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## 1 Basic

### 1.1 Run

```
1 #use -> sh run.sh {name}
2 g++ -O2 -std=c++14 -Wall -Wextra -Wshadow -o $1 $1.cpp
3 ./ $1 < t.in > t.out
```

### 1.2 Binary Search

```
1 lower_bound(a, a + n, k); //最左邊 ≥ k 的位置
2 upper_bound(a, a + n, k); //最左邊 > k 的位置
3 upper_bound(a, a + n, k) - 1; //最右邊 ≤ k 的位置
4 lower_bound(a, a + n, k) - 1; //最右邊 < k 的位置
5 [lower_bound, upper_bound) //等於 k 的範圍
6 equal_range(a, a + n, k);
```

### 1.3 Ternary Search

```
1 const double EPS = 1e-6;
2 // target function
3 double f(double x) { return x * x; }
4 double ternarySearch() {
5     double L = -1e5, R = 1e5;
6     while (R - L > EPS) {
7         double mr = (L + R) / 2.0;
8         double ml = (L + mr) / 2.0;
9         if (f(ml) < f(mr)) {
10             R = mr;
```

```
11     } else {
12         L = ml;
13     }
14 }
15 return L;
16 }
```

## 2 Data Structure

### 2.1 BIT RARSQ

```
1 // 1-base
2 #define lowbit(k) (k & -k)
3 int n;
4 vector<int> B1, B2;
5 void add(vector<int> &tr, int id, int val) {
6     for (; id <= n; id += lowbit(id)) {
7         tr[id] += val;
8     }
9 }
10 void range_add(int l, int r, int val) {
11     add(B1, l, val);
12     add(B1, r + 1, -val);
13     add(B2, l, val * (1 - 1));
14     add(B2, r + 1, -val * r);
15 }
16 int sum(vector<int> &tr, int id) {
17     int ret = 0;
18     for (; id >= 1; id -= lowbit(id)) {
19         ret += tr[id];
20     }
21     return ret;
22 }
23 int prefix_sum(int id) {
24     return sum(B1, id) * id - sum(B2, id);
25 }
26 int range_sum(int l, int r) {
27     return prefix_sum(r) - prefix_sum(l - 1);
28 }
```

### 2.2 zkw RMQ

```
1 // 0-base
2 const int INF = 1e9;
3 const int MAXN = ;
4 int n;
5 int a[MAXN], tr[MAXN << 1];
6 // !!! remember to call this function
7 void build() {
8     for (int i = 0; i < n; i++) {
9         tr[i + n] = a[i];
10     }
11     for (int i = n - 1; i > 0; i--) {
12         tr[i] = max(tr[i << 1], tr[i << 1 | 1]);
13     }
14 }
15 void update(int id, int val) {
16     for (tr[id += n] = val; id > 1; id >>= 1) {
17         tr[id >> 1] = max(tr[id], tr[id ^ 1]);
18     }
19 }
20 int query(int l, int r) { // [l, r)
21     int ret = -INF;
22     for (l += n, r += n; l < r; l >>= 1, r >>= 1) {
23         if (l & 1) {
24             ret = max(ret, tr[l++]);
25         }
26         if (r & 1) {
27             ret = max(ret, tr[--r]);
28         }
29     }
30     return ret;
31 }
```

