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STABILITY ISSUES FOR VEHICLE PLATOONING IN AUTOMATED HIGHWAY SYSTEMS

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Abstract

This paper discusses notions and definitions pertinent to the stability of systems operating in platoon structures. This paper reviews some of the existing stability definitions (i.e. string stability), and discusses how the available platoon information and separation policies influences the stability results. It turns out that some of these notions and control policies do not suffice to ensure safety (collision avoidance) of the platoon. This is illustrated via simulation examples showing that there exists, in general, a non empty set of initial condition that results in a colliding situation. This set being not necessarily small. In connection with this, we propose a new way to formulate the platooning problem aiming at solving some of these difficulties.

Keywords: Vehicle control, string stability, platooning, longitudinal control.

1. Introduction

Controlling a platoon of vehicles is appropriate in several scenarios such as: increasing traffic capacity while maintaining a high level of security (automated highway systems), fuel consumption reduction (heavy duty vehicles platooning), decreasing the driver's stress in urban conditions (stop-and-go strategy), and partially automated vehicle capabilities (intelligent or (ACC) adaptive cruise control).

Although the desired inter-vehicle distances and the accessible information may be different to each of the scenarios mentioned above, all them have the primary and the common goal of using feedback to enlarge the closed-loop system bandwidth up to a level which is impossible to be achieved by the human driver alone. This capability allows for instance: to correctly regulate and reduce inter-vehicle distances to only a few meters.

Automated systems comprising platoon must exhibit both individual stability and stability as a group while avoiding collisions among the individual systems. These properties should also hold in the presence of physical system limitation on the vehicle acceleration and breaking capabilities.

Optimal error regulation for a string of moving vehicles was initially studied by Levine and Athans [4]. The authors designed a LQ regulator for the linearized vehicle inter distance error equations. The notion of *string stability* has been introduced in early works of Peppard [7] (see also references therein), in connection with moving-cell systems. Peppard defined string stability as the property of the vehicle string to attenuate disturbances as they propagated down to the string.

Condition for string stability were already provided in the works of Peppard [7], and Sheikholeslam and Desoer [9], in terms of the norm magnitude ($|G(\omega j) < 1|$) and the impulse response ($g(t) > 0$) of the linear operator, $G(s)$, where $G(s)$ maps the deviation in the assigned distances between vehicle i, and $i-1$.

String stability is further formalized in the works of Swaroop [10]. He introduced mathematical definitions for: *string stability, asymptotically string stability,* and *l_p-string stability,* where string stability implies uniform boundedness of the system states if the initial conditions are uniformly bounded. It is also shown that for a general interconnected system only *l_2-string stability* can be guaranteed, which is a weaker property than string stability.

The relevant questions of what type of vehicle information (backwards-looking, forwards-looking, backwards-forward looking, etc), and inter-vehicle spacing strategy (constant, time-varying, velocity dependent) should be used to ensure string stability have been debated by several authors (i.e. [11], [13], [14]). For instant, works in Swaroop [11], and Yanakiev and Kanellakapoulos [13], study the norm condition on $G(s)$ as a function of different vehicle information strategies and spacing policies. They conclude the impossibility of achieving string stability under autonomous operation when the desired inter vehicle spacing is constant.

Chien, and Ioannou [3] showed that string stability can be achieved by using speed-dependent spacing strategy. As underlying in Yanakiev and Kanellakapoulos [13], the effect of this strategy is equivalent to introduce additional damping in the map $G(s)$, which allows the poles to be moved independently from the zeroes of $G(s)$. Similar results were already presented in the paper of Sheikholeslam, and Desoer [9] which proposed a controller based on a constant spacing strategy, but using the additional leading vehicle velocity and acceleration information. The relevance of using the vehicle leading information for relaxing the string stability condition, was already pointed out by Shladover [8].

Nonlinear control design for simultaneous lateral and longitudinal control in a platoon of vehicles, has been studied in Canudas-de-Wit and NDoudi-Likoho [2], and Canudas-de-Wit [1], where back-stepping design was used. These nonlinear feedbacks collapse to linear laws (similar to the ones studied here), in the particular case of longitudinal control.

It is worth to mention that the string stability definition introduced in the literature does not provide a formal warranty of collision avoidance among the vehicles. Only few studies address this type of problems. In works of Li, Alvarez and Horowitz [6], [5], they consider bounds on the vehicle acceleration (or inputs), and find a region in the state space within which initial condition can be taken resulting in safety operation. Their results are obtained by looking at the explicit solution of the closed-loop linear equations.

