dots, H_{v_N}(R)$$

$$R \approx \beta_1 A_1 + \beta_2 A_2 + \dots + \beta_N A_N$$

where R is the real-value activation

- A_1, A_2, \dots, A_N is the base to represent the real-valued activations



- ApproxConv is expected to approximate the conventional full-precision convolution with linear combination of binary convolutions
- The right part is the overall block structure of the convolution in ABC-Net. The input is binarized using different functions H_{v1}, H_{v2}, H_{v3}

$$\text{Conv}(\mathbf{W}, \mathbf{R}) \approx \text{Conv} \left(\sum_{m=1}^M \alpha_m \mathbf{B}_m, \sum_{n=1}^N \beta_n \mathbf{A}_n \right) = \sum_{m=1}^M \sum_{n=1}^N \alpha_m \beta_n \text{Conv} (\mathbf{B}_m, \mathbf{A}_n)$$



Read the paper³if you want to learn the specific details of the algorithm

Towards Accurate Binary Convolutional Neural Network

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³Xiaofan Lin, Cong Zhao, and Wei Pan (2017). "Towards accurate binary convolutional neural network". In: *Advances in Neural Information Processing Systems*, pp. 345–353.



1 Minimize the Quantization Error

2 Improve Network Loss Function

3 Reduce the Gradient Error



Motivation

- Only focusing on the **local layers** can hardly promise the exact final output passed through a series of layers.
- It is highly required that the network training should **globally** take the **binarization** as well as the **task-specific objective** into account.

Intuition

- Finding the desired loss function contribute to **guide the learning of parameter with restriction**



Training binary neural networks with real-to-binary convolutions

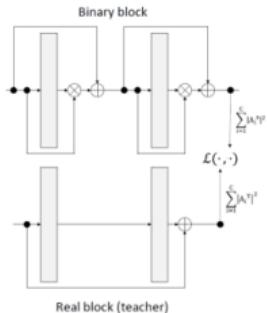


Contribution

- Use an attention matching strategy called “a sequence of teacher-student pairs”, so that the real-valued network can more closely guide the binary network during optimization
- Use the real-valued activations of the binary network to compute scale factors that are used to re-scale the activations right after the application of the binary convolution.



- Proposed Real-to-Bin Block



Supervision is injected at the end of each binary block

Loss Term

- Compare attention maps between real-valued and binary network
- Gradients do not have to travel the whole network and suffer degradation

$$\mathcal{L}_{att} = \sum_{j=1}^{\mathcal{J}} \left\| \frac{\mathcal{Q}_S^j}{\|\mathcal{Q}_S^j\|_2} - \frac{\mathcal{Q}_T^j}{\|\mathcal{Q}_T^j\|_2} \right\| \text{ where } \mathcal{Q}^j = \sum_{i=1}^c |A_i|^2$$



Progressive Teacher-Student

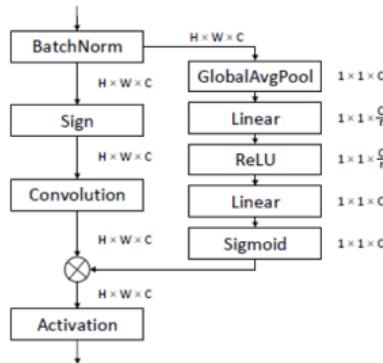
- Step1
teacher: real-valued network with standard ResNet architecture
student: real-valued network with the same architecture as the binary ResNet-18
- Step2
teacher: student network from step1
student: binary ResNet-18 with binary activations and real-valued weights
- Step3
teacher: student network from step2
student: binary ResNet-18 with binary activations and binary weights



Data-driven Channel Rescaling

- To solve limited representation problem
- Rely on the full-precision activation signal to predict the scaling factors used to re-scale the output of the binary convolution channel-wise

$$\mathcal{A} * \mathcal{W} \approx (\text{sign}(\mathcal{A}) \otimes \text{sign}(\mathcal{W})) \odot \alpha \odot G(\mathcal{A}; \mathcal{W}_G)$$



The proposed data-driven channel re-scaling approach.



Read the paper⁴ if you want to learn the specific details of the algorithm

TRAINING BINARY NEURAL NETWORKS WITH REAL-TO-BINARY CONVOLUTIONS

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⁴Brais Martinez et al. (2020). "Training binary neural networks with real-to-binary convolutions". In: *arXiv preprint arXiv:2003.11535*.



