Large-Scale Robust Sequential Path Planning

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Abstract

To be written.

I. Introduction

Focus on the following contributions:

- Present a powerful theory for robust path planning. Focus on explicitly pointing out the limitations of the intruder Method-1 and how we are addressing them in the current paper.
- Demonstrate the scaling of the theory.
- Provide some more intuition about the solution that emerge out of theory– Space-time separation, buffer region between vehicles, trajectories, etc.
- Explaining how this solution change with the space structure (city vs bay area) and with the disturbance bounds.
- Zero communication overhead due to the presence of a feedback law! (Important based on IoT Journal description).

II. Problem Formulation

Directly form the full problem that includes the intruder.

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III. Background

We need to include following things in the background:

- Time-varying reachability
- Vanilla SPP
- Robust trajectory tracking algorithm

IV. Response to Intruders

Most of the assumption/notation content in this section will be moved to the problem formulation section. In Section **??**, we made the basic SPP algorithm more robust by taking into account disturbances and considering situations in which vehicles may not have complete information about the control strategy of the other vehicles. However, if a vehicle not in the set of SPP vehicles enters the system, or even worse, if this vehicle is an adversarial intruder, the original plan can lead to vehicles entering into each other's danger zones. If vehicles do not plan with an additional safety margin that takes a potential intruder into account, a vehicle trying to avoid the intruder may effectively become an intruder itself, leading to a domino effect. In this section, we propose a method to allow vehicles to avoid an intruder while maintaining the SPP structure.

In general, the effect of an intruder on the vehicles in structured flight can be entirely unpredictable, since the intruder in principle could be adversarial in nature, and the number of intruders could be arbitrary. Therefore, for our analysis to produce reasonable results, two assumptions about the intruders must be made.

Assumption 1: At most one intruder (denoted as Q_I here on) affects the SPP vehicles at any given time. The intruder exits the altitude level affecting the SPP vehicles after a duration of t^{IAT}.

Let the time at which intruder appears in the system be $\underline{t}$ and the time at which it disappears be $\bar{t}$. Assumption 1 implies that $\bar{t} \leq \underline{t} + t^{\text{IAT}}$. Thus, any vehicle Q_i would need to avoid the intruder Q_I for a maximum duration of t^{IAT}. This assumption can be valid in situations where intruders are rare, and that some fail-safe or enforcement mechanism exists to force the intruder out of the altitude level affecting the SPP vehicles. Note that we do not make any assumptions about $\underline{t}$; however, we assume that once the intruder appears, it stays for a maximum duration of t^{IAT}.

Assumption 2: The dynamics of the intruder are known and given by $\dot{x}_I = f_I(x_I, u_I, d_I)$. The initial state of the intruder is given by x_I^0.

Assumption 2 is required for HJ reachability analysis. In situations where the dynamics of the intruder are not known exactly, a conservative model of the intruder may be used instead.

Based on the above assumptions, we aim to design a control policy that ensures separation with the intruder and with other SPP vehicles, and ensures a successful transit to the destination. However, depending on the initial state of the intruder, its control policy, and the disturbances in the dynamics of a vehicle and the intruder, a vehicle may arrive at different states after avoiding the intruder. Therefore, a control policy that ensures a successful transit to the destination needs to account for all such possible states, which is a path planning problem with multiple (infinite, to be precise) initial states and a single destination, and is hard to solve in general. Thus, we divide the intruder avoidance problem into two sub-problems: (i) we first design a control policy that ensures a successful transit to the destination if no intruder appears and that successfully avoid the intruder, if it does. (ii) after the intruder disappears at $\bar{t}$, we replan the trajectories of the affected vehicles. Following the same theme, authors in [] present an algorithm to avoid an intruder in SPP formulation; however, once the intruder disappears, the algorithm might need to replan the trajectories for all SPP vehicles. Since the replanning is done in real-time, it should be fast and scalable with the number of SPP vehicles, rendering the method in [] unsuitable for practical implementation.

In this work, we propose a novel intruder avoidance algorithm, which will need to replan trajectories only for a fixed number of vehicles, irrespective of the total number of SPP vehicles. Moreover, this number is a design parameter, which can be chosen based on the resources available during the run time. Intuitively, one can think about dividing the flight space of vehicles such that at any given time, any two vehicles are far enough from each other so that an intruder can only affect atmost k vehicles in a duration of t^{IAT} despite its best efforts. The advantage of this approach is that after the intruder disappears, we only have to replan the trajectory of k vehicles regardless of the number of total vehicles in the system, which makes this approach particularly suitable for practical systems. In this method, we build upon this intuition and show that such a division of space is indeed possible. Thus the proposed method guarantees that *atmost* k vehicles are affected by the presence of intruder, regardless of the number of SPP vehicles, and hence the replanning can be efficiently done in real-time.

