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A Study on Decentralized Receding Horizon Control for Decoupled Systems

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Abstract— **We consider a set of decoupled dynamical systems and an optimal control problem where cost function and constraints couple the dynamical behavior of the systems. The coupling is described through a connected graph where each system is a node and, cost and constraints of the optimization problem associated to each node are only function of its state and the states of its neighbors. For such scenario, we propose a framework for designing decentralized Receding Horizon Control (RHC) control schemes.**

In these decentralized schemes, a centralized RHC controller is broken into distinct RHC controllers of smaller sizes. Each RHC controller is associated to a different node and computes the local control inputs based only on the states of the node and of its neighbors. The proposed decentralized control schemes are formulated in a rigorous mathematical framework. Moreover, we highlight the main issues involved in guaranteeing stability and constraint fulfillment for such schemes and the degree of conservativeness that the decentralized approach introduces.

I. Introduction

The interest in decentralized control goes back to the seventies. Probably Wang and Davison were the first in [1] to envision the "increasing interest in decentralized control systems" when "control theory is applied to solve problems for large scale systems". Since then the interest has grown more than exponentially despite some non-encouraging results on the complexity of the problem [2]. Decentralized control techniques today can be found in a broad spectrum of applications ranging from robotics and formation flight to civil engineering. Such a wide interest makes a survey of all the approaches that have appeared in the literature very difficult and goes also beyond the scope of this paper.

Approaches to decentralized control design differ from each other in the assumptions they make on: (*i*) the kind of interaction between different systems or different components of the same system (dynamics, constraints, objective), (*ii*) the model of the system (linear, nonlinear, constrained, continuous-time, discrete-time), (*iii*) the model of information exchange between the systems, (*iv*) the control design technique used.

Dynamically coupled systems have been the most studied [1], [3], [4]. In this paper we focus on *decoupled systems*. In a descriptive way, the problem of decentralized control for decoupled systems can be formulated as follows. A dynamical system is composed of (or can be decomposed into) distinct dynamical subsystems that can be independently actuated. The subsystems are dynamically decoupled but have common objectives and constraints which make them interact with each other. Typically the *interaction* is local, i.e. the goal and the constraints of a subsystem are function of only a subset of other subsystems' states. The interaction will be represented by an "interaction graph", where the nodes represent the subsystems and an arc between two nodes denotes a coupling term in the goal and/or in the constraints associated to the nodes. Also, typically it is assumed that the *exchange of information* has a special structure, i.e., it is assumed that each subsystem can sense and/or exchange information with only a subset of other subsystems. Often the *interaction graph* and the *information exchange graph* coincide. A decentralized control scheme consists of distinct controllers, one for each subsystem, where the inputs to each subsystem are computed only based on local information, i.e., on the states of the subsystem and its neighbors.

Our interest in decentralized control for dynamically decoupled systems arises from the abundance of networks of independently actuated systems and the necessity of avoiding centralized design when this becomes computationally prohibitive. Networks of vehicles in formation, production units in a power plant, cameras at an airport, an array of mechanical actuators for deforming a surface are just a few examples. Each network has its peculiarity. In formation flight for instance the coupling constraints model collision avoidance. The interaction graph is full (each vehicle has to avoid all the other vehicles) but it is often approximated with a time-varying graph based on a "closest spatial neighbors" model.

We will make use of Receding Horizon Control (RHC) schemes [5]. Recently, centralized RHC schemes applied to formation flight have appeared in [6], [7]. In [8] decentralized RHC and potential functions have been used for flying multiple autonomous helicopters in a dynamical environment.

In this paper we take explicitly into account constraints and use the model of the neighbors to predict their behavior. In this respect, the "boids" control strategy [9] can be seen as a special case of decentralized RHC when the prediction horizon is one. We describe a framework for designing *decentralized* RHC control schemes, where a centralized

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RHC controller is broken into distinct RHC controllers of smaller sizes. Each RHC controller is associated to a different node and computes the local control inputs based only on the states of the node and of its neighbors. In general, computation is distributed over the nodes and the number of RHC controllers is smaller than the total number of nodes.

