Safe Platooning of Unmanned Aerial Vehicles via Reachability

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Abstract— **Recently, there has been immense interest in using unmanned aerial vehicles (UAV) for civilian operations such as package delivery, firefighting, and fast disaster response. As a result, UAV traffic management systems are needed to support potentially thousands of UAVs flying simultaneously in the airspace, in order to ensure their liveness and safety requirements are met. Hamilton-Jacobi (HJ) reachability is a powerful framework for providing conditions under which these requirements can be met, and for synthesizing the optimal controller for meeting them. However, due to the curse of dimensionality, HJ reachability is only tractable for a small number of vehicles if their set of maneuvers are unrestricted. In this paper, we define a platoon to be a group of UAVs in a single-file formation. We model each vehicle as a hybrid system with modes corresponding to its role in the platoon, and specify the set of allowed maneuvers in each mode to make the analysis tractable. We propose several liveness controllers based on HJ reachability, and wrap a safety controller, also based on HJ reachability, around the liveness controllers. For a single altitude range, our approach guarantees safety for one safety breach; in the unlikely event of multiple safety breaches, safety can be guaranteed over multiple altitude ranges. We demonstrate the satisfaction of liveness and safety requirements through simulations of three common scenarios.**

I. Introduction

Unmanned aerial vehicle (UAV) systems have in the past been mainly used for military operations [1]. Recently, however, there has been an immense surge of interest in using UAVs for civil applications. Through projects such as Amazon Prime Air and Google Project Wing, companies are looking to send UAVs into the airspace to deliver packages [2], [3]. As a rough estimate, suppose in a city of 2 million people, each person requests a drone delivery every 2 months on average and each delivery requires a 30 minute trip for the drone. This would equate to thousands of simultaneous vehicles in the air. As a result, government agencies such as the Federal Aviation Administration (FAA) and National Aeronautics and Administration (NASA) of the United States are also investigating air traffic control for autonomous vehicles in order to prevent collisions among potentially numerous UAVs [4]. Applications of UAVs extend beyond package delivery; they can also be used, for example, to provide supplies or to firefight in areas that are difficult to reach but require prompt response [5].

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Optimal control and game theory are powerful tools for providing liveness and safety guarantees to controlled dynamical systems under disturbances, and various formulations [6], [7], [8] have been successfully used to analyze problems involving a small number of vehicles [9], [10], [11]. These formulations are based on Hamilton-Jacobi (HJ) reachability, which computes the reachable set, defined as the set of states from which a system is guaranteed to have a control strategy to reach a target set of states under the worst case disturbance (This is not the definition of reachable set we use in designing neither the liveness nor the safety controller? How about: the set of states from which a system can reach a target set under certain conditions?). Reachability is a powerful tool because reachable sets can be used for synthesizing both controllers that steer the system towards a set of goal states (liveness controllers), and controllers that steer the system away from a set of unsafe states (safety controllers). Furthermore, the HJ formulations are flexible in terms of system dynamics, enabling the analysis of nonlinear systems, and provides the optimal controller given the target set.

The power and success of HJ reachability analysis in previous applications cannot be denied, especially since numerical tools are readily available to solve the associated HJ Partial Differential Equation (PDE) [12], [13], [14]. However, the computation is done on a grid, making the problem complexity scale exponentially with the number of states, and therefore with the number of vehicles. This makes the computation intractable for large numbers of vehicles.

A considerable body of work has been done on the platooning of vehicles; [15] provides a broad overview. For example, [16] investigated the feasibility of vehicle platooning in terms of tracking errors in the presence of disturbances, taking into account complex nonlinear dynamics of each vehicle. [17] explored several control techniques for performing various platoon maneuvers such as lane changes, merge procedures, and split procedures. In [18], the authors modeled vehicles in platoons as hybrid systems, synthesized safety controllers, and analyzed throughput. Finally, reachability analysis was used in [19] to analyze a platoon of two trucks in order to minimize drag by minimizing the following distance while maintaining collision avoidance safety guarantees.

In the context of HJ reachability, putting vehicles into platoons is desirable because of the additional structure that platoons impose on its members. With additional structure, pairwise safety guarantees of vehicles can be more easily translated into safety guarantees of all the vehicles in the platoon. In this paper, assuming a simple single-file platoon, we first propose a hybrid systems model of UAVs that participate in platooning. We then show how HJ reachability can be used to synthesize *liveness controllers* that enable vehicles to reach a set of desired states, and wrap *safety controllers* around the liveness controllers in order to prevent dangerous configurations such as collisions. Finally, we show simulation results of quadrotors forming a platoon, quadrotors in a platoon responding to a malfunctioning platoon member, and quadrotors in a platoon responding to an intruder outside of the platoon to illustrate the use of reachable sets and the behavior of vehicles in these example scenarios.

II. Problem Formulation

A. Vehicle Dynamics

Consider a UAV whose dynamics are given by

$$\dot{x} = f(x, u) \tag{1}$$

where $x \in \mathbb{R}^n$ represents the state, and $u \in \mathbb{R}^{n_u}$ represents the control action. In this paper, we will assume that each vehicle has the following simple model of a quadrotor:

$$\begin{aligned} \dot{p}_x &= v_x, \qquad \dot{p}_y = v_y \\ \dot{v}_x &= u_x, \qquad \dot{v}_y = u_y \\ \underline{u} &\leq |u_x|, |u_y| \leq \bar{u} \end{aligned} \tag{2}$$

where the state $x = (p_x, v_x, p_y, v_y) \in \mathbb{R}^4$ represents the quadrotor's position in the x-direction, its velocity in the x-direction, and its position and velocity in the y-direction, respectively. For convenience, we will denote the position and velocity $p = (p_x, p_y), v = (v_x, v_y)$, respectively. We will consider a group of N such quadrotors which we denote $Q_i, i = 1 \ldots, N$.

In general, the problem of collision avoidance among N vehicles cannot be tractably solved using traditional dynamic programming approaches because the computation complexity of these approaches scales exponentially with the number of vehicles. Thus, in our present work, we will consider the situation where N quadrotors form a platoon. The structure imposed by the platoon enables us to analyze the liveness and safety of the quadrotors in a tractable manner.

