Large-Scale Robust Sequential Path Planning

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Abstract

To be written.

I. INTRODUCTION

Focus on the following contributions:

- Present a powerful theory for robust path planning. Focus on explicitly pointing out the limitations of the intruder Method-1 and how we are addressing them in the current paper.
- Demonstrate the scaling of the theory.
- Provide some more intuition about the solution that emerge out of theory– Space-time separation, buffer region between vehicles, trajectories, etc.
- Explaining how this solution change with the space structure (city vs bay area) and with the disturbance bounds.
- Zero communication overhead due to the presence of a feedback law! (Important based on IoT Journal description).

II. PROBLEM FORMULATION

Directly form the full problem that includes the intruder.

This work has been supported in part by NSF under CPS:ActionWebs (CNS-931843), by ONR under the HUNT (N0014-08-0696) and SMARTS (N00014-09-1-1051) MURIs and by grant N00014-12-1-0609, by AFOSR under the CHASE MURI (FA9550-10-1-0567). The research of M. Chen and J. F. Fisac have received funding from the "NSERC" program and "la Caixa" Foundation, respectively.

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III. Background

We need to include following things in the background:

- Time-varying reachability
- Vanilla SPP
- Robust trajectory tracking algorithm

IV. Response to Intruders

Most of the assumption/notation content in this section will be moved to the problem formulation section. In Section **??**, we made the basic SPP algorithm more robust by taking into account disturbances and considering situations in which vehicles may not have complete information about the control strategy of the other vehicles. However, if a vehicle not in the set of SPP vehicles enters the system, or even worse, if this vehicle is an adversarial intruder, the original plan can lead to vehicles entering into each other's danger zones. If vehicles do not plan with an additional safety margin that takes a potential intruder into account, a vehicle trying to avoid the intruder may effectively become an intruder itself, leading to a domino effect. In this section, we propose a method to allow vehicles to avoid an intruder while maintaining the SPP structure.

In general, the effect of an intruder on the vehicles in structured flight can be entirely unpredictable, since the intruder in principle could be adversarial in nature, and the number of intruders could be arbitrary. Therefore, for our analysis to produce reasonable results, two assumptions about the intruders must be made.

Assumption 1: At most one intruder (denoted as Q_I here on) affects the SPP vehicles at any given time. The intruder exits the altitude level affecting the SPP vehicles after a duration of t^{IAT}.

Let the time at which intruder appears in the system be $\underline{t}$ and the time at which it disappears be $\bar{t}$. Assumption 1 implies that $\bar{t} \leq \underline{t} + t^{\text{IAT}}$. Thus, any vehicle Q_i would need to avoid the intruder Q_I for a maximum duration of t^{IAT}. This assumption can be valid in situations where intruders are rare, and that some fail-safe or enforcement mechanism exists to force the intruder out of the altitude level affecting the SPP vehicles. Note that we do not make any assumptions about $\underline{t}$; however, we assume that once the intruder appears, it stays for a maximum duration of t^{IAT}.

Assumption 2: The dynamics of the intruder are known and given by $\dot{x}_I = f_I(x_I, u_I, d_I)$. The initial state of the intruder is given by x_I^0.

Assumption 2 is required for HJ reachability analysis. In situations where the dynamics of the intruder are not known exactly, a conservative model of the intruder may be used instead.

Based on the above assumptions, we aim to design a control policy that ensures separation with the intruder and with other SPP vehicles, and ensures a successful transit to the destination. However, depending on the initial state of the intruder, its control policy, and the disturbances in the dynamics of a vehicle and the intruder, a vehicle may arrive at different states after avoiding the intruder. Therefore, a control policy that ensures a successful transit to the destination needs to account for all such possible states, which is a path planning problem with multiple (infinite, to be precise) initial states and a single destination, and is hard to solve in general. Thus, we divide the intruder avoidance problem into two sub-problems: (i) we first design a control policy that ensures a successful transit to the destination if no intruder appears and that successfully avoid the intruder, if it does. (ii) after the intruder disappears at $\bar{t}$, we replan the trajectories of the affected vehicles. Following the same theme, authors in [] present an algorithm to avoid an intruder in SPP formulation; however, once the intruder disappears, the algorithm might need to replan the trajectories for all SPP vehicles. Since the replanning is done in real-time, it should be fast and scalable with the number of SPP vehicles, rendering the method in [] unsuitable for practical implementation.