In Sections IV-A and IV-B, we compute a space division of state-space such that atmost k vehicles need to apply the avoidance maneuver regardless of the initial state of the intruder. However, we still need to ensure that vehicles do not collide with each other while avoiding the intruder. The induced obstacles that reflect this possibility are computed in Section **??**. Intruder avoidance control and re-planning are discussed in Section **??**.

A. Separation Region

Depending on the information known to a lower-priority vehicle Q_i about Q_j's control strategy, we can use one of the three methods described in Section 5 in First journal paper should be cited here to compute the "base" obstacles $\mathcal{M}_j(t)$, the obstacles that would have been induced by Q_j in the absence of an intruder.

Given $\mathcal{M}_j(t)$, we want to compute the set of all initial states of the intruder for which vehicle Q_j may have to apply an avoidnace maneuver. We refer to this set as *separation region* here on, and denote it as $\mathcal{S}_j(t)$. The significance of $\mathcal{S}_j(t)$ is that as long as the intruder is outside $\mathcal{S}_j(t)$, that is $x_I(t) \in (\mathcal{S}_j(t))^c$, Q_j can apply any control at time t and still guaranteed to not collide with the intruder. $\mathcal{S}_j(t)$ can be conveniently computed using the relative dynamics between Q_j and Q_I.

We define relative dynamics of the intruder Q_I with state x_I with respect to Q_i with state x_i:

$$\begin{aligned} x_{I,i} &= x_I - x_i \\ \dot{x}_{I,i} &= f_r(x_{I,i}, u_i, u_I, d_i, d_I) \end{aligned} \tag{1}$$

Given the relative dynamics, we compute the set of states from which the joint states of Q_I and Q_j can enter danger zone $\mathcal{Z}_{jI}$ in a duration of t^{IAT} despite the best efforts of Q_j to avoid Q_I. Under the relative dynamics (1), this set of states is given by the backwards reachable set $\mathcal{V}_j^{\text{S}}(\tau, t^{\text{IAT}})$:

$$\begin{aligned} \mathcal{V}_j^{\text{S}}(\tau, t^{\text{IAT}}) = &\{y : \forall u_j(\cdot) \in \mathbb{U}_j, \exists u_I(\cdot) \in \mathbb{U}_I, \exists d_j(\cdot) \in \mathbb{D}_j, \\ &\exists d_I(\cdot) \in \mathbb{D}_I, x_{I,j}(\cdot) \text{ satisfies (1)}, \\ &\exists s \in [\tau, t^{\text{IAT}}], x_{I,j}(s) \in \mathcal{L}_j^{\text{S}}, x_{I,j}(\tau) = y\}, \end{aligned} \tag{2}$$

where

$$\begin{aligned} \mathcal{L}_j^{\text{S}} &= \{x_{I,j} : \|p_{I,j}\|_2 \le R_c\} \\ H_j^{\text{S}}(x_{I,j}, \lambda) &= \max_{u_j \in \mathcal{U}_j} \left(\min_{u_I \in \mathcal{U}_I, d_i \in \mathcal{D}_i, d_I \in \mathcal{D}_I} \lambda \cdot f_r(x_{I,j}, u_j, u_I, d_j, d_I) \right) \end{aligned} \tag{3}$$

The interpretation of set $\mathcal{V}_j^{\text{S}}(0, t^{\text{IAT}})$ is that as long as Q_I is outside the boundary of this set (in relative coordinates), then Q_I and Q_j cannot enter the danger zone $\mathcal{Z}_{jI}$, irrespective of control applied by them. We can now transform $\mathcal{V}_j^{\text{S}}(0, t^{\text{IAT}})$ to absolute coordinates to obtain sets $\mathcal{S}_j(\cdot)$ as follows:

$$\mathcal{S}_j(t) = \mathcal{M}_j(t) + \mathcal{V}_j^{\text{S}}(0, t^{\text{IAT}}), \tag{4}$$

where the "+" in (4) denotes the Minkowski sum[1].

B. *Buffer Region*

In section IV-A, we computed sets $\mathcal{S}_j(\cdot)$ such that Q_j avoids the intruder only if $x_I(t) \in \mathcal{S}_j(t)$. But to ensure that atmost $\bar{k}$ vehicles need to replan their trajectories after the intruder disappears, we need to make sure that the intruder can cause atmost $\bar{k}$ vehicles to deviate from their planned trajectories. Equivalently, we want to ensure that atmost $\bar{k}$ vehicles need to avoid the intruder.