B. Quadrotors in a Platoon

We consider a platoon of quadrotors to be a group of M quadrotors $Q_{P_1}, \ldots, Q_{P_M}$ in a single-file formation. Not all of the N quadrotors need to be in a platoon: $\{P_j\}_{j=1}^M \subseteq \{i\}_{i=1}^N$. Q_{P_1} is the leader of the platoon, and $Q_{P_2}, \ldots, Q_{P_M}$ are the followers. We will assume that the quadrotors in a platoon travel along an air highway, which is defined by as a path inside a pre-defined altitude range. The quadrotors maintain a separation distance of b. In order to allow for close proximity of the quadrotors and the ability to resolve multiple simultaneous safety breaches, we assume that in the event of a malfunction, a quadrotor will be able to exit the altitude range of the highway within t_{internal} seconds. Such a requirement may be implemented practically as an

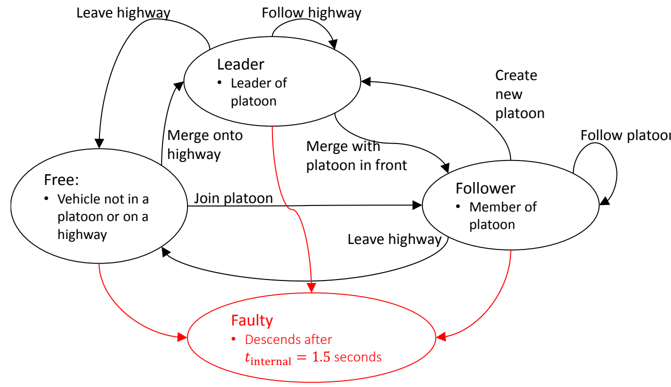

Fig. 1: Hybrid modes for vehicles in platoons.

emergency landing procedure to which the quadrotors revert when a malfunction is detected.

As part of a platoon, each quadrotor must be capable of performing a number of essential cooperative maneuvers. In this paper, we consider the following:

- safely merging onto an air highway;
- safely joining a platoon;
- reacting to a malfunctioning vehicle in the platoon;
- reacting to an intruder vehicle;
- following the highway, a curve defined in space at constant altitude, at a specified speed;
- maintaining a constant relative position and velocity with the leader of a platoon.

C. Vehicles as Hybrid Systems

UAVs in general may be in a number of modes of operations, depending on whether it is part of a platoon, and in the affirmative case, whether it is a leader or a follower. Therefore, it is natural to model quadrotors as hybrid systems [18], [20]. In this paper, we restrict the available maneuvers of each quadrotor depending on the mode. We assume that each quadrotor in the airspace has the following modes:

- Free: Vehicle that's not in a platoon. Available maneuvers: merge onto a highway, join a platoon on a highway.
- Leader: Leader of platoon (could be by itself). Available maneuvers: travel along the highway at a pre-specified speed, merge current platoon with a platoon in front, leave the highway.
- Follower: Vehicle in a platoon following the leader. Available maneuvers: follow a platoon, create a new platoon.
- Faulty: Vehicle in a platoon that has malfunctioned: reverts to default behavior and descends after $t_{\text{internal}} = 1.5$ seconds.

The available maneuvers and associated mode transitions are shown in Figure 1.

D. Objectives

Using the previously-mentioned modeling assumptions, we would like to address the following questions:

1) How do vehicles form platoons?
2) How can the safety of the vehicles be ensured during normal operation and when there is a malfunctioning vehicle within platoon?
3) How can the platoon respond to intruders such as unresponsive UAVs, birds, or other aerial objects.

The answers to these questions can be broken down into the maneuvers listed in Section II-B. In general, the control strategies of each quadrotor have a liveness component, which specifies a set of states towards which the quadrotor aims to reach, and a safety component, which specifies a set of states that it must avoid. In this paper, we address both the liveness component and safety component using reachability analysis.

III. Hamilton-Jacobi Reachability

A. General Framework

We consider a differential game between two players described by the system

$$\dot{x} = f(x, u, d), t \in [-T, 0] \tag{3}$$

where $x \in \mathbb{R}^n$ is the system state, $u \in \mathcal{U}$ is the control of Player 1, and $d \in \mathcal{D}$ is the control of Player 2. We assume $f : \mathbb{R}^n \times \mathcal{U} \times \mathcal{D} \to \mathbb{R}^n$ is uniformly continuous, bounded, and Lipschitz continuous in x for fixed u, d, and the control functions $u(\cdot) \in \mathbb{U}, d(\cdot) \in \mathbb{D}$ are drawn from the set of measurable functions[1]. Player 2 is only allowed to use nonanticipative strategies [21], [22] γ, defined by

$$\begin{aligned} &\gamma \in \Gamma := \{\mathcal{N} : \mathbb{U} \to \mathbb{D} \mid \\ &\quad u(r) = \hat{u}(r) \text{ for almost every } r \in [t, s] \\ &\quad \Rightarrow \mathcal{N}[u](r) = \mathcal{N}[\hat{u}](r) \text{ for almost every } r \in [t, s]\} \end{aligned} \tag{4}$$

In our differential game, the goal of Player 2 is to drive the system into some target set $\mathcal{L}$, and the goal of Player 1 is to drive the system away from it. The set $\mathcal{L}$ is represented as the zero sublevel set of a bounded, Lipschitz continuous function $l : \mathbb{R}^n \to \mathbb{R}$. We call $l(\cdot)$ the implicit surface function representing the set $\mathcal{L}$.

$$\mathcal{L} = \{z \in \mathbb{R}^n \mid l(z) \leq 0\} \tag{5}$$

Given the dynamics (3) and the target set $\mathcal{L}$, we would like to compute the backwards reachable set, $\mathcal{V}(t)$:

$$\begin{aligned} \mathcal{V}(t) := \{&z \in \mathbb{R}^n \mid \exists \gamma \in \Gamma \text{ such that} \\ &\forall u(\cdot) \in \mathbb{U}, \exists s \in [t, 0], f(\xi_f(s; z, t, u(\cdot), \gamma[d](\cdot))\} \end{aligned} \tag{6}$$

Should this be:

$$\begin{aligned} \mathcal{V}(t) := \{&z \in \mathbb{R}^n \mid \exists \gamma \in \Gamma \text{ such that} \\ &\forall u(\cdot) \in \mathbb{U}, \exists s \in [t, 0], \xi_f(s, t, z, u(\cdot), \gamma[u](\cdot)) \in \mathcal{L}\} \end{aligned} \tag{7}$$

[1]A function $f : X \to Y$ between two measurable spaces (X, Σ_X) and (Y, Σ_Y) is said to be measurable if the preimage of a measurable set in Y is a measurable set in X, that is: $\forall V \in \Sigma_Y, f^{-1}(V) \in \Sigma_X$, with Σ_X, Σ_Y σ-algebras on X,Y.

where ξ_f represents the state transition function such that for the system with dynamics (3) starting at x_0 at t_0 with input $u(\cdot)$ and disturbance $d(\cdot)$ the state at time t_1 is $x(t_1) = \xi_f(t_1, t_0, x_0, u(\cdot), d(\cdot))$. ?