In this work, we propose a novel intruder avoidance algorithm, which will need to replan trajectories only for a fixed number of vehicles, irrespective of the total number of SPP vehicles. Moreover, this number is a design parameter, which can be chosen based on the resources available during the run time. Intuitively, one can think about dividing the flight space of vehicles such that at any given time, any two vehicles are far enough from each other so that an intruder can only affect atmost k vehicles in a duration of t^{IAT} despite its best efforts. The advantage of this approach is that after the intruder disappears, we only have to replan the trajectory of k vehicles regardless of the number of total vehicles in the system, which makes this approach particularly suitable for practical systems. In this method, we build upon this intuition and show that such a division of space is indeed possible. Thus the proposed method guarantees that *atmost* k vehicles are affected by the presence of intruder, regardless of the number of SPP vehicles, and hence the replanning can be efficiently done in real-time.

In Sections IV-A and IV-B, we compute a space division of state-space such that atmost k vehicles need to apply the avoidance maneuver regardless of the initial state of the intruder. However, we still need to ensure that vehicles do not collide with each other while avoiding the intruder. The induced obstacles that reflect this possibility are computed in Section IV-C. Intruder avoidance control and replanning are discussed in Sections IV-D and IV-E respectively.

A. Separation Region

Depending on the information known to a lower-priority vehicle Q_i about Q_j's control strategy, we can use one of the three methods described in Section 5 in First journal paper should be cited here to compute the "base" obstacles $\mathcal{M}_j(t)$, the obstacles that would have been induced by Q_j in the absence of an intruder.

Given $\mathcal{M}_j(t)$, we want to compute the set of all initial states of the intruder for which vehicle Q_j may have to apply an avoidnace maneuver. We refer to this set as *separation region* here on, and denote it as $\mathcal{S}_j(t)$. The significance of $\mathcal{S}_j(t)$ is that as long as the intruder is outside $\mathcal{S}_j(t)$, that is $x_I(t) \in (\mathcal{S}_j(t))^c$, Q_j can apply any control at time t and still guaranteed to not collide with the intruder. $\mathcal{S}_j(t)$ can be conveniently computed using the relative dynamics between Q_j and Q_I.

We define relative dynamics of the intruder Q_I with state x_I with respect to Q_i with state x_i:

$$\begin{aligned} x_{I,i} &= x_I - x_i \\ \dot{x}_{I,i} &= f_r(x_{I,i}, u_i, u_I, d_i, d_I) \end{aligned} \tag{1}$$

Given the relative dynamics, we compute the set of states from which the joint states of Q_I and Q_j can enter danger zone $\mathcal{Z}_{jI}$ in a duration of t^{IAT} despite the best efforts of Q_j to avoid Q_I. Under the relative dynamics (1), this set of states is given by the backwards reachable set $\mathcal{V}_j^{\text{S}}(\tau, t^{\text{IAT}})$:

$$\begin{aligned} \mathcal{V}_j^{\text{S}}(\tau, t^{\text{IAT}}) =& \{y : \forall u_j(\cdot) \in \mathbb{U}_j, \exists u_I(\cdot) \in \mathbb{U}_I, \exists d_j(\cdot) \in \mathbb{D}_j, \\ & \exists d_I(\cdot) \in \mathbb{D}_I, x_{I,j}(\cdot) \text{ satisfies (1)}, \\ & \exists s \in [\tau, t^{\text{IAT}}], x_{I,j}(s) \in \mathcal{L}_j^{\text{S}}, x_{I,j}(\tau) = y\}, \end{aligned} \tag{2}$$

where

$$\begin{aligned} \mathcal{L}_j^{\text{S}} &= \{x_{I,j} : \|p_{I,j}\|_2 \leq R_c\} \\ H_j^{\text{S}}(x_{I,j}, \lambda) &= \max_{u_j \in \mathcal{U}_j} \left(\min_{u_I \in \mathcal{U}_I, d_j \in \mathcal{D}_j, d_I \in \mathcal{D}_I} \lambda \cdot f_r(x_{I,j}, u_j, u_I, d_j, d_I) \right) \end{aligned} \tag{3}$$