## 2.3 Segment Tree RARMQ

```

1 struct Node {
2     int val, tag;
3     Node *lc, *rc;
4     Node() : lc(nullptr), rc(nullptr), tag(0) {}
5     void pull() {
6         if (!lc) {
7             val = rc->val;
8         } else if (!rc) {
9             val = lc->val;
10        } else {
11            val = max(lc->val, rc->val);
12        }
13    }
14    void push() {
15        if (lc) {
16            lc->tag += tag;
17            lc->val += tag;
18        }
19        if (rc) {
20            rc->tag += tag;
21            rc->val += tag;
22        }
23        tag = 0;
24    }
25 };
26 struct SegmentTree {
27     Node *root;
28     SegmentTree() : root(nullptr) {}
29     void build(Node* &T, int l, int r, const
30         vector<int> &o) {
31         T = new Node();
32         if (l == r) {
33             T->val = o[l];
34             return;
35         }
36         int mid = (l + r) / 2;
37         build(T->lc, l, mid, o);
38         build(T->rc, mid + 1, r, o);
39         T->pull();
40     }
41     void update(Node* &T, int l, int r, int ql, int qr,
42         int v) {
43         if (ql <= l && r <= qr) {
44             T->val += v;
45             T->tag += v;
46             return;
47         }
48         T->push();
49         int mid = (l + r) / 2;
50         if (qr <= mid) {
51             update(T->lc, l, mid, ql, qr, v);
52         } else if (mid < ql) {
53             update(T->rc, mid + 1, r, ql, qr, v);
54         } else {
55             update(T->lc, l, mid, ql, mid, v);
56             update(T->rc, mid + 1, r, mid + 1, qr, v);
57         }
58         T->pull();
59     }
60     int query(Node* &T, int l, int r, int ql, int qr) {
61         if (ql <= l && r <= qr) {
62             return T->val;
63         }
64         T->push();
65         int mid = (l + r) / 2;
66         if (qr <= mid) {
67             return query(T->lc, l, mid, ql, qr);
68         } else if (mid < ql) {
69             return query(T->rc, mid + 1, r, ql, qr);
70         } else {
71             return max(query(T->lc, l, mid, ql, mid),
72                 query(T->rc, mid + 1, r, mid + 1, qr));
73         }
74     }
75 };

```

## 2.4 Treap

```

1 struct Treap {
2     int val, pri, sz;
3     Treap *lc, *rc;
4     Treap() {}
5     Treap(int _val) {
6         val = _val;
7         pri = rand();
8         sz = 1;
9         lc = rc = NULL;
10    }
11 };
12 int getSize(Treap *a) { return (a == NULL ? 0 :
13     a->sz); }
14 void split(Treap *t, Treap *&a, Treap *&b, int k) {
15     if (t == NULL) {
16         a = b = NULL;
17         return;
18     }
19     if (getSize(t->lc) < k) {
20         a = t;
21         split(t->rc, a->rc, b, k - getSize(t->lc) - 1);
22     } else {
23         b = t;
24         split(t->lc, a, b->lc, k);
25     }
26 }
27 Treap *merge(Treap *a, Treap *b) {
28     if (!a || !b) {
29         return (a ? a : b);
30     }
31     if (a->pri > b->pri) {
32         a->rc = merge(a->rc, b);
33         return a;
34     } else {
35         b->lc = merge(a, b->lc);
36         return b;
37     }
38 }
39 void Insert(Treap *&t, int x, int p) {
40     Treap *a, *b;
41     split(t, a, b, x);
42     t = merge(a, merge(new Treap(p), b));
43 }
44 void Delete(Treap *&t, int x) {
45     Treap *a, *b, *c;
46     split(t, b, c, x);
47     split(b, a, b, x - 1);
48     t = merge(a, c);
49 }
50 /*
51 Usage
52 Treap *root = NULL; // declare
53 root = merge(root, new Treap(val)); // push back
54 Insert(root, x, y); // insert y after x-th element
55 Delete(root, x); // delete x-th element
56 */

```

## 3 Graph

### 3.1 Directed MST

```

1 // 0-base
2 const LL INF = 1e18;
3 const int MAXN = ;
4
5 struct Edge {
6     int from;
7     int to;
8     LL cost;
9     Edge(int u, int v, LL c) : from(u), to(v), cost(c) {}
10 }

