Module 2F: Backwardation, Contango and the Roll Yield

Learning Outcomes

By working on this module you should be able to

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Background

In previous modules we have taken the view that hedgers and speculators are the primary participants in commodity futures markets. Historically that was largely true but in the early 2000s that changed when it was recognized that commodities such as oil and soybeans provide both a good hedge against inflation and help to diversity risk within a typical financial portfolio. Strong promotion of these benefits created an accelerated flow of institutional-managed funds into various commodity-indexed based instruments. Commodity hedge funds became popular with wealthy investors who had an appetite for leveraged investments in commodities, and who didn't mind paying the high fees and commissions set by the managers of these funds.

The past year (2021) has provided commodity investors with very high returns as commodities across the board surged with the easing of COVID lock down restrictions. According to an October 18, 2021 report in the Globe and Mail titled "Energy and commodities hedge funds post big gains as prices skyrocket", Westbeck Capital Management hedge fund generated a 17.2 percent return for September and a 94 percent return for the year-to-date. Not surprisingly, large institutional investors are further increasing their holdings of commodity futures (mostly energy funds). For example, in a September 1, 2021 newsletter by Alberta-based Auspice Capital Management (http://www.auspicecapital.com/alt-invest/2021/9/1/institutional-investors-adding-commodities-big-time) it was noted that the Ontario Teachers Pension Plan, which held assets valued at about $220 billion at the beginning of 2021, recently increased their commodity holdings to about 12 percent, up from 8 percent the previous year.

Read the first few pages of a 2012 paper (https://www.princeton.edu/~wxiong/papers/commodity.pdf) titled "Index Investment and the Financialization of Commodities" by Ke Tang and Wei Xiong. These authors note that large-scale institutional investment in various commodity index–related instruments began in the early 2000s. They also noted that between 2008 and 2013 investment in these instrumens surged from about $15 billion to about $200 billion. This surge meant that commodities no longer existed as a separate asset class. Indeed, prior to the early 2000s there was little correlation between commodity prices and the broader index of stock market activity. With the strong level of institutional investment in commodity index-linked instruments the correlation between the returns from these instruments with the returns from the broader stock market strengthened considerably. Over this same time period there was much less strengthening of the correlation between the returns of commodities which were not indexed and the returns of the broader stock market. Based on this differential impact Tang and Xiaong concluded that institutional investment in commodity index-related instruments has indeed contributed toward what has come to be known as the financialization of commodities.

To dig deeper into this claim, Figure 1 from Tang and Xiong (2012) (https://www.princeton.edu/~wxiong/papers/commodity.pdf) show the one-year rolling return correlations of oil with various commodities, together with the 95% confidence levels. The two indexed commodities are soybeans and copper, and the two non-indexed commodities are cotton and live cattle. Notice that the strengthening of the

correlation began in about 2004 for the two indexed commodities, which is when large scale investment in commodity futures began. A significant strengthening of the correlation for the non-indexed commodities began several several years later (e.g., 2006 for cotton and 2008 for live cattle). Tang and Xiong provide more rigorous evidence in their paper to support their conjecture concerning the financialization of commodities.

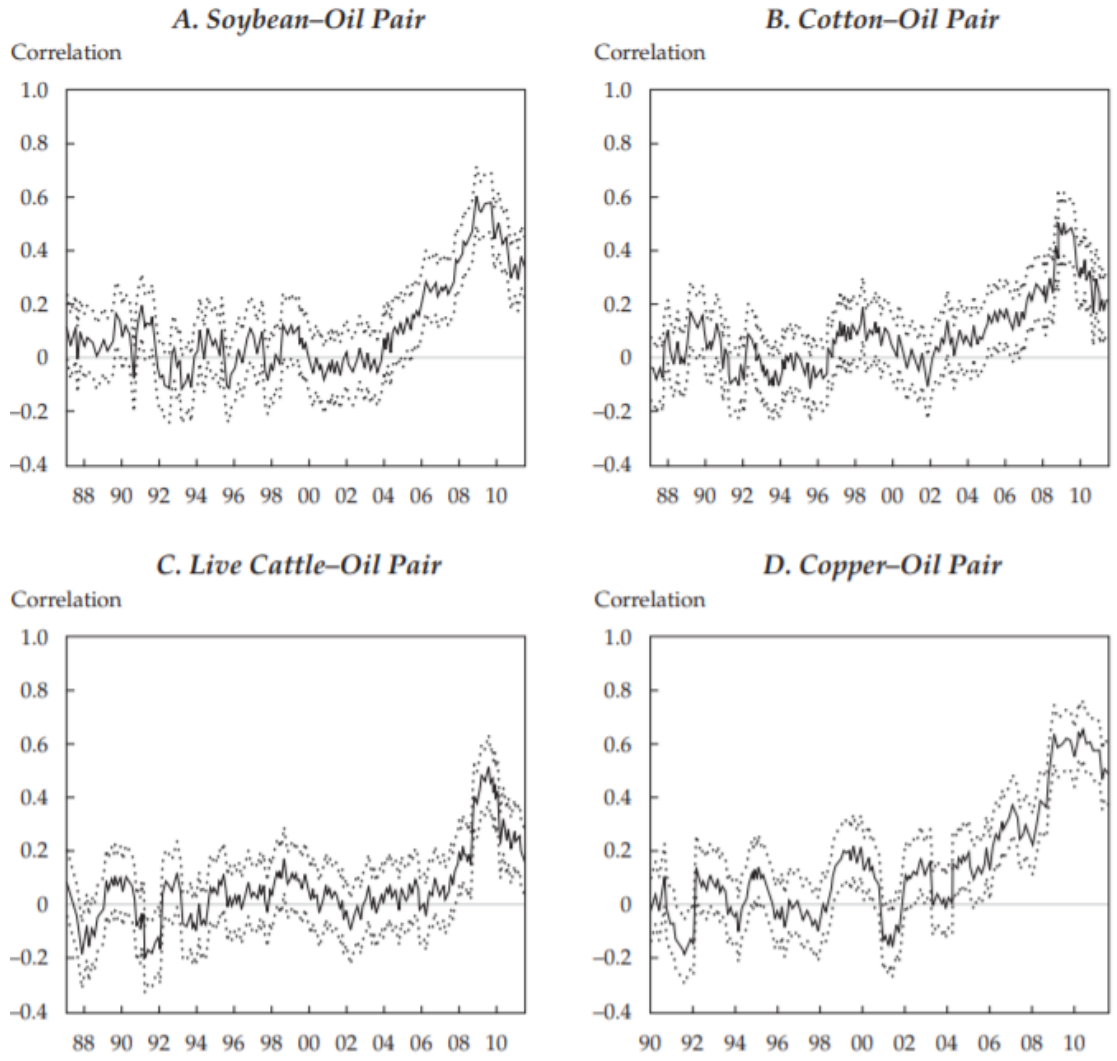

Figure 1: One year rolling return correlations

Purpose of this Module

In this module we will not focus on the incentives for institutions to invest in commodities and the associated diversification benefits. Rather, we will look into the mechanics of index investing and examine the determinants of returns for long term passive investors in commodity futures. You may have already detected an inconsistency between the claim in the previous modules that futures prices are expected to remain flat over time (thus generating a zero expected return for investors) and the stylized facts that investors are including commodity futures into their portfolios at what appears to be an increasing rate. Empirical evidence is very clear that the long run average return to holding futures is close to zero. This being the case, is it only the diversification benefits which have attracted investors into holding commodity futures?

