Multiplayer Reach-Avoid Games, Single Defender versus Two Attackers

***Abstract*— We consider a multiplayer reach-avoid game with an equal number of attackers and defenders moving with simple dynamics on a two-dimensional domain possibly with obstacles. The attacking team attempts to win the game by sending at least m attackers to a certain target location quickly while the defenders aim to capture the attackers to prevent the attacking team from reaching its goal. The analysis of problems like this plays an important role in collision avoidance, motion planning, and aircraft control, among other applications. Computing optimal solutions for such multiplayer games is intractable due to the curse of dimensionality. This paper provides a first attempt to address such computational intractability by combining maximum matching in graph theory with the classical Hamilton-Jacobi-Isaacs approach. In addition, our solution provides an initial step to take cooperation into account by computing maximum matching in real time.**

I. Introduction

II. Problem Formulation

We consider a reach-avoid game between two attackers, P_{A_1}, P_{A_2}, and a defenders, P_D. Each player is confined in a bounded, open domain $\Omega \subset \mathbb{R}^2$, which can be partitioned as follows: $\Omega = \Omega_{free} \cup \Omega_{obs}$. Ω_{free} is a compact set representing the free space in which the players can move, while $\Omega_{obs} = \Omega \backslash \Omega_{free}$ represents the obstacles that obstruct movement in the domain. Let $x_{A_1}, x_{A_2}, x_D \in \mathbb{R}^2$ denote the states of the players P_{A_1}, P_{A_2}, P_D, respectively. Initial conditions of the players are denoted by $x^0_{A_1}, x^0_{A_2}, x_D \in \Omega_{free}$. We assume that the dynamics of the players are defined by the following decoupled system for $t \geq 0$:

$$\begin{aligned} \dot{x}_{A_1}(t) &= v_{A_1} a_1(t), \quad & x_{A_1}(0) &= x^0_{A_1}, \\ \dot{x}_{A_2}(t) &= v_{A_2} a_2(t), \quad & x_{A_2}(0) &= x^0_{A_2}, \\ \dot{x}_D(t) &= v_D d(t), & x_D(0) &= x^0_D \end{aligned} \tag{1}$$

where $a_1(\cdot), a_2(\cdot), d(\cdot)$ represent the control functions of P_{A_1}, P_{A_2}, and P_D, respectively. The attackers P_{A_1}, P_{A_2} have respective maximum speeds v_{A_1} and v_{A_2} and the defender P_D has maximum speed v_D. We assume that the control functions $a_1(\cdot), a_2(\cdot), d(\cdot)$ are drawn from the set $\Sigma = \{\sigma\colon [0,\infty) \to \overline{B}_n \mid \sigma \text{ is measurable}\}$, where $\overline{B}_n$ denotes the closed unit ball in $\mathbb{R}^2$. As a clarification on the notation and terminology, the control functions (with a dot notation, e.g. $a_i(\cdot), d_i(\cdot), u(\cdot)$ etc.) which are the entire control trajectories, are distinguished from the control inputs (such as $a_i, a_i(t), d_i, d_i(t)$ etc.) which are the instantaneous control inputs. Furthermore, given $x^0_{A_i} \in \Omega_{free}$, we define the admissible control function set for P_{A_i} to be the the set of all control functions such that $x_{A_i}(t) \in \Omega_{free}, \forall i$, $\forall t \geq 0$. The admissible control function set for defenders $P_{D_i}, i = 1, 2, \ldots, N$ is defined similarly, given that $x^0_{D_i} \in \Omega_{free}$. The joint state of all the players is denoted by $\mathbf{x} = (x_{A_1}, \ldots x_{A_N}, x_{D_1}, \ldots, x_{D_N})$. The joint initial condition is denoted by $\mathbf{x^0} = (x^0_{A_1}, \ldots, x^0_{A_N}, x^0_{D_1}, \ldots, x^0_{D_N})$.

In this reach-avoid game, the attacking team aims to reach the target $\mathcal{T} \subset \Omega_{free}$, a compact subset of the domain, without getting captured by the defenders. The capture conditions are formally described by the capture sets $\mathcal{C}_{ij} \subset \Omega^{2N}$ for the pairs of the players $\left(P_{A_i}, P_{D_j}\right), i, j = 1, \ldots, N$. In general, $\mathcal{C}_{ij}$ can be an arbitrary compact subset of Ω^{2N}, which represents the set of the joint player states $\mathbf{x}$ at which P_{A_i} is captured by P_{D_j}. Hence, in the general case, the interpretation of capture is given by the set $\mathcal{C}_{ij}$, which in turn depends on the specific situation one wishes to model. In this paper, we define the capture sets to be $\mathcal{C}_{ij} = \left\{\mathbf{x} \in \Omega^{2N} \mid \|x_{A_i} - x_{D_j}\|_2 \leq R_C\right\}$, the interpretation of which is that P_{A_i} is captured by P_{D_j} if P_{A_i}'s position is within R_C of P_{D_j}'s position.

In our multiplayer reach-avoid game, the team of the attackers $\{P_{A_i}\}_{i=1}^N$ wins when at least m attackers reach the target $\mathcal{T}$ without being captured. The team of the defenders $\{P_{D_i}\}_{i=1}^N$ wins the game if they can delay at least $N - m + 1$ attackers from reaching the target indefinitely. An illustration of the game setup is shown in Figure **??**.

With the above definitions, we now state the main question that this paper answers. Given the minimum number m of the attackers that need to reach the target, the joint initial state $\mathbf{x^0}$ of all the players in domain Ω with obstacles Ω_{obs}, the target set $\mathcal{T}$, and the capture sets $\mathcal{C}_{ij}$, can the defenders be guaranteed to win?

III. Solution

IV. Computation Results

V. Conclusions and Future Work

References

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