The interpretation of set $\mathcal{V}_j^{\text{S}}(0, t^{\text{IAT}})$ is that as long as Q_I is outside the boundary of this set (in relative coordinates), then Q_I and Q_j cannot enter the danger zone $\mathcal{Z}_{jI}$, irrespective of control applied by them. We can now transform $\mathcal{V}_j^{\text{S}}(0, t^{\text{IAT}})$ to absolute coordinates to obtain sets $\mathcal{S}_j(\cdot)$ as follows:

$$\mathcal{S}_j(t) = \mathcal{M}_j(t) + \mathcal{V}_j^{\text{S}}(0, t^{\text{IAT}}), \tag{4}$$

where the "+" in (4) denotes the Minkowski sum[1].

B. Buffer Region

In section IV-A, we computed sets $\mathcal{S}_j(\cdot)$ such that Q_j avoids the intruder only if $x_I(t) \in \mathcal{S}_j(t)$. But to ensure that atmost $\bar{k}$ vehicles need to replan their trajectories after the intruder disappears, we need to make sure that the intruder can cause atmost $\bar{k}$ vehicles to deviate from their planned trajectories. Equivalently, we want to ensure that atmost $\bar{k}$ vehicles need to avoid the intruder.

Intuitively, we want to make sure that at any given time the separation regions of any two vehicles are far enough from each other (that is, there is a "buffer" region between two separation regions) such that it will take at least $t^{\text{BRD}} := t^{\text{IAT}}/\bar{k}$ time for the intruder to go from the separation region of one vehicle to that of the other vehicle. This means that there is a "buffer" time interval of t^{BRD} between any t_1, $t_2 \in [\underline{t}, \bar{t}]$ where Q_I is in the separation regions of two different vehicles at t_1 and t_2, e.g. $x_I(t_1) \in \mathcal{S}_j(t)$ and $x_I(t_2) \in \mathcal{S}_i(t)$, $i \neq j$. Thus, in the worst case, the intruder can force atmost $\bar{k}$ vehicles to apply avoidance maneuver in a duration of t^{IAT}.

We next focus on computing the buffer region between any two vehicles Q_j and Q_i, $j < i$. Without loss of generality, we can assume that the intruder appears at the separation region of a vehicle at $t = \underline{t}$, because if it doesn't then the vehicles need not account for intruders until it reaches the boundary of the separation region of a vehicle. To compute the buffer region, we consider the following two cases:

1) Case1- $x_I^0 \in \mathcal{S}_j(\underline{t})$: Given the relative dynamics $x_{i,I}$ in (1), we compute the set of states from which the joint states of Q_I and Q_i can enter danger zone $\mathcal{Z}_{iI}$ within a duration of t^{BRD} when both Q_i and Q_I are using *optimal control to collide* with each other. This set of states is given by the backwards reachable set $\mathcal{V}_i^{\text{B}}(\tau, t^{\text{BRD}})$:

$$\begin{aligned} \mathcal{V}_i^{\text{B}}(t, t^{\text{BRD}}) = &\{y : \exists u_i(\cdot) \in \mathbb{U}_i, u_I(\cdot) \in \mathbb{U}_I, d_i(\cdot) \in \mathbb{D}_i, \\ &d_I(\cdot) \in \mathbb{D}_I, x_{i,I}(\cdot) \text{ satisfies (1)}, \\ &\exists s \in [t, t^{\text{BRD}}], x_{i,I}(s) \in \mathcal{L}_i^{\text{B}}, x_{i,I}(t) = y\}, \end{aligned} \tag{5}$$