There is another reason why investment in commodity futures may be strong despite a lack of evidence that such investments lack long run profitablity. Guided by professional trading blog sites and direct advice from financial advisors, many investors have come to incorrectly believe that the process of rolling an expiring contract into a new contract generates an instantaneous profit or loss. In years when strong short term demand cause short term

futures prices to rise faster than longer term futures prices, the market goes into a stage of “backwardation” and roll yields become positive. In contrast, in years when weak short term demand cause short term futures prices to fall faster than longer term future prices, the market goes into a state of “contango” and roll yields become negative. Commodity investors earn a positive return in the first case because of rising commodity prices and not because of the positive roll yield. Conversely, investors earn a negative return in the second case because of falling commodity prices and not because of a negative roll yield. This conjeture is elaborated on later in this module.

There exists a widespread and highly incorrect belief about the relationship between investment returns and the sign and size of the roll. The so-called *roll yield myth* postulates that rolling futures contracts forward in a backwardated market creates an instantaneous gain which is proportional to the size of the roll. Similarly, rolling futures contracts forward in a contango market creates an instantaneous loss which is proportional to the size of the roll.

The first part of this module is used to explain the theory of roll yield and to demonstrate why roll yield should not be interpreted as a financial yield. The second part of this module uses data from the U.S. crude oil market to demonstrate empirically that roll yield is measuring the return from holding a futures contract relative to holding the spot commodity. After accounting for carrying costs, both the return from holding the spot commodity and the return from rolling futures through time are, in principle, identical and equal to zero.

Before examining the mechanics of index investing and roll yield, and demonstrating the falsehood of the roll yield myth, several popular commodity ETFs are examined.

Agricultural Commodity ETFs

Ordinary retail investors can gain exposure to agricultural commodity futures by investing in specialized agricultural commodity exchange traded funds (ETFs). The following list (https://etfdb.com/etfs/natural-resources/agriculture/) shows that the most popular agriculture-specific ETF is the Invesco DB Agricultural Fund (https://etfdb.com/etf/DBA/#etf-ticker-profile), formerly named Powershares DB Agricultural Fund. Created in 2007, and holding roughly $100 million in assets, this ETF has generated a strong return in 2021 (about 22 percent). The Invesco DB Agriculture ETF is designed to follow a specialized non-traded index, and it is this index which is comprised of the specific returns from rolling long futures positions through time.The weights within the non-traded index are approximately 12 percent each for corn, live cattle, soybeans, sugar, cocoa and coffee, and the remaining percentage is allocated to hogs, wheat and cotton.

An important ETF which serves as both a direct investment and an index which other ETFs follow is the Bloomberg Commodity Index (https://www.bloomberg.com/quote/BCOM:IND). According to a Bloomberg press release (https://www.bloomberg.com/press-releases/2021-11-09/bloomberg-commodity-index-2022-target-weights-announced) assets which track the Bloomberg index are valued at roughly $100 billion. The Bloomberg index tracks the prices of 23 physical commodities, about a third of which are agricultural-based.

Figure 2 shows the current (December 2021) and proposed new composition of the Bloomberg Commodity Index (see press release for more details). Notice that about 20 percent of the index is allocated to grains, about 22.5 percent to the soft commodities and about 5 percent to livestock.

Bloomberg

below:

Group	Commodity	Ticker	2022 Target Weight	2021 Target Weight
	WTI Crude Oil	CL	8.0368820%	8.1448320%
	Natural Gas	NG	7.9548670%	8.0720060%
	Brent Crude Oil	CO	6.9631180%	6.8551680%
Energy	Low Sulphur Gas Oil	QS	2.6496240%	2.6415190%
	RBOB Gasoline	XB	2.1728010%	2.1791840%
	ULS Diesel	HO	2.0526330%	2.0820140%
			29.83%	29.97%
	Corn	C	5.5899030%	5.5866490%
	Soybeans	S	5.7888440%	5.8174090%
	Soybean Meal	SM	3.5200260%	3.5987640%
Grains	Wheat	W	2.8463610%	2.8850050%
	Soybean Oil	BO	3.1716110%	3.1955900%
	HRW Wheat	KW	1.6636530%	1.5713880%
			22.58%	22.65%
	Copper	HG	5.3982920%	5.3937680%
	Aluminum	LA	4.2457680%	4.2083970%
Industrial Metals	Zinc	LX	3.1189270%	3.2468830%
	Nickel	LN	2.7134270%	2.7139540%
			15.48%	15.56%
	Gold	GC	15.0000000%	14.6459560%
Precious Metals	Silver	SI	4.7468930%	4.3539140%
			19.75%	19.00%
	Sugar	SB	2.7943260%	2.9870850%
Softs	Coffee	KC	2.7333550%	2.7366190%
	Cotton	CT	1.5032870%	1.5110980%
			7.03%	7.23%
	Live Cattle	LC	3.5807520%	3.8464030%
Livestock	Lean Hogs	LH	1.7546500%	1.7263950%
			5.34%	5.57%

Figure 2: Bloomberg Commodity Index Component Weights

A highly specialized agricultural commodity investment is the Teucrium Corn ETF (https://www.teucrium.com/etfs/corn), which is designed to track corn futures only. This relatively small ETF had a return of nearly 40 percent on a base of about $135 million in net assets as of early December 2021. As of early December 2021 the Teucrium Corn ETF held (https://www.teucrium.com/holdings/corn) roughly an equal number of long positions in the March 2022, May 2022 and December 2022 corn contracts. The lack of diversification of this asset makes it relatively high risk to hold.

To conclude this section it is useful to examine the historical performance of the Bloomberg Commodity Index. Figure 3 shows the ETF price of the Bloomberg Commodity Index (black) and the S&P 500 Index (orange). It is clear that commodities performed quite poorly relative to the market as a whole between 2017 to early 2020. The strong growth in the Bloomberg index in 2021 has erased some of that poor performance but its overall performance relative to the S&P 500 is still relatively poor.