```

```

10 };
11
12 struct DMST {
13     int n;
14     int vis[MAXN], pre[MAXN], id[MAXN];
15     LL in[MAXN];
16     vector<Edge> edges;
17     void init(int _n) {
18         n = _n;
19         edges.clear();
20     }
21     void add_edge(int from, int to, LL cost) {
22         edges.eb(from, to, cost);
23     }
24     LL run(int root) {
25         LL ret = 0;
26         while (true) {
27             for (int i = 0; i < n; i++) {
28                 in[i] = INF;
29             }
30
31             // find in edge
32             for (auto &e : edges) {
33                 if (e.cost < in[e.to] && e.from != e.to) {
34                     pre[e.to] = e.from;
35                     in[e.to] = e.cost;
36                 }
37             }
38
39             // check in edge
40             for (int i = 0; i < n; i++) {
41                 if (i == root) {
42                     continue;
43                 }
44                 if (in[i] == INF) {
45                     return -1;
46                 }
47             }
48
49             int nodelist = 0;
50             memset(id, -1, sizeof(id));
51             memset(vis, -1, sizeof(vis));
52             in[root] = 0;
53
54             // find cycles
55             for (int i = 0; i < n; i++) {
56                 ret += in[i];
57                 int v = i;
58                 while (vis[v] != i && id[v] == -1 && v !=
                    root) {
59                     vis[v] = i;
60                     v = pre[v];
61                 }
62                 if (id[v] == -1 && v != root) {
63                     for (int j = pre[v]; j != v; j = pre[j]) {
64                         id[j] = nodelist;
65                     }
66                     id[v] = nodelist++;
67                 }
68             }
69
70             // no cycle
71             if (nodelist == 0) {
72                 break;
73             }
74
75             for (int i = 0; i < n; i++) {
76                 if (id[i] == -1) {
77                     id[i] = nodelist++;
78                 }
79             }
80
81             // grouping the vertices
82             for (auto &e : edges) {
83                 int to = e.to;
84                 e.from = id[e.from];
85                 e.to = id[e.to];

```

```

86         if (e.from != e.to) {
87             e.cost -= in[to]; //!!!
88         }
89     }
90
91     n = nodelist;
92     root = id[root];
93 }
94 return ret;
95 }
96 };

```

### 3.2 LCA

```

1 const int LOG = 20;
2 vector<int> tin(MAXN), tout(MAXN), depth(MAXN);
3 int par[MAXN][LOG];
4 int timer = 0;
5 vector<int> G[MAXN];
6 void dfs(int u, int f) {
7     tin[u] = ++timer;
8     par[u][0] = f;
9     for (int v : G[u]) {
10         if (v != f) {
11             depth[v] = depth[u] + 1;
12             dfs(v, u);
13         }
14     }
15     tout[u] = ++timer;
16 }
17 void Doubling(int n) {
18     for (int j = 1; j < LOG; ++j) {
19         for (int i = 1; i <= n; ++i) {
20             par[i][j] = par[par[i][j - 1]][j - 1];
21         }
22     }
23 }
24 bool anc(int u, int v) { return tin[u] <= tin[v] &&
    tout[v] <= tout[u]; }
25 int LCA(int u, int v) {
26     if (depth[u] > depth[v]) {
27         swap(u, v);
28     }
29     if (anc(u, v)) {
30         return u;
31     }
32     for (int j = LOG - 1; j >= 0; --j) {
33         if (!anc(par[u][j], v)) u = par[u][j];
34     }
35     return par[u][0];
36 }
37 int dis(int u, int v) {
38     int lca = LCA(u, v);
39     return depth[u] + depth[v] - 2 * depth[lca];
40 }
41
42 /*
43 dfs(root, root);
44 Doubling(n);
45 */

```

### 3.3 Euler Circuit

七橋問題根據起點與終點是否相同，分成 Euler path (不同) 及 Euler circuit (相同)。

- 判斷法
  - 無向圖部分，將點分成奇點 (度數為奇數) 和偶點 (度數為偶數)。
    - Euler path: 奇點數為 0 或 2
    - Euler circuit: 沒有奇點
  - 有向圖部分，將點分成出點 (出度 - 入度 = 1) 和入點 (入度 - 出度 = 1) 還有平衡點 (出度 = 入度)。