The main issue regarding decentralized schemes is that the inputs computed locally are, in general, not guaranteed to be globally feasible and to stabilize the overall team. In general, stability and feasibility of decentralized schemes are very difficult to prove and/or too conservative. A scheme with stability guarantees has been proposed in [10] for dynamically coupled systems, with information exchange between nodes and contractive stability constraints in the distributed RHC subproblems.

We will formulate decentralized control schemes in a rigorous mathematical framework, without giving any proof of feasibility and stability. Instead, we will highlight the main issues involved in guaranteeing stability and constraint fulfillment for such schemes and briefly discuss their conservativeness. We will show the applicability of the proposed approach when decentralized schemes are used for controlling a set of vehicles in formation flight. Simulation examples will be used to investigate the effect of cost weights and horizon lengths on the feasibility of the decentralized RHC schemes. We will also point out some interesting behaviors of the decentralized scheme which are different from what is observed in standard centralized RHC control theory.

II. Problem Formulation

Consider a set of N_v decoupled dynamical systems, the i-th system being described by the discrete-time time-invariant state equation:

$$x_{k+1}^i = f^i(x_k^i, u_k^i) \tag{1}$$

where $x_k^i \in \mathbb{R}^{n^i}$, $u_k^i \in \mathbb{R}^{m^i}$, $f^i : \mathbb{R}^{n^i} \times \mathbb{R}^{m^i} \to \mathbb{R}^{n^i}$ are state, input and state update function of the i-system, respectively. Let $\mathcal{X}^i \subseteq \mathbb{R}^{n^i}$ and $\mathcal{U}^i \subseteq \mathbb{R}^{m^i}$ denote the set of feasible inputs and states of the i-th system, respectively:

$$x_k^i \in \mathcal{X}^i, \quad u_k^i \in \mathcal{U}^i, \ k \geq 0 \tag{2}$$

We will refer to the set of N_v constrained systems as *team system*. Let $\tilde{x}_k \in \mathbb{R}^{N_v \times n^i}$ and $\tilde{u}_k \in \mathbb{R}^{N_v \times m^i}$ be the vectors which collect the states and inputs of the team system at time k, i.e. $\tilde{x}_k = [x_k^1, \dots, x_k^{N_v}]$, $\tilde{u}_k = [u_k^1, \dots, u_k^{N_v}]$, with

$$\tilde{x}_{k+1} = f(\tilde{x}_k, \tilde{u}_k) \tag{3}$$

We denote by (x_e^i, u_e^i) the equilibrium pair of the i-th system and $(\tilde{x}_e, \tilde{u}_e)$ the corresponding equilibrium for the team system.

So far the systems belonging to the team system are completely decoupled. We consider an optimal control problem for the team system where cost function and constraints couple the dynamic behavior of individual systems. We use a graph topology to represent the coupling in the following way. We associate the i-th system to the i-th node of the graph, and if an edge (i, j) connecting the i-th and j-th node is present, then the cost and the constraints of the optimal control problem will have a component which is a function of both x^i and x^j. The graph will be *undirected*, i.e. $(i,j) \in \mathcal{A} \Rightarrow (j,i) \in \mathcal{A}$. Before defining the optimal control problem, we need to define a graph

$$\mathcal{G} = \{\mathcal{V}, \mathcal{A}\} \tag{4}$$

where $\mathcal{V}$ is the set of nodes $\mathcal{V} = \{1, \dots, N_v\}$ and $\mathcal{A} \subseteq \mathcal{V} \times \mathcal{V}$ the sets of arcs (i,j) with $i \in \mathcal{V}, \ j \in \mathcal{V}$.

Once the graph structure has been fixed, the optimization problem is formulated as follows. Denote with $\tilde{x}^i$ the states of all neighboring systems of the i-th system, i.e. $\tilde{x}^i = \{x^j \in \mathbb{R}^{n^j} | (j,i) \in \mathcal{A}\}$, $\tilde{x}^i \in \mathbb{R}^{\tilde{n}^i}$ with $\tilde{n}^i = \sum_{j|(j,i)\in\mathcal{A}} n^j$. Analogously, $\tilde{u}^i \in \mathbb{R}^{\tilde{m}^i}$ denotes the inputs to all the neighboring systems of the i-th system. Let