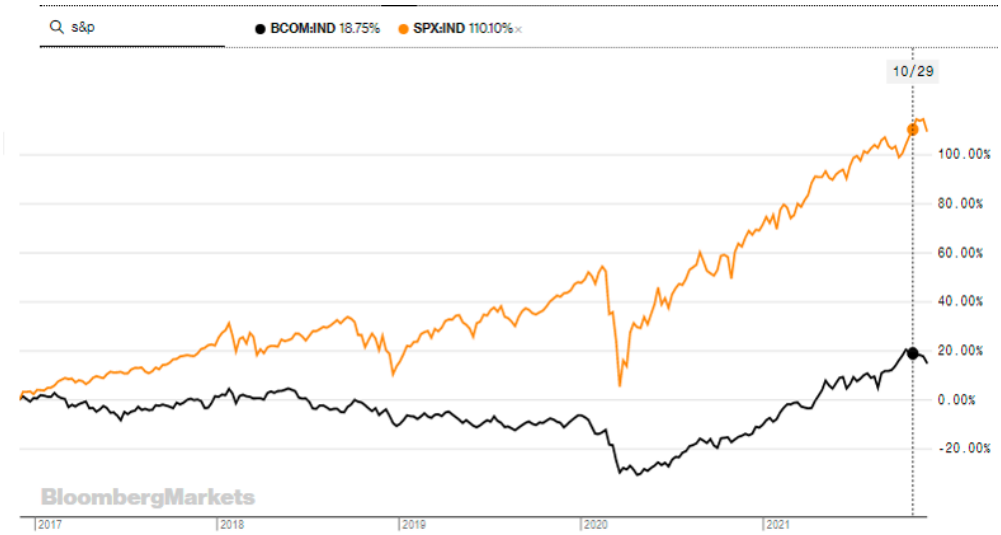

Figure 3: Bloomberg Commodity Index and S&P500

A Simplified Way to Calculate Roll Yield

To model roll yield and the roll yield myth, it is useful to focus on crude oil because unlike agricultural commodities there are 12 monthly futures contracts which trade for crude oil. The even spacing between rolls over the course of a year (i.e., one month) simplifies the calculations as compared to a corn contract which rolls with either a two month or three month span. The second reason why crude oil is easier to work with for the purpose of calculating roll yield is that there is no seasonality in prices. The lack of seasonality means that the size of the roll adjustment is expected to be relatively stable over time.With a storable agricultural commodity with annual production, the recurring seasonality results in the roll switching signs and varying in size over the course of a marketing year.

The first simplification is that we will assume the crude oil ETF follows an index which measures the price changes in the next-to-expire futures contract of a single commodity (e.g., WTI crude). Moreover, we will assume the roll fully takes place on the last day before the futures contract expires. In real world markets, a crude oil ETF tracks an index which measures the prices of multiple futures contracts and rolls well before the futures contract expires. For example, the ProShares (https://www.proshares.com/funds/oilk_daily_holdings.html?gclid=Cj0KCQiAqbyNBhC2ARIsALDwAsDhGrKZ-promUu7Qn6-xpUD2wEiOwMnP_0V4Pc8kSC8Vvx456eSuxIaAs4dEALw_wcB) K-1 Free Crude Oil Strategy ETF tracks the Bloomberg Commodity Balanced WTI Crude Oil Index. As of December 7, 2021 this index had equal weightings in the March 2022, June 2022 and December 2022 crude oil futures. The specific rolling of this index is described as follows:

“The Bloomberg Commodity Balanced WTI Crude Oil Index (ticker: BCBCLI) aims to track the performance of three separate contract schedules for WTI crude oil futures, which are reset on a semiannual basis. One third of the index follows a monthly roll schedule that rolls from the current futures contract (which expires one month out) into the following month’s contract; the second third of the index is designated to be in a June contract and follows an annual roll schedule in March of each year; and the remaining third is designated to be in a December contract and follows an annual roll schedule in September of each year.”

The simplified roll structure that we assume allows us to easily measure the roll yield and show how roll yield is a measure of the differential performance of the spot price of crude oil and the futures price. Like most economic models, the goal is to remove the clutter which makes analysis complicated and adds very little by way of economic insight.

The Roll Yield Myth

In this article (https://www.etf.com/publications/journalofindexes/joi-articles/12274-better-beta-in-commodities-indexing.html), the author assumes a simplified roll structure like the one used for this module. Specifically, the index measures the earnings of a hypothetical investor who rolls over each month the next-to-expire crude oil futures contract. Figure 4 shows the specific roll structure. For example, the second row shows that on January 14th the February futures is sold and the March futures is purchased. The third row shows that on February 15th the March futures is sold and the April futures is purchased. The last colum shows the calculated roll yield, which is the price of the expiring contract minus the price of the new contract.

Figure 2

iPath S&P GSCI Crude Oil ETN (NYSE Arca: OIL) Contract Roll Model

Date	Contract Held/Sold	Close Price	Contract Bought	Close Price	Roll Yield
12/31/10	Feb (CLG11)	91.38	began year holding CLG11		
1/14/11	Feb (CLG11)	91.54	Mar (CLH11)	92.57	(1.03)
2/15/11	Mar (CLH11)	84.32	Apr (CLJ11)	87.57	(3.25)
3/15/11	Apr (CLJ11)	97.18	May (CLK11)	97.98	(0.80)
4/15/11	May (CLK11)	109.66	Jun (CLM11)	110.22	(0.56)
5/16/11	Jun (CLM11)	97.37	Jul (CLN11)	97.85	(0.48)
6/15/11	Jul (CLN11)	94.81	Aug (CLQ11)	95.26	(0.45)
7/15/11	Aug (CLQ11)	97.24	Sep (CLU11)	97.60	(0.36)
8/15/11	Sep (CLU11)	87.88	Oct (CLV11)	88.14	(0.26)
9/15/11	Oct (CLV11)	89.40	Nov (CLX11)	89.59	(0.19)
10/14/11	Nov (CLX11)	86.80	Dec (CLZ11)	87.00	(0.20)
11/15/11	Dec (CLZ11)	99.37	Jan (CLF12)	99.43	(0.06)
12/15/11	Jan (CLF12)	93.87	Feb (CLG12)	94.07	(0.20)
12/30/11	Feb (CLG12)	98.83	ended year holding CLG12		

Net Roll Yield (7.84) or -8.58%

2011 % Change Of Nearby Crude Oil: 8.15%
2011 Performance Of OIL ETF: -1.91%
NAV 12/31/2010 25.61 NAV 12/30/2011 25.12

Sources: Barchart.com and Longview Funds Management

Figure 4: Bloomberg Commodity Index and S&P500

In this particular case, the market is in contango since the price of the new contract is consistently higher than the price of the old contract. You should recognize that a market in contango is equivalent to a market having an upward sloping forward curve. The opposite scenario is market backwardation. In this case the price of the new contract is consistently lower than the price of the old contract, and this scenario is equivalent to a market with a downward sloping forward curve.

At the bottom of Figure 4 the authors add up the monthly roll yields and obtain a net roll yield equal to -$7.84/barrel. They interpret this value as representing a -8.58 percent financial loss for the hypothetical investor. The authors conclude that holding a crude oil ETF when the market is in a state of contango typically leads to losses. In contrast, in those years when the market was in a state of backwardation the roll yield is positive and investors should a expect a positive return.

Academics have pointed out that the logic in the previous paragraph is incorrect. The roll yield does not measure a financial loss or gain but rather is a measure of the performance of the futures price relative to the spot price. Specifically, when the roll yield is negative in a contango market the index of joined futures prices lies below the spot price and so the futures market is under performing relative to the spot price. In the opposite case where the roll yield is positive in a backwardated market, the index of joined futures lies above the spot price and so the futures market is over performing relative to the spot market. Some trading professionals who understand the correct interpretation of roll yield as a differential measure of performance in the futures and spot market use this differential to link poor returns to contango markets and strong returns to backwarded markets. This line of thinking is also faulty because it fails to account for the carrying charges which are being incurred by those who hold the spot commodity (more on this below).

In summary, there are two incorrect beliefs about roll yield. The first is highlighted in Figure 4 where the negative roll yield is interpreted directly as a financial loss. The second is associating poor returns with a contango market because the joined futures price is below the spot price, and associating strong returns with a backwardated market because the joined futures price is above the spot price. Academics have collectively labeled both of these beliefs “the roll yield myth”. In the analysis below both versions of the roll yield myth are highlighted.

