

# Guaranteeing Safety and Liveness of Unmanned Aerial Vehicle Platoons on Air Highways

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**Abstract.**

## Nomenclature

(Nomenclature entries should have the units identified)

$x$  = System state

## I. Introduction

## II. Air Highways

### A. Problem Formulation

We consider air highways to be line segments of constant altitude over a region. On the highway, a number of platoons may be present. The concept of platoons will be proposed in Section ?? For now, denote the line segments  $\mathbb{H}(s)$ ,  $s \in [0, 1]$ . In a horizontal plane of fixed altitude, the end points of the line segment are given by  $x_0 = \mathbb{H}(0)$  and  $y_0 = \mathbb{H}(1)$ . We assign a speed of travel  $v_{\mathbb{H}}$  and specify the direction of travel to be the direction from  $\mathbb{H}(0)$  to  $\mathbb{H}(1)$ , denoted using a unit vector  $\hat{d} = \frac{\mathbb{H}(1) - \mathbb{H}(0)}{\|\mathbb{H}(1) - \mathbb{H}(0)\|_2}$ .

Air highways not only provide structure to make the analysis of a large number of vehicles tractable, but also allow vehicles reach their destinations. Thus, given a origin-destination pair (eg. two cities), air highways must connect the two points while potentially satisfying other criteria. We call this the “air highway placement problem”, and propose to address it in the following way:

1. Establish a cost map over a region of space. This cost map represents the aggregate cost of placing air highways over any point in space. The costs of placing air highways may include interference with commercial air spaces, cost of accidents, noise pollution, etc., and can be designed by government regulation bodies.
2. Compute the cost-minimizing paths  $\mathcal{P}_{ij}$  from an origin  $z_i$  to multiple destinations  $z_j$ .
3. Convert the cost-minimizing path into a sequence of air highways. The endpoints of the resulting air highways can be thought of as waypoints in the airspace.

### B. The Eikonal Equation – Cost-Minimizing Path

### C. From Paths to Waypoints

Each of the cost-minimizing paths  $\mathcal{P}_{ij}$  computed from the solution to the Eikonal equation consists of a set of points which are close to adjacent points. To sparsify this set of points to obtain a collection of waypoints,  $\mathcal{W}_{ij}$ , which are also endpoints of air highways, we first add the destination point to the collection and note the path’s heading. Next, we add to the collection of waypoints the first point on the path at which the heading changes by some threshold  $\theta_C$ . This process is repeated along the entire path. Finally, we add the origin points to the collection. We create a collection of these points for all the cost-minimizing paths.

If there is a large change in heading within a small section of the cost-minimizing path, then the collections  $\mathcal{W}_{ij}$  may contain many points which are close together. In addition, there may be multiple paths that are very close to each other (in fact, this behavior is desirable), also cluttering the airspace with too many waypoints. We propose to sparsify the waypoints by clustering the points in  $\mathcal{W}_{ij}$ . Each

cluster contains points that are within a certain distance to the closest point in the cluster, and all points in each cluster are replaced with a single point located at the centroid of the cluster.

## D. Results

To illustrate our air highway placement proposal, we used the San Francisco Bay Area as an example, and classified each point on the map into three different region. Each region has an associated cost, reflecting the desirability of flying a vehicle over an area in the region. In general, these costs can be arbitrary and determined by government regulation agencies. For illustration purposes, we assumed the following categories and costs:

- Region around airports:  $c_{\text{airports}} = b$ ,
- Cities:  $c_{\text{cities}} = 1$ ,
- Water:  $c_{\text{water}} = b^{-2}$ ,
- Other:  $c_{\text{other}} = b^{-1}$ .

This assumption assigns costs in descending order to the categories “region around airports”, “cities”, “water”, and “other”. Flying a UAV in each category is more costly by a factor of  $b$  compared to the next most important category. The factor  $b$  is a tuning parameter that we adjusted to vary the relative importance of the different categories, and we used  $b = 4$  in the figures below.

### 1. Cost-Minimizing Paths

Fig. ?? shows the San Francisco Bay Area (geographic) map, cost map, cost-minimizing paths, and contours of the value function. The region enclosed by the black boundary represent “region around airports”, which have the highest cost. The dark blue, yellow, and light blue regions represent the “cities”, the “water”, and the “other” categories, respectively. We assumed that the origin corresponds to the city “Concord”, and chose a number of other major cities as destinations.