- Euler path: 出點和入點個數同時為 0 或 1。
- Euler circuit: 只有平衡點。
- 求出一組解
- 用 DFS 遍歷整張圖，設  $S$  為離開的順序，無向圖的答案為  $S$ ，有向圖的答案為反向的  $S$ 。
- DFS 起點選定：
  - Euler path: 無向圖選擇任意一個奇點，有向圖選擇出點。
  - Euler circuit: 任意一點。

```

1 // Code from Eric
2 #define ll long long
3 #define PB push_back
4 #define EB emplace_back
5 #define PII pair<int, int>
6 #define MP make_pair
7 #define all(x) x.begin(), x.end()
8 #define maxn 50000+5
9 //structure
10 struct Euler {
11     vector<PII> adj[maxn];
12     vector<bool> edges;
13     vector<PII> path;
14     int chk[maxn];
15     int n;
16     void init(int _n) {
17         n = _n;
18         for (int i = 0; i <= n; i++) adj[i].clear();
19         edges.clear();
20         path.clear();
21         memset(chk, 0, sizeof(chk));
22     }
23     void dfs(int v) {
24         for (auto i : adj[v]) {
25             if (edges[i.first] == true) {
26                 edges[i.first] = false;
27                 dfs(i.second);
28                 path.EB(MP(i.second, v));
29             }
30         }
31     }
32     void add_Edge(int from, int to) {
33         edges.PB(true);
34         // for bi-directed graph
35         adj[from].PB(MP(edges.size() - 1, to));
36         adj[to].PB(MP(edges.size() - 1, from));
37         chk[from]++;
38         chk[to]++;
39         // for directed graph
40         // adj[from].PB(MP(edges.size()-1, to));
41         // check[from]++;
42     }
43     bool eular_path() {
44         int st = -1;
45         for (int i = 1; i <= n; i++) {
46             if (chk[i] % 2 == 1) {
47                 st = i;
48                 break;
49             }
50         }
51         if (st == -1) {
52             return false;
53         }
54         dfs(st);
55         return true;
56     }
57     void print_path(void) {
58         for (auto i : path) {
59             printf("%d %d\n", i.first, i.second);
60         }
61     }
62 };

```

```

1 // Code from allen(lexicographic order)
2 #include <bits/stdc++.h>
3 using namespace std;
4 const int ALP = 30;
5 const int MXN = 1005;
6 int n;
7 int din[ALP], dout[ALP];
8 int par[ALP];
9 vector<string> vs[MXN], ans;
10 bitset<MXN> vis, used[ALP];
11 void djsInit() {
12     for (int i = 0; i != ALP; ++i) {
13         par[i] = i;
14     }
15 }
16 int Find(int x) { return (x == par[x] ? (x) : (par[x]
    = Find(par[x]))); }
17 void init() {
18     djsInit();
19     memset(din, 0, sizeof(din));
20     memset(dout, 0, sizeof(dout));
21     vis.reset();
22     for (int i = 0; i != ALP; ++i) {
23         vs[i].clear();
24         used[i].reset();
25     }
26     return;
27 }
28 void dfs(int u) {
29     for (int i = 0; i != (int)vs[u].size(); ++i) {
30         if (used[u][i]) {
31             continue;
32         }
33         used[u][i] = 1;
34         string s = vs[u][i];
35         int v = s[s.size() - 1] - 'a';
36         dfs(v);
37         ans.push_back(s);
38     }
39 }
40 bool solve() {
41     int cnt = 1;
42     for (int i = 0; i != n; ++i) {
43         string s;
44         cin >> s;
45         int from = s[0] - 'a', to = s.back() - 'a';
46         ++din[to];
47         ++dout[from];
48         vs[from].push_back(s);
49         vis[from] = vis[to] = true;
50         if ((from = Find(from)) != (to = Find(to))) {
51             par[from] = to;
52             ++cnt;
53         }
54     }
55     if ((int)vis.count() != cnt) {
56         return false;
57     }
58     int root, st, pin = 0, pout = 0;
59     for (int i = ALP - 1; i >= 0; --i) {
60         sort(vs[i].begin(), vs[i].end());
61         if (vs[i].size()) root = i;
62         int d = dout[i] - din[i];
63         if (d == 1) {
64             ++pout;
65             st = i;
66         } else if (d == -1) {
67             ++pin;
68         } else if (d != 0) {
69             return false;
70         }
71     }
72     if (pin != pout || pin > 1) {
73         return false;
74     }
75     ans.clear();
76     dfs((pin ? st : root));