Unfortunately, the roll yield myth is widespread. For example, in December of 2021 if you Google “contango and investor returns” the first link which is returned is a newsletter by Fidelity (https://www.fidelity.com/learning-center/investment-products/etf/commodity-etfs-contango-backwardation) titled “Commodity ETFs: A guide to

contango and backwardation". This newsletter notes the following:

"If each subsequent month on the futures"curve" is priced higher than preceding months, a commodity is said to be in contango. The opposite situation—when subsequent months are priced lower than preceding months—is called backwardation…. An ETF that employs a basic strategy of investing in the front-month futures contract of a given commodity, for example, will either see its returns decrease in the case of contango or increase in the case of backwardation… These roll costs can be substantial. A 1% monthly cost comes to a nearly 13% cost on an annualized basis. That could wipe out any gains in the spot price, or similarly, exacerbate any losses in the spot price. "

A Formal Definition of Backwardation and Contango

In earlier modules we referred extensively to the slope of the forward curve. In this module we follow the industry naming convention:

- A market with a downward sloping forward curve is backwardated market.
- A market with an upward sloping forward curve is a contango market.

For the case of a storable agricultural commodities with a once per year harvest (e.g., corn) it is often difficult to distinguish between a backwardated market and a contango market due to the saw-toothed pricing pattern which is typically embedded in the forward curve. Specifically, the forward curve typicall shows short term backwardation with the arrival of new harvest and short term contango in the post-harvest period. We are interested in the To avoid this problem we will use data from the crude oil market to calculate and compare roll yield in a backwardated market versus a contango market.

Recall that a futures price is a measure of the spot price in Chicago (to avoid relocation costs) when the futures contract expires. Figure 5 shows the case of a backwardated market. Notice that we expect the spot price to decline over time because of a negative carrying charge (i.e., a high convenience yield). Times T_1, T_2 and T_3 on the horizontal axis represent the expire dates of the nearest three futures contracts. The vertical axis shows a mapping of the expected spot prices at these three dates. These expected spot prices are also the date 0 futures prices. This means the prices along the vertical axis is a graph (of sorts) of the forward curve. The fact that futures prices are lower for contracts with more distant expiry dates (i.e., the forward curve slopes down) confirms that this market is in backwardation.

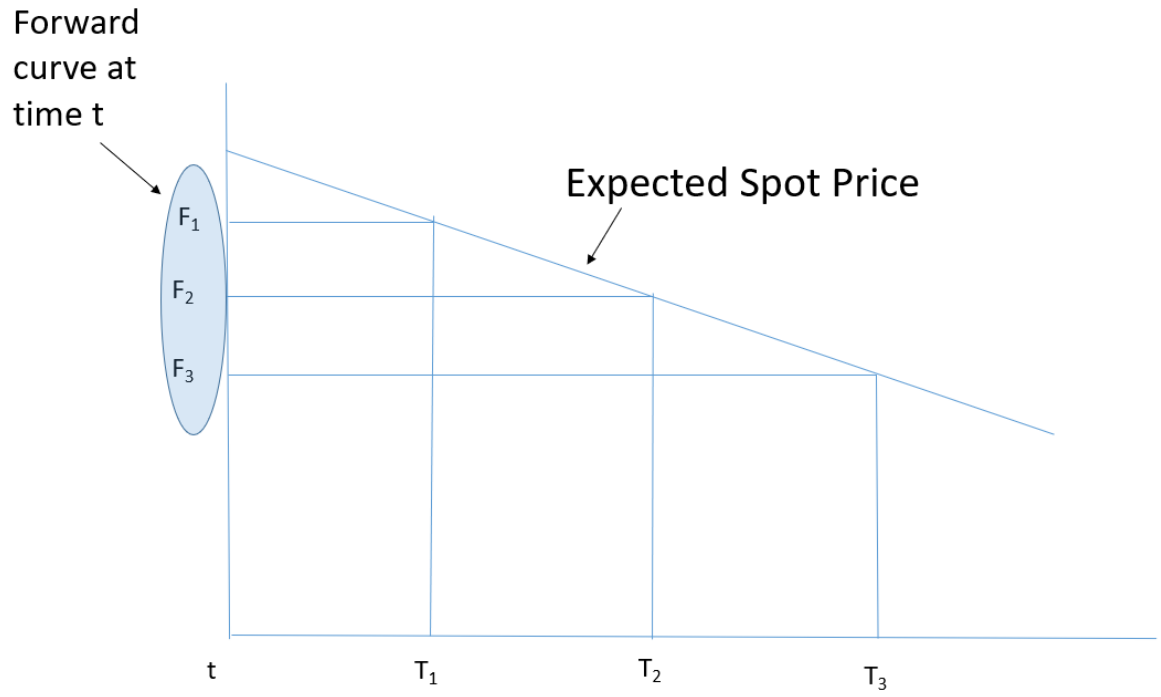

Figure 5: Expected Spot Price Path and Forward Curve with Backwardation

Figure 6 shows the case of a contango market. The graph is the same as the previous case except now we expect the spot price to rise over time due to a positive carrying charge. In this case the forward curve which is mapped on the vertical axis shows a higher futures price for contracts with a more distant expiry. This upward sloping pattern confirms the existence of a contango market.

