

Relational Algebra and Calculus

Answer 4.3 In the answers below RA refers to Relational Algebra, TRC refers to Tuple Relational Calculus and DRC refers to Domain Relational Calculus.

1. ■ RA

$$\pi_{sname}(\pi_{sid}((\pi_{pid}\sigma_{color='red'}Parts) \bowtie Catalog) \bowtie Suppliers)$$

■ SQL

```
SELECT S.sname
FROM   Suppliers S, Parts P, Catalog C
WHERE  P.color='red' AND C.pid=P.pid AND C.sid=S.sid
```

2. ■ RA

$$\pi_{sid}(\pi_{pid}(\sigma_{color='red'\vee color='green'}Parts) \bowtie catalog)$$

■ SQL

```
SELECT C.sid
FROM   Catalog C, Parts P
WHERE  (P.color = 'red' OR P.color = 'green')
AND P.pid = C.pid
```

3. ■ RA

$$\begin{aligned} &\rho(R1, \pi_{sid}((\pi_{pid}\sigma_{color='red'}Parts) \bowtie Catalog)) \\ &\rho(R2, \pi_{sid}\sigma_{address='221PackerStreet'}Suppliers) \\ &R1 \cup R2 \end{aligned}$$

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■ SQL

```
SELECT S.sid
FROM   Suppliers S
WHERE  S.address = '221 Packer street'
      OR S.sid IN ( SELECT C.sid
                    FROM   Parts P, Catalog C
                    WHERE  P.color='red' AND P.pid = C.pid )
```

4. ■ RA

$$\rho(R1, \pi_{sid}((\pi_{pid\sigma_{color='red'}} Parts) \bowtie Catalog))$$
$$\rho(R2, \pi_{sid}((\pi_{pid\sigma_{color='green'}} Parts) \bowtie Catalog))$$
$$R1 \cap R2$$

■ SQL

```

SELECT C.sid
FROM   Parts P, Catalog C
WHERE  P.color = 'red' AND P.pid = C.pid
      AND EXISTS ( SELECT P2.pid
                   FROM   Parts P2, Catalog C2
                   WHERE  P2.color = 'green' AND C2.sid = C.sid
                   AND P2.pid = C2.pid )

```

5. ■ RA

$$(\pi_{sid,pid}Catalog)/(\pi_{pid}Parts)$$

■ SQL

```

SELECT C.sid
FROM   Catalog C
WHERE  NOT EXISTS (SELECT P.pid
                  FROM   Parts P
                  WHERE  NOT EXISTS (SELECT C1.sid
                                    FROM   Catalog C1
                                    WHERE  C1.sid = C.sid
                                    AND C1.pid = P.pid))

```

6. ■ RA

$$(\pi_{sid,pid}Catalog)/(\pi_{pid}\sigma_{color='red'}Parts)$$

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■ SQL

```

SELECT C.sid
FROM   Catalog C
WHERE  NOT EXISTS (SELECT P.pid
                   FROM   Parts P
                   WHERE  P.color = 'red'
                   AND (NOT EXISTS (SELECT C1.sid
                                   FROM   Catalog C1
                                   WHERE  C1.sid = C.sid AND
                                         C1.pid = P.pid)))

```

7. ■ RA

$$(\pi_{sid, pid} Catalog) / (\pi_{pid} \sigma_{color='red' \vee color='green'} Parts)$$

■ SQL

```

SELECT C.sid
FROM   Catalog C
WHERE  NOT EXISTS (SELECT P.pid
                   FROM   Parts P
                   WHERE  (P.color = 'red' OR P.color = 'green')
                   AND (NOT EXISTS (SELECT C1.sid
                                   FROM   Catalog C1
                                   WHERE  C1.sid = C.sid AND
                                         C1.pid = P.pid)))

```

8. ■ RA

$$\begin{aligned}
 &\rho(R1, ((\pi_{sid, pid} Catalog) / (\pi_{pid} \sigma_{color='red'} Parts))) \\
 &\rho(R2, ((\pi_{sid, pid} Catalog) / (\pi_{pid} \sigma_{color='green'} Parts))) \\
 &R1 \cup R2
 \end{aligned}$$