$$g^{i,j}(x^i, u^i, x^j, u^j) \leq 0 \tag{5}$$

define the interconnection constraints between the i-th and the j-th systems, with $g^i : \mathbb{R}^{n^i} \times \mathbb{R}^{m^i} \times \mathbb{R}^{n^j} \times \mathbb{R}^{m^j} \to \mathbb{R}^{nc_{i,j}}$. We will often use the following shorter form of the interconnection constraints defined between the i-th system and all its neighbors:

$$g^i(x^i, u^i, \tilde{x}^i, \tilde{u}^i) \leq 0 \tag{6}$$

with $g^i : \mathbb{R}^{n^i} \times \mathbb{R}^{m^i} \times \mathbb{R}^{\tilde{n}^i} \times \mathbb{R}^{\tilde{m}^i} \to \mathbb{R}^{nc_i}$.

Consider the following cost

$$l(\tilde{x}, \tilde{u}) = \sum_{i=1}^{N_v} l^i(x^i, u^i, \tilde{x}^i, \tilde{u}^i) \tag{7}$$

where $l^i : \mathbb{R}^{n^i} \times \mathbb{R}^{m^i} \times \mathbb{R}^{\tilde{n}^i} \times \mathbb{R}^{\tilde{m}^i} \to \mathbb{R}$ is the cost associated to the i-th system and is a function of its states and the states of its neighbor nodes. Assume that l is a positive convex function and that $l^i(x_e^i, u_e^i, \tilde{x}_e^i, \tilde{u}_e^i) = 0$ and consider the infinite time optimal control problem

$$\tilde{J}_\infty^*(\tilde{x}) \triangleq \min_{\{\tilde{u}_0, \tilde{u}_1, \dots\}} \sum_{k=0}^{\infty} l(\tilde{x}_k, \tilde{u}_k) \tag{8}$$

$$\text{subj. to} \begin{cases} x_{k+1}^i = f^i(x_k^i, u_k^i), \\ \quad i = 1, \dots, N_v, \ k \geq 0 \\ g^{i,j}(x_k^i, u_k^i, x_k^j, u_k^j) \leq 0, \\ \quad i = 1, \dots, N_v, \ k \geq 0, \\ \quad (i,j) \in \mathcal{A} \\ x_k^i \in \mathcal{X}^i, \ u_k^i \in \mathcal{U}^i, \\ \quad i = 1, \dots, N_v, \ k \geq 0 \\ \tilde{x}_0 = \tilde{x} \end{cases} \tag{9}$$

For all $\tilde{x} \in \mathbb{R}^{N_v \times n^i}$, if problem (9) is feasible, then the optimal input $\tilde{u}_0^*, \tilde{u}_1^*, \dots$ will drive the N_v systems to their equilibrium points x_e^i while satisfying state, input and interconnection constraints.

Remark 1: Throughout the paper we assume that a solution to problem (9) exists and it generates a feasible and stable trajectory for the team system. Our assumption is not restrictive. If there is no infinite time centralized optimal control problem fulfilling the constraints, then there is no reason to look for a decentralized receding horizon controller with the same properties.

With the exception of a few cases, solving an infinite horizon optimal control problem is computationally prohibitive. An infinite horizon controller can be designed by repeatedly solving finite time optimal control problems in a receding horizon fashion as described next. At each sampling time, starting at the current state, an open-loop optimal control problem is solved over a finite horizon. The optimal command signal is applied to the process only during the following sampling interval. At the next time step a new optimal control problem based on new measurements of the state is solved over a shifted horizon. The resultant controller is often referred to as Receding Horizon Control (RHC). More specifically, assume at time t the current state $\tilde{x}_t$ to be available and consider the following constrained finite time optimal control problem

$$\tilde{J}_N^*(\tilde{x}_t) \triangleq \min_{\{U_t\}} \sum_{k=0}^{N-1} l(\tilde{x}_{k,t}, \tilde{u}_{k,t}) + l_N(\tilde{x}_{N,t}) \tag{10a}$$