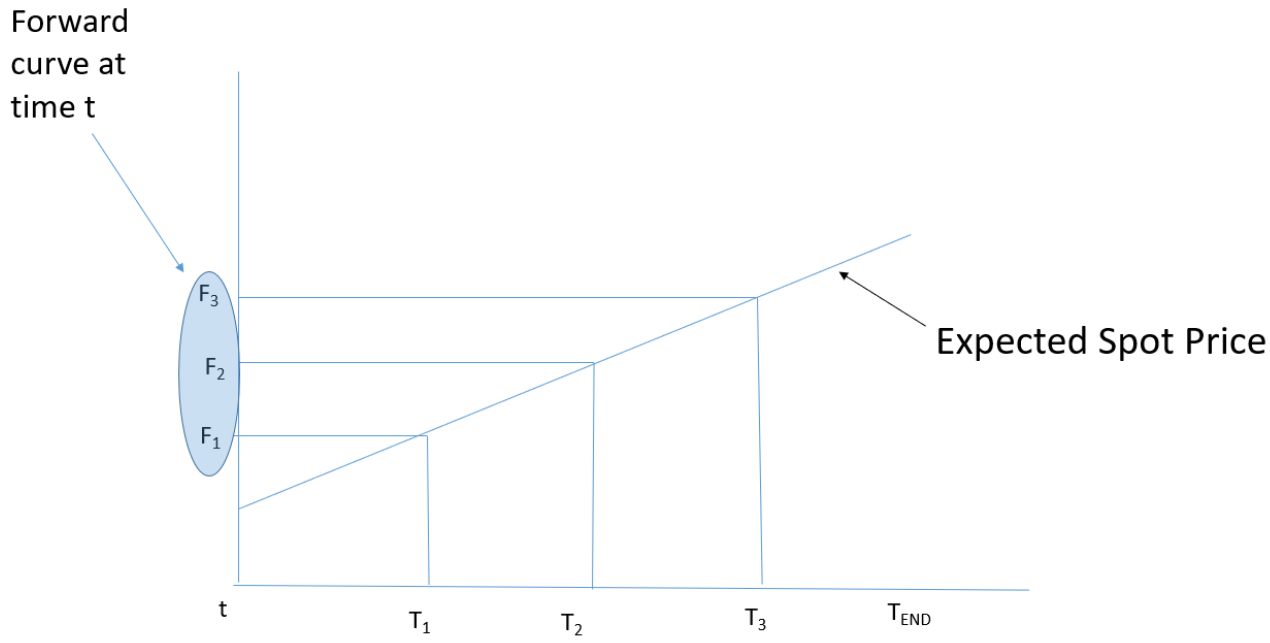

Figure 6: Expected Spot Price Path and Forward Curve with Contango

Backwardation and Contango in the Corn Market

Even though the case study below features crude oil it is useful to discuss two recent cases of pronounced backwardation and contango in eh corn market. Figure 7 shows the futures prices of the eight closest corn contracts on July 2, 2020. The months the contracts expire the labels on the horizonal axis. Note that “Jul_0” stands for the “old crop” July contract (i.e., July 2020) and “Jul_N” stands for the “new crop” July contract (i.e., July

2021). The other labels have a similar interpretation. Figure 8 shows the futures prices of the eight closest corn contracts on July 2, 2021. In this case the horizontal axis labels correspond to the old crop contracts in 2021 and the new crop contracts in 2022.

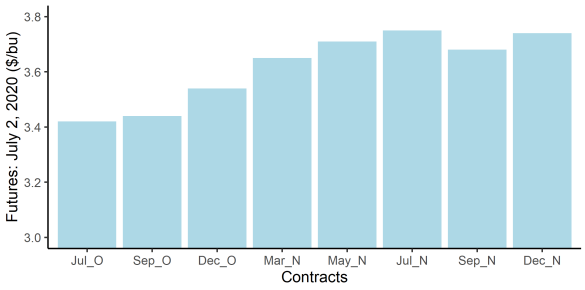

Figure 7: Contango (July 2020) in CME Corn Futures

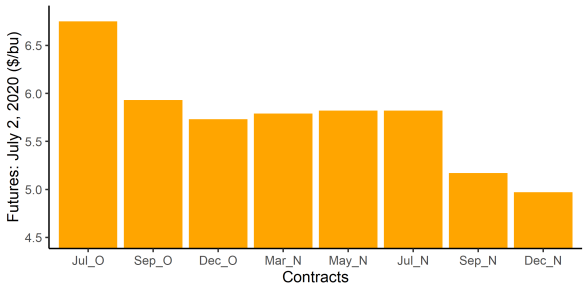

Figure 8: Backwardation (June 2021) in CME Corn Futures

It is obvious that the corn market was in a state of contanto in July of 2020, and is in a state of backwardation in June of 2021. There are easy to identify reasons for these two different outcomes. In July of 2020 the reality of the COVID-19 was sinking in and the global demand for commodities had slumped. Indeed, in June of 2020 the price of oil had recently traded in negative territory for the first time in history. The contango in the corn market reflected the relative glut of corn in the old crop year, and the cost of carrying his glut of corn foward to the new crop year. Another way of viewing this problem is that demand for corn was expected to gradually strengthen and this belief revealed itself as an upward sloping forward curve.

In contrast, Figure 8 shows that the corn market was in a state of backwardation in June of 2021. In this case the global lockdown from COVID-19 was easing and there was strong pent up demand for commodities. The high short term demand resulted in high convenience yield, which is a market signal that corn should be sold immediately rather than stored for future sale. Another way of viewing this problem is that corn stocks were tight in the spring of 2021 due to the high demand, but with the arrival of the new corn crop in the fall of 2021 prices were expected to fall. This gradual decrease in demand reveals itself as downward sloping forward curve.

Roll Yield Case Study: Crude Oil

In this section we will use spot and futures prices from the WTI crude oil market to calculate the roll yield. Once we have calculated the roll yield we will demonstrate that roll yield is nothing more than a measure of the arbitrage-enforced relationship between the prices in the spot and futures markets. After establishing what the roll yield actually is we will show what it is not, and have some interesting discussion about the roll yield myth.

Data

Spot prices for oil come from the U.S. E.I.A. (https://www.eia.gov/dnav/pet/hist/rwtcD.htm). Futures prices come the price history tab of Barchart. Two relatively short periods of time were chosen for the analysis. The first was period runs from October of 2018 to February of 2019 (five months) and the second period runs from July of 2021 to November of 2021 (five months). The spot price is measured FOB Cushings Nebraska, and this is also the delivery location for the WTI crude oil futures. This means that no location adjustment is required when comparing the spot price to the futures price. Arbitrage ensures that the spot price and futures price are equal when a futures contract expires. This means we can interpret the spot price as the price of an expiring futures contract. With this interpretation the spot price is the first price in the crude oil forward curve.

Crude Oil Market in Contango

Figure 9 shows the March 2021 crude oil futures as well as the basis which is calculated with the March 2019 contract for the first of the two time periods (i.e., October of 2018 to February of 2019). Over the first half of this period the March 2019 futures fall from about $70/barrel to $45/barrel and stayed largely flat thereafter. More importantly, Figure 9 shows that the basis was negative over the full time period. A negative basis implies that the price of expiring futures contract is less than the price of the March 2019 contract. Thus, we should expect this market to be in contango with an upward sloping forward curve. Figure 10 shows that this is indeed the case. The scale of the vertical axis can be deceiving. In reality the slope of the crude oil forward curve during this time period is very small.

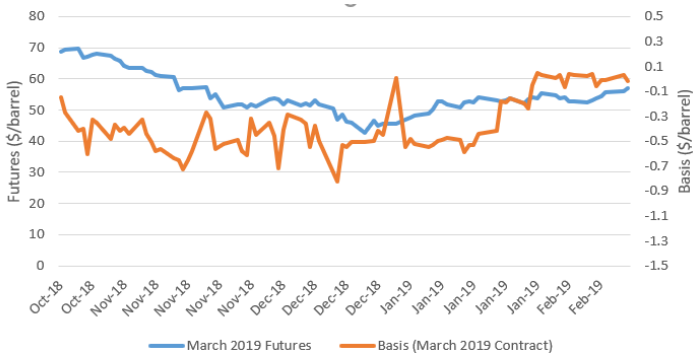

Figure 9: WTI Crude Oil Basis **with Contango**

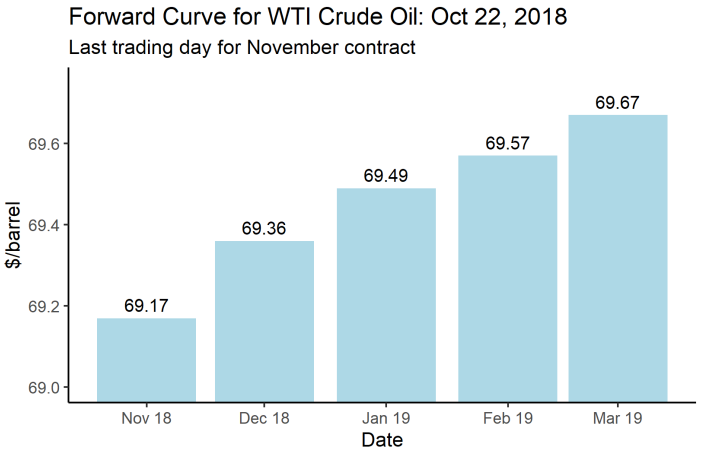

Figure 10: WTI Crude Oil Futures **with Contango**

Crude Oil Market in Backwardation

Figure 11 shows the WTI Basis and March 2021 futures for the July 2021 to November 2021 period. The price of crude oil surged upward earlier in 2021 and it is remaining at around $70/barrel throughout the July to November time period. Previously we showed that the corn market was in backwardation during this time period, and we explained the reasons for the backwardation. The positive crude oil basis in Figure 11 and the downward sloping crude oil forward curve in Figure 12 confirms that indeed the crude oil market was backwardated between July and November of 2020. The reasons for the backwardation are similar to those identified for the backwardated corn market.