Many methods involving solving HJ PDEs [7] and HJ variational inequalities (VI) [6], [8], [9] have been developed for computing the reachable set. These HJ PDEs and HJ VIs can be solved using well-established numerical methods. For this paper, we use the formulation in [7], which has shown that the backwards reachable set $\mathcal{V}(t)$ can be obtained as the zero sublevel set of the viscosity solution [23] $V(t, x)$ of the following terminal value Hamilton-Jacobi-Isaacs (HJI) PDE:

$$\begin{aligned} D_t V(t, x) + \min\{0, \max_{u \in \mathcal{U}} \min_{d \in \mathcal{D}} D_x V(t, x) \cdot f(x, u, d)\} &= 0 \\ V(0, x) &= l(x) \end{aligned} \tag{8}$$

from which we obtain $\mathcal{V}(t) = \{x \in \mathbb{R}^n \mid V(t, x) \leq 0\}$. From the solution $V(t, x)$, we can also obtain the optimal controls for both players via the following:

$$\begin{aligned} u^*(t) &= \arg \max_{u \in \mathcal{U}} \min_{d \in \mathcal{D}} D_x V(t, x) \cdot f(x, u, d) \\ d^*(t) &= \arg \min_{d \in \mathcal{D}} D_x V(t, x) \cdot f(x, u^*, d) \end{aligned} \tag{9}$$

In the special case where there is only one player, we obtain an optimal control problem for a system with dynamics

$$\dot{x} = f(x, d), t \in [-T, 0]. \tag{10}$$

The reachable set in this case would be given by the Hamilton-Jacobi-Bellman (HJB) PDE (I changed the ‘d’ to ‘u’ in the 2 eqns below, since it’s the control for the liveness controller?)

$$\begin{aligned} D_t V(t, x) + \min\{0, \min_{u \in \mathcal{U}} D_x V(t, x) \cdot f(x, u)\} &= 0 \\ V(0, x) &= l(x) \end{aligned} \tag{11}$$

where the optimal control is given by

$$u^*(t) = \arg \min_{u \in \mathcal{U}} D_x V(t, x) \cdot f(x, u) \tag{12}$$

For our application, we will use a several decoupled system models and utilize the decoupled HJ formulation in [**?**] (Add reference), which enables real time 4D reachable set computations and tractable 6D reachable set computations.

B. Relative Dynamics and Augmented Relative Dynamics

Besides Equation (3), we will also consider the relative dynamics between two quadrotors Q_i, Q_j. These dynamics can be obtained by defining the relative variables

$$\begin{aligned} p_{x,r} &= p_{x,i} - p_{x,j}, \qquad & p_{y,r} &= p_{y,i} - p_{y,j} \\ v_{x,r} &= v_{x,i} - v_{x,j}, \qquad & v_{y,r} &= v_{y,i} - v_{y,j} \end{aligned} \tag{13}$$

We treat Q_i as Player 1, the evader who wishes to avoid collision, and we treat Q_j as Player 2, the pursuer, or disturbance, that wishes to cause a collision. In terms of the relative variables given in Equation (13), we have the following:

$$\begin{aligned} \dot{p}_{x,r} &= v_{x,r}, & \dot{p}_{y,r} &= v_{y,r} \\ \dot{v}_{x,r} &= u_{x,i} - u_{x,j}, & \dot{v}_{y,r} &= u_{y,i} - u_{y,j} \end{aligned} \tag{14}$$

We also consider relative dynamics augmented by the velocity of quadrotor Q_i, given in Equation (15). These dynamics are needed to impose a velocity limit on the quadrotor.

$$\begin{aligned} \dot{p}_{x,r} &= v_{x,r}, & \dot{p}_{y,r} &= v_{y,r} \\ \dot{v}_{x,r} &= u_{x,i} - u_{x,j}, & \dot{v}_{y,r} &= u_{y,i} - u_{y,j} \\ \dot{v}_{x,i} &= u_{x,i}, & \dot{v}_{y,i} &= u_{y,i} \end{aligned} \tag{15}$$

IV. Liveness Controllers

A. Merging onto a Highway

We model the merging of a vehicle onto an air highway as a path planning problem, where in addition to the target position, we also specify a target velocity such that the magnitude (the speed) is given by the highway specification, and the direction is along the direction of the highway. Thus, for a quadrotor, the objective would be to drive the system in Equation (3) to a specific state $\bar{x} = (\bar{p}_x, \bar{v}_x, \bar{p}_y, \bar{v}_y)$, or a small range of states defined by the set

$$\begin{aligned} \mathcal{L}_H = \{x \mid &|p_x - \bar{p}_x| \leq r_{p_x}, |v_x - \bar{v}_x| \leq r_{v_x}, \\ &|p_y - \bar{p}_y| \leq r_{p_y}, |v_y - \bar{v}_y| \leq r_{v_y}\}. \end{aligned} \tag{16}$$

In this reachability problem, $\mathcal{L}_H$ is the target set, represented by the zero sublevel set of the function $l_H(x)$, which specifies the terminal condition of the HJB PDE we would need to solve, Equation (11). The solution we obtain, $V_H(t,x)$, is the implicit surface function representing the reachable set $\mathcal{V}(t)$; $V_H(-T,x) \leq 0$, then, specifies the reachable set $\mathcal{V}_H(T)$, the set of states from which the system can be driven to the target $\mathcal{L}_H$ within a duration of T. This gives the following algorithm for a quadrotor merging onto the highway:

1) Move towards $\bar{x}$ in a straight line, with some velocity, until $V_H(-T,x) \leq 0$.
2) Apply the optimal control extracted from $V_H(-T,x)$ according to Equation (12) until the quadrotor reaches $\mathcal{L}_H$.