$$\text{subj. to} \begin{cases} x^i_{k+1,t} = f^i(x^i_{k,t}, u^i_{k,t}), \\ \quad i = 1, \ldots, N_v, \quad k \geq 0 \\ g^{i,j}(x^i_{k,t}, u^i_{k,t}, x^j_{k,t}, u^j_{k,t}) \leq 0, \\ \quad i = 1, \ldots, N_v, (i,j) \in \mathcal{A}, \\ \quad k = 1, \ldots, N-1 \\ x^i_{k,t} \in \mathcal{X}^i, \quad u^i_{k,t} \in \mathcal{U}^i \\ \quad i = 1, \ldots, N_v, \\ \quad k = 1, \ldots, N-1 \\ \tilde{x}_{N,t} \in \mathcal{X}_f, \\ \tilde{x}_{0,t} = \tilde{x}_t \end{cases} \tag{10b}$$

where N is the prediction horizon, $\mathcal{X}_f \subseteq \mathbb{R}^{N_v \times n^i}$ is a terminal region, l_N is the cost on the terminal state. In (10) we denote with $U_t \triangleq [\tilde{u}_{0,t}, \ldots, \tilde{u}_{N-1,t}]' \in \mathbb{R}^s$, $s \triangleq N_v \times mN$ the optimization vector, $x^i_{k,t}$ denotes the state vector of the i-th node predicted at time $t+k$ obtained by starting from the state x^i_t and applying to system (1) the input sequence $u^i_{0,t}, \ldots, u^i_{k-1,t}$. The tilded vectors will denote the prediction vectors associated to the team system.

Let $U_t^* = \{\tilde{u}^*_{0,t}, \ldots, \tilde{u}^*_{N-1,t}\}$ be the optimal solution of (10) at time t and $\tilde{J}_N^*(\tilde{x}_t)$ the corresponding value function. Then, the first sample of U_t^* is applied to the team system (3)

$$\tilde{u}_t = \tilde{u}^*_{0,t}. \tag{11}$$

The optimization (10) is repeated at time $t+1$, based on the new state x_{t+1}.

It is well known that stability is not ensured by the RHC law (10)–(11). Usually the terminal cost l_N and the terminal constraint set $\mathcal{X}_f$ are chosen to ensure closed-loop stability. A treatment of sufficient stability conditions goes beyond the scope of this work and can be found in the surveys [5], [11]. We assume that the reader is familiar with the basic concept of RHC and its main issues, we refer to [5] for a comprehensive treatment of the topic. In this paper we will assume that terminal cost l_N and the terminal constraint set $\mathcal{X}_f$ have been appropriately chosen in order to ensure the stability of the closed-loop system.

In general, the optimal input u^i_t to the i-th system computed by solving (10) at time t, will be a function of the overall state information $\tilde{x}_t$. The main objective of this work is to describe how problem (10) can be decomposed into smaller subproblems whose independent computation can be distributed over the graph nodes. We propose a decentralized control scheme where problem (10) is decomposed into N_v finite time optimal control problems, each one associated to a different node. The i-th subproblem will be a function of the states of the i-th node and the states of its neighbors. The solution of the i-th subproblem will yield a control policy for the i-th node of the form $u^i_t = f^i(x^i_t, \tilde{x}^i_t)$.

Remark 2: The techniques presented next will be meaningful only if the graph $\mathcal{G}$ is not a full graph. Often, the interconnection graph is not fully connected because of the nature of the problem. For instance, each node could represent a production unit of a certain plant and the production of a node could be related to only a few other units of the plant. Also the interconnection graph is not fully connected because some constraints associated to certain arcs are implicitly satisfied by interconnection constraints associated to other arcs. In formation flight, $\mathcal{G}$ is a full graph which describes the constraints between each node (since each vehicle has to keep a certain distance form all the other vehicles of the formation). Rigid graph topology [7], [12] can be used to implicitly enforce constraints between two vehicles not connected by any arc of the graph. More often, time-varying graph topology based on a closest neighbor principle is used. In this work we focus on fixed graph topology. Time-varying graph topologies have been studied in [13].

Remark 3: In the formulation above, we are assuming that the equilibrium $(\tilde{x}_e, \tilde{u}_e)$ of the formation is known a priori. The equilibrium of the formation can be defined in several other different ways. For instance, we can assume that there is a leader (real or virtual) which is moving and the equilibrium is given in terms of distances of each vehicle from the leader. Also, it is possible to formulate the equilibrium by using relative distances between vehicles and signed areas [7]. The approach of this paper does not depend on the way the formation equilibrium is defined, as long as this is known a priori. In some formation control schemes, the equilibrium is not known a priori, but is the result of the evolutions of decentralized control laws. The approach of the paper is not applicable to such schemes.