1 Minimize the Quantization Error

2 Improve Network Loss Function

3 Reduce the Gradient Error



Motivation

- Although STE is often adopted to estimate the gradients in BP, there exists obvious gradient mismatch between the gradient of the binarization function
- With the restriction of STE, the parameters outside the range of $[-1 : +1]$ will not be updated.



Bi-real net: Enhancing the performance of 1-bit CNNs with improved representational capability and advanced training algorithm



Naive Binarization Function

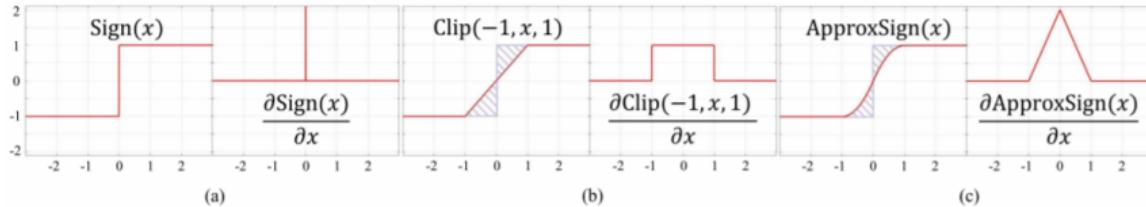
- Recall the partial derivative calculation in back propagation

$$\frac{\partial \mathcal{L}}{\partial \mathbf{A}_r^{l,t}} = \frac{\partial \mathcal{L}}{\partial \mathbf{A}_b^{l,t}} \frac{\partial \mathbf{A}_b^{l,t}}{\partial \mathbf{A}_r^{l,t}} = \frac{\partial \mathcal{L}}{\partial \mathbf{A}_b^{l,t}} \frac{\partial \text{Sign}(\mathbf{A}_r^{l,t})}{\partial \mathbf{A}_r^{l,t}} \approx \frac{\partial \mathcal{L}}{\partial \mathbf{A}_b^{l,t}} \frac{\partial F(\mathbf{A}_r^{l,t})}{\partial \mathbf{A}_r^{l,t}}$$

- Sign* function is a non-differentiable function, so F is an approximation differentiable function of *Sign* function



$$\frac{\partial \mathcal{L}}{\partial \mathbf{A}_r^{l,t}} = \frac{\partial \mathcal{L}}{\partial \mathbf{A}_b^{l,t}} \frac{\partial \mathbf{A}_b^{l,t}}{\partial \mathbf{A}_r^{l,t}} = \frac{\partial \mathcal{L}}{\partial \mathbf{A}_b^{l,t}} \frac{\partial \text{Sign}(\mathbf{A}_r^{l,t})}{\partial \mathbf{A}_r^{l,t}} \approx \frac{\partial \mathcal{L}}{\partial \mathbf{A}_b^{l,t}} \frac{\partial F(\mathbf{A}_r^{l,t})}{\partial \mathbf{A}_r^{l,t}}$$



Approximation of *Sign* function

- Naive Approximation $F(x) = \text{clip}(x, 0, 1)$, see fig(b)
- More Precious Approximation in Bi-Real, see fig(c)

$$\text{Approxsign}(x) = \begin{cases} -1, & \text{if } x < -1 \\ 2x + x^2, & \text{if } -1 \leq x < 0 \\ 2x - x^2, & \text{if } 0 \leq x < 1 \\ 1, & \text{otherwise} \end{cases} \quad \frac{\partial \text{Approxsign}(x)}{\partial x} = \begin{cases} 2 + 2x, & \text{if } -1 \leq x < 0 \\ 2 - 2x, & \text{if } 0 \leq x < 1 \\ 0, & \text{otherwise} \end{cases}$$



Read the paper⁵ if you want to learn the specific details of the algorithm

Bi-Real Net: Enhancing the Performance of 1-bit CNNs With Improved Representational Capability and Advanced Training Algorithm

Zechun Liu¹ , Baoyuan Wu² , Wenhan Luo², Xin Yang^{3*}, Wei Liu², and
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¹ Hong Kong University of Science and Technology

