Safe Platooning of Unmanned Aerial Vehicles via Reachability

Abstract—

I. Introduction

Unmanned aerial vehicle (UAV) systems have in the past been mainly used for military operations [?]. Recently, however, there has been an immense increase of interest in using UAVs for civil applications. Through projects such as Amazon Prime Air and Google Project Wing, companies are looking to send UAVs into the airspace to deliver packages. As a rough estimate, suppose in a city of 2 million people, each person requests a drone delivery every 2 months on average and each delivery requires a 30 minute trip for the drone. This would equate to thousands of simultaneous vehicles in the air. As a result of, government agencies such as the Federal Aviation Administration (FAA) and National Aeronautics and Administration (NASA) of the United States are also investigating air traffic control for autonomous vehicles in order to prevent collisions among numerous UAVs [?]. Applications of UAVs extend beyond simply package delivery; they can also be used, for example, to provide supplies or to firefight in areas that are difficult to reach but require prompt response [?].

Optimal control and game theory are powerful tools for providing liveness and safety guarantees to controlled dynamical systems under disturbances, and various formulations have been successfully used to analyze relatively small problems involving a small number of vehicles []. These formulations are based on Hamilton-Jacobi (HJ) reachability, which computes the reachable set, defined as the set of states from which a system is guaranteed to have a control strategy to reach a set of target states under the worst case disturbance. Reachability is a powerful tool because reachable sets can be used for synthesizing both controllers that steer the system towards a set of goal states (liveness controllers), and controllers that steer the system away from a set of unsafe states (safety controllers). Furthermore, the HJ formulations are flexible in terms of system dynamics, enabling the analysis of non-linear systems, and provides the optimal controller given the target set.

Despite its power and success in previous applications, HJ reachability analysis involves solving an HJ Partial Differential Equation (PDE) on a grid, making the computation complexity scale exponentially with the number of states,

This work has been supported in part by NSF under CPS:ActionWebs (CNS-931843), by ONR under the HUNT (N0014-08-0696) and SMARTS (N00014-09-1-1051) MURIs and by grant N00014-12-1-0609, by AFOSR under the CHASE MURI (FA9550-10-1-0567). The research of J.F. Fisac has received funding from the "la Caixa" Foundation.

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and therefore the number of vehicles. This makes the computation intractable for large numbers of vehicles.

A considerable body of work has been done on the platooning of vehicles []. For example, blah blah blah [].

In the context of HJ reachability, putting vehicles into platoons is desirable because of the additional structure that platoons impose on its members. In this paper, we will consider a platoon of vehicles to be a group of vehicles in a single file along an air highway. With this additional structure, pairwise safety guarantees of vehicles can be more easily translated into safety guarantees of all the vehicles in the platoon.

In this paper, assuming the simple single-file platoon mentioned above, and we first propose a hybrid systems model of vehicles that participate in platooning. We then show how HJ reachability can be used to synthesize live controllers that enable vehicles to reach a set of desired states, and how HJ reachability can also be used to wrap safety controllers around liveness controllers in order to prevent dangerous configurations such as collisions. Finally We show simulation results of vehicles forming a platoon, vehicles in a platoon responding to a malfunctioning platoon member, and vehicles in a platoon responding to an intruder outside of the platoon to illustrate the use of reachable sets and the behavior of the vehicles in these example scenarios.

II. PROBLEM FORMULATION

A. Vehicle Dynamics

Consider an unmanned aerial vehicle whose dynamics are given by

$$\dot{x} = f(x, u) \tag{1}$$

where $x \in \mathbb{R}^n$ represents the state of the unmanned aerial vehicle, and $u \in \mathbb{R}^{n_u}$ represents the control action. In this paper, we will assume that each vehicle has the following simple model of a quadrotor:

$$\begin{split} \dot{p}_x &= v_x \\ \dot{v}_x &= u_x \\ \dot{p}_y &= v_y \\ \dot{v}_y &= u_y \\ \underline{u} &\leq |u_x|, |u_y| \leq \bar{u} \end{split} \tag{2}$$

where the state $x=(\dot{p}_x,\dot{v}_x,\dot{p}_y,\dot{v}_y)\in\mathbb{R}^4$ represents the quadrotor's position in the x-direction, its velocity in the x-direction, and its position and velocity in the y-direction, respectively. For convenience, we will denote the position and velocity $p=(p_x,p_y),v=(v_x,v_y)$, respectively.