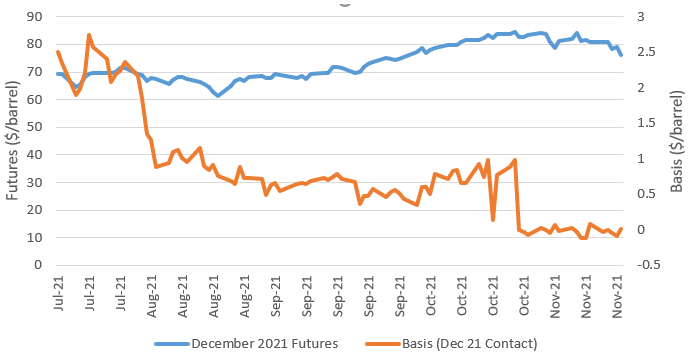

Figure 11: WTI Crude Oil Basis **with Backwardation**

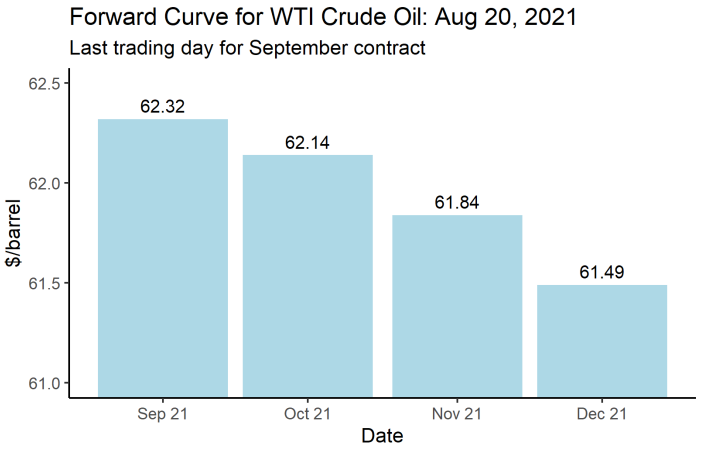

Figure 12: WTI Crude Oil Futures **with Backwardation**

Constructing a Joined Futures Price Series

Our goal in this section is to join together the prices of the five monthly futures contracts which trade in the October 2018 to February 2019 period, and in the July to November 2021 period. The joined price series is equivalent to an index which tracks the earnings of a hypothetical investor who rolls a series of long positions through time. For the example, on October 18, 2018 the investor would take a long position in the November crude oil futures. On October 22nd the November contract is closed and a new position in the December contract is pick up, which we refer to as rolling the long position from the back (expiring) contract to the new (front contract). On November 20th the long position is rolled into the January contract, and so forth. Suppose over the same period a different investor is holding the joined futures series that we will construct below. The properties of the joined futures series must be such that the earnings of the two investors are the same.

Illustrative Example

Table 1 uses a simple example to show how the roll yield and joined futures price series is created. The highly artificial time span is seven trading days. On days 1 through 3 *Fut1* is the front contract and this is the beginning of the joined futures series. On day 3 we see that expiring *Fut1* becomes the back contract and *Fut2* becomes the front contract. It is on this day that the first roll takes place. In the third row of the third (Roll12) column we enter the difference between the day 3 back price (69.17) and the day 3 front price (68.49), which is equal to 0.68. This 0.68 value is the roll adjustment between the first and second contracts. This roll adjustment is copied down until the day before the next roll adjustment.

Table 1: Example of Roll Yield Calculation

TradeDay	Fut1	Roll12	Fut2	Roll23	Fut3	Roll	CumRoll	Front	Joined
1	68.65					0.00	0.00	68.65	68.65
2	69.12					0.00	0.00	69.12	69.12
3	69.17	0.68	68.49			0.68	0.68	68.49	69.17
4		0.68	65.62			0.68	0.68	65.62	66.30
5			65.96	1.05	64.91	1.05	1.73	64.91	66.64
6				1.05	65.34	1.05	1.73	65.34	67.07
7				1.05	64.94	1.05	1.73	64.94	66.67

The next roll adjustment in Table 1 takes place on day 5. In the fifth row of the fifth column (Roll23) we enter the difference between the day 5 back price (*Fut2*=65.96) and the day 5 front price (*Fut3*=64.91), which is equal to 1.05. This roll adjustment is copied down to the end of the data series on day 7. This 1.05 value is the roll adjustment between the second and third contracts. The middle column in Table 1, which is titled *Roll*, combines the days 1 and 2 roll yields, which are equal to zero, the days 3 and 4 roll yields from the *Roll12* column and the days 5, 6 and 7 roll yields from the **Roll22* column.

There are three remaining steps to calculate the complete roll yield and joined futures series. The third column from the right in Table 1, titled *CumRoll*, is the cumulative roll yield. It is calculated by maintaining a running total of the individually-calculated roll yields from the *Roll* column. The second column from the right, called *Front*, contains the prices of the current (i.e., next to expire) futures contracts. In other words, the second column

contains all of the most recent front prices. Most importantly, the last column is the desired joined futures price sequence. It is calculated by adding the values in the *CumRoll* column to the front futures prices in the *Front* column.

Earlier we claimed that if one investor rolled a long futures position through the investment period and a second investor held the joined futures, which was made available to investors as an index, the return which is earned by the two investors must be equal. We can use the example in Table 1 to verify this outcome. Table 2 shows the contract-by-contract gains and losses for the investor who rolls the long position through the seven day time period. The first buy-sell transaction is for the investor to takes a long position in *Fut1* on day 1 a price 68.65 and offsets it on day 3 at price 69.17. On this same day the investor takes a long position in *Fut2* at price 68.49. This rolling of the long futures continues until the *Fut3* contract is offset on day 7.

The last column of Table 1 shows the profit on each buy-sell transaction. If we add these individual profits and losses together we see that the investor would have lost 1.98 per barrel. Now consider the investor who holds the joined futures series in the form of an index. The investor is assume to buy into the index on day 1 at price at 68.65 and to sell her position in the index on day 7 at price 66.71. Subtracting the first value from the latter value shows that this investor would have earned -1.98 per barrel. As predicted, the gain/loss is identical for the two investors. The reason for this outcome is explained below.