```

```

77     return true;
78 }
79 int main() {
80     int t;
81     cin >> t;
82     while (t--) {
83         cin >> n;
84         init();
85         if (!solve()) {
86             cout << "***\n";
87             continue;
88         }
89         for (int i = ans.size() - 1; i >= 0; --i) {
90             cout << ans[i] << ".\n"[i == 0];
91         }
92     }
93 }

```

## 4 Connectivity

### 4.1 Articulation Point

```

1 // from aizu
2 typedef long long int ll;
3 typedef unsigned long long int ull;
4 #define BIG_SIZE 2000000000
5 #define MOD 1000000007
6 #define EPS 0.000000001
7 using namespace std;
8 #define SIZE 100000
9 vector<int> G[SIZE];
10 int N;
11 bool visited[SIZE];
12 int visited_order[SIZE], parent[SIZE], lowest[SIZE],
   number;
13 void dfs(int cur, int pre_node) {
14     visited_order[cur] = lowest[cur] = number;
15     number++;
16     visited[cur] = true;
17     int next;
18     for (int i = 0; i < G[cur].size(); i++) {
19         next = G[cur][i];
20         if (!visited[next]) {
21             parent[next] = cur;
22             dfs(next, cur);
23             lowest[cur] = min(lowest[cur], lowest[next]);
24         } else if (visited[next] == true && next !=
           pre_node) {
25             lowest[cur] = min(lowest[cur],
               visited_order[next]);
26         }
27     }
28 }
29 void art_points() {
30     for (int i = 0; i < N; i++) visited[i] = false;
31     number = 1;
32     dfs(0, -1);
33     int tmp_parent, root_num = 0;
34     vector<int> V;
35     for (int i = 1; i < N; i++) {
36         tmp_parent = parent[i];
37         if (tmp_parent == 0) {
38             root_num++;
39         } else if (visited_order[tmp_parent] <=
           lowest[i]) {
40             V.push_back(tmp_parent);
41         }
42     }
43     if (root_num >= 2) {
44         V.push_back(0);
45     }
46     sort(V.begin(), V.end());
47     V.erase(unique(V.begin(), V.end()), V.end());
48     for (int i = 0; i < V.size(); i++) {

```

```

49         printf("%d\n", V[i]);
50     }
51 }
52 int main() {
53     int E;
54     scanf("%d %d", &N, &E);
55     int from, to;
56     for (int i = 0; i < E; i++) {
57         scanf("%d %d", &from, &to);
58         G[from].push_back(to);
59         G[to].push_back(from);
60     }
61     art_points();
62 }

```

### 4.2 Bridges

```

1 // from aizu
2 typedef long long int ll;
3 typedef unsigned long long int ull;
4 #define BIG_NUM 2000000000
5 #define MOD 1000000007
6 #define EPS 0.000000001
7 using namespace std;
8 struct Edge {
9     bool operator<(const struct Edge &arg) const {
10         if (s != arg.s) {
11             return s < arg.s;
12         } else {
13             return t < arg.t;
14         }
15     }
16     int s, t;
17 };
18 struct Info {
19     Info(int arg_to, int arg_edge_id) {
20         to = arg_to;
21         edge_id = arg_edge_id;
22     }
23     int to, edge_id;
24 };
25 int V, E, number;
26 int order[100000], lowlink[100000];
27 bool visited[100000];
28 Edge edge[100000];
29 vector<Info> G[100000];
30 void recursive(int cur) {
31     order[cur] = number++;
32     lowlink[cur] = order[cur];
33     int next;
34     for (int i = 0; i < G[cur].size(); i++) {
35         next = G[cur][i].to;
36         if (order[next] == -1) {
37             visited[G[cur][i].edge_id] = true;
38             recursive(next);
39             lowlink[cur] = min(lowlink[cur], lowlink[next]);
40         } else if (visited[G[cur][i].edge_id] == false) {
41             lowlink[cur] = min(lowlink[cur], order[next]);
42         }
43     }
44 }
45 int main() {
46     scanf("%d %d", &V, &E);
47     for (int i = 0; i < E; i++) {
48         scanf("%d %d", &edge[i].s, &edge[i].t);
49         if (edge[i].s > edge[i].t) {
50             swap(edge[i].s, edge[i].t);
51         }
52         G[edge[i].s].push_back(Info(edge[i].t, i));
53         G[edge[i].t].push_back(Info(edge[i].s, i));
54     }
55     sort(edge, edge + E);
56     number = 0;
57     for (int i = 0; i < V; i++) {
58         order[i] = -1;
59         lowlink[i] = -1;