[1]The Minkowski sum of sets A and B is the set of all points that are the sum of any point in A and B.

where

$$\mathcal{L}_i^{\mathrm{B}} = \{x_{i,I} : \|p_{i,I}\|_2 \leq R_c\}$$
$$H_i^{\mathrm{B}}(x_{i,I}, \lambda) = \min_{u_i \in \mathcal{U}_i, u_I \in \mathcal{U}_I, d_i \in \mathcal{D}_i, d_I \in \mathcal{D}_I} \lambda \cdot f_r(x_{i,I}, u_i, u_I, d_i, d_I) \tag{6}$$

The interpretation of set $\mathcal{V}_i^{\mathrm{B}}(0, t^{\mathrm{BRD}})$ is that if the separation region of Q_i is outside the boundary of this set and Q_I is at the boundary of $\mathcal{L}_i^{\mathrm{B}}$ (in relative coordinates), then Q_I and Q_i cannot enter the danger zone $\mathcal{Z}_{iI}$ for a duration of t^{BRD}, irrespective of control applied by them. If we augment this set on the separation region of the Q_j, then we get the same property in the state space of Q_i:

$$\mathcal{B}_{ij}(t) = \mathcal{S}_j(t) + \mathcal{V}_i^{\mathrm{B}}(0, t^{\mathrm{BRD}}). \tag{7}$$

Finally, during the path planning of Q_i, we need to ensure that Q_i is far enough from the boundary of $\mathcal{B}_{ij}(t)$ such that Q_I and Q_i cannot enter the danger zone $\mathcal{Z}_{iI}$ for the remaining duration of $t^{\mathrm{BRD}} := t^{\mathrm{IAT}} - t^{\mathrm{BRD}}$. Thus, during the path planning of Q_i, we need to ensure that veh_i is outside the augmented buffer region:

$$\tilde{\mathcal{B}}_{ij}(t) = \mathcal{B}_{ij}(t) + \mathcal{V}_i^{\mathrm{S}}(0, t^{\mathrm{BRD}}), \tag{8}$$

where $\mathcal{V}_i^{\mathrm{S}}(0, t^{\mathrm{BRD}})$ can be computed as described in Section IV-A.

2) Case2- $x_I^0 \in \mathcal{S}_i(\underline{t})$: This case can be treated in a similar fashion as Section IV-B1. We can now look at the same problem from Q_i's perspective and compute the augmented buffer region $\tilde{\mathcal{B}}_{ji}(t)$ as:

$$\tilde{\mathcal{B}}_{ij}(t) = \mathcal{M}_j(t) + \mathcal{V}_j^{\mathrm{S}}(0, t^{\mathrm{BRD}}) + \mathcal{V}_j^{\mathrm{B}}(0, t^{\mathrm{BRD}}) + \mathcal{V}_i^{\mathrm{S}}(0, t^{\mathrm{IAT}}). \tag{9}$$

During the path planning of Q_i, we need to ensure that Q_i is outside $\tilde{\mathcal{B}}_{ji}(t)$.

C. Obstacle Computation

In sections IV-A and IV-B, we computed a separation between any two vehicles, such that itruder can affect atmost $\bar{k}$ vehicles during a duration of t^{IAT}. However, we need to make sure that while applying avoidance control a vehicle does not enter the danger zone of other vehicle. In this section, we compute the obstacles that reflect this possibility. In particular, we want to find the set of states that a lower priority vehicle Q_i needs to avoid to avoid entering in the danger zone of a higher priority vehicle Q_j, $j < i$. To find such states, we consider the following five mutually exclusive and exhaustive cases:

1) Intruder does not affect Q_j or Q_i during their flight.

2) Intruder affects Q_j, but not Q_i.
3) Intruder affects Q_i, but not Q_j.
4) Intruder first affects Q_j and then Q_i.
5) Intruder first affects Q_i and then Q_j.