Now, consider a group of N quadrotors, $Q_i, i=1, ..., N$. Each quadrotor Q_i has collision sets given by $\mathcal{C}_{ij}=\{x_i,x_j\mid |p_{x_i}-p_{x_j}|\leq d, |p_{y_i}-p_{y_j}|\leq d\}$, whose interpretation is that Q_i,Q_j are considered to be in collision if their states $x_i,x_j\in\mathcal{C}_{ij}$.

In general, the problem of collision avoidance among N vehicles cannot be tractably solved using traditional dynamic programming approaches because the computation complexity of these approaches scales exponentially with the number of vehicles. Thus, in our present work, we will consider the situation where N quadrotors form a platoon. The structure imposed by the platoon enables us to analyze the liveness and safety of the quadrotors in a tractable manner.

B. Platoon Definition

A platoon of vehicles can impose a variety of requirements on the behavior of the vehicles. We will assume that the quadrotors in a platoon travel along an air highway, which is defined by as a path inside a pre-defined altitude range. The quadrotors maintain a single-file formation behind the platoon leader, with a separation distance of $3\sqrt(2)d$. In order to allow for close proximity of the quadrotors, we assume that in the event of a malfunction, a quadrotor will be able to exit the altitude range of the highway within two seconds. Such a requirement may be implemented practically as an emergency landing procedure which the quadrotors revert to when a malfunction is detected.

As part of a platoon, each quadrotor must be capable of performing a number of essential cooperative maneuvers. In this paper, we consider the following:

- safely merging onto an air highway;
- safely joining a platoon;
- reacting to a malfunctioning vehicle in the platoon;
- reacting to an intruder vehicle;
- following the highway, a curve defined in space at constant altitude, at a specified speed;
- maintaining a constant relative position and velocity with the leader of a platoon.

C. Vehicles as Hybrid Systems

UAVs in general may be in a number of modes of operations, depending on whether it's part of a platoon, and in the case that it is part of a platoon, whether it is a leader or a follower. Therefore, it is natural to model quadrotors as hybrid systems []. In this paper, we restrict the control actions of each quadrotor depending on the mode. We assume that each quadrotor has the following modes:

- Free: Vehicle that's not in a platoon. Available control actions:
 - Any control action, but must be more than 3 seconds away from collision with all other quadrotors, assuming the worst case disturbance
- Leader: Leader of platoon (could be by itself). Available control actions:
 - Travel along the highway at a pre-specified speed,
 - Merge current platoon with a platoon in front; this changes the mode to "Follower".

- Follower: Vehicle in a platoon following the leader. Available control actions:
 - Follow a platoon; must be more than 1.5 seconds away from collision with the quadrotors in front and behind, assuming the worst case disturbance.
 - Create a new platoon; this changes the mode to "Leader".
 - Leave the highway; this changes the mode to "Free".
- Faulty: Vehicle in a platoon that has malfunctioned: reverts to default behavior and descends after 2 seconds.

D. Objectives

Using the previously-mentioned modeling assumptions, we would like to address the following questions:

- 1) How do vehicles form platoons?
- 2) How to ensure safety of the vehicles during normal operation and when there is a malfunctioning vehicle within platoon?
- 3) How can the platoon respond to intruder UAVs?

The answers to these questions can be broken down into the maneuvers listed in Section II-B. In general, the control strategies of each quadrotor have a liveness component, which specifies a set of states towards which the quadrotor aims to reach, and a safety component, which specifies a set of states that it must avoid. In this paper, we address both the liveness component and safety component using reachability analysis.

III. HAMILTON-JACOBI-ISAACS REACHABILITY

A. General Framework

We consider a differential game between two players described by the system

$$\dot{x} = f(x, u, d), t \in [-T, 0]$$
 (3)

where $x \in \mathbb{R}^n$ is the system state, $u \in \mathcal{U}$ is the control of Player 1, and $d \in \mathcal{D}$ is the control of Player 2. We assume $f: \mathbb{R}^n \times \mathcal{U} \times \mathcal{D} \to \mathbb{R}^n$ is uniformly continuous, bounded, and Lipschitz continuous in x for fixed u,d, and the control functions $u(\cdot) \in \mathbb{U}, d(\cdot) \in \mathbb{D}$ are drawn from the set of measurable functions. Player 2 is only allowed to use nonanticipative strategies γ , defined by

$$\begin{split} \gamma \in \Gamma &:= \{\mathcal{N} : \mathbb{U} \to \mathbb{D} \mid \\ u(r) &= \hat{u}(r) \text{ for almost every } r \in [t,s] \\ &\Rightarrow \mathcal{N}[u](r) = \mathcal{N}[\hat{u}](r) \text{for almost every} r \in [t,s] \} \end{split}$$