■ SQL

```

SELECT C.sid
FROM   Catalog C
WHERE  (NOT EXISTS (SELECT P.pid
                     FROM   Parts P
                     WHERE  P.color = 'red' AND
                     (NOT EXISTS (SELECT C1.sid
                                 FROM   Catalog C1
                                 WHERE  C1.sid = C.sid AND
                                       C1.pid = P.pid))))
OR ( NOT EXISTS (SELECT P1.pid
                 FROM   Parts P1
                 WHERE  P1.color = 'green' AND
                 (NOT EXISTS (SELECT C2.sid
                             FROM   Catalog C2
                             WHERE  C2.sid = C.sid AND
                                   C2.pid = P1.pid))))

```

9. ■ RA

$$\rho(R1, Catalog)$$

$$\rho(R2, Catalog)$$

$$\pi_{R1.sid, R2.sid}(\sigma_{R1.pid=R2.pid \wedge R1.sid \neq R2.sid \wedge R1.cost > R2.cost}(R1 \times R2))$$

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■ SQL

```
SELECT C1.sid, C2.sid
FROM   Catalog C1, Catalog C2
WHERE  C1.pid = C2.pid AND C1.sid ≠ C2.sid
      AND C1.cost > C2.cost
```

10. ■ RA

$$\begin{aligned} & \rho(R1, Catalog) \\ & \rho(R2, Catalog) \\ & \pi_{R1.pid} \sigma_{R1.pid=R2.pid \wedge R1.sid \neq R2.sid} (R1 \times R2) \end{aligned}$$

■ SQL

```
SELECT C.pid
FROM   Catalog C
WHERE  EXISTS (SELECT C1.sid
               FROM   Catalog C1
               WHERE  C1.pid = C.pid AND C1.sid ≠ C.sid )
```

11. ■ RA

$$\begin{aligned} & \rho(R1, \pi_{sid} \sigma_{sname='YosemiteSham'} Suppliers) \\ & \rho(R2, R1 \bowtie Catalog) \\ & \rho(R3, R2) \\ & \rho(R4(1 \rightarrow sid, 2 \rightarrow pid, 3 \rightarrow cost), \sigma_{R3.cost < R2.cost} (R3 \times R2)) \\ & \pi_{pid} (R2 - \pi_{sid, pid, cost} R4) \end{aligned}$$

■ SQL

```
SELECT C.pid
FROM   Catalog C, Suppliers S
WHERE  S.sname = 'Yosemite Sham' AND C.sid = S.sid
      AND C.cost ≥ ALL (Select C2.cost
                        FROM   Catalog C2, Suppliers S2
                        WHERE  S2.sname = 'Yosemite Sham'
                        AND C2.sid = S2.sid)
```


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Exercise 4.5 Consider the following relations containing airline flight information:

```
Flights(flno: integer, from: string, to: string,  
        distance: integer, departs: time, arrives: time)  
Aircraft(aid: integer, aname: string, cruisingrange: integer)  
Certified(eid: integer, aid: integer)  
Employees(eid: integer, ename: string, salary: integer)
```

Note that the Employees relation describes pilots and other kinds of employees as well; every pilot is certified for some aircraft (otherwise, he or she would not qualify as a pilot), and only pilots are certified to fly.

Write the following queries in relational algebra, tuple relational calculus, and domain relational calculus. Note that some of these queries may not be expressible in relational algebra (and, therefore, also not expressible in tuple and domain relational calculus)! For such queries, informally explain why they cannot be expressed. (See the exercises at the end of Chapter 5 for additional queries over the airline schema.)

1. Find the *eids* of pilots certified for some Boeing aircraft.
2. Find the *names* of pilots certified for some Boeing aircraft.
3. Find the *aids* of all aircraft that can be used on non-stop flights from Bonn to Madras.
4. Identify the flights that can be piloted by every pilot whose salary is more than \$100,000.
5. Find the names of pilots who can operate planes with a range greater than 3,000 miles but are not certified on any Boeing aircraft.