We will compute the set of states that Q_i needs to avoid to avoid a collision with Q_j for each of the five cases. Let ${}^k\mathcal{O}_i^j(\cdot)$ denotes the set of obstacles corresponding to case k above.

1) Case1: When the intruder does not affect any of the two vehicles, Q_i simply needs to avoid the set of base obstacles $\mathcal{M}_j(t)$. Therfore, ${}^1\mathcal{O}_i^j(t) = \mathcal{M}_j(t)$.

2) Case2: To compute the obstacles that Q_i needs to avoid at time t for the remaining four cases, it is sufficient to consider the scenarios where $\underline{t} \in [t - t^{\text{IAT}}, t]$. This is because if $\underline{t} < t - t^{\text{IAT}}$, then Q_i and/or Q_j will already be in the replanning phase at time t (see assumption 1) and hence the two vehicles cannot be in conflict at time t. On the other hand, if $\underline{t} > t$, then we need not account for the intruder as it has not appeared in the system yet. Actually, we compute obstacles at time t' in a way such that their effect doesn't propagate to $t < t'$, but not sure if we need to menbtion this.

The induced obstacles for Case2 at time t are given by the states that Q_j can reach while avoiding the intruder, starting from some state in $\mathcal{M}_j(\underline{t}), \underline{t} \in [t - t^{\text{IAT}}, t]$. These states can be obtained by computing a FRS from the base obstacles.

$$\begin{aligned} \mathcal{W}_j^{\mathcal{O}}(0, \tau) = \{ & y : \exists u_j(\cdot) \in \mathbb{U}_j, \exists d_j(\cdot) \in \mathbb{D}_j, \\ & x_j(\cdot) \text{ satisfies } (\textbf{??}), x_j(0) \in \mathcal{M}_j(t - \tau), \\ & x_j(\tau) = y \}. \end{aligned} \tag{10}$$

$\mathcal{W}_j^{\mathcal{O}}(0, \tau)$ represents the set of all possible states that Q_j can reach after a duration of τ starting from inside $\mathcal{M}_j(t - \tau)$. This FRS can be obtained by solving the HJ VI in (**??**) with the following Hamiltonian:

$$H_j^{\mathcal{O}}(x_j, \lambda) = \max_{u_j \in \mathcal{U}_j} \max_{d_j \in \mathcal{D}_j} \lambda \cdot f_j(x_j, u_j, d_j). \tag{11}$$

Since $\tau \in [0, t^{\text{IAT}}]$, the induced obstacles in this case can be obtained as:

$${}^2\mathcal{O}_i^j(t) = \bigcup_{\tau \in [0, t^{\text{IAT}}]} \mathcal{W}_j^{\mathcal{O}}(0, \tau). \tag{12}$$

Note that by the definition of base obstacles, $\mathcal{M}_j(t + \tau_2) \subset \mathcal{W}_j^{\text{BO}}(0, \tau_2 - \tau_1)\ \forall t$ and $\tau_2 > \tau_1$, where $\mathcal{W}_j^{\text{BO}}(0, \tau_2 - \tau_1)$ denotes the FRS of $\mathcal{M}_j(t + \tau_1)$ computed for a duration of $\tau_2 - \tau_1$.

Therefore, we have that $\mathcal{W}_j^{\mathcal{O}}(0,\tau) \subset \mathcal{W}_j^{\mathcal{O}}(0,t^{\text{IAT}})\ \forall \tau \in [0,t^{\text{IAT}})$. Thus, (**??**) can be equivalently written as:

$$^2\mathcal{O}_i^j(t) = \mathcal{W}_j^{\mathcal{O}}(0,t^{\text{IAT}}). \tag{13}$$