III. Decentralized Control Scheme

Consider the overall problem: systems (1), graph $\mathcal{G}$, and RHC policy (10)-(11). Consider the i-th system and the following finite time optimal control problem $\mathcal{P}_i$:

$$\min_{\tilde{U}_t^i} \sum_{k=0}^{N-1} l^i(x_{k,t}^i, u_{k,t}^i, \tilde{x}_{k,t}^i, \tilde{u}_{k,t}^i) + l_N^i(x_{N,t}^i, \tilde{x}_{N,t}^i) \quad \text{(12a)}$$

$$\text{subj. to} \begin{cases} x_{k+1,t}^i = f^i(x_{k,t}^i, u_{k,t}^i),\ k \geq 0 \\ x_{k,t}^i \in \mathcal{X}^i,\quad u_{k,t}^i \in \mathcal{U}^i, \\ \quad k = 1, \dots, N-1 \\ x_{k+1,t}^j = f^j(x_{k,t}^j, u_{k,t}^j), \\ \quad (j,i) \in \mathcal{A},\ k \geq 0 \\ x_{k,t}^j \in \mathcal{X}^j,\quad u_{k,t}^j \in \mathcal{U}^j, \\ \quad (j,i) \in \mathcal{A}, \\ \quad k = 1, \dots, N-1 \\ g^{i,j}(x_{k,t}^i, u_{k,t}^i, x_{k,t}^j, u_{k,t}^j) \leq 0, \\ \quad (i,j) \in \mathcal{A}, \\ \quad k = 1, \dots, N-1 \\ g^{q,r}(x_{k,t}^q, u_{k,t}^q, x_{k,t}^r, u_{k,t}^r) \leq 0, \\ \quad (q,i) \in \mathcal{A},\ (r,i) \in \mathcal{A}, \\ \quad k = 1, \dots, N-1 \\ x_{N,t}^i \in \mathcal{X}_f^i,\quad x_{N,t}^j \in \mathcal{X}_f^j, (i,j) \in \mathcal{A} \\ x_{0,t}^i = x_t^i,\quad \tilde{x}_{0,t}^i = \tilde{x}_t^i, \end{cases} \quad \text{(12b)}$$

where $\tilde{U}_t^i \triangleq [u_{0,t}^i, \tilde{u}_{0,t}^i, \dots, u_{N-1,t}^i, \tilde{u}_{N-1,t}^i]' \in \mathbb{R}^s$, $s \triangleq (\tilde{m}^i + m^i)N$ denotes the optimization vector. Denote by $\tilde{U}_t^{i*} = [u_{0,t}^{*i}, \tilde{u}_{0,t}^{*i}, \dots, u_{N-1,t}^{*i}, \tilde{u}_{N-1,t}^{*i}]$ the optimizer of problem $\mathcal{P}_i$. Note that problem $\mathcal{P}_i$ involves only the state and input variables of the i-th node and its neighbors.

We will define the following decentralized RHC control scheme.

1) The i-th node at time t measures its state x_t^i and the state of all its neighbors $\tilde{x}_t^i$.
2) Each node i solves problem $\mathcal{P}_i$.
3) Each node i implements the first sample of $\tilde{U}_t^{i*}$

$$u_t^i = u_{0,t}^{*i}. \quad \text{(13)}$$

4) Each node repeats steps 2 to 4 at time $t+1$, based on the new state information x_{t+1}^i, $\tilde{x}_{t+1}^i$.

Steps one to four describe a decentralized strategy that uniquely defines the control inputs to the team system. Each node knows its current states, its neighbors' current states, its terminal region, its neighbors' terminal regions and models and constraints of its neighbors. Based on such information, each node computes its optimal inputs and its neighbors' optimal inputs. The input to the neighbors will only be used to predict their trajectories and then discarded, while the first component of the i-th optimal input of problem $\mathcal{P}_i$ will be implemented on the i-th node.

