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Split and Join of Vehicle Formations doing Obstacle Avoidance

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Abstract—**In this paper, we study a scenario where a set of vehicles having different origins and/or destinations move in a common region. The goal is to have the vehicles join and leave formations in a completely decentralized manner. When a vehicle traveling along its own path finds itself moving close to another vehicle it automatically switches into follower mode. The vehicle stays in follower mode as long as the path of the other vehicle is beneficial to it. If, at some point, the leader is not moving towards the destination of the follower, the follower will leave the leader and head of on its own.**

We address this problem for a group of dynamic unicycle robots. Incorporating the split and join capability into a Receding Horizon Control approach to obstacle avoidance we are able to show safety as well as convergence of all vehicles to their destinations under general nonconvex obstacle assumptions.

We illustrate the method with a simulation example.

I. Introduction

The two separate areas of obstacle avoidance and formation control are both well studied, see e.g. [1], [2], [5], [6], [9], [10], [11], [13]. The former has been an active branch for a long time, while the later is fairly new, and has received an increasing amount of attention in the last couple of years. In the light of this development it is only natural that the merge of the two branches is also gaining interest within the robotics community.

The coordinated motion of multiple robots can loosely be divided into two categories, formations and swarms. Formations are groups of vehicles where a particular geometric shape is prescribed. Swarms on the other hand are groups that exhibit some kind of cohesion without having prescribed distances or other precise geometrical relations. Schools of fish and flocks of birds are examples of swarming behavior.

The early papers that addressed obstacle avoidance of multiple vehicles either took an approach based on planning and optimal control, [13], or a classical reactive approach [8]. The optimal control approaches usually suffer from extensive computational demands, while the purely reactive schemes are often heuristic or dependent on specialized obstacle assumptions. More recent work on flocks include [17],[20] and formation studies can be found in [16] and [18]. Most of these studies use appealingly minimalistic controllers to achieve quite remarkable group behavior. Such controllers can also be motivated by observations of biological systems. But there is a prize to pay for this simplicity. In many cases some specialized obstacle assumptions, in particular convexity, must be made. These assumptions are furthermore seldom valid in either man made or natural environments. The approach presented here is different in that it is founded in a more elaborate obstacle avoidance scheme, designed to handle general nonconvex obstacles.

The obstacle avoidance approach we use is both reactive and deliberate. The reactive part consists of a short-horizon, discretized (and therefore tractable), optimal control scheme that can avoid newly discovered obstacles. The deliberate part relies on a solution to a shortest path problem on a graph approximation of the obstacle-free space. This solution is used to form a navigation function [7], which in turn is used to construct a Lyapunov function guaranteeing convergence.

The key observation of this paper is that the cost function of the optimization described above is only used to boost navigation performance. The properties of safety and goal convergence are guaranteed by the constraints of the optimization. Therefore it is possible to exchange the utility function for a new one reflecting e.g. formation fitness. In this way we can switch between the short term goals of formation maintenance and optimal routing towards the destination, without compromising the long term properties of safe arrival at the individual destinations.

The organization of the paper is as follows. In Section II we review our earlier work and briefly present the Convergent Dynamic Window Approach to obstacle avoidance [3], [4]. In Section III we describe how to design a hybrid switching between optimization of formation maintenance and individual goal achievement. Then, in Section IV we apply our method to a simulation example and draw conclusions in Section V.

II. Preliminaries

In this section we discuss the obstacle avoidance scheme this paper builds upon and see an example of how this scheme was used in a previous study, with complementary focus, on formation obstacle avoidance. First however, we specify the models and notation used.

The robot model we consider is the dynamic unicycle [12]. This model is accurate for many indoor robots such as the Nomadic Technologies Super Scout as well

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as all caterpillar-type outdoor vehicles. The equations of motions are

$$\begin{aligned}\dot{y}_1 &= v\cos\theta,\\ \dot{y}_2 &= v\sin\theta,\\ \dot{\theta} &= \omega,\\ \dot{v} &= F/m,\\ \dot{\omega} &= \tau/J.\end{aligned}$$

where y_1, y_2 is the position, θ the orientation, v the translational velocity, ω the angular velocity and F/m and τ/J are force per mass and torque per moment of inertia, respectively. A kinematic version of this model (where v and ω are the controls) was used in [8], [13]. It was shown in [11] that the dynamics of the position $r \in \mathbb{R}^2$ of an off-wheel axis point of this model can be feedback linearized to $\ddot{r} = u$, i.e. a two-dimensional double integrator (which is the model used in [2], [5], [10]).