3) Case3: In this case, we need to ensure that Q_i doesn't collide with the obstacle set $\mathcal{M}_j(t)$ even when it is avoiding the intruder. In particular, we can compute a region around the obstacles $\mathcal{M}_j(\cdot)$ such that for all disturbances, Q_i can avoid colliding with obstacles for t^{IAT} seconds regardless of its avoidance control, if Q_i starts outside this region. To ensure that a vehicle does not collide with the obstacle $\mathcal{M}_j(t_1+t')$ at time $t=t_1+t'$ starting at $t=t_1$, regardless of its control $u_i(s)$ and disturbance $d_i(s)$ for the time interval $s\in[t_1,t_1+t']$, it suffices to avoid the t'-horizon BRS of $\mathcal{M}_i(t_1+t')$. This argument applies for all $t'\in[0,t^{\text{IAT}}]$. Mathematically,

$$^3\mathcal{O}_i^j(t) = \bigcup_{\tau\in[0,t^{\text{IAT}}]} \mathcal{V}_i^{\mathcal{G}}(0,\tau) \tag{14}$$

where $\mathcal{V}_i^{\mathcal{G}}(0,\tau)$ represents BRS of $\mathcal{M}_j(t+\tau)$ computed backwards for τ seconds. Formally,

$$\begin{aligned} \mathcal{V}_i^{\mathcal{G}}(0,\tau) = & \{y : \exists u_i(\cdot)\in\mathbb{U}_i, \exists d_i(\cdot)\in\mathbb{D}_i, \\ & x_i(\cdot) \text{ satisfies } (\textbf{??}), x_i(0)=y, \\ & \exists s\in[0,\tau], x_i(s)\in\mathcal{M}_i(t)\}. \end{aligned} \tag{15}$$

The Hamiltonian $H_i^{\mathcal{G}}$ to compute $\mathcal{V}_i^{\mathcal{G}}(\cdot)$ is given by:

$$H_i^{\mathcal{G}}(x_i,\lambda) = \min_{u_i\in\mathcal{U}_i}\min_{d_i\in\mathcal{D}_i} \lambda\cdot f_i(x_i,u_i,d_i) \tag{16}$$

4) Case4: start from here

D. Optimal Avoidance Controller

Given the relative dynamics, we compute the set of states from which the joint states of Q_I and Q_i can enter danger zone $\mathcal{Z}_{iI}$ despite the best efforts of Q_i to avoid Q_I. This set of states is given by the backwards reachable set $\mathcal{V}_i^{\text{S}}(\tau,t^{\text{IAT}}),\ \tau\in[0,t^{\text{IAT}}]$, given by (2).

Once the value function $V_i^{\text{S}}(\cdot)$ is computed, the optimal avoidance control u_i^{S} can be obtained as:

$$u_i^{\text{S}} = \arg\max_{u_i\in\mathcal{U}_i}\left(\min_{u_I\in\mathcal{U}_I, d_i\in\mathcal{D}_i, d_I\in\mathcal{D}_I} \lambda\cdot f_r(x_{I,i},u_i,u_I,d_i,d_I)\right) \tag{17}$$

Under normal circumstances when the intruder Q_I is far away, we have $V_i^{\text{S}}(0,x_{I,i}(t))>0$; as Q_I gets closer to Q_i, $V_i^{\text{S}}(0,x_{I,i}(t))$ decreases. If Q_i applies the control u_i^{S} when $V_i^{\text{S}}(0,x_{I,i}(t)) =$

0, then collision avoidance between Q_i and Q_I is guaranteed for a duration of t^{IAT} under the worst-case intruder control strategy u_I.

In addition, obstacle augmentation (**??**) ensures that Q_i does not collide with $\mathcal{G}_i(\cdot)$ during the avoidance maneuver. The overall control policy for avoiding the intruder and collision with other vehicles is thus given by:

$$u_i^{\text{A}}(t) = \begin{cases} u_i^{\text{AO}}(t) & t \leq \underline{t} \\ u_i^{\text{S}}(t) & \underline{t} \leq t \leq \bar{t} \end{cases}$$

