On the air highways, platoons of UAVs modeled by hybrid systems were considered. It was shown how various required platoon functions (merging onto an air highway, changing platoons, etc.) can be implemented using only the free, leader, and follower modes of operation. Using HJ reachability, goal satisfaction controllers were proposed that guarantee the success of all mode transitions and wrapped a safety controller around goal satisfaction controllers to ensure no collision between the UAVs can occur. Under the assumption that faulty vehicles can descend after a prespecified duration, the safety controller guarantees that no collisions will occur in a single altitude level as long as, at most, one safety breach occurs for each vehicle in the platoon. Additional safety breaches can be handled by multiple altitude ranges in the airspace.

14

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15

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16

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