A couple of important observations can be made here. First, the cost-minimizing paths to the various destinations in general overlap, and only split up when they are very close to entering their destination cities. This is intuitively desirable because the number of air highways is kept low. In addition, the cost of flying in the airspace according to the cost map is minimized. Secondly, the spacing of the contours, which correspond to level curves of the value function, have a spacing corresponding to the cost map. This provides insight into the placement of air highways to destinations that were not shown in this example.

Fig. ?? shows the result of converting the cost-minimizing paths to a small number of waypoints. The left subfigure shows

## III. Unmanned Aerial Vehicle Platooning

### A. Problem Formulation

#### 1. Vehicle Dynamics

Consider a UAV whose dynamics are given by

$$\dot{x} = f(x, u) \quad (1)$$

where  $x \in \mathbb{R}^n$  represents the state, and  $u \in \mathbb{R}^{n_u}$  represents the control action. In this paper, we will assume that each vehicle has the following simple model of a quadrotor:

$$\begin{aligned} \dot{p}_x &= v_x, & \dot{p}_y &= v_y \\ \dot{v}_x &= u_x, & \dot{v}_y &= u_y, & |u_x|, |u_y| &\leq u_{\max} \end{aligned} \quad (2)$$

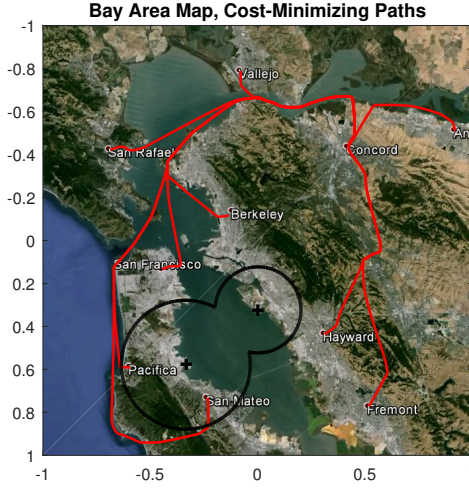
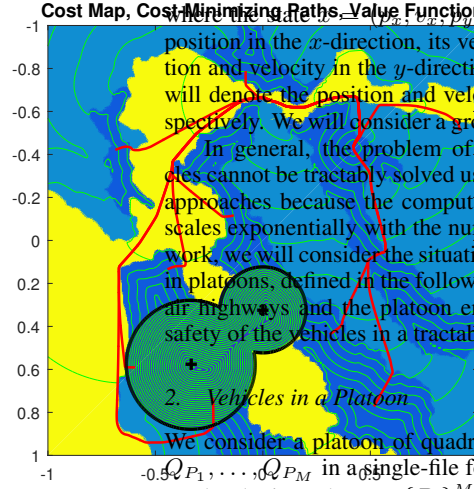


Figure 1. Cost-minimizing paths computed by the Fast Marching Method based on the assumed cost map of the San Francisco Bay Area



where the state  $x = (p_x, v_x, p_y, v_y) \in \mathbb{R}^4$  represents the quadrotor's position in the  $x$ -direction, its velocity in the  $x$ -direction, and its position and velocity in the  $y$ -direction, respectively. For convenience, we will denote the position and velocity  $p = (p_x, p_y)$ ,  $v = (v_x, v_y)$ , respectively. We will consider a group of  $N$  quadrotors  $Q_i, i = 1 \dots, N$ . In general, the problem of collision avoidance among  $N$  vehicles cannot be tractably solved using traditional dynamic programming approaches because the computation complexity of these approaches scales exponentially with the number of vehicles. Thus, in our present work, we will consider the situation where UAVs travel on air highways in platoons, defined in the following sections. The structure imposed by air highways and the platoon enables us to analyze the liveness and safety of the vehicles in a tractable manner.

## 2. Vehicles in a Platoon

We consider a platoon of quadrotors to be a group of  $M$  quadrotors  $Q_{P_1}, \dots, Q_{P_M}$  in a single-file formation. Not all of the  $N$  quadrotors need to be in a platoon:  $\{P_j\}_{j=1}^M \subseteq \{i\}_{i=1}^N$ .  $Q_{P_1}$  is the leader of the platoon, and  $Q_{P_2}, \dots, Q_{P_M}$  are the followers. We will assume that the quadrotors in a platoon travel along an air highway, which is defined by as a path inside a pre-defined altitude range. The quadrotors maintain a separation distance of  $b$ . In order to allow for close proximity of the quadrotors and the ability to resolve multiple simultaneous safety breaches, we assume that in the event of a malfunction, a quadrotor will be able to exit the altitude range of the highway within a duration of  $t_{\text{internal}} = 1.5$ . Such a requirement may be implemented practically as an emergency landing procedure to which the quadrotors revert when a malfunction is detected.