6. Find the *eids* of employees who make the highest salary.
7. Find the *eids* of employees who make the second highest salary.
8. Find the *eids* of employees who are certified for the largest number of aircraft.
9. Find the *eids* of employees who are certified for exactly three aircraft.
10. Find the total amount paid to employees as salaries.
11. Is there a sequence of flights from Madison to Timbuktu? Each flight in the sequence is required to depart from the city that is the destination of the previous flight; the first flight must leave Madison, the last flight must reach Timbuktu, and there is no restriction on the number of intermediate flights. Your query must determine whether a sequence of flights from Madison to Timbuktu exists for *any* input Flights relation instance.

Answer 4.5 In the answers below RA refers to Relational Algebra, TRC refers to Tuple Relational Calculus and DRC refers to Domain Relational Calculus.

1. ■ RA

$$\pi_{eid}(\sigma_{aname='Boeing'}(Aircraft \bowtie Certified))$$

- SQL

```
SELECT C.eid
FROM   Aircraft A, Certified C
WHERE  A.aid = C.aid AND A.aname = 'Boeing'
```

2. ■ RA

$$\pi_{ename}(\sigma_{aname='Boeing'}(Aircraft \bowtie Certified \bowtie Employees))$$

■ SQL

```
SELECT E.ename
FROM   Aircraft A, Certified C, Employees E
WHERE  A.aid = C.aid AND A.aname = 'Boeing' AND E.eid = C.eid
```

3. ■ RA

$$\rho(BonnToMadrid, \sigma_{from='Bonn' \wedge to='Madrid'}(Flights))$$

$$\pi_{aid}(\sigma_{cruisingrange > distance}(Aircraft \times BonnToMadrid))$$

■ SQL

```
SELECT A.aid
FROM   Aircraft A, Flights F
WHERE  F.from = 'Bonn' AND F.to = 'Madrid' AND
       A.cruisingrange > F.distance
```

4. ■ RA

$$\pi_{fno}(\sigma_{distance < cruisingrange \wedge salary > 100,000}(Flights \bowtie Aircraft \bowtie$$

$$Certified \bowtie Employees)))$$

■ SQL

```
SELECT E.ename
FROM   Aircraft A, Certified C, Employees E, Flights F
WHERE  A.aid = C.aid AND E.eid = C.eid AND
       distance < cruisingrange AND salary > 100,000
```

5. ■ RA $\rho(R1, \pi_{eid}(\sigma_{cruisingrange > 3000}(Aircraft \bowtie Certified)))$
 $\pi_{ename}(Employees \bowtie (R1 - \pi_{eid}(\sigma_{aname='Boeing'}(Aircraft \bowtie Certified))))$

=

■ SQL

```
SELECT E.ename
FROM   Certified C, Employees E, Aircraft A
WHERE  A.aid = C.aid AND E.eid = C.eid AND A.cruisingrange > 3000
AND E.eid NOT IN ( SELECT C2.eid
FROM Certified C2, Aircraft A2
WHERE C2.aid = A2.aid AND A2.aname = 'Boeing' )
```

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6. ■ RA

The approach to take is first find all the employees who do not have the highest salary. Subtract these from the original list of employees and what is left is the highest paid employees.

$$\begin{aligned} &\rho(E1, Employees) \\ &\rho(E2, Employees) \\ &\rho(E3, \pi_{E2.eid}(E1 \bowtie_{E1.salary > E2.salary} E2)) \\ &(\pi_{eid} E1) - E3 \end{aligned}$$

■ SQL

```
SELECT E.eid
FROM   Employees E
WHERE  E.salary = ( Select MAX (E2.salary)
                  FROM   Employees E2 )
```

7. ■ RA

The approach taken is similar to the solution for the previous exercise. First find all the employees who do not have the highest salary. Remove these from the original list of employees and what is left is the highest paid employees. Remove the highest paid employees from the original list. What is left is the second highest paid employees together with the rest of the employees. Then find the highest paid employees of this new list. This is the list of the second highest paid employees.