Even if we assume N to be infinite, the approach described so far does not guarantee that solutions computed locally are globally feasible and stable (i.e. feasible for problem (10)). The reason is simple: At the i-th node the prediction of the neighboring state x_k^j is done independently from the prediction of problem $\mathcal{P}_j$. Therefore, the trajectory of x^j predicted by problem $\mathcal{P}_i$ and the one predicted by problem $\mathcal{P}_j$, based on the same initial conditions, are different (since, in general, $\mathcal{P}_i$ and $\mathcal{P}_j$ will be different). This will imply that constraint fulfillment will be ensured by the optimizer u_t^{*i} for problem $\mathcal{P}_i$ but not for the overall problem (10).

There are three main issues that arise in the decentralized control scheme. In order to ensure central feasibility and stability of the decentralized control scheme,

- *Decoupled Terminal Cost.* How does one choose the terminal cost l_N^i for each problem $\mathcal{P}_i$?
- *Decoupled Terminal Region.* How does one choose the terminal region $\mathcal{X}_f^i$ for each problem $\mathcal{P}_i$?
- *Feasibility Issue.* Is it enough to choose the right decoupled terminal cost and terminal region?

We can anticipate here that the answer to the "feasibility issue" is negative. That is, a good choice of l_N^i and $\mathcal{X}_f^i$ is, in general, not sufficient to ensure stability and feasibility of the decentralized scheme.

A. Decoupled Terminal Costs

If performance of the decentralized RHC is not critical, then stability is not the major issue in decentralized RHC schemes for dynamically decoupled systems. One can always sacrifice optimality of the centralized problem (10) in order to guarantee stability. In fact, in the worst case the cost can be chosen to be decoupled as well, i.e. $l^i(x^i, u^i, \tilde{x}^i, \tilde{u}^i) = l^i(x^i, u^i)$ and each subsystem dynamics will converge to its equilibrium. In doing so we have completely neglected the coupling term in the cost. In general, if one writes $l^i(x^i, u^i, \tilde{x}^i, \tilde{u}^i) = l_1^i(x^i, u^i) + \alpha l_2^i(\tilde{x}^i, \tilde{u}^i)$, then it can be easily proven that one can always choose $\alpha \in [0,1]$ such that stability is guaranteed. With $\alpha = 1$ we recover the original cost for each node and how close α can be to one is a function of the error between predictions of neighbor's behavior and their real behavior. For more details we refer the reader to the full version of this paper [14].

B. Decoupled Terminal Regions

The problem of the terminal set can be approached in two different ways. One can start from the terminal set $\mathcal{X}_f$ in problem (10) and decompose it into N_v non-empty sets $\mathcal{X}_f^i \subset \mathbb{R}^{n^i}$ which will be used in (12). The N_v sets $\mathcal{X}_f^i \subset \mathbb{R}^{n^i}$ can be also computed without taking into consideration the original invariant set $\mathcal{X}_f$. Often the latter route is preferable for two main reasons; *(i)* it can be computationally prohibitive to compute the invariant set $\mathcal{X}_f$ in (10) for a large team of systems, *(ii)* it is difficult to decompose the invariant set $\mathcal{X}_f$ into N_v terminal sets, which used in (12) will guarantee the feasibility of the decentralized control schemes.

We propose the following construction of the sets $\mathcal{X}_f^i$. For each vehicle, we compute an hyper-rectangular inner approximation of the feasible space defined by the interconnection constraints which contains the equilibria $(x_e^i, \tilde{x}_e^i)$ as follows. Consider the i-th node and the set $S^{i,j} \subset \mathbb{R}^{n^i+n^j}$ for $(i,j) \in \mathcal{A}$ defined by the coupling constraints $g^{i,j}$:

$$S^{i,j} = \{x^i \in \mathbb{R}^{n^i},\ x^j \in \mathbb{R}^{n^j} \mid g^{i,j}(x^i, x^j) \leq 0\}.$$

Compute the sets $I^i_{i,j}$ and $I^j_{i,j}$ satisfying

$$I^i_{i,j} \times I^j_{i,j} \subseteq S^{i,j}$$

Let $I^i = \bigcap_{(i,j)\in\mathcal{A}} I^i_{i,j}$ and $\mathcal{X}^i_f$ be a controlled invariant set of the i-th system (1), subject to input and state constraints (2) and to the additional constraint $x^i_k \in I^i\ \forall k \geq 0$.