A different approach to design a controller achieving collision free motions, will be discussed in this paper. The approach taken here consists in reformulating the problem in the context of constrained system (nonlinear system subject to inequality constraints), which incorporate the collision free restriction via virtual state space constraints. Thus, the purpose of this paper is to review, to put in perspective, and to propose a different formulation for the string stability problem ensuring safety vehicle operation.

Our aim is also to set up a more general framework seeking to enlarge the stability analysis allowing us also to study the condition under which vehicle collisions are avoided. In addition to this, we expect this new framework to help in designing a new class of controllers, possibly nonlinear, reaching these goals.

2. Unconstrained and constrained models

In stability studies concerning a platoon of vehicles, it is customary to model (eventually after linearization), each of the vehicle dynamics as a second order system. In other words it is assumed that each vehicle can be represented by a controlled point mass, where the control is a force input.

2.1. Unconstrained model

The set of vehicle constituting the platoon is thus represented as:

$$\Sigma_1 : \begin{cases} \dot{x}_1 & = & v_1 \\ \dot{v}_1 & = & u_1, \\ \dot{x}_2 & = & v_2 \\ \dot{v}_2 & = & u_2, \\ & \vdots & \\ \dot{x}_n & = & v_n \\ \dot{v}_n & = & u_n \end{cases} \tag{1}$$

where $x = [x_1, x_2, \ldots x_n]$, and $v = [v_1, v_2, \ldots v_n]$, are the vehicle positions and velocities, respectively. The control input is $u = [u_1, u_2, \ldots u_n]$. The leading vehicle is described by the pair (x_1, v_1), whereas the last vehicle in the platoon is described by the pair (x_n, v_n). Model Σ_1, is an unconstrained model since the state variables x, v, belong to R^{2n} without any particular restriction.

Let us introduce the inter-vehicle space error, e, as

$$e_i(x, v, t) = x_i - x_{i+1} - L(x, v, t)$$

for all $i = 1, 2, \ldots n-1$. $L(x, v, t) > 0$ is the desired inter-vehicle distance. In general, it may depend on velocity, position, and possibly on time. Different inter-space strategies can be formulated in this way. As it will be reviewed later, every particular choice has an important impact in terms of string stability conditions.

The control goal is thus to design a feedback law, such that it attenuates disturbances as they propagate down to the string. Or equivalently (string stability) find small gain conditions for each of the $n-1$ maps $G_i(\cdot) : e_i \mapsto e_{i+1}$, $\forall, i = 1, 2, \ldots n-2$. A complementary goal is to find an initial condition set $(e(0), \dot{e}(0))$, such that the vehicles do not collide with each other.

2.2. Constrained model

The model in (1) possesses the advantage of providing a tractable set of equations for the design of stable control inputs. It is however obviously a very simplified representation of the real system. Firstly, it may well be that each vehicle cannot be modeled through a double integrator, but that a further step requires to incorporate higher-order dynamics as well as some nonlinearities. Secondly, a platoon of carts, cars, or trucks is not a linear system: indeed the vehicles cannot interpenetrate each other. This puts a set of restrictions on the configuration space of the system, and allows one to recast such systems in the framework of *nonsmooth hybrid dynamical systems*, see [15] for more details. These restrictions take the form of unilateral constraints on the state. A constrained model can thus be written as:

$$\Sigma_2 : \begin{cases} \dot{x}_1 &= v_1 \\ \dot{v}_1 &= u_1, \\ \dot{x}_2 &= v_2 \\ \dot{v}_2 &= u_2, \\ &\vdots \\ \dot{x}_n &= v_n \\ \dot{v}_n &= u_n \end{cases} \tag{2}$$

subject to:

$$\varphi_1(x) \geq 0, \quad \varphi_2(x,v) \geq 0 \tag{3}$$

The $n-1$ vector of inequalities $\varphi_1(x) \geq 0$ accounts for the physical constraints, i.e. the vehicles cannot interpenetrate (which, by the way, does not imply that they are supposed to be perfectly rigid bodies): hence the distance between the vehicles must remain non-negative. The second inequality may be imposed by the designer (this may be seen as a virtual, or control, constraint), and may incorporate as well velocities. It is important at this stage to realize that the addition of such unilateral constraints drastically modifies the dynamics of the system in (1). This in turn implies that the control design should satisfy either: $\varphi_1(\underline{x}) > 0$ and $\varphi_2(\underline{x}, \underline{v}) \geq 0$, $t \geq 0$ (collision free case) in which case one says that the input is *viable*, or that possible collisions do not destroy the stability properties. The vectors $\underline{x}$ and $\underline{v}$ denote the solution of the controlled system.