B. Merging into a Platoon

We again pose the merging of a quadrotor into a platoon on an air highway as a reachability problem. Here, we would like quadrotor Q_i to merge onto the highway and follow another quadrotor Q_j in a platoon. Thus, we would like to drive the system given by Equation (15) to a specific $\bar{x} = (\bar{p}_{r,x}, \bar{v}_{x,r}, \bar{p}_{r,y}, \bar{v}_{y,r})$, or a small range of relative states defined by the set

$$\begin{aligned} \mathcal{L}_P = \{x \mid &|p_{x,r} - \bar{p}_{x,r}| \leq r_{p_x}, |v_{x,r} - \bar{v}_{x,r}| \leq r_{v_x}, \\ &|p_{y,r} - \bar{p}_{y,r}| \leq r_{p_y}, |v_{y,r} - \bar{v}_{y,r}| \leq r_{v_y}\} \end{aligned} \tag{17}$$

The target set $\mathcal{L}_P$ is represented by the implicit surface function $l_P(x)$, which specifies the terminal condition of the HJI PDE (8). The zero sublevel set of the solution to (8), $V_P(-T,x)$, gives us the set of relative states from which Q_i can reach the target and join the platoon following Q_j within a duration of T. We assume that Q_j moves along the highway at constant speed, so that $u_j(t)$ = 0. Similar to the last section, the following is a suitable algorithm for a quadrotor merging onto a highway and joining a platoon to follow Q_j:

1) Move towards $\bar{x}$ in a straight line, with some velocity, until $V_P(-T,x) \leq 0$.
2) Apply the optimal control extracted from $V_P(-T,x)$ according to Equation (9) until the quadrotor reaches $\mathcal{L}_P$.

C. Other Quadrotor Maneuvers

Reachability was used in Sections IV-A and IV-B for the relatively complex maneuvers of merging onto a highway and joining a platoon. For the simpler maneuvers of traveling long a highway and following a platoon, we resort to simpler controllers described below.

1) Traveling along a highway: We use a model-predictive controller (MPC) for traveling along a highway; this controller allows the leader to travel along a highway at a prespecified speed. Here, the goal is for a leader quadrotor to track a constant-altitude path, defined as a curve $\bar{p}(s)$ parametrized by $s \in [0,1]$ in $p = (p_x, p_y)$ space (position space), while maintaining a velocity $\bar{v}(s)$ that corresponds to constant speed in the direction of the highway. Assuming that the initial position on the highway, $s_0 = s(t_0)$ is specified, such a controller can be obtained from the following optimization problem over the time horizon $[t_0, t_1]$:

$$\begin{aligned} \text{minimize} \quad & \int_{t_0}^{t_1} \big\{ \|p(t) - \bar{p}(s(t))\|_2 + \\ & \qquad \|v(t) - \bar{v}(s(t))\|_2 + 1 - s \big\} dt \\ \text{subject to} \quad & \dot{x} = f(x,u) \text{ where } f \text{ is given in (3)} \\ & \underline{u} \leq |u_x|, |u_y| \leq \bar{u} \\ & \underline{v} \leq |v_x|, |v_y| \leq \bar{v} \\ & s(t_0) = s_0 \\ & \dot{s} \geq 0 \end{aligned} \tag{18}$$

If we discretize time, and assume that $\bar{p}(\cdot)$ is linear, then the above optimization is convex, and can be quickly solved.

2) Following a Platoon: Follower vehicles use a simple feedback control law tracking a nominal position and velocity in the platoon, with an additional feed-forward term given by the leader's acceleration input; here, for simplicity, we will assume perfect communication between the leader and the follower vehicles. This following law enables smooth vehicle trajectories in the relative platoon frame, while allowing the platoon as a whole to perform agile maneuvers by transmitting the leader's acceleration command $u_{P_1}(t)$ to all vehicles.

The i-th member of the platoon, Q_{P_i}, is expected to track a relative position in the platoon $r^i = (r^i_x, r^i_y)$ with respect to the leader's position p_{P_1}, and the leader's velocity v_{P_1} at all times. The resulting control law has the form:

$$u^i(t) = k_p\big[p^L(t)+r^i(t)-p^i(t)\big]+k_v\big[v^L(t)-v^i(t)\big]+u^L(t), \tag{19}$$

for some $k_p, k_v > 0$. The leader can modify the nominal position of vehicles in the platoon, for example to command the formation to turn. In particular, a simple rule for determining $r^i(t)$ in a single-file platoon is given for the Q_{P_i} as:

$$r^i(t) = -(i-1)b\frac{\bar{v}(s(t))}{\|\bar{v}(s(t))\|_2}, \tag{20}$$

where b is the chosen spacing between vehicles along the platoon.

V. Safety Controllers

A. Wrapping Reachability Around Existing Controllers

A quadrotor, whether in a platoon or not, can only use a liveness controller when it is not in any danger of collision with other quadrotors or obstacles. However, if the quadrotor could potentially be involved in a collision within the next short period of time, it must switch to a safety controller. In this section, we will demonstrate how HJ reachability can be used to both detect imminent danger and synthesize a controller that guarantees safety within a specified time horizon. For our safety analysis, we will use the model in Equation (15).

We begin by defining the target set $\mathcal{L}_S$, which characterizes the configurations in the relative coordinates for which quadrotors Q_i and Q_j are considered to be in collision for any i, j.

$$\begin{aligned}\mathcal{L}_S = \{x \,||p_{x,r}|, |p_{y,r}| \leq d \\ \text{or } |v_{x,i}| \geq v_{\max} \text{ or } |v_{y,i}| \geq v_{\max}\}\end{aligned} \tag{21}$$

With this definition, Q_i is considered to be unsafe if Q_i and Q_j are within a distance d in both x- and y-directions simultaneously, or if Q_i has exceeded some maximum speed $v_{\max}$ in either x- or y-direction. For illustration purposes, we choose $d = 2$ meters, and $v_{\max} = 5$ m/s.

We can now define the implicit surface function $l_S(x)$ corresponding to $\mathcal{L}_S$, and solve the HJI PDE (8) using $l_S(x)$ as the terminal condition. As before, the zero sublevel set of the solution $V_S(t,x)$ specifies the reachable set $\mathcal{V}_S(t)$, which characterizes the states in the augmented relative coordinates, as defined in (15), from which Q_i avoid $\mathcal{L}_S$ for a time period of t, despite the worst possible control of Q_j. The safety controller can be synthesized according to Equation (9).

To wrap our safety controller around liveness controllers, we use the following algorithm:

1) For a specified time horizon t, evaluate $V_S(t, x_i - x_j)$ for all $j \in \mathcal{Q}(i)$.
   $\mathcal{Q}(i)$ is the set of quadrotors with which quadrotor i checks safety against. We will discuss $\mathcal{Q}(i)$ in the next section.
2) Use the safety or liveness controller depending on the values $V_S(t, x_i - x_j), j \in \mathcal{Q}(i)$:
   If $\exists j \in \mathcal{Q}(i)$ such that $V_S(t, x_i - x_j) \leq 0$, then Q_i, Q_j are in potential conflict, and Q_i must use a safety controller; otherwise Q_i use a liveness controller.