Intuitively, we want to make sure that at any given time the separation regions of any two vehicles are far enough from each other such that it will take at least $t^{\text{BRD}} := t^{\text{IAT}}/\bar{k}$ time for the intruder to go from the separation region of one vehicle to that of the other vehicle. This means that there is a "buffer" time interval of t^{BRD} between any t_1, $t_2 \in [\underline{t}, \bar{t}]$ where Q_I is in the separation regions of two different vehicles at t_1 and t_2, e.g. $x_I(t_1) \in \mathcal{S}_j(t)$ and $x_I(t_2) \in \mathcal{S}_i(t)$, $i \neq j$. Thus, in the worst case, the intruder can force atmost $\bar{k}$ vehicles to apply avoidance maneuver in a duration of t^{IAT}.

start from here

But to ensure that atmost one vehicle avoids the intruder, we also need to make sure that no other vehicle $Q_i, i > j$, avoids the intruder if it appears inside $sep_j(\underline{t})$. This can be achieved by ensuring that vehicle Q_i is far enough from $\mathcal{S}_j(\cdot)$ (that is, there is a "buffer" region between Q_i and $\mathcal{S}_j(\cdot)$), such that intruder appearing inside $sep_j(\cdot)$ cannot enter the danger zone $\mathcal{Z}_{iI}$. We denote this buffer region as $\mathcal{B}^i(t)$, and the separation region augmented with the buffer region as $\tilde{\mathcal{S}}_i^j(t)$.

$\mathcal{B}^i(t)$ can be computed using the relative dynamics

However, to ensure that atmost one vehicle needs to apply the avoidance maneuver, we need to make sure that no other vehicle $Qi, i \neq j$, needs to avoid the intruder if intruder starts inside $\mathcal{V}_{\text{C}}$. This can be achieved by ensuring that $\mathcal{V}_{\text{C},i}$ and $\mathcal{V}_{\text{C},j}$ do not intersect.

[1]The Minkowski sum of sets A and B is the set of all points that are the sum of any point in A and B.

The folliwng need to be moved to the buffer region section when both Q_j and Q_I are using *optimal control to collide* with each other for a duration of t^{IAT}.

$$\begin{aligned}\mathcal{V}_j^{\text{S}}(t, t^{\text{IAT}}) =&\{y : \exists u_j(\cdot) \in \mathbb{U}_j, u_I(\cdot) \in \mathbb{U}_I, d_j(\cdot) \in \mathbb{D}_j,\\ &d_I(\cdot) \in \mathbb{D}_I, x_{I,j}(\cdot) \text{ satisfies (1)},\\ &\exists s \in [t, t^{\text{IAT}}], x_{I,j}(s) \in \mathcal{L}_j^{\text{S}}, x_{I,j}(t) = y\},\end{aligned} \tag{5}$$

where

$$\begin{aligned}\mathcal{L}_j^{\text{S}} &= \{x_{I,j} : \|p_{I,j}\|_2 \leq R_c\}\\ H_j^{\text{S}}(x_{I,j}, \lambda) &= \min_{u_j \in \mathcal{U}_j, u_I \in \mathcal{U}_I, d_i \in \mathcal{D}_i, d_I \in \mathcal{D}_I} \lambda \cdot f_r(x_{I,j}, u_j, u_I, d_j, d_I)\end{aligned} \tag{6}$$

To-Dos:

- A remark about the single vehicle replanning property of Method-2. Moreover, Method-2 can, in theory, handle multiple intruders as long as they are affecting different vehicles. Though, we have to replan for several vehicles in that case.
- Once the replanning is complete, another intruder can appear in the system. So strictly speaking we are making an assumption that atmost one intruder is in the system *at any given time* as opposed to throughout the trajectory.
- For method-2 results, it may be helpful to include a figure which is showing the division of space among vehicles at some time (probably right before the intruder enters).

V. Simulations

Focus on the following aspects:

- Demonstration of theory (that it avoids collision w/ other vehicles and intruders, and we reach our destinations).
- Scaling of SPP.
- Reactivity of controller to the actual disturbance (Claire: be very detailed about explaining the setup of simulation)
- Illustrate the structure that emerge out of SPP algorithm (Almost straight line path w/ different starting times)
- Illustrate how this structure change with change in disturbance bounds (Straight line trajectories become curvy?)

Also mention the technical details for the simulations, like RTT parameters, relative co-ordinate dynamics, rotation and translation of obstacles, union for obstacles, etc.

Let's pick speed $1.5Km/min$ (or $2.5Decametre/s$) and turnrate to be $120rad/min$ ($2rad/s$). Let's use grid to be $[0, 500]Dm$.