In our differential game, the goal of Player 2 is to drive the system into some target set \mathcal{L} , and the goal of Player 1 is to drive the system away from it. The set \mathcal{L} is represented as the zero sublevel set of a bounded, Lipschitz continuous function $l: \mathbb{R}^n \to \mathbb{R}$:

$$\mathcal{L} = \{ z \in \mathbb{R}^n \mid l(z) \le 0 \} \tag{5}$$

Given the dynamics (3) and the target set l, we would like to compute the backwards reachable set, V(t), defined below:

$$\mathcal{V}(t) := \{ z \in \mathbb{R}^n \mid \exists \gamma \in \Gamma \text{ such that} \\ \forall u(\cdot) \in \mathbb{U}, \exists s \in [t, 0], f(\xi_f(s; z, t, u(\cdot), \gamma[d](\cdot)) \}$$
 (6)

Many methods involving solving HJI PDEs [] and HJI variational inequalities (VI) [] have been developed for computing the reachable set. These HJI PDEs and HJI VIs can be solved using well-established numerical methods. For this paper, we use the formulation in [?], which has shown that the backwards reachable set $\mathcal{V}(t)$ can be obtained as the zero sublevel set of the viscosity solution [] V(t,x) of the following terminal value HJI PDE:

$$D_t V(t, x) + \min\{0, \max_{u \in \mathcal{U}} \min_{d \in \mathcal{D}} D_x V(t, x) \cdot f(x, u, d)\} = 0$$
$$V(0, x) = l(x)$$

from which we obtain $V(t) = \{x \in \mathbb{R}^n \mid V(t,x) \leq 0\}$. From the solution V(t,x), we can also obtain the optimal controls for both players via the following:

$$u^{*}(t) = \arg \max_{u \in \mathcal{U}} \min_{d \in \mathcal{D}} D_{x} V(t, x) \cdot f(x, u, d)]$$

$$d^{*}(t) = \arg \min_{d \in \mathcal{D}} D_{x} V(t, x) \cdot f(x, u^{*}, d)]$$
(8)

In the special case where there is only one player, we obtain an optimal control problem for a system with dynamics

$$\dot{x} = f(x, d), t \in [-T, 0]$$
 (9)

$$D_t V(t, x) + \min\{0, \min_{d \in \mathcal{D}} D_x V(t, x) \cdot f(x, d)\} = 0$$

$$V(0, x) = l(x)$$
(10)

where the optimal control is given by

$$d^*(t) = \arg\min_{d \in \mathcal{D}} D_x V(t, x) \cdot f(x, d)$$
 (11)

in which we would like to compute the backwards reachable set and the control that will optimally drive the system into the target set \mathcal{L} . The corresponding HJB PDE we obtain becomes

For our application, the formulation in [?] is desirable due to a recent formulation for decoupled systems in [], which not only speeds some of our computations, and makes some of our computations tractable.

B. Relative Dynamics and Augmented Relative Dynamics

Besides, Equation (3), we will also consider the relative dynamics between two quadrotors Q_i, Q_j , given in Equation (??). These dynamics can be obtained by defining the relative variables

$$\begin{aligned} p_{x,r} &= p_{x,i} - p_{x,j} \\ v_{x,r} &= v_{x,i} - v_{x,j} \\ p_{y,r} &= p_{y,i} - p_{y,j} \\ v_{y,r} &= v_{y,i} - v_{y,j} \end{aligned} \tag{12}$$

We treat Q_i as Player 1, the evader who wishes to avoid collision, and we treat Q_j as Player 2, the pursuer, or disturbance, that wishes to cause a collision. In terms of the relative variables given in Equation (12), we have the following:

$$\dot{p}_{x,r} = v_{x,r}
\dot{v}_{x,r} = u_{x,i} - u_{x,j}
\dot{p}_{y,r} = v_{y,r}
\dot{v}_{y,r} = u_{y,i} - u_{y,j}$$
(13)