Notice that there may be in general two basic design philosophies for the second case: either design a "free-motion" controller that is robust with respect to "accidental" collisions, or a controller that directly incorporates some "programmed" shocks between the vehicles as a tool for stabilization. Although the latter may seem strange, one should not forget that impacts dissipate energy. If this property is used in a suitable manner, impacts are rather stabilizing than destabilizing events [15].

Remark 1 *The model in (2) is not complete. In order to render the domain $\Phi = \{x : \varphi_1(x) \geq 0\}$ invariant,one must add a collision rule that plays the role of a re-initialization of the state (x, v) when the boundary $\partial\Phi$ is attained transversely. Furthermore so-called* complementarity relations *between $\varphi_1(x)$ and a contact force vector (a Lagrange multiplier) $\lambda \geq 0$ also hold: $\varphi_1(x)^T\lambda = 0$, and are necessary to monitor the detachment of the vehicles (i.e. the deactivation of the constraints $\varphi_{1,i}(x) = 0$). Moreover let us recall that the hybrid nature of the model in (2) comes from the fact that this system may evolve in 2^{n-1} different modes, each mode corresponding to having certain constraints $\varphi_{1,i}(x)$ active and the rest passive. In summary, one faces a complex non-smooth hybrid dynamical system that is a mixture of ODEs, DAEs (Differential Algebraic Equations), and MDEs (Measure Differential Equations), plus a finite automaton that governs the switches between the modes.*

3. String stability

String stability is usually defined as the requirement that disturbances on the spacing errors e_i are attenuated as they propagate back through the platoon.

This definition is motivated by the relation:

$$e_j(s) = \prod_{l=1}^{j-1} G_l(s) \cdot e_1(s) = G(s) \cdot e_1(s), \forall j = 2, 3, \ldots n.$$

that gives the transfer function of the backward error propagation, where $G_i(\cdot) : e_i \mapsto e_{i+1}$. From this relation we have that:

$$||e_i||_\infty \leq ||g * e_1||_\infty \leq ||g||_1 ||e_1||_\infty$$

where $g(t)$ is the inverse Laplace transform of $G(s)$. Clearly, a necessary and sufficient condition for error attenuation is that:

$$||g||_1 \leq 1$$

If in addition, $g(t)$ is positive, then the above condition is equivalent to:

$$||G||_\infty = \max_\omega |G(j\omega)| \leq 1, \quad g(t) \geq 0$$

If $g(t)$ is not positive, then $||G||_\infty \leq 1$ only gives L_2 stability, since:

$$||e_i||_2 \leq ||G||_\infty ||e_1||_2$$

These conditions were already presented in works of Sheikholeslam and Desoer [9].

The notion of string stability has been formalized and generalized in [10]. Consider the error closed-loop system in the $z^T = [e^T, \dot{e}^T]^T$ error vector coordinates of dimension $2n-2$, then the origin $z = 0$ of this system is said:

Definition 1 String stable, *if given any $\epsilon > 0$, there exist a $\delta > 0$ such that:*

$$||z_i(0)||_\infty < \delta, \Longrightarrow \sup_i ||z_i(\cdot)||_\infty < \epsilon.$$

Definition 2 Asymptotically (exponentially) string stable, *if the error system is string stable, and $z_i(t) \to 0$ asymptotically (exponentially) for all i.*

Definition 3 l_p-String stable, *if given any $\epsilon > 0$, there exist a $\delta > 0$ such that:*

$$||z_i(0)||_p < \delta, \Longrightarrow ||z_i(t)||_p = \sup_i \left(\int_o^\infty |z_i(t)|^p \right)^{1/p} < \epsilon.$$

for all $p \in [1, \infty)$.