```

```

60 }
61 for (int i = 0; i < E; i++) {
62     visited[i] = false;
63 }
64 recursive(0);
65 int from, to;
66 for (int i = 0; i < E; i++) {
67     from = edge[i].s;
68     to = edge[i].t;
69     if (order[edge[i].s] > order[edge[i].t]) {
70         swap(from, to);
71     }
72     if (order[from] < lowlink[to]) {
73         printf("%d %d\n", edge[i].s, edge[i].t);
74     }
75 }
76 return 0;
77 }

```

## 5 Flow & Matching

### 5.1 Relation

```

1 | 1. 一般圖
2 | |最大匹配| + |最小邊覆蓋| = |V|
3 | |最大獨立集| + |最小點覆蓋| = |V|
4 | |最大圖| = |補圖的最大獨立集|
5 | 2. 二分圖
6 | |最大匹配| = |最小點覆蓋|
7 | |最大獨立集| = |最小邊覆蓋|
8 | |最大獨立集| = |V| - |最大匹配|
9 | |最大圖| = |補圖的最大獨立集|

```

### 5.2 Bipartite Matching

```

1 // 0-base
2 const int MAXN = ;
3 int n;
4 vector<int> G[MAXN];
5 int vy[MAXN], my[MAXN];
6 bool match(int u) {
7     for (int v : G[u]) {
8         if (vy[v]) {
9             continue;
10        }
11        vy[v] = true;
12        if (my[v] == -1 || match(my[v])) {
13            my[v] = u;
14            return true;
15        }
16    }
17    return false;
18 }
19 int sol() {
20     int cnt = 0;
21     memset(my, -1, sizeof(my));
22     for (int i = 0; i < n; i++) {
23         memset(vy, 0, sizeof(vy));
24         if (match(i)) {
25             cnt++;
26         }
27     }
28     return cnt;
29 }

```

### 5.3 KM

```

1 const int INF = 1e9;
2 const int MAXN = ;

```

```

3 struct KM { //1-base
4     int n, G[MAXN][MAXN];
5     int lx[MAXN], ly[MAXN], my[MAXN];
6     bool vx[MAXN], vy[MAXN];
7     void init(int _n) {
8         n = _n;
9         for (int i = 1; i <= n; i++) {
10             for (int j = 1; j <= n; j++) {
11                 G[i][j] = 0;
12             }
13         }
14     }
15     bool match(int i) {
16         vx[i] = true;
17         for (int j = 1; j <= n; j++) {
18             if (lx[i] + ly[j] == G[i][j] && !vy[j]) {
19                 vy[j] = true;
20                 if (!my[j] || match(my[j])) {
21                     my[j] = i;
22                     return true;
23                 }
24             }
25         }
26         return false;
27     }
28     void update() {
29         int delta = INF;
30         for (int i = 1; i <= n; i++) {
31             if (vx[i]) {
32                 for (int j = 1; j <= n; j++) {
33                     if (!vy[j]) {
34                         delta = min(delta, lx[i] + ly[j] -
35                                     G[i][j]);
36                     }
37                 }
38             }
39             for (int i = 1; i <= n; i++) {
40                 if (vx[i]) {
41                     lx[i] -= delta;
42                 }
43                 if (vy[i]) {
44                     ly[i] += delta;
45                 }
46             }
47         }
48     }
49     int run() {
50         for (int i = 1; i <= n; i++) {
51             lx[i] = ly[i] = my[i] = 0;
52             for (int j = 1; j <= n; j++) {
53                 lx[i] = max(lx[i], G[i][j]);
54             }
55         }
56         for (int i = 1; i <= n; i++) {
57             while (true) {
58                 for (int i = 1; i <= n; i++) {
59                     vx[i] = vy[i] = 0;
60                 }
61                 if (match(i)) {
62                     break;
63                 } else {
64                     update();
65                 }
66             }
67         }
68         int ans = 0;
69         for (int i = 1; i <= n; i++) {
70             ans += lx[i] + ly[i];
71         }
72         return ans;
73     };