Through the procedure described above one can independently compute N_v terminal sets $\mathcal{X}^i_f$ which will be used in problem (12). Such sets have the following property. If each system enters its associated terminal set, we are ensured that all the interconnection constraints are satisfied and that there exists a decentralized control law which keeps each one in its respective terminal set. The sum of the ratios between the volumes of $I^i_{i,j} \times I^j_{i,j}$ and $S^{i,j}$ for all $(i,j) \in \mathcal{A}$ will be a measure of the conservativeness of the method. The smaller this sum is, the smaller will be the region of attraction of the decentralized control scheme. Note that the sets I^i and I^j will be convex even if $S^{i,j}$ is not convex.

C. Ensuring Feasibility

We have mentioned that feasibility of the decentralized trajectories is the main issue in decentralized control schemes. In this section we discuss some modification to the original problem which can ensure feasibility.

1) Robust Constraint Fulfillment: Consider the coupling constraints of problem $\mathcal{P}_i$ at step k

$$g^i(x^i_{k,t}, \tilde{x}^i_{k,t}) \leq 0 \tag{14}$$

and by using the state update equations

$$\begin{aligned} x^i_{k+1,t} &= f^i(x^i_{k,t}, u^i_{k,t}), \quad k \geq 0 \\ x^j_{k+1,t} &= f^j(x^j_{k,t}, u^j_{k,t}), \quad (j,i) \in \mathcal{A},\ k \geq 0 \end{aligned} \tag{15}$$

rewrite them as

$$g^i_k(x^i_t, \tilde{x}^i_t, u^i_{[0,\ldots,k-1]}, \tilde{u}^i_{[0,\ldots,k-1]}) \leq 0 \tag{16}$$

where $u^i_{[0,\ldots,k-1]} \triangleq \{u^i_0, \ldots, u^i_{k-1}\}$ and $\tilde{u}^i_{[0,\ldots,k-1]} \triangleq \{\tilde{u}^i_0, \ldots, \tilde{u}^i_{k-1}\}$. In order to ensure the feasibility of the team system, a possible approach is to "robustify" the constraints (16) for all vehicles at all time steps. In other words, we can require that the coupling constraints at each node are satisfied for *all* possible behaviors of the neighboring nodes, once their initial condition is known. Therefore, the vector $\tilde{u}^i_{[0,\ldots,k-1]}$ can be considered as a disturbance which can lead to possible infeasibility of constraint (16). There are two possible schemes: open-loop and closed-loop constraint fulfillment. An open-loop robust constraint fulfillment is formulated next. Substitute the functions g^i_k with $\bar{g}^i_k : \mathbb{R}^{n^i} \times \mathbb{R}^{\tilde{n}^i} \times \mathbb{R}^{m^i} \to \mathbb{R}^{n^i}$ where

1) For all $x^i_t, \tilde{x}^i_t, u^i_{[0,\ldots,k-1]}$ which satisfy

$$\bar{g}^i_k(x^i_t, \tilde{x}^i_t, u^i_{[0,\ldots,k-1]}) \leq 0 \tag{17}$$

we have

$$g^i_k(x^i_t, \tilde{x}^i_t, u^i_{[0,\ldots,k-1]}, \tilde{u}^i_{[0,\ldots,k-1]}) \leq 0$$

for all admissible[1] $\tilde{u}^i_{[0,\ldots,k-1]}$.

2) The sets described by

$$\bar{g}^i_k(x^i_t, \tilde{x}^i_t, u^i_{[0,\ldots,k-1]}) \leq 0 \tag{18}$$

for $i = 1, \ldots, N_v$ $k = 1, \ldots, N-1$ are nonempty.

Robust closed-loop formulation [15] is less conservative but more computationally involved. We will not describe the details of the robust closed-loop formulation for a simple reason. "Robust constraint fulfillment" applied to decentralized control schemes results in a very conservative approach even for the closed-loop case. For instance, consider the case of formation flight. Assume we have only two aircraft and we want to design a local controller on the first aircraft using robust constraint fulfillment. The worst case scenario will include, in most cases, the collision of the two aircraft if they are not very far from each other and if they have the same dynamics and constraints. However in reality, neighboring vehicles collaborate between each other to fly in formation.