Remark 2 *String stability implies uniform boundedness of $z(t)$ for all times, if the initial states $z(0)$ are uniformly bounded. In addition to the signal boundedness property, this definition implicitly seeks –via the use of l_∞-norms– to ensure that there exist non empty sets of initial conditions for $z(0)$, such that collisions* may *be avoided. For instance, for $\epsilon < L$, then $\delta(L)$ defines such a set. Nevertheless, explicit determination of this safe set remains an open problem.*

Remark 3 *As mentioned previously, if $g(t) \geq 0$, and $|G(j\omega)| \leq 1$, results in string stability as defined above. However, if $g(t)$ is not positive, then the condition $|G(j\omega)| \leq 1$, results in l_2-string stability, which only provides signals with bounded mean energy, but not necessarily with bounded magnitude. l_2-string stability is thus less suitable for this type of applications.*

4. Linear control strategies

In order to illustrate the limitations and capabilities of string stability while using linear controllers, we present in this section some examples of control strategies that have been used in the literature. In connection with some of these strategies, we present the collision-free initial condition regions obtained throughout simulations.

In what follows, we will discuss some of the following control policies (see [13], [11] for details and further discussion on these control strategies) having different stability properties:

- Unidirectional operation with constant spacing policy (string unstable).
- Bidirectional operation with constant spacing policy (l_2-string stable)
- Unidirectional operation with speed-dependent inter-vehicle spacing (string stable)

where unidirectional (bidirectional) operation implies that each vehicle is controlled using only information about the relative distances and velocities of the preceding (preceding and following) vehicle. Constant spacing policy indicates that the desired inter-vehicle distance $L(\cdot) = L$ is constant, whereas speed-dependent spacing indicates that $L(\cdot) = L(v) = hv$, with h being the time headway.

4.1. String unstable controllers

Unidirectional operation with constant spacing policy is an example of a strategy yielding string instability. The leading vehicle control is determined by an arbitrarily acceleration time-profile $u_1 = a_1^d(t)$, whereas rest of the controllers are of the form:

$$u_{i+1} = k_{i+1}e_i + c_{i+1}\dot{e}_i$$

with the inter-vehicle error distance defined as:

$$e_i = x_i - x_{i+1} - L, \quad \dot{e}_i = v_i - v_{i+1}$$

$\forall i = 1, 2, \ldots, n-1$. The error equations are of the form:

$$\ddot{e}_i = u_i - u_{i+1} = c_i\dot{e}_{i-1} + k_i e_{i-1} - (c_{i+1}\dot{e}_i + k_{i+1}e_i)$$

which leads to the following transfer functions:

$$G_i(s) = \frac{e_{i+1}(s)}{e_i(s)} = \frac{c_{i+1}s + k_{i+1}}{s^2 + c_{i+2}s + k_{i+2}}, \quad \forall i = 1, 2, \ldots n-2.$$

It is well known in the literature that if all the proportional and derivative gains are chosen equal ($c_j = c$, $k_j = k$, $\forall j = 2, 3, \ldots n$), then all the transfer functions are the same, and therefore there exists no possible choice for k and c such that $|G_i(j\omega)| = |G(j\omega)| \leq 1$, $\forall \omega \geq 0$.

Remark 4 *It is however interesting to note that when each of the vehicles is allowed to have different gains, it is possible to find control gains such that $|G_i(j\omega)| \leq 1$, for all $i = 1, 2, \ldots n-2$. As an example we can consider a 4-vehicle platoon, having a 3-dimensional inter-vehicle error vector e. The system has thus 2 associated transfer functions G_1, G_2, given as:*

$$\begin{aligned} G_1(s) &= \frac{e_2(s)}{e_1(s)} = \frac{c_2 s + k_2}{s^2 + c_3 s + k_3} \\ G_2(s) &= \frac{e_3(s)}{e_2(s)} = \frac{c_3 s + k_3}{s^2 + c_4 s + k_4} \end{aligned}$$

It can be shown the a sufficient small gain conditions is:

$$k_4 > k_3 > k_2, \quad \textit{and}, \quad c_4^2 > c_3^2 + 2k_3 > c_2^2 + 2k_4 + 2k_3$$

Intuitively, this condition indicates that the dynamics of the error system equations should be increased as the vehicle's location in the platoon goes downwards. In other words, the control gains of the vehicle should be increased as its location with respect to the leading vehicle becomes further back. Note also that this modification yields, at least, l_2-string stability (the condition $g(t) > 0$ need to be studied further).

Simulation using this controller for a 4-vehicle platoon have been performed to study the performance of this operation mode. Simulation gains where $c_2 = 1, c_3 = 2, c_4 = 3$, $k_2 = 1, k_3 = 1.2, k_4 = 1.5$, and $m = 1$, $a_1^d = 0$, and initial conditions were: $e_2(0) = -e_1(0)$, $\dot{e}_2(0) = -\dot{e}_1(0)$, $e_3(0) = \dot{e}_3(0) = 0$, and the pair $(e_1(0), \dot{e}_1(0))$ was varied in order to investigate the regions on $(e(0), \dot{e}(0))$, as a function of different values of the inter-vehicle distance L, leading to collision-free motions.