```

### 5.4 Dinic

```

1 #define eb emplace_back

```

```

2  const LL INF = 1e18;
3  const int MAXN = ;
4  struct Edge {
5      int to;
6      LL cap;
7      int rev;
8      Edge(int v, LL c, int r) : to(v), cap(c), rev(r) {}
9  };
10 struct Dinic {
11     int n;
12     int level[MAXN], now[MAXN];
13     vector<Edge> G[MAXN];
14     void init(int _n) {
15         n = _n;
16         for (int i = 0; i <= n; i++) {
17             G[i].clear();
18         }
19     }
20     void add_edge(int u, int v, LL c) {
21         G[u].eb(v, c, G[v].size());
22         // directed graph
23         G[v].eb(u, 0, G[u].size() - 1);
24         // undirected graph
25         // G[v].eb(u, c, G[u].size() - 1);
26     }
27     bool bfs(int st, int ed) {
28         fill(level, level + n + 1, -1);
29         queue<int> q;
30         q.push(st);
31         level[st] = 0;
32         while (!q.empty()) {
33             int u = q.front();
34             q.pop();
35             for (const auto &e : G[u]) {
36                 if (e.cap > 0 && level[e.to] == -1) {
37                     level[e.to] = level[u] + 1;
38                     q.push(e.to);
39                 }
40             }
41         }
42         return level[ed] != -1;
43     }
44     LL dfs(int u, int ed, LL limit) {
45         if (u == ed) {
46             return limit;
47         }
48         LL ret = 0;
49         for (int &i = now[u]; i < G[u].size(); i++) {
50             auto &e = G[u][i];
51             if (e.cap > 0 && level[e.to] == level[u] + 1) {
52                 LL f = dfs(e.to, ed, min(limit, e.cap));
53                 ret += f;
54                 limit -= f;
55                 e.cap -= f;
56                 G[e.to][e.rev].cap += f;
57                 if (!limit) {
58                     return ret;
59                 }
60             }
61         }
62         if (!ret) {
63             level[u] = -1;
64         }
65         return ret;
66     }
67     LL flow(int st, int ed) {
68         LL ret = 0;
69         while (bfs(st, ed)) {
70             fill(now, now + n + 1, 0);
71             ret += dfs(st, ed, INF);
72         }
73         return ret;
74     }
75 };

```

## 5.5 MCMF

```

1  // 0-base
2  const LL INF = 1e18;
3  const int MAXN = ;
4  struct Edge {
5      int u, v;
6      LL cost;
7      LL cap;
8      Edge(int _u, int _v, LL _c, LL _cap) : u(_u),
9          v(_v), cost(_c), cap(_cap) {}
10 };
11 struct MCMF { // inq times
12     int n, pre[MAXN], cnt[MAXN];
13     LL ans_flow, ans_cost, dis[MAXN];
14     bool inq[MAXN];
15     vector<int> G[MAXN];
16     vector<Edge> edges;
17     void init(int _n) {
18         n = _n;
19         edges.clear();
20         for (int i = 0; i < n; i++) {
21             G[i].clear();
22         }
23     }
24     void add_edge(int u, int v, LL c, LL cap) {
25         // directed
26         G[u].pb(edges.size());
27         edges.eb(u, v, c, cap);
28         G[v].pb(edges.size());
29         edges.eb(v, u, -c, 0);
30     }
31     bool SPFA(int st, int ed) {
32         for (int i = 0; i < n; i++) {
33             pre[i] = -1;
34             dis[i] = INF;
35             cnt[i] = 0;
36             inq[i] = false;
37         }
38         queue<int> q;
39         bool negcycle = false;
40
41         dis[st] = 0;
42         cnt[st] = 1;
43         inq[st] = true;
44         q.push(st);
45
46         while (!q.empty() && !negcycle) {
47             int u = q.front();
48             q.pop();
49             inq[u] = false;
50             for (int i : G[u]) {
51                 int v = edges[i].v;
52                 LL cost = edges[i].cost;
53                 LL cap = edges[i].cap;
54
55                 if (dis[v] > dis[u] + cost && cap > 0) {
56                     dis[v] = dis[u] + cost;
57                     pre[v] = i;
58                     if (!inq[v]) {
59                         q.push(v);
60                         cnt[v]++;
61                         inq[v] = true;
62
63                         if (cnt[v] == n + 2) {
64                             negcycle = true;
65                             break;
66                         }
67                     }
68                 }
69             }
70         }
71
72         return dis[ed] != INF;
73     }
74     LL sendFlow(int v, LL curFlow) {
75         if (pre[v] == -1) {