B. Platoon Safety Guarantees

Under normal operations in a single platoon, each follower quadrotor $Q_i, i = P_2, \ldots, P_{N-1}$ in a platoon checks whether it is in the safety reachable set with respect to $Q_{P_{i-1}}$ and $Q_{P_{i+1}}$. So $\mathcal{Q}(i) = \{P_{i+1}, P_{i-1}\}$ for $i = P_2, \ldots, P_{N-1}$. Assuming there are no nearby quadrotors outside of the platoon, the platoon leader Q_{P_1} checks safety against Q_{P_2}, and the platoon trailer Q_{P_N} checks safety against $Q_{P_{N-1}}$. So $\mathcal{Q}(P_1) = \{P_2\}, \mathcal{Q}(P_N) = \{P_{N-1}\}$.

When all quadrotors are using liveness controllers to perform their allowed maneuvers, no pair of quadrotors should be in an unsafe configuration if the liveness controllers are well-designed. However, occasionally a quadrotor Q_k may behave unexpectedly due to faults or malfunctions, in which case it may come into an unsafe configuration with another quadrotor.

With our choice of $\mathcal{Q}(i)$, and with the assumption that the platoon is in a single-file formation, some quadrotor Q_i would get into an unsafe configuration with Q_k, where Q_k is likely to be the quadrotor in front or behind of Q_i. In this case, a "safety breach" occurs. Our synthesis of the safety controller via HJ reachability guarantees that between every pair of quadrotors Q_i, Q_k, as long as $V_S(t, x_i - x_k) > 0$, $\exists u_i$ to keep Q_i from colliding with Q_k for a desired time horizon t, despite the worst case (an adversarial) control from Q_k. Therefore, as long as the number of "safety breaches" is at most one, Q_i can simply use the optimal control to avoid Q_k and avoid collision for the time horizon of t. Since quadrotors in platoons are able to exit the altitude range of the highway within t_{internal} seconds, if we choose $t = t_{\text{internal}}$, the safety breach would always end before any collision can occur.

Within a duration of t_{internal}, after the first safety breach, there is a possibility, although a small one, that additional safety breaches may occur. However, as long as the number of safety breaches on each quadrotor is not more than one, collision avoidance of all the quadrotors can be guaranteed for the duration t_{internal}. In the event that multiple safety breaches occur for some of the quadrotors due to a malfunctioning quadrotor within the platoon or an intruding quadrotor outside of the platoon, those quadrotors with more than one safety breach still have the option of exiting the highway altitude range in order to avoid collisions. Every extra altitude range reduces the number of simultaneous safety breaches by 1, so K simultaneous safety breaches can be resolved using $K-1$ different altitude ranges. However, as the our simulation results show, the likelihood of multiple safety breaches is low during a quadrotor malfunction and the presence of one intruder vehicle.

VI. Scenarios Case Study

In this section, we consider several situations that quadrotors in a platoon on an air highway may commonly encounter, and show via simulations the behaviors that emerge from the controllers we defined in Sections IV and V.

A. *Forming a Platoon*

We first consider the scenario in which a number of quadrotors are trying to merge onto an initially unoccupied highway. In order to do this, each quadrotor first checks for safety with respect to the other quadrotors, and uses the safety controller if necessary, according to Section V. Otherwise, the quadrotor uses the liveness controller described in Section IV.

For the simulation example, the highway is specified by the line $p_y = 0.5p_x$, the point of entry on the highway is chosen to be $(\bar{p}_x, \bar{p}_y) = (4, 2)$, and the velocity on the highway is chosen to be $(\bar{v}_x, \bar{v}_y) = \frac{\bar{v}}{\sqrt{0.5^2+1^2}}(0.5, 1)$. The velocity simply states that the quadrotors must travel at a speed $\bar{v}$ along the direction of the highway. This forms the target state $\bar{x} = (\bar{p}_x, \bar{v}_x, \bar{p}_y, \bar{v}_y)$, from which we define the target set $\mathcal{L}_H$ as in Section IV-A.

The first quadrotor that completes the merging onto an empty highway creates a platoon and becomes its leader, while subsequent quadrotors form a platoon behind the leader in a pre-specified order. The process of joining a platoon is described in Section IV-B. Here, we choose $(\bar{p}_{r,x}, \bar{p}_{r,y})$ to be a distance b behind the last quadrotor in the platoon, and $(\bar{v}_{x,r}, \bar{v}_{y,r}) = (0, 0)$. This gives us the target set $\mathcal{L}_P$ that we need.

Figures 2 and 3 show the simulation results. Since the liveness reachable sets are in 4D and the safety reachable sets are in 6D, we compute and plot their 2D slices based on the quadrotors velocities and relative velocities.

Figure 2 illustrates the use of liveness and safety reachable sets using just the first two quadrotors to reduce visual clutter. The first quadrotor Q_1 (red disk) first travels in a straight line towards the highway merging point $\bar{x}$ (red circle) at $t = 1.5$, because it is not yet in the liveness reachable set for merging onto the highway (red dotted boundary). When it is within the liveness reachable set boundary at $t = 2.8$, it is "locked-in" to the target state $\bar{x}$, and follows the optimal control in (12) to $\bar{x}$. During the entire time, the first quadrotor Q_1 checks whether it may collide with Q_2 within a time horizon of 3 seconds. However, since Q_1 never goes into the boundary of the safety reachable set (red dashed boundary), it is able to use the liveness controller the entire time.