In spite of the conceptual limitations of the l_2-string stability, several simulation trials resulted in stable motions leading to collision-free solutions. Figure 1 shows a projection in the 2-dimensional $e_1(0) - \dot{e}_1(0)$ plane, of the safe regions (area inside of the contours) for different values of L. As expected, the safe regions shrink as the distance L diminishes.

4.2. l_2-string stable controllers

An other example of a l_2-string stable control structure, is a bidirectional feedback with constant spacing policy (see early works in [7]).

The control law is:

$$u_1 = -ke_1 - c\dot{e}_1 + a_1^d(t) \tag{4}$$

$$u_i = -ke_{i-1} - c\dot{e}_{i-1} - (ke_i + c\dot{e}_i) \tag{5}$$

$$u_n = -ke_n - c\dot{e}_n \tag{6}$$

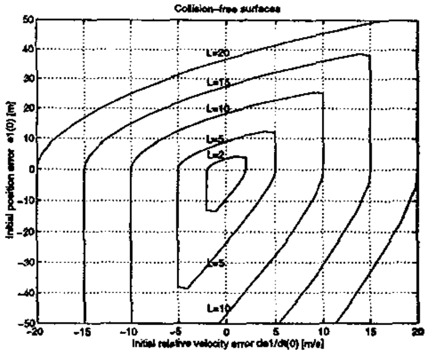

Figure 1: Safe regions (area inside of the contours) in the $e_1(0) - \dot{e}_1(0)$ plane for different values of L. Unidirectional constant spacing strategy with different control gains for each vehicle.

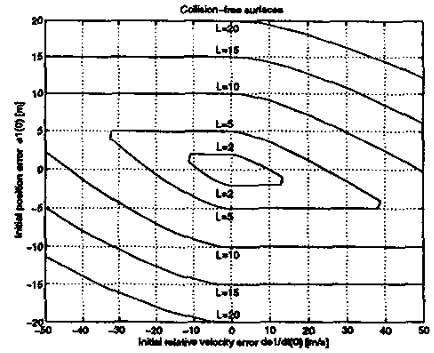

Figure 2: Safe regions (area inside of the contours) $e_1(0) - \dot{e}_1(0)$ plane for different values of L. Bidirectional constant spacing strategy.

for $i = 2, 3, \cdot n-1$. As before $a_1^d(t)$ is arbitrarily acceleration time-profile, and the error e is defined as:

$$e_i = x_i - x_{i+1} - L, \quad \dot{e}_i = v_i - v_{i+1}$$

$\forall i = 1, 2, \ldots, n-1$. which results in the following transfer functions:

$$G_{n-2}(s) = \frac{e_{n-1}(s)}{e_{n-2}(s)} = \frac{c_s + k}{s^2 + 2cs + 2k}, \qquad (7)$$

$$G_i(s) = \frac{e_{i+1}(s)}{e_i(s)} = \frac{G_n(s)}{1 - G_{i-1}(s)G_n(s)}. \qquad (8)$$

$\forall i = 1, 2, \ldots n-2$. As described in [12], although it is possible to find conditions, such that the magnitudes of these transfer functions are smaller than one, their impulse response are not positive for all t. Hence, only l_2-string stability can be guaranteed.

Figure 2 shows a projection in the 2-dimensional $e_1(0) - \dot{e}_1(0)$ plane, of the safe regions (area inside of the contours) for different values of L. As before, the safe regions shrink as the distance L diminishes. Simulation gains where $c = 1.2, k = 1, m = 1$.

4.3. String stable controllers

One of the control strategies leading to string stable controllers (among others) is the unidirectional operation with speed-dependent inter-vehicle spacing (see [9], and [3]).