The problem we consider in this paper is to control a set of n vehicles $\ddot{r}_i = u_i$, $||u_i|| \leq u_{max}$, $i = 1 \ldots n$ moving, where applicable, in formation, towards individual goal points without colliding with obstacles. Before studying the multi vehicle case we look at the single vehicle obstacle avoidance scheme used in this paper.

A. Obstacle Avoidance

The problem of robotic motion planning is a well-studied one, see for instance [7]. Apart from the classical approaches of histograms and vector addition, a few somewhat more recent schemes have emerged. One of them is the Dynamic Window Approach [1], [2]. We will use a revised version of the latter that can be viewed as a synthesis inspired by Primbs et. al., [15], of the performance-oriented approaches [1], [2] and the convergence-oriented method of exact navigation by artificial potentials presented by Rimon et. al. in [14].

In [3], a Provable Convergent Dynamic Window Approach is proposed. The original approach is reformulated as a combined Receding Horizon Control and Control Lyapunov Function scheme. The problem is stated as

$$\begin{aligned} u(\cdot) = \text{argmin} \quad & V(x(t+T)) && (1)\\ \text{s.t.} \quad & \ddot{r} = u && (2)\\ & \dot{V}(x, u) \leq -\epsilon||\dot{r}|| && (3)\\ & r(\cdot) \text{ collision free in } [t, t+T] && (4)\end{aligned}$$

where $x = (r, \dot{r})$ is the state, argmin is short for 'the argument that minimizes', V is a control Lyapunov function $V(x) = \frac{1}{2}\dot{r}^T\dot{r} + \frac{k}{\sqrt{2}}NF(r)$. $NF(r)$ is the Navigation Function, a continuous approximation of the length of the shortest obstacle-free path to the goal. Finally, T is the planning horizon of the Optimization in Receding Horizon Control scheme. Below we restate the main results of [3] but refer to that paper for proofs and details.

Theorem 2.1 (Asymptotic Stability): If the control scheme in (1)-(4) is used and if there is a traversable path from start to goal in the occupancy grid. Then the robot will reach the goal position.

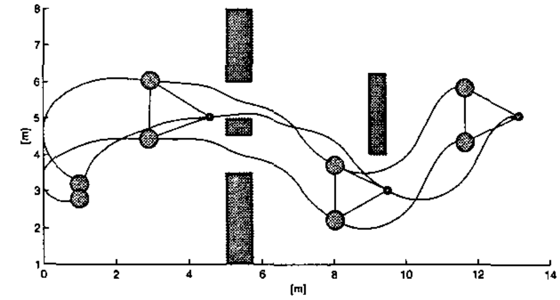

Fig. 1. Earlier complementary work, compare with Figure 4. Trajectories of the three robots traveling in a rigid formation. The smaller circle denotes the leader. All vehicles start near (1,3).

Remark 2.2: Note that this excludes all so-called local minima problems present in some navigation schemes.

Theorem 2.3 (Safety): If the control scheme in (1)-(4) is used and if the robot starts at rest in an unoccupied position. Then, the robot will not run into an obstacle.

The key observation is that both these theorems hold if we replace the utility function, (1), but not the constraints, (2), (3), (4), above. This fact will be exploited in Section III, but before going into the details we mention an earlier and complementary study on formation obstacle avoidance.

B. Earlier Formation Work

In the paper [19] it was shown how to guide a non-flexible formation through an obstacle environment. In that case the formation itself was given higher priority than the navigation. The majority of the vehicles did nothing but leader following, and the job of the leader was to guide the whole group towards the destination in such a way that no follower was forced to collide while trying to stay in formation. A typical scenario is shown in Figure 1.

The problem formulation of the present study is the reverse. Here all robots top priority is to reach their own destinations. If however, situations arise where parts of the journey can be traversed in the company of others, as part of a formation, these opportunities are taken advantage of. Such a flexible scheme is necessary in cases where the obstacles can not be passed by a rigid formation.

III. Split and Join

In this section we will see how switching between the short term goals of formation maintenance on the one hand and optimized navigation on the other hand can be done, without compromising the long term goal of safe arrival at the individual destinations.