We also consider relative dynamics augmented by the velocity of quadrotor Q_i , given in Equation (14). These dynamics are needed to impose a velocity limit on the quadrotor.

$$\dot{p}_{x,r} = v_{x,r}
\dot{v}_{x,r} = u_{x,i} - u_{x,j}
\dot{v}_{x,i} = u_{x,i}
\dot{p}_{y,r} = v_{y,r}
\dot{v}_{y,r} = u_{y,i} - u_{y,j}
\dot{v}_{y,i} = u_{y,i}$$
(14)

IV. LIVENESS CONTROLLERS

A. Merging onto a Highway

We model the merging of a vehicle onto an air highway as a path planning problem, where in addition to the target position, we also specify a target velocity such that the magnitude (the speed) is given by the highway specification, and the direction is along the direction of the highway. Thus, for a quadrotor, the objective would be to drive the system in Equation (3) to a specific state $\bar{x}=(\bar{p}_x,\bar{v}_x,\bar{p}_y,\bar{v}_y)$, or a small range of states defined by the set

$$\mathcal{L} = \{ x \mid |p_x - \bar{p}_x| \le r_{p_x}, |v_x - \bar{v}_x| \le r_{v_x}, \\ |p_y - \bar{p}_y| \le r_{p_y}, |v_y - \bar{v}_y| \le r_{v_y} \}$$
 (15)

Note that for now, we assume that there are no nearby vehicles on the highway.

In this reachability problem, \mathcal{L} is the target set, represented by the zero sublevel set of the function l(x), which specifies the terminal condition of the HJB PDE we would need to solve, Equation (10). The solution we obtain, V(t,x); $V(-T,x) \leq 0$, then, specifies the reachable set $\mathcal{V}(T)$, the set of states from which the system can be driven to the target \mathcal{L} within a duration of T. This gives the following algorithm for a quadrotor merging onto the highway:

- 1) Move towards \bar{x} in a straight line, with some velocity, until V(-T,x) < 0.
- Apply the optimal control according to Equation (11) until the quadrotor reaches L.

B. Merging into a Platoon

We again pose the merging of a vehicle onto an air highway near a platoon as a reachability problem. Here, we would like quadrotor Q_i to merge onto the highway and follow another quadrotor Q_j in a platoon. Thus, we would like to drive the system given by Equation (14) to a specific $\bar{x}=(\bar{p}_{r,x},\bar{v}_{x,r},\bar{p}_{r,y},\bar{v}_{y,r})$, or a small range of relative states defined by the set

$$\mathcal{L} = \{ x \mid |p_{x,r} - \bar{p}_{x,r}| \le r_{p_x}, |v_{x,r} - \bar{v}_{x,r}| \le r_{v_x}, \\ |p_{y,r} - \bar{p}_{y,r}| \le r_{p_y}, |v_{y,r} - \bar{v}_{y,r}| \le r_{v_y} \}$$
(16)

The target set \mathcal{L} is represented as the zero sublevel set of a function l(x), which specifies the terminal condition of the HJI PDE (7). The zero sublevel set of the solution to (7), V(t,x), gives us the set of relative states from which Q_i can reach the target and join the platoon following Q_j within a duration of t. We assume that Q_j moves along the highway at constant speed, so that $u_j(t)=0$. Similar to the last section, the following is a suitable algorithm for a quadrotor merging onto a highway and joining a platoon to follow Q_j :

- 1) Move towards \bar{x} in a straight line, with some velocity, until V(-T,x) < 0.
- 2) Apply the optimal control according to Equation (8) until the quadrotor reaches \mathcal{L} .

C. Other Quadrotor Maneuvers

Reachability was used in Sections IV-A and IV-B for the relatively complex maneuvers of merging onto a highway and joining a platoon. For the simpler maneuvers of traveling long a highway and following a platoon, we resort to simpler controllers described below.