$$\begin{aligned} &\rho(E1, Employees) \\ &\rho(E2, Employees) \\ &\rho(E3, \pi_{E2.eid}(E1 \bowtie_{E1.salary > E2.salary} E2)) \\ &\rho(E4, E2 \bowtie E3) \\ &\rho(E5, E2 \bowtie E3) \\ &\rho(E6, \pi_{E5.eid}(E4 \bowtie_{E1.salary > E5.salary} E5)) \\ &(\pi_{eid} E3) - E6 \end{aligned}$$

■ SQL

```

SELECT E.eid
FROM   Employees E
WHERE  E.salary = (SELECT MAX (E2.salary)
                  FROM   Employees E2
                  WHERE  E2.salary ≠ (SELECT MAX (E3.salary)
                                      FROM   Employees E3 ))

```

8. This cannot be expressed in relational algebra (or calculus) because there is no operator to count, and this query requires the ability to count up to a number that depends on the data. The query can however be expressed in SQL as follows:

```

SELECT Temp.eid
FROM   ( SELECT   C.eid AS eid, COUNT (C.aid) AS cnt,
              FROM   Certified C
              GROUP BY C.eid) AS Temp
WHERE  Temp.cnt = ( SELECT   MAX (Temp.cnt)
                  FROM     Temp)

```

9. ■ RA

The approach behind this query is to first find the employees who are certified for at least three aircraft (they appear at least three times in the Certified relation). Then find the employees who are certified for at least four aircraft. Subtract the second from the first and what is left is the employees who are certified for exactly three aircraft.

$$\begin{aligned}
 &\rho(R1, \text{Certified}) \\
 &\rho(R2, \text{Certified}) \\
 &\rho(R3, \text{Certified}) \\
 &\rho(R4, \text{Certified}) \\
 &\rho(R5, \pi_{eid}(\sigma_{(R1.eid=R2.eid=R3.eid) \wedge (R1.aid \neq R2.aid \neq R3.aid)}(R1 \times R2 \times R3))) \\
 &\rho(R6, \pi_{eid}(\sigma_{(R1.eid=R2.eid=R3.eid=R4.eid) \wedge (R1.aid \neq R2.aid \neq R3.aid \neq R4.aid)}(R1 \times R2 \times R3 \times R4)))
 \end{aligned}$$

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$$(R1 \times R2 \times R3 \times R4)))$$
$$R5 - R6$$

■ SQL

```
SELECT C1.eid
FROM   Certified C1, Certified C2, Certified C3
WHERE  (C1.eid = C2.eid AND C2.eid = C3.eid AND
        C1.aid ≠ C2.aid AND C2.aid ≠ C3.aid AND C3.aid ≠ C1.aid)

EXCEPT
SELECT C4.eid
FROM   Certified C4, Certified C5, Certified C6, Certified C7,
WHERE  (C4.eid = C5.eid AND C5.eid = C6.eid AND C6.eid = C7.eid AND
        C4.aid ≠ C5.aid AND C4.aid ≠ C6.aid AND C4.aid ≠ C7.aid AND
        C5.aid ≠ C6.aid AND C5.aid ≠ C7.aid AND C6.aid ≠ C7.aid )
```

This could also be done in SQL using COUNT.

10. This cannot be expressed in relational algebra (or calculus) because there is no operator to sum values. The query can however be expressed in SQL as follows:

```
SELECT SUM (E.salaries)
FROM   Employees E
```

11. This cannot be expressed in relational algebra or relational calculus or SQL. The problem is that there is no restriction on the number of intermediate flights. All of the query methods could find if there was a flight directly from Madison to Timbuktu and if there was a sequence of two flights that started in Madison and ended in Timbuktu. They could even find a sequence of n flights that started in Madison and ended in Timbuktu as long as there is a static (i.e., data-independent) upper bound on the number of intermediate flights. (For large n , this would of course be long and impractical, but at least possible.) In this query, however, the upper bound is not static but dynamic (based upon the set of tuples in the Flights relation).

In summary, if we had a static upper bound (say k), we could write an algebra or SQL query that repeatedly computes (upto k) joins on the Flights relation. If the upper bound is dynamic, then we cannot write such a query because k is not known when writing the query.