After Q_1 has reached $\bar{x}$, it forms a platoon, becomes the platoon leader, and continues to travel along the highway. Q_2 (blue disk) now, at $t = 7$, begins joining the platoon behind Q_1, by moving towards the target $\bar{x}$ relative to the position of Q_1. Note that $\bar{x}$ moves with Q_1 as $\bar{x}$ is defined in terms of the relative states of the two quadrotors. When Q_2 moves inside the liveness reachable set boundary for joining the platoon (blue dotted boundary), it is "locked-in" to the target relative state $\bar{x}$, and begins following the optimal control in

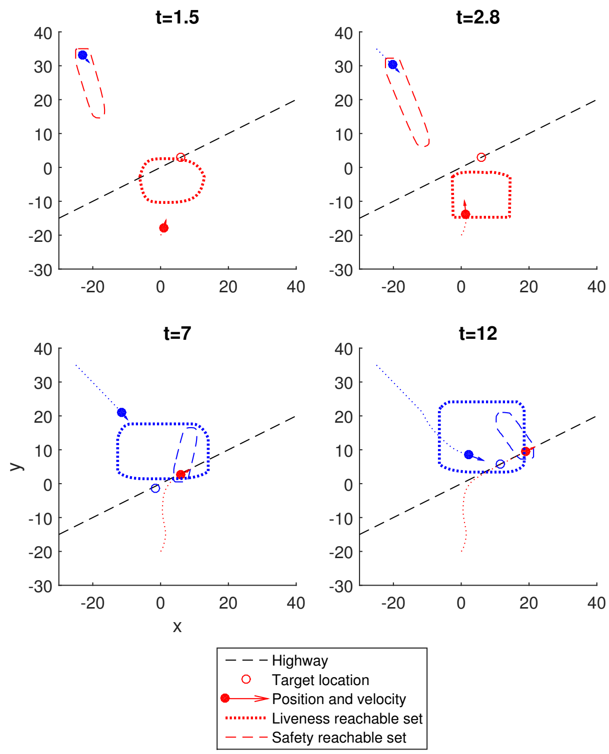

Fig. 2: Reachable sets used to merge onto a highway to form a platoon (top subplots) and to join a platoon on the highway (buttom subplots).

(9) towards the target as long as it stays out of the safety reachable set (blue dashed boundary).

Figure 3 shows the behavior of all 5 quadrotors which eventually form a platoon and travel along the highway together. The liveness controllers allow the quadrotors to optimally and smoothly enter the highway and join platoons, while the safety controllers prevent collisions from occurring.

B. *Malfunctioning Vehicle in Platoon*

We now consdier a scenario where a platoon is traveling down a highway, and then a quadrotor in the platoon malfunctions. To best demonstrate the behavior of the other quadrotors in the platoon, this simulation shows a quadrotor in the center of the platoon malfunction, and reverse direction on the highway. When the quadrotor becomes faulty, all of the other quadrotors in the platoon begin checking safety against the faulty quadrotor. In addition, the faulty quadrotor is removed from the platoon, causing the other quadrotors to treat it as an intruder and close the space the faulty quadrotor once occupied. The trailing quadrotors must leave the highway to avoid colliding with the faulty quadrotor.

The highway is specified by the line $p_y = 0.5p_x$.

Figure 6 shows a platoon of 5 quadrotors, $Q_i, i = 1, ..., 5$ with $P_i = i$, traveling along the highway at nominal velocity. At t=0, Q_3 malfuctions and begins to track the highway in reverse. Once Q_3 malfunctions, it is removed from the platoon and treated as an intruder (Q_0). The platoon is then restructured with the faulty quadrotor removed ($Q_n =$

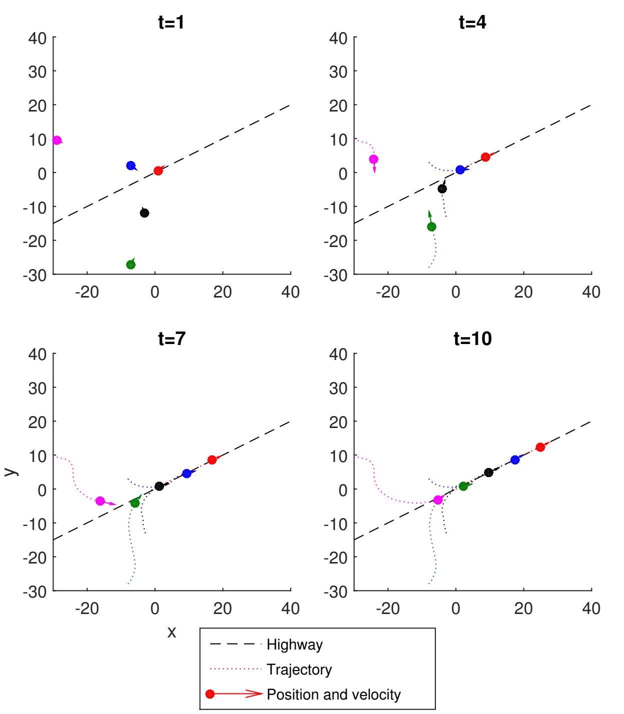

Fig. 3: Five quadrotors merging onto a highway. The first quadrotor forms a platoon on the highway and becomes the platoon leader; the rest of the quadrotors join the platoon behind the first quadrotor.

Q_{n-1} for n=3,...,end). $Q_{n,...,end}$ accelerate to reach their new platoon positions.

Q_1 and Q_2 are unaffected by the malfunctioning quadrotor and continue down the highway using the liveness controller. As shown in figure 6, at t=???, Q_3 applies the safe controller to avoid entering the reachible set for Q_0. Q_4 follows Q_3 without needing to avoid the reachable set for Q_0. Once Q_3 has cleared the faulty quadrotor, it merges back to the highway, with Q_4 trailing. The platoon is now under normal operation again.

In the real implementation, the faulty quadrotor would descend out of reach in the z direction within 2 seconds. Here, we allow the quadrotor to remain at the highway altitude throughout the simulation.

C. Intruder Vehicle

We now consider the scenario in which a platoon of quadrotors encounters an intruder vehicle. To avoid collision, each quadrotor checks for safety with respect to the intruder and any quadrotor in front and behind in the platoon. If necessary, the quadrotor uses the safety controller, otherwise it uses the appropriate liveness controller, depending on whether it is a leader or follower.