1) Traveling along a highway: We use a model predictive controller for traveling along a highway; this controller allows the leader to travel along a highway at a pre-specified speed. Here, the goal is for a leader quadrotor to track a constant-altitude path, defined as a curve $\bar{p}(s)$ parametrized by $s \in [0,1]$ in $p=(p_x,p_y)$ space, while maintaining a velocity $\bar{v}(s)$ that corresponds to constant speed in the direction of the highway. Assuming that the initial position on the highway, $s_0=s(t_0)$ is specified in the time horizon $[t_0,t_1]$, Such a controller can be obtained from the following optimization problem:

$$\begin{aligned} & \text{maximize } \int_{t_0}^{t_0+T} \left\{ \|p(t) - \bar{p}(s(t))\|_2 + \\ & \|v(t) - \bar{v}(s(t))\|_2 + 1 - s \right\} dt \\ & \text{subject to } \dot{x} = f(x,u) \text{ where } f \text{ is given in (3)} \\ & \underline{u} \leq |u_x|, |u_y| \leq \bar{u} \\ & \underline{v} \leq |v_x|, |v_y| \leq \bar{v} \\ & s(t_0) = s_0 \\ & \dot{s} > 0 \end{aligned} \tag{17}$$

If we discretize time, and assume that $\bar{p}(\cdot)$ is linear, then the above optimization is convex, and can be quickly solved.

2) Following a Platoon:

V. SAFETY CONTROLLERS: WRAPPING REACHABILITY AROUND EXISTING CONTROLLERS

A quadrotor, whether in a platoon or not, can only use a liveness controller when it is not in any danger of collision with other quadrotors or obstacles. However, if the quadrotor could potentially be involved in a collision within the next short period of time, it must switch to a safety controller. In this section, we will demonstrate how HJI reachability can be used to both detect imminent danger and synthesize a controller that guarantees safety within a specified time horizon. For our safety analysis, we will use the model in Equation (14).

We begin by defining the target set \mathcal{L} , which characterizes the configurations in the relative coordinates for which quadrotors Q_i and Q_j are considered to be in collision.

$$\mathcal{L} = \{ x \mid |p_{x,r}|, |p_{y,r}| \le d, |v_{x,i}|, |v_{y,i}| \ge v_{\text{max}} \}$$
 (18)

With this definition, Q_i is considered to be unsafe if Q_i and Q_j are within a distance d in either of the x- and y-directions, or if Q_i has exceeded some maximum speed $v_{\rm max}$. For illustration purposes, we choose d=2 meters, and $v_{\rm max}=5$ m/s.

We can now define the function l(x), whose zero sublevel set coincides with \mathcal{L} , and solve the HJI PDE (7) using l(x) as the terminal condition. As before, the zero sublevel set of the solution V(t,x) specifies the reachable set $\mathcal{V}(t)$, which characterizes the states, as defined in (14), from which Q_i avoid \mathcal{L} for a time period of t, despite the worst possible control of Q_j . The safety controller can be synthesizes according to Equation (8).

To wrap our safety controller around liveness controllers, we use the following algorithm:

- 1) For a specified time horizon t, evaluate V(t, x).
- 2) Use the safety or liveness controller depending on the value V(t,x):
 - If $V(t,x) \le 0$, use the safety controller, specified by Equation (8).
 - If V(t, x) > 0, use a liveness controller.

VI. ANALYSIS

Safety:

- There always exists a safety control to keep the system safe for a prescribed time horizon despite the worst case disturbances
- Pairwise safety is translated into safety among all vehicles in the platoon thanks to linear chain and platoon assumptions. (Main proposition?)
- Safety with respect to intruder vehicles: under what exact conditions? What can we prove?

VII. SCENARIOS CASE STUDY

In this section, we consider several situations that a quadrotor in platoon on an air highway may commonly encounter, and show via simulations the behaviors that emerge from the controllers we defined in Section ??.

A. Forming a Platoon

We first consider the scenario in which a number of quadrotors are trying to merge onto an initially unoccupied highway. In order to do this, each quadrotor first checks for safety with respect to the other quadrotors, and uses the safety controller if necessary, according to Section ??. Otherwise, the quadrotor uses the liveness controller described in Section ??.

For the simulation example, the highway is specified by the line $p_y=0.5p_x$, the point of entry on the highway is chosen to be $(\bar{p}_x,\bar{p}_y)=(4,2)$, and the velocity on the highway is chosen to be $\bar{v}_x,\bar{v}_y)=\frac{\bar{v}}{\sqrt{0.5^2+1^2}}(0.5,1)$. The velocity simply states that the quadrotors must travel at a speed \bar{v} along the direction of the highway. This forms the target state $\bar{x}=(\bar{p}_x,\bar{v}_x,\bar{p}_y,\bar{v}_y)$, from which we define the target set \mathcal{L} as in Section $\ref{eq:condition}$.