Table 2: Profits on Individual Futures Transactions

TradeDay	Fut1	Fut2	Fut3	Buy	Sell	Profit
1	68.65			68.65		
2	69.12					
3	69.17	68.49		68.49	69.17	0.52
4		65.62				
5		65.96	64.91	64.91	65.96	-2.53
6			65.34			
7			64.94		64.94	0.03

Backwardation

We will first examine roll yield in a backwardated crude oil market. Following the procedure used in the illustrative example, step one is to construct the joined futures series. Step 2 is to verify that we constructed the joined price series correctly by comparing its return to that of a hypothetical investor who continually rolls forward a long futures position. Step 3 is to add the spot price and Step 4 is to compare the spot price return and the joined futures return as a measure of the roll yield.

Step 1: Construct Joined Futures

To focus on a backwardated crude oil market we restrict our attention to the July to November 2021 period. Table 3 shows the daily prices with various rows hidden to limit the size of the table. On July 15,2021 (top row) the joined series in the right column begins tracking the price of the August 2021 futures contract, The August contract

expires five days later (on July 20th) and so this is the day within which the first roll yield is calculated, and when the September futures replaces the August futures as the front contract. Working down the columns, the roll from the September to October contract takes place on August 20th and the roll from the October to November contract takes place on September 21. The last roll from the November to the December contract takes place on October 20.

Table 3: Joined Futures with Backwardation Data

Time	Spot	Aug 21	Sep 21	Oct 21	Nov 21	Dec 21	Cum Roll	Joined
2021-07-15	71.67	71.65	71.38	70.64	69.86	69.16	0	71.65
2021-07-16	71.76	71.81	71.56	70.82	70.07	69.41	0	71.81
2021-07-19	66.45	66.42	66.35	65.76	65.12	64.55	0	66.42
2021-07-20	67.32	67.42	67.20	66.62	65.95	65.34	0.22	67.42
2021-07-21	70.26		70.30	69.57	68.78	68.05	0.22	70.52
2021-08-19	63.69		63.69	63.50	63.15	62.78	0.22	63.91
2021-08-20	62.25		62.32	62.14	61.84	61.49	0.4	62.54
2021-08-23	65.65			65.64	65.34	64.96	0.4	66.04
2021-09-20	70.41			70.29	70.14	69.73	0.4	70.69
2021-09-21	70.51			70.56	70.49	70.14	0.47	70.96
2021-09-22	72.37				72.23	71.89	0.47	72.70
2021-10-19	83.19				82.96	82.44	0.47	83.43
2021-10-20	84.40				83.87	83.42	0.92	84.34
2021-10-21	82.64					82.50	0.92	83.42
2021-11-19	76.11					76.10	0.92	77.02

The second last column in Table 3 shows the cumulative roll adjustments. The first adjustment of 0.22 coorresponds to the August to September futures contract roll. The second adjustment of 0.40 - 0.22 = 0.28 corresponds to the September to October roll. The final adjustment on October 20th, which corresponds to the November to December contract roll, brings the cumulative roll adjustment to 0.92. Following the procedure in the simplifying example, the cumulative roll adjustment is added to the current front futures prices to obtain the sequence of joined futures prices, which are shown in the last column of Table 3. The positive values of the cumulative roll adjustments make it clear that the crude oil market is in backwardation throughout the July to November 2021 period.

Steo 2: Verify Joined Futures Returns

In this section we want to verify that if an investor holds an exchange traded fund (ETF) which perfectly tracks the joined futures price which resides in the last column of Table 3 then than investor must earn the same as another investor who continually rolls long futures contracts through time. Suppose Investor A takes a long position in the joined futures at the July 15, 2021 beginning date and offsets this position at the November 19, 2021 ending date. In contrast, investor B rolls a long futures position through the five futures contracts, replacing the long position in the back contract with a long position in the front contract when the back contract expires.

Tables 4 and 5 show how profits are calculated for the two investors, and also verifies that each earns the same amount, 5.37, over the July to November time period.

Table 4: Investor A

Date	Buy	Sell	Gain
Jul 15, 2021	71.65		
Nov 19, 2021		77.02	
Cumulative Gain			**5.37**

Table 5: Investor B

Date	Buy	Sell	Gain
Aug 21	71.65	67.42	-4.23
Sep 21	67.2	62.32	-4.88
Oct 21	62.14	70.56	8.42
Nov 21	70.49	83.87	13.38
Dec 21	83.42	76.1	-7.32
Cumulative Gain			**5.37**

Step 3: Incorporate the Spot Price

The second column of Table 3 shows the spot price of crude oil over the July to November 2021 backwardation period. Arbitrage ensures that the spot price is equal to the price of the expiring futures contract. To verify that this is the case, notice that on July 15, the 71.67 spot price is very close to the 71.65 price of the expiring August futures contract. Similarly, on August 20 the 62.25 spot price is very close to the 62.32 price of the expiring September futures contract. When the time period is over on November 19, the spot price and the expiring December futures price are both equal to 76.1.

Table 6 compares the starting and ending values of the crude oil spot price with the starting and ending values of the joined crude oil futures price. It is clear that the two price series are equal at the starting point but not at the ending point. We can see that the joined futures finishes above the spot commodity. Calculations below Table 6 show that an investor who was able to hold the spot commodity without any carrying charges would earn a profit of 4.44 whereas the investor holding the joined futures would earn a profit of 5.37.

Table 6: Comparing Investor A's and B's return

Time	Spot	Aug 21	Sep 21	Oct 21	Nov 21	Dec 21	Cum Roll	Joined
2021-07-15	71.67	71.65	71.38	70.64	69.86	69.16	0.00	71.65
2021-07-16	71.76	71.81	71.56	70.82	70.07	69.41	0.00	71.81
2021-11-18	78.92					79.01	0.92	79.93
2021-11-19	76.11					76.1	0.92	77.02

An investor who held spot crude oil would earn 76.11 - 71.67 = 4.44

An investor who held joined futures would earn 77.02 - 71.65 = 5.37

Step 4: Calculate Roll Yield

From Table 7, we see that the profits from holding joined futures minus the profits from holding spot (e.g. 5.37 - 4.44 = 0.93) is approximately equal to the cumulative roll of 0.92 (in bold).

Table 7: Roll Yield Calculation

Time	Spot	Aug 21	Sep 21	Oct 21	Nov 21	Dec 21	Cum Roll	Joined
2021-07-15	71.67	71.65	71.38	70.64	69.86	69.16	0	71.65
2021-10-19	83.19				82.96	82.44	0.47	83.43
2021-10-20	84.4				83.87	83.42	**0.92**	84.34
2021-10-21	82.64					82.5	0.92	83.42

It can now be seen why roll yield was defined as such. If you had an opportunity to roll over long crude oil futures positions, which is equivalent to holding the joined futures, and holding the spot commodity at zero cost, the relative yield you would earn from holding futures is measured by the roll yield. However, by construction, the roll yield is the sum of in the individual rolls, where an individual role is a measure of the back futures price minus the front futures price when the back futures price is expiring. The spot price must equal the futures price at the time of the rollover and so it must be the case that the cumulative roll adjustment explains the gap between the ending value of the joined futures and the ending value of the spot price series.