Figure 6 shows the simulation result. At $t = 0$, a platoon of 4 quadrotors, $Q_{P_i}, i = 1, \dots, 4$ with $P_i = i$, travels along the highway at its nominal velocity. The highway is defined by the line $p_y = p_x$. An intruder vehicle Q_0 (red disk) starts from position $(p_x, p_y) = (40, 30)$ and heads toward bottom-left of the grid. As before, platoon members incur a

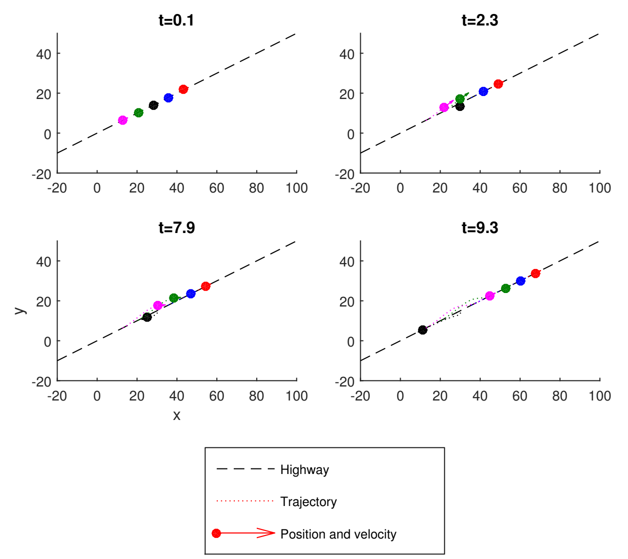

Fig. 4: A quadrotor in the platoon becomes faulty. The rest of the platoon safely avoids the quadrotor and continues normal operation as a platoon once the faulty quadrotor has been cleared.

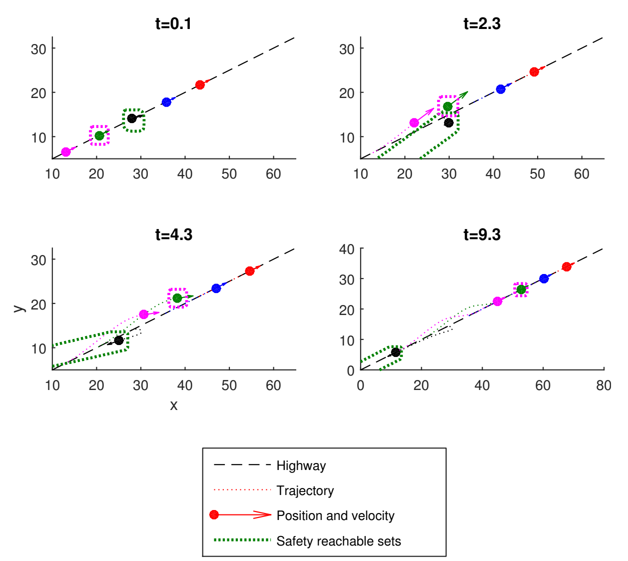

Fig. 5: Reachables sets used by trailing quadrotors to avoid colliding with the faulty quadrotor.

safety breach if $V(t_{\text{internal}}, x_{P_i}, x_{P_j}) \leq 0$ with $t_{\text{internal}} = 1.5$ seconds. However, with respect to Q_0, platoon members incur a safety breach if $V(t_{\text{external}}, x_{P_i} - x_0) \leq 0$ with $t_{\text{external}} = 3$ seconds.

The platoon leader Q_{P_1}'s (black disk) safety is unaffected by the intruder, thus it simply follows its original path on the highway using the liveness controller described in Section IV-C.1. Followers Q_{P_2} (blue disk), Q_{P_3} (green disk) and Q_{P_4} (cyan disk), on the other hand, must use the safety controller in order to avoid collision with the intruder ($t = 3.3, 6.2$). This causes their paths to deviate off the highway. Once each quadrotor is safe relative to the intruder, they merge back onto the highway, join the original platoon and continue traveling along it ($t = 12.4$).

Figure 7 illustrates the use of safety reachable sets in this scenario using only Q_{P_2} as an example to reduce visual clutter. The safety reachable sets of Q_{P_2} with respect to the intruder Q_0 (red dashed line), Q_{P_1} (black dashed line) and Q_{P_3} (green dashed line) are shown. With respect to the intruder, Q_{P_2}'s safety is considered to be breached if $V_S(t_{\text{external}}, x_{P_2} - x_0) \leq 0$, where we have chosen $t_{\text{external}} = 3$ seconds. To avoid possible collision with the intruder in the next 3 seconds, Q_{P_2} must remain outside the safety reachable set with respect to the intruder. The same applies to collision avoidance with Q_{P_1} and Q_{P_3}. Note that safety reachable sets with Q_{P_1} and Q_{P_3} are much smaller than that with the intruder. This is because under normal conditions, vehicles in the same platoon have near-zero relative velocity. For this reason, they may only check possible collisions within the next 1.5 seconds, instead of 3 seconds for vehicles outside of the platoon. This allows vehicles to travel closer together within a platoon and thus increasing throughput on the air highway.

Initially, Q_{P_2} ($P_2 = 2$) is a follower and is outside all 3 safety reachable sets. Hence it is free to use the liveness controller to follow the platoon as described in Section IV-C.2. At time $t = 0.6$, Q_2 came to the boundary of the safety set with respect to the intruder and therefore must apply the safety control law to avoid possible future collision. Thus it splits the original platoon and becomes the leader of a new platoon consisting of itself, Q_3 and Q_4. Q_2 keeps using the safety controller until it is safe with respect to the intruder again at $t = 3$, since if it tried to merge back to the highway before this time, it would enter the safety set with respect to the intruder and lose the $t_{\text{externa}} = 3$ seconds safety guarantee. After $t = 3$, Q_2 is safe to use the liveness controller again to merge back onto the highway and join the original platoon as a follower. Note that during this process, Q_2 maintains safety with respect to the intruder, Q_1 and Q_3 by always staying outside of all 3 safety reachable sets.

D. *Malfunction During Merging*

VII. Conclusions and Future Work

References

[1] B. P. Tice, "Unmanned aerial vehicles – the force multiplier of the 1990s," *Airpower Journal*, Spring 1991.

[2] Amazon.com, Inc. (2014) Amazon prime air. [Online]. Available: http://www.amazon.com/b?node=8037720011

[3] J. Stewart. (2014) Google tests drone deliveries in Project Wing trials. [Online]. Available: http://www.bbc.com/news/technology-28964260

[4] Jointed Planning and Development Office (JPDO), "Unmanned aircraft systems (UAS) comprehensive plan – a report on the nation's UAS path forward," Federal Aviation Administration, Tech. Rep., September 2013.

[5] W. M. Debusk, "Unmanned aerial vehicle systems for disaster relief: Tornado alley," in *Infotech@Aerospace Conferences*, 2010.

[6] O. Bokanowski, N. Forcadel, and H. Zidani, "Reachability and minimal times for state constrained nonlinear problems without any controllability assumption," *SIAM Journal on Control and ...*, pp. 1–24, 2010.

[7] I. Mitchell, A. Bayen, and C. Tomlin, "A time-dependent Hamilton-Jacobi formulation of reachable sets for continuous dynamic games," *IEEE Transactions on Automatic Control*, vol. 50, no. 7, pp. 947–957, July 2005.

