Multi-Vehicle Collision Avoidance via Reachability and Mixed Integer Programming

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***Abstract*—**

I. Introduction

- motivation – UAVs for civil purposes [1], [2], [3], [4], [5], [6], [7]. For example, package delivery, aerial surveillance, disaster response, ... – difficult problem

- previous methods for dealing with multi-agent systems – limitations (especially with respect to HJ formulation)

In [8], the author discuss a class of three-player differential game in which two of the players oppose the third player in order to neutralize the third player's attempts to alter their joint cost function.

[9] considers multiple agent games in which communication follows a graphical structure.

In [10], the authors propose a control law derived from a differential game that models the protection of a target from a homing missile.

Most recently, [11] uses time-dependent Hamilton-Jacobi reachability analysis to approximate the solution to a pursuit-evasion game in which a third player, the defender, attempts to capture the pursuer.

However, all of the above makes assumptions such as linear dynamics and the assumption of agent roles, and describes non-cooperative situations, making them insufficient for addressing the multi-vehicle collision avoidance problem

- HJ background – references – theory [12] – success in applications [13], [14] – normal limitations

- HJ for multi-UAV systems – [15], [16], [17] – assumed structure (platooning, SPP)

- summary

II. Problem Formulation

Each vehicle

$$\begin{aligned} \dot{x}_i &= g_i(x_i, u_i) \\ i &= 1, \dots, N \end{aligned} \tag{1}$$

Relative dynamics between i and j:

$$\begin{aligned} \dot{x}_{ij} &= f_{ij}(x_{ij}, u_i, d_i) \\ i, j &= 1, \dots, N, i \neq j \end{aligned} \tag{2}$$

- Existence of solution

Target sets for each vehicle: $\mathcal{T}_i$. Each vehicle avoids $\mathcal{D}_{ij}$

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III. Hamilton-Jacobi Reachability

IV. Mixed Integer Programming

V. Results

VI. Conclusions and Future Work

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