As in the previously described unidirectional strategy, the leading vehicle control is determined by an arbitrarily acceleration time-profile $u_1 = a_1^d(t)$, whereas rest of the controllers are of the form:

$$u_{i+1} = ke_i + c(v_i - v_{i+1}), \quad \forall i = 2, 3 \ldots, n-1$$

and the inter-vehicle error distance is now velocity dependent:

$$e_i = x_i - x_{i+1} - hv_{i+1}, \quad \dot{e}_i = v_i - v_{i+1} - h\dot{v}_{i+1}$$

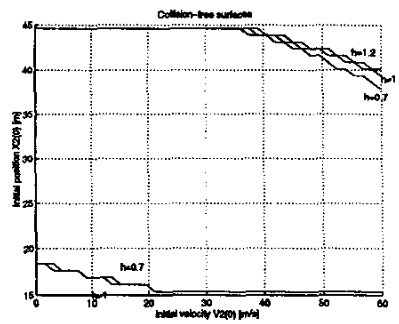

Figure 3: Safe regions (area inside of the contours) in the $x_2(0) - v_2(0)$ plane for different values of h. Unidirectional variable spacing strategy

$\forall i = 1, 2, \ldots, n-1$. This control gives the following transfer functions:

$$G_i(s) = \frac{e_{i+1}(s)}{e_i(s)} = \frac{cs + k}{s^2 + (c + hk)s + k}, \quad \forall i = 1, 2, \ldots n-2$$

As indicated by many authors, h acts as a absolute additional damping allowing the verification of both condition (under a particular choice of c and k) $|G_i(j\omega)| = |G(j\omega)| \leq 1$, and $g(t) > 0$. This control results in string stable behaviour.

Simulations were performed with: $v_1(0) = v_3(0) = v_4(0) = v$, $v = 30[m/s]$ $x_1(0) = 3hv$, $x_3(0) = hv$, $x_4(0) = 0$, and $c = 2$, $k = 1$. $x_2(0)$ was varied between hv and $3hv$, and $v_2(0)$ between 0 and $2v$. Figure 3 shows the safe region in the $x_2(0) - v_2(0)$ plane, for different values of h. As expected, the safe regions shrink as the distance h diminishes.

5. Constrained control design

As we saw in subsection 2.2, working with the model in (2) may imply to reconsider the controller design and stability analysis (although the above discussed controllers may satisfy the required criteria, this remains to be proved). The effects of collisions and the avoidance of such events have been investigated in [6] [5] [19]. These works mainly consist of studying join and split tasks under acceleration constraints, assuming that the platoon themselves are stable [19]. The set of initial data that ensure a safe maneuver is calculated analytically [6] [5], and numerically in [19]. However the problem of designing viable (or safe) controllers for the internal dynamics of the platoon seems not being solved up to date.

From a general point of view, it remains true that the study of the controllability and stabilizability properties of such nonsmooth hybrid systems is a relatively new and open field. Preliminary works in this direction can be found in [16] [18] [15], and concern the control of manipulators and juggling systems. A system theoretical approach of unilaterally constrained dynamical systems has been proposed in [17]. It constitutes an extension of positive invariance theory for systems $\dot{x} = Ax + Bu$, $y = Cx$, to systems $\dot{x} = f(x) + g(x)u$, $y = h(x)$ subject to unilateral constraints $y \geq 0$ (notice that the constraints may be purely virtual, not truly "hard" physical constraints). It mainly consists of classifying the set $\partial\Phi = \{y \equiv 0\}$ into particular subsets (for instance the subset of states $h(x) = 0$ that can be attained smoothly, or such that the system can leave $\partial\Phi$, with or without control). The setting in [17] supersedes that of controllable mechanical systems since $h(x)$ has a general form. The collision maps are shown to be necessary for the integration in case of certain functions $h(\cdot)$ (because no bounded input is able to keep the state within the admissible domain Φ if $\partial\Phi$ is attained transversally) and are introduced as mappings between particular subsets of $\partial\Phi$. Such studies may be seen as a preliminary step towards the analysis of controllability properties of unilaterally constrained systems. They might be used to get a better characterization of the surfaces $\varphi_1(x) = 0$ and $\varphi_2(x, v) = 0$, in terms of the existence of controllers that guarantee safety, for instance. One should keep in mind at this stage that the double integrators in (2) might eventually be replaced by more complex dynamics, where u would no longer represent a force input but some other signal.

6. Conclusions

In this paper we have discussed several stability notions of vehicle operating in a platoon. It turn out, that the notions of string stability, may not be sufficient to guarantee safe operation of the platoon. It has been shown by numerical simulations, that all the control structures, have a region in the initial state space, that lead to vehicle collisions. We have proposed a new formulation that seeks to explicitly account for vehicle collisions. We believe that that approach will allow to better characterize the controllability and stabilizability properties of such a system, aiming at the design of a new class of controllers.

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