E. Replanning after intruder avoidance

After the intruder disappears, liveness controllers which ensure that the vehicles reach their destinations can be obtained by solving a new SPP problem, where the starting states of the vehicles are now given by the states they end up in, denoted $\tilde{x}_j^0$, after avoiding the intruder. The set of all vehicles Q_j for whom we need to replan the trajectories, $\mathcal{N}^{\text{RP}}$, can be obtained by checking if a vehicle Q_j applied any avoidnace control during $[\underline{t}, \bar{t}]$, e.g.,

$$\mathcal{N}^{\text{RP}} = \{j \in \{1, \ldots, N\} : V_j^{\text{S}}(0, x_{I,j}(t)) \leq 0, t \in [\underline{t}, \bar{t}]\}. \tag{18}$$

Let the optimal control policy corresponding to this liveness controller be denoted $u_j^{\text{L}}(t)$. The overall control policy that ensures intruder avoidance, collision avoidance with other vehicles, and successful transition to the destination for vehicles $j \in \mathcal{N}^{\text{RP}}$ is given by:

$$u_j^*(t) = \begin{cases} u_j^{\text{A}}(t) & t \leq \bar{t} \\ u_j^{\text{L}}(t) & t > \bar{t} \end{cases}$$

Note that in order to replan using a SPP method, we need to determine feasible t^{STA}s for all vehicles $j \in \mathcal{N}^{\text{RP}}$. This can be done by computing an FRS:

$$\begin{aligned} \mathcal{W}_j^{\text{RP}}(\bar{t}, t) = \{ & y \in \mathbb{R}^{n_j} : \exists u_j(\cdot) \in \mathbb{U}_j, \forall d_j(\cdot) \in \mathbb{D}_j, \\ & x_j(\cdot) \text{ satisfies } (\textbf{??}), x_j(\bar{t}) = \tilde{x}_j^0, \\ & x_j(t) = y\}, \end{aligned} \tag{19}$$

where $\tilde{x}_j^0$ represents the state of Q_j at $t = \bar{t}$. The FRS in (18) can be obtained by solving the HJ VI in (**??**) with the following Hamiltonian:

$$H_j^{\text{RP}}(x_j, \lambda) = \max_{u_j \in \mathcal{U}_j} \min_{d_j \in \mathcal{D}_j} \lambda \cdot f_j(x_j, u_j, d_j). \tag{20}$$

The new t^{STA} of Q_j is now given by the earliest time at which $\mathcal{W}_j^{\text{RP}}(\bar{t}, t)$ intersects the target set $\mathcal{L}_j$, $t_j^{\text{STA}} := \arg\inf_t \{\mathcal{W}_j^{\text{RP}}(\bar{t}, t) \cap \mathcal{L}_j \neq \emptyset\}$. Intuitively, this means that there exists a control policy which will steer the vehicle to its destination by that time, despite the worst case disturbance it might experience.

This does not seem correct. We have to compute STA for each vehicle in the order of priority and take into account the obstacles that they might experience while computing FRS.

To-Dos:

- A remark about the single vehicle replanning property of Method-2.
- Once the replanning is complete, another intruder can appear in the system. So strictly speaking we are making an assumption that atmost one intruder is in the system *at any given time* as opposed to throughout the trajectory.
- For method-2 results, it may be helpful to include a figure which is showing the division of space among vehicles at some time (probably right before the intruder enters).

V. SIMULATIONS

Focus on the following aspects:

- Demonstration of theory (that it avoids collision w/ other vehicles and intruders, and we reach our destinations).
- Scaling of SPP.
- Reactivity of controller to the actual disturbance (Claire: be very detailed about explaining the setup of simulation)
- Illustrate the structure that emerge out of SPP algorithm (Almost straight line path w/ different starting times)
- Illustrate how this structure change with change in disturbance bounds (Straight line trajectories become curvy?)

Also mention the technical details for the simulations, like RTT parameters, relative co-ordinate dynamics, rotation and translation of obstacles, union for obstacles, etc.

Let's pick speed $1.5Km/min$ (or $2.5Decametre/s$) and turnrate to be $120rad/min$ ($2rad/s$). Let's use grid to be $[0, 500]Dm$.