The obstacle avoidance control algorithm is, as discussed above, described by equations (1)-(4). Here equation (1) is a greedy, i.e. sort term, minimization of the Lyapunov Function $V(\cdot)$. This Lyapunov Function furthermore appears in the second constraint, (3), which implies that $V(\cdot)$ must be nonincreasing over time. Since it is this property that is vital in the proofs of Theorem 2.1

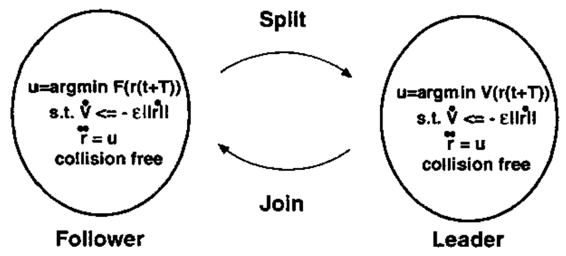

Fig. 2. The hybrid control combination of the two control laws. Note that the constraints are the same in both cases.

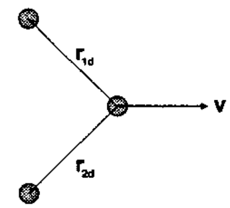

Fig. 3. The ideal formation used in the examples below. Note that the direction of r_{id} is relative to the leader velocity.

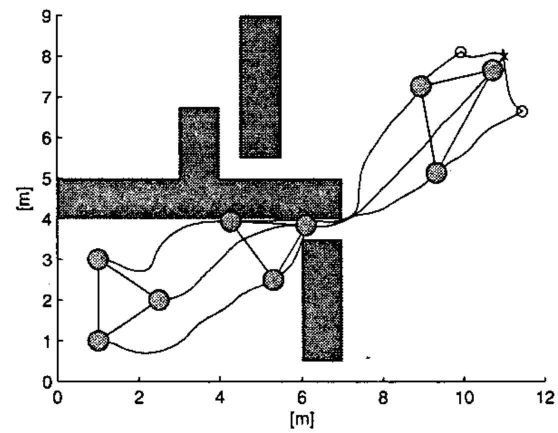

Fig. 4. The two leftmost vehicles immediately switch into follower mode. They then squeeze through the gap, reestablish the formation and then later, at the small circles, switch back into leader mode to reach their destinations.

and 2.3 we can in fact, at times, replace $V(\cdot)$ in (1) with some other function $F(\cdot)$, measuring the formation error. With this choice the vehicle would strive to minimize F with the constraint of not worsening its state from a navigation perspective. In fact, as long as the vehicle is moving with a velocity above some threshold, $||\dot{r}|| \geq v_t$, the constraint (3) implies $\dot{V} \leq -\epsilon v_t$ which makes sure that the destinations are reached, see [3] for details.

Replacing the elaborate Lyapunov Function V, which is carefully designed to be local minima free, with some function F, which by definition only incorporates inter vehicle relations brings back the risk of the vehicle getting stuck in a local minimum. We propose the following solution to this problem. If the vehicle is captured by a local minima the velocity approaches zero. In particular it drops below the threshold, v_t, discussed above. This can be interpreted as a sign that the formation keeping has failed and can be remedied by switching back to focusing on the individual navigation towards the destination, i.e. minimizing V. Apart from velocity, it is also reasonable to let the switching be governed by how big the formation error, F, is; this idea was originally proposed in [18]. We formalize these arguments in the hybrid control framework seen in Figure 2.

The rules of switching from one control law to the other are as follows.

- **Split** if
$$||\dot{r}|| \leq v_t \bigvee F(r) \geq F_t,$$
i.e. if the velocity is too low or if the formation is too poor.
- **Join** if
$$||\dot{r}|| \geq v_t \bigwedge ||\dot{r}_{leader}|| \geq v_t \bigwedge F(r) \leq F_t,$$
i.e. if the velocity of both the leader and the follower vehicles are high and they already are in a reasonably good formation position.

Above $\bigvee$, $\bigwedge$ denote logical (and), (or) respectively.

Remark 3.1: Note that we do not address the issue of inter-vehicle collisions in this work. Such features could however be implemented in the framework presented here, using the ideas of local networks from [21] to make collision checks of the local trajectories planned in the Receding Horizon Control loop.

We are now ready to look at the results of some simulation runs.

IV. Simulation Example

To illustrate the approach we chose a setting with three large obstacles in a 9 by 12 meter area, as seen in Figure 4. We used parameters for the Nomadic Technologies XR4000 robot obtained from [2], $||u|| \leq u_{max} = 1.5m/s^2$, $||\dot{r}|| \leq v_{max} = 1.2m/s$.