The first quadrotor that completes the merging onto an empty highway creates a platoon and becomes its leader, while subsequent quadrotors form a platoon behind the leader in a pre-specified order. The process of joining a platoon is described in Section ??. Here, we choose $(\bar{p}_{r,x},\bar{p}_{r,y})$ to be a distance $3\sqrt(2)d$ behind the last quadrotor in the platoon, and $(\bar{v}_{x,r},\bar{v}_{y,r})=(0,0)$. This gives us the target set $\mathcal L$ that we need.

Figures 1 and 2 show the simulation results. Since the liveness reachable sets are in 4D and the safety reachable sets are in 6D, we plot their 2D slices based on the quadrotors velocities and relative velocities.

Figure 1 illustrates the use of liveness and safety reachable sets using just the first two quadrotors to reduce visual clutter. The first quadrotor Q_1 (red disk) first travels in a straight line towards the highway merging point \bar{x} (red circle) at t=1.5, because it is not yet in the liveness reachable set for merging onto the highway (red dotted boundary). When it is travels within the liveness reachable set boundary at t=2.8, it is "locked-in" to the target position \bar{x} , and follows the optimal control in (11) to \bar{x} . During the whole time, the first quadrotor Q_1 checks whether it may collide with Q_2 within a time horizon of 3 seconds. However, since Q_1 never goes into the boundary of the safety reachable set (red dashed boundary), it is able to use the liveness controller the entire time.

After Q_1 has reached \bar{x} , it forms a platoon, becomes the platoon leader, and continues to travel along the highway. Q_2 (blue disk) now, at t=7, begins joining the platoon behind Q_1 , by moving towards the target \bar{x} relative to the position of Q_1 . Note that \bar{x} moves with Q_1 as \bar{x} is defined in terms of the relative states of the two quadrotors. When Q_2 moves inside the liveness reachable set boundary for joining the platoon (blue dotted boundary), it is "locked-in" to the target relative position \bar{x} , and begins following the optimal control in (8) towards the target as long as it stays out of the safety reachable set (blue dashed boundary).

Figure 2 shows the behavior of all 5 quadrotors which eventually form a platoon and travel along the highway together. The liveness controllers allow the quadrotors to

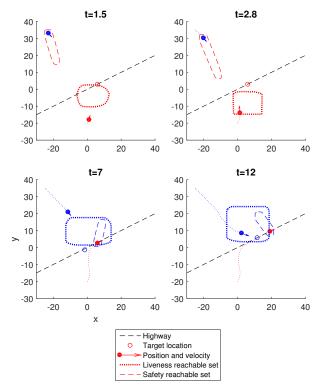


Fig. 1: Reachable sets used to merge onto a highway to form a platoon (top subplots) and to join a platoon on the highway (buttom subplots).

optimally and smoothly enter the highway and join platoons, while the safety controllers prevent collisions from occurring.

B. Malfunctioning Vehicle in Platoon

- · No fault detected
 - Normal operation
- Fault detected (2 seconds period)
 - Detect faulty QR
 - All QR check safety against fault
 - Split platoon with QR following the fault becoming leader of the 2nd platoon
 - Treat fault as an intruder
 - Fault will resolve by descending (2 seconds)
- · After fault resolved
 - 2nd Platoon rejoins 1st platoon with faulty quadrotor removed

C. Intruder Vehicle

D. Malfunction During Merging

VIII. CONCLUSIONS AND FUTURE WORK

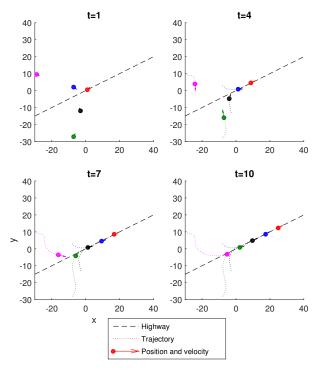


Fig. 2: Five quadrotors merging onto a highway. The first quadrotor forms a platoon on the highway and becomes the platoon leader; the rest of the quadrotors join the platoon behind the first quadrotor.