To conclude this section, Figure 13 shows the time path of the spot price (blue) and joined futures (orange) over the last two thirds of the period in question (the graph was truncated in order to better show the gap between the two price series). This graph shows the gap between the joined futures and the spot price is growing over time due to the ever increasing roll yield. The final gap between the spot and joined futures is equal to 77.02 - 76.11 = 0.91, which is roughly equal to the 0.92 cumulative roll yield shown in Table 7.

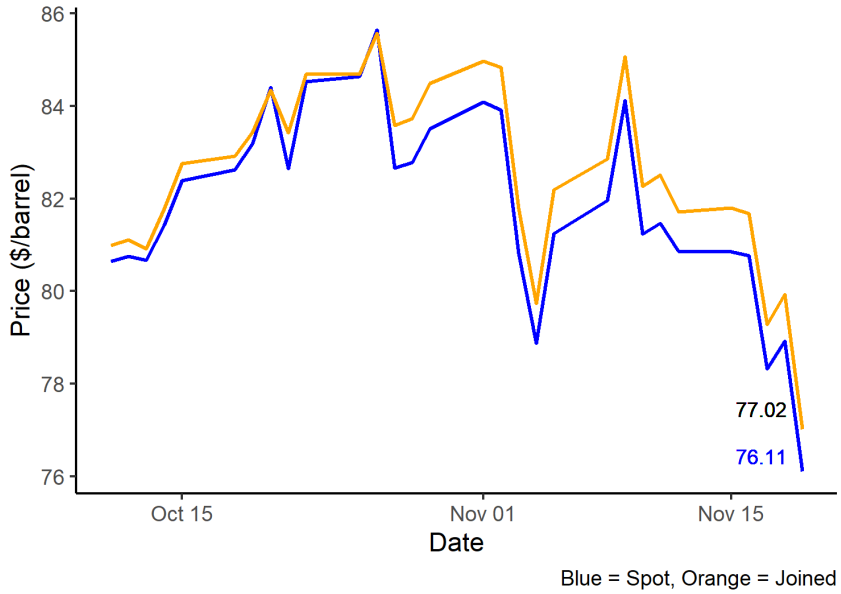

Figure 13: Joined Futures vs. Spot Price in a Backwardated Market (July 2021 - Nov 2021)

Contango Market

We will not repeat all the steps for deriving the joined futures price series and the roll yield for the October 2018 to February 2019 time period, which is what when the crude oil market was in a state of contango. The summary statistics are as follows:

- On October 18, 2018 (beginning of period) the November futures is about to expire, and so the 68.63 spot price is very close to the 68.65 joined futures price
- The roll adjustments are equal to: -0.19 for the November to December roll, -0.44 for the December to January roll, -0.97 for the January to February roll and -0.44 for the February to March roll.
- The cumulative roll adjustment totaled -2.04 over the October to February time period.
- The spot price finished the time period at 56.90, and the joined futures finished the time period at 54.88.
- The roll yield, which is the gain in the joined futures minus the gain in the spot price over the full period, is equal to (54.88 - 68.65) - (56.90 - 68.63) = -2.04
- The -2.04 roll yield is equal to the -2.04 cumulative roll adjustment, which is what theory predicts.

Figure 14 shows the time path of the crude oil joined futures price series and the crude oil spot price over the last two thirds of the October 2018 to February to 2019 time period (the first third of the graph as omitted to better emphasize the gap between the two price series). As was expected given the summary statistics presented above, in this contango market the spot price dominates the joined futures price and the gap is growing over time. RBy the end of the time period (February 2019) the joined futures price minus the spot price is given by 54.88 - 56.90, which is -2.04. This gap is of course our measure of the final roll yield, and it matches the cumulative roll adjustment, which is shown in the summary statistics (see above).

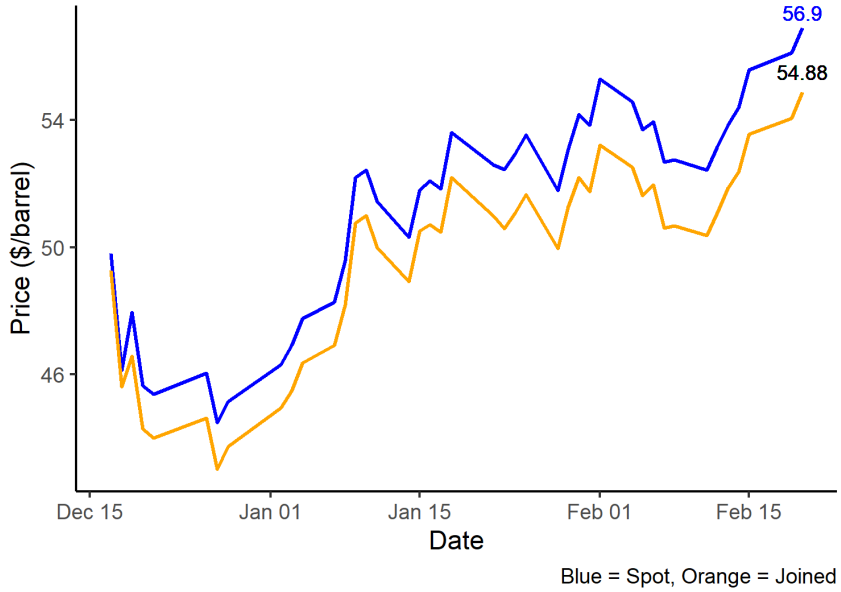

Figure 14: Joined Futures vs. Spot Price in a Contango Market (Oct 2018 - Feb 2019)

Figure 14 has an alternative interpretation. In the previous modules we emphasized that the expected futures price does not change over time because there is essentially no cost of holding futures for a risk neutral investor. In contrast, in a contango market we saw that the expected spot price increases over time in order to cover the carrying cost of grain merchants who were storing the commodity. We noted that the gap between the expected futures price and the Chicago spot price was a measures of the carrying cost from the current time period until the futures contract expires.

We can use this same logic to interpret Figure 14. If we were to shift the spot price schedule down (parallel) so that the spot price and joined futures price were equal at the end of the time period, then our revised graph would have the -2.04 gap at the beginning of the time series rather than at the end of the time series. This -2.04 gap at the beginning of the time series can then be interpreted as the carrying cost between the October 2018 begining of the time period to the February 2019 end of the time period. We now have a new result to empahsize:

- In a contango market the roll yield which measures the change in the joined futures price minus the change in the spot price is equal to cumulative roll adjustment, which itself is equal to the negative of the cost of carrying the commodity.

A different version of this same result is as follows:

- In a contango market, the difference in the return for an investor holding the spot commodity versus an investor who is rolling long futures through time is equal to the cost of carrying the commodity through time.

This last result emphasizes that roll yield is no more than a dynamic equilibrium condition which eliminates the possibility for arbitrage across the spot and futures markets. Indeed, after accounting for the carrying costs the investor who holds the spot commodity earns exactly the same return as the investor who holds an exchange traded fund (ETF) which tracks the changes in the joined futures price over time.

There is an analogous result in the backwardation market but the interpretation is less clear cut. In a contango market, after accounting for the convenience yield of the grain merchant, the return from holding the commodity in the spot market is exactly equal to the return from holding an ETF which tracks the movements in the joined futures price over time.

Roll Yield Myth Revisited

Summary and Conclusions