For these examples we furthermore set $v_t = 0.2m/s$, $F_t = 3m$ and choose a simple function F to reflect the formation shown in Figure 3.

$$F_i(r) = \min_j ||r_i - r_{id} - r_j||,$$

where r_{id} is of length $\sqrt{2}m$ and direction as defined in Figure 3. The minimization is furthermore over all potential leader vehicles r_j. For simplicity we restricted r_j to always be the same vehicle in the examples below.

Running the algorithm we get the trajectories of Figure 4. At the start the two leftmost vehicles switch into follower mode and start tracking the rightmost one. The common destination is marked by a cross at (11,8). At the narrow passage the followers are trying to keep the formation but the constraint of no collisions force them to squeeze through the gap. The follower starting at (1,3) is going along a straight path through the gap and comes out just behind the leader. The other follower however needs to make a sharp right braking turn and therefore looses speed and comes out somewhat behind the other

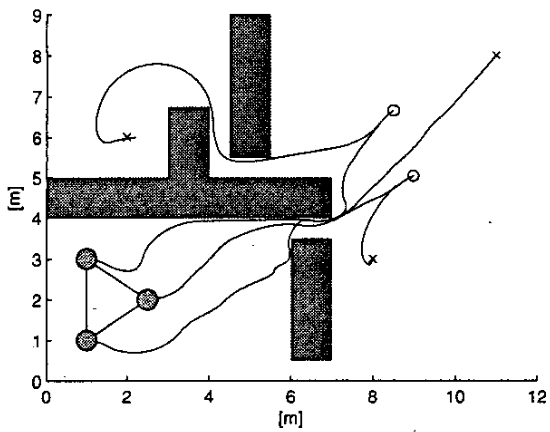

Fig. 5. Three different destinations are marked by x. The two leftmost vehicles stay in follower mode as long as possible without breaking the constraints.

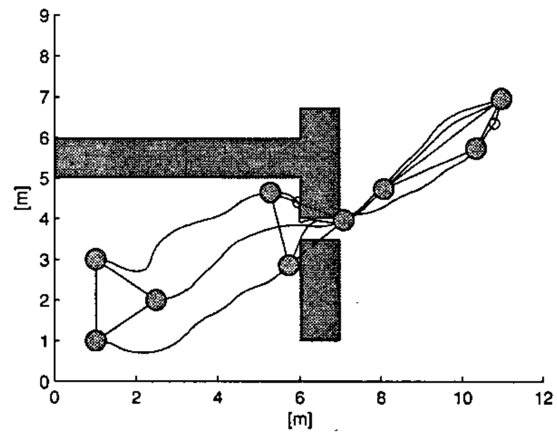

Fig. 6. The up most vehicle is trapped in a local minima of F which is resolved by switching out of follower mode.

two. When approaching the goal the leader slows down and stops. This makes the followers slow down too and at the positions marked by the small circles they switch back into leader mode, thus splitting the formation, and carry on towards the common goal point.

Remark 4.1: Note that the obstacles in the Figures are expanded to account for the size of the robots. Thus a trajectory grazing the obstacle is still collision free, even though the circle symbolizing the robot may intersect the obstacle.

The scenario in Figure 5 is almost the same as Figure 4, but now all three vehicles have different destinations. However, the first part of their routes coincide and thus they move in a formation similar to the one described above. After passing the gap both the followers are still trying to stay in formation, but constraint (3) forces them to brake hard as they travel away from their destinations. At the locations of the small circles near (9,6) they have reached a speed below the threshold v_t and thus split the formation and continue towards their individual destinations. Note how the vehicle traveling towards (2,6) smoothly navigates the narrow passage.

Figure 6 depicts another situation similar to the first one. The difference now is that above the gap there is a tricky nonconvex corner. The upper follower is forced into this spot when trying to stay in formation and since the leader is continuing towards the northeast the attempts to follow fail. The vehicle is trapped in a local minima of F. At this point the low velocity rule makes the vehicle switch into leader mode and start heading off on its own. The obstacle avoidance scheme handles the nonconvexity and the vehicle traces out a trajectory similar to the first one. Note that if the vehicle had been faster than the original leader it would have caught up and rejoined the formation had there been enough time.

V. Conclusions

We have shown how to do safe and convergent obstacle avoidance while exploiting opportunities for formation traveling.

This was done by switching the utility function in the optimization of a new convergent dynamic window approach. The fact that the constraints where always the same made the properties of safety and goal attainment carry over to this switching scheme.

The approach is illustrated by a simulation example.

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