2) Reducing Conservativeness: A less conservative approach for ensuring feasibility of the decentralized scheme has to take into consideration that systems in a team are cooperating, and therefore the trajectory that a node is predicting should not be extremely different form what its neighbors are executing. This idea can be formulated in several ways. For instance, one could allow the exchange of optimizers between the nodes in order to try to be as close as possible to what the neighboring system has predicted about a certain node. Another possibility is to tighten the coupling constraints (6) by a quantity which is an indirect measure of the cooperativeness of the team [10]

$$g^i_k(x^i_t, \tilde{x}^i_t, u^i_{[0,\ldots,k-1]}, \tilde{u}^i_{[0,\ldots,k-1]}) \leq \epsilon^i_k \tag{19}$$

where $\epsilon^i_k \leq 0$ is a new optimization variable. Finding efficient methods to compute ϵ^i_k off-line, based on a priori knowledge of the team behavior is a focus of our current, ongoing research. Also, the idea of tightening these constraints (19) can be exploited in a two-stage process. In the first stage of the optimization problems (12), the coupling constraints are substituted with the one in (19). Their parametric solution [16] with respect to ϵ^i_k yields the optimizer function $u^{*i}(\epsilon^i_0, \ldots, \epsilon^i_N)$. In a second stage the nodes communicate between themselves in order to agree on a set of $\bar{\epsilon}^i_k$ for $i = 1, \ldots, N_v$, $k = 1, \ldots, N$ which ensures feasibility of the decentralized trajectories. If the agreement algorithm ends with a positive answer, each vehicle will implement $u^{*i}(\bar{\epsilon}^i_0, \ldots, \bar{\epsilon}^i_N)$.

IV. Simulations and Final Remarks

In [14], [17] the reader can find interesting simulation examples where the proposed decentralized scheme is applied to formation flight. We simulate formation flight of vehicles flying at a certain altitude where each vehicle is modeled as a point mass in two dimensions with constraints on states

[1] admissible inputs have to satisfy constraints (2)

and inputs. The coupling between vehicles stems from the common objective of the team (moving in formation) and its constraints (vehicles are not allowed to violate each others protection zones). For brevity we report here only the main features and observations.

We use a linear model of the vehicle, piecewise-linear cost functions and parallelepipedal protection zones so that we can rewrite problem (12) as a Mixed Integer Linear Program (MILP) [18], [19], for which efficient branch-and-bound solvers are available [20]. Furthermore, assuming a modest number of neighboring vehicles, explicit solutions of the underlying MILP problem can be computed off-line, which reduces the required number of calculations to a function evaluation [21].

As in most simple decentralized schemes, even though vehicles have only a limited knowledge of their neighbors, the simulations show signs of a collective behavior that could be attributed to a centralized solution. Extensive simulations have shown that a decentralized scheme can find reasonable solutions to cooperative problems even though feasibility can be compromised depending on initial conditions of the vehicles. The sizes of the protection zones have a significant influence on overall feasibility and the "quality" of solutions as well.

The role of the prediction horizon length is quite different from what standard RHC theory would suggest, mainly because of the decentralized nature of the problem. This means, for instance, that longer horizon lengths do not necessarily provide a better solution in general [10], since predictions about the future behavior of neighboring vehicles can be completely inaccurate. In some cases feasibility of the decentralized scheme was lost with the use of longer horizons.

Also, feasibility of the decentralized problem without terminal cost and constraints is a function of the vehicles' "strategy". This can be influenced by selecting appropriate weights in the cost function. For instance a larger weight on the relative positions implies that vehicles are prompted to reach their desired relative states (formation) and resolve associated conflicts within a time frame that is comparable to their horizon lengths. Once the formation is attained, the remaining common goal of each vehicle is to "drift" to their target points. This at the same time becomes a much simpler objective to accomplish even in a decentralized way.

A formal stability and feasibility proof of a particular scheme in the proposed framework which is not too conservative is still under investigation. Our experience is that the more complex the decentralized control scheme is, the more difficult it is to give any stability or feasibility proofs. As in most of the RHC literature, such decentralized schemes work very well in practice even without any "theoretical stability proof".

Simulation examples show that the decentralized approach to formation flight can provide feasible solutions even in challenging scenarios. A number of alternative decentralized RHC approaches which ensure feasibility in a decentralized way are currently under investigation [13], [22].

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