As part of a platoon, each quadrotor must be capable of performing a number of essential cooperative maneuvers. In this paper, we consider the following:

- safely merging onto an air highway;
- safely joining a platoon;
- reacting to a malfunctioning vehicle in the platoon;
- reacting to an intruder vehicle;
- following the highway, a curve defined in space at constant altitude, at a specified speed;
- maintaining a constant relative position and velocity with the leader of a platoon.

## 3. Vehicles as Hybrid Systems

A UAV in general can be in a number of modes of operations, depending on whether it is part of a platoon, and in the affirmative case, whether it is a leader or a follower. Therefore, it is natural to model quadrotors as hybrid systems [2, 21]. In this paper, we restrict the available maneuvers of each quadrotor depending on the mode. We assume that each quadrotor in the airspace has the following modes:

- Free: Vehicle not in a platoon. Available maneuvers: merge onto a highway, join a platoon on a highway.
- Leader: Leader of platoon (could be by itself). Available maneuvers: travel along the highway at a pre-specified speed, merge current platoon with a platoon in front, leave the highway.
- Follower: Vehicle following the platoon leader. Available maneuvers: follow a platoon, create a new platoon.

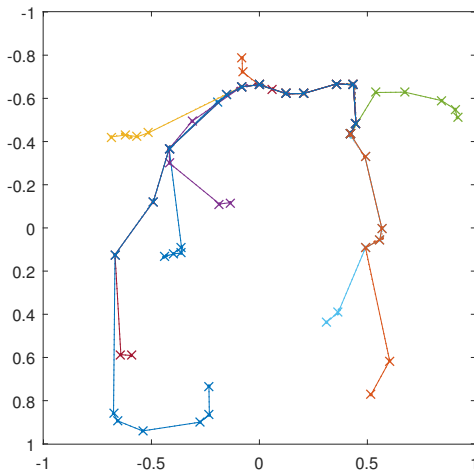
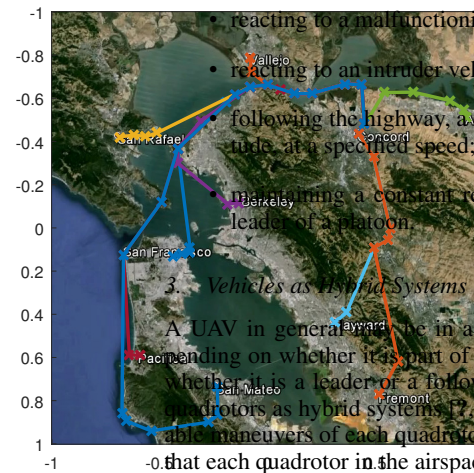


Figure 2. Results of conversion from cost-minimizing paths to highway way points.



- Faulty: Malfunctioned vehicle in a platoon: reverts to default behavior and descends after a duration of  $t_{\text{internal}}$ .

The available maneuvers and associated mode transitions are shown in Figure ??.

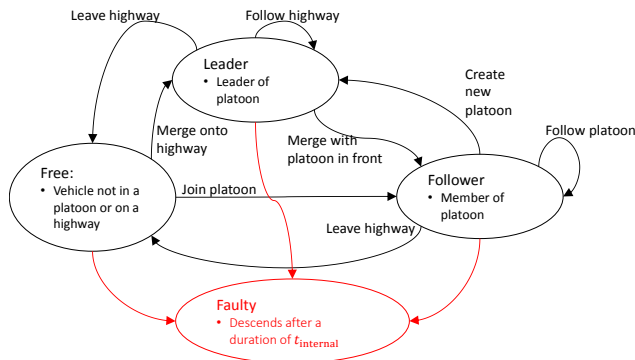


Figure 3. Hybrid modes for vehicles in platoons.

#### 4. Objectives

Using the previously-mentioned modeling assumptions, we would like to address the following questions:

1. How do vehicles form platoons?
2. How can the safety of the vehicles be ensured during normal operation and when there is a malfunctioning vehicle within the platoon?
3. How can the platoon respond to intruders such as unresponsive UAVs, birds, or other aerial objects?

The answers to these questions can be broken down into the maneuvers listed in Section ?. In general, the control strategies of each quadrotor have a liveness component, which specifies a set of states towards which the quadrotor aims to reach, and a safety component, which specifies a set of states that it must avoid. In this paper, we address both the liveness and safety component using reachability analysis.

#### B. Hamilton-Jacobi Reachability

#### C. Intra-Platoon Controllers

#### D. Inter-Platoon Controllers

#### E. Safety Analysis

#### F. Numerical Simulations

### IV. Conclusions

### Appendix

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