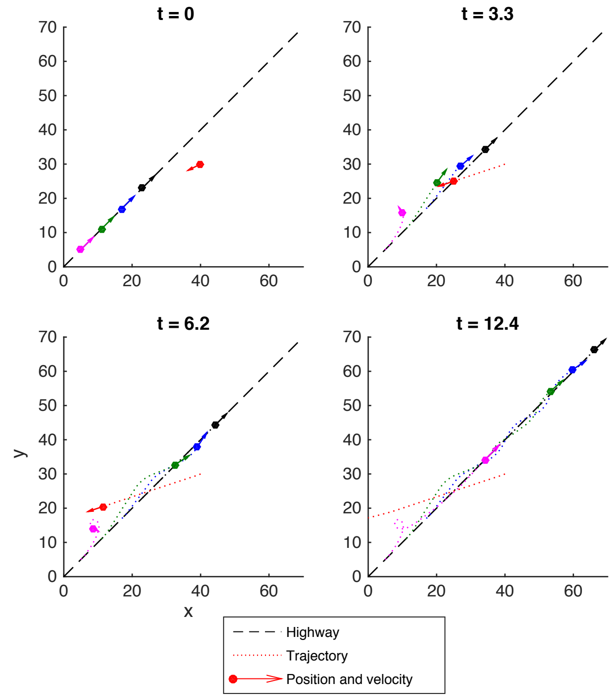

Fig. 6: Reaction of a platoon of 4 quadrotors on a highway to an intruder. Quadrotors in the platoon avoid collision with the intruder and return to follow their path on highway once they are safe.

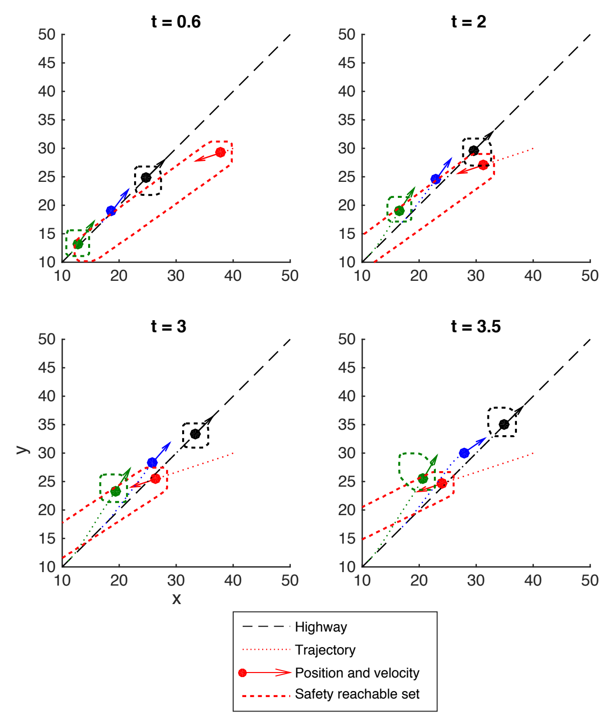

Fig. 7: Reachable sets used by quadrotor Q_2 to avoid collision with respect to the intruder and quadrotors Q_1 and Q_3 in front and behind it respectively.

[8] E. Barron and H. Ishii, “The Bellman equation for minimizing the maximum cost,” *Nonlinear Analysis: Theory, Methods & Applications*, 1989.

[9] “Reach-avoid problems with time-varying dynamics, targets and constraints.”

[10] M. Chen, Z. Zhou, and C. Tomlin, “Multiplayer reach-avoid games via low dimensional solutions and maximum matching,” in *Proceedings of the American Control Conference*, 2014.

[11] J. Ding, J. Sprinkle, S. S. Sastry, and C. J. Tomlin, “Reachability calculations for automated aerial refueling,” in *IEEE Conference on Decision and Control*, Cancun, Mexico, 2008.

[12] I. Mitchell, *A Toolbox of Level Set Methods*, 2009, http://people.cs.ubc.ca/ mitchell/ToolboxLS/index.html.

[13] S. Osher and R. Fedkiw, *Level Set Methods and Dynamic Implicit Surfaces*. Springer-Verlag, 2002, ISBN: 978-0-387-95482-0.

[14] J. A. Sethian, “A fast marching level set method for monotonically advancing fronts,” *Proceedings of the National Academy of Sciences*, vol. 93, no. 4, pp. 1591–1595, 1996.

[15] P. Kavathekar and Y. Chen, “Vehicle platooning: A brief survey and categorization,” vol. 3, pp. 829–845, 2011.

[16] D. McMahon, J. Hedrick, and S. Shladover, “Vehicle modelling and control for automated highway systems,” in *American Control Conference, 1990*, May 1990, pp. 297–303.

[17] J. Hedrick, G. Zhang, V. Narendran, K. Chang, P. for Advanced Transit, H. (Calif.), and B. I. o. T. S. University of California, *Transitional Platoon Maneuvers in an Automated Highway System*, ser. PATH research report. California PATH Program, Institute of Transportation Studies, University of California at Berkeley, 1992. [Online]. Available: http://books.google.com/books?id=eOPrHAAACAAJ

[18] J. Lygeros, D. Godbole, and S. Sastry, “Verified hybrid controllers for automated vehicles,” *Automatic Control, IEEE Transactions on*, vol. 43, no. 4, pp. 522–539, Apr 1998.

[19] A. Alam, A. Gattami, K. H. Johansson, and C. J. Tomlin, “Establishing safety for heavy duty vehicle platooning: A game theoretical approach,” in *18th IFAC World Congress*, Milan, Italy, August 2011.

[20] S. S. John Lygeros and C. Tomlin, *Hybrid Systems: Foundations, advanced topics and applications*. Springer Verlag, 2012.

[21] L. C. Evans and P. E. Souganidis, “Differential games and representation formulas for solutions of Hamilton-Jacobi-Isaacs equations,” *Indiana University Mathematics Journal*, vol. 33, no. 5, pp. 773–797, 1984.

[22] P. Varaiya, “On the existence of solutions to a differential game,” *SIAM Journal on Control*, vol. 5, no. 1, pp. 153–162, 1967.

[23] M. G. Crandall, L. C. Evans, and P. L. Lions, “Some properties of viscosity solutions of hamilton-jacobi equations,” *Transactions of the American Mathematical Society*, vol. 282, no. 2, p. 487, Apr. 1984.