² Tencent AI lab

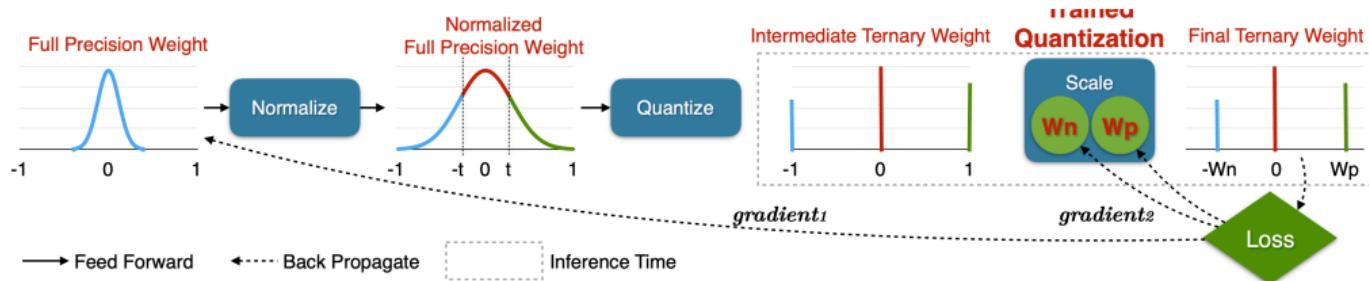
³ Huazhong University of Science and Technology

⁵Zechun Liu et al. (2018). “Bi-real net: Enhancing the performance of 1-bit cnns with improved representational capability and advanced training algorithm”. In: *Proceedings of the European conference on computer vision (ECCV)*, pp. 722–737.



Trained ternary quantization

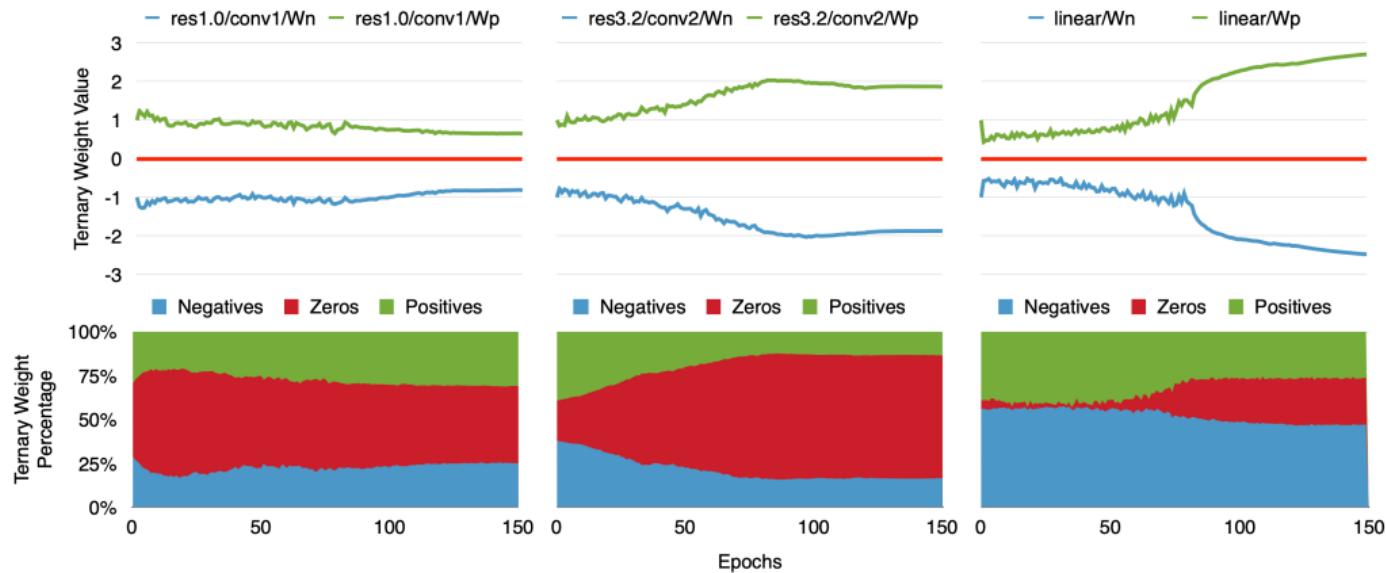
Trained Ternary Quantization⁶



Overview of the trained ternary quantization procedure.

⁶Chenzhuo Zhu et al. (2017). “Trained ternary quantization”. In: *Proc. ICLR*.

Trained Ternary Quantization⁶



Ternary weights value (above) and distribution (below) with iterations for different layers of ResNet-20 on CIFAR-10.

⁶Chenzhuo Zhu et al. (2017). "Trained ternary quantization". In: *Proc. ICLR*.



- Hyeonuk Kim et al. (2017). “A Kernel Decomposition Architecture for Binary-weight Convolutional Neural Networks”. In: *Proc. DAC*, 60:1–60:6
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- Zhaowei Cai et al. (2017). “Deep learning with low precision by half-wave gaussian quantization”. In: *Proceedings of the IEEE conference on computer vision and pattern recognition*, pp. 5918–5926