```

```

75     return curFlow;
76 }
77 int i = pre[v];
78 int u = edges[i].u;
79 LL cost = edges[i].cost;
80
81 LL f = sendFlow(u, min(curFlow, edges[i].cap));
82
83 ans_cost += f * cost;
84 edges[i].cap -= f;
85 edges[i ^ 1].cap += f;
86 return f;
87 }
88 pair<LL, LL> run(int st, int ed) {
89     ans_flow = ans_cost = 0;
90     while (SPFA(st, ed)) {
91         ans_flow += sendFlow(ed, INF);
92     }
93     return make_pair(ans_flow, ans_cost);
94 }
95 };

```

## 6 String

### 6.1 Manacher

```

1 int p[2 * MAXN];
2 int Manacher(const string &s) {
3     string st = "@#";
4     for (char c : s) {
5         st += c;
6         st += '#';
7     }
8     st += '$';
9     int id = 0, mx = 0, ans = 0;
10    for (int i = 1; i < st.length() - 1; i++) {
11        p[i] = (mx > i ? min(p[2 * id - i], mx - i) : 1);
12        for (; st[i - p[i]] == st[i + p[i]]; p[i]++);
13        if (mx < i + p[i]) {
14            mx = i + p[i];
15            id = i;
16        }
17        ans = max(ans, p[i] - 1);
18    }
19    return ans;
20 }

```

### 6.2 Trie

```

1 const int MAXL = ;
2 const int MAXC = ;
3 struct Trie {
4     int nex[MAXL][MAXC];
5     int len[MAXL];
6     int sz;
7     void init() {
8         memset(nex, 0, sizeof(nex));
9         memset(len, 0, sizeof(len));
10        sz = 0;
11    }
12    void insert(const string &str) {
13        int p = 0;
14        for (char c : str) {
15            int id = c - 'a';
16            if (!nex[p][id]) {
17                nex[p][id] = ++sz;
18            }
19            p = nex[p][id];
20        }
21        len[p] = str.length();
22    }
23    vector<int> find(const string &str, int i) {

```

```

24    int p = 0;
25    vector<int> ans;
26    for (; i < str.length(); i++) {
27        int id = str[i] - 'a';
28        if (!nex[p][id]) {
29            return ans;
30        }
31        p = nex[p][id];
32        if (len[p]) {
33            ans.pb(len[p]);
34        }
35    }
36    return ans;
37 }
38 };

```

## 7 DP

### 7.1 Huge Knapsack

```

1 // from aizun
2 #include <bits/stdc++.h>
3 typedef long long int ll;
4 typedef unsigned long long int ull;
5 #define BIG_NUM 2000000000
6 #define HUGE_NUM 9999999999999999
7 #define MOD 1000000007
8 #define EPS 0.000000001
9 using namespace std;
10 #define SIZE 25
11 struct Info {
12     Info() { value = 0, weight = 0; }
13     Info(ll arg_value, ll arg_weight) {
14         value = arg_value;
15         weight = arg_weight;
16     }
17     bool operator<(const struct Info &arg) const {
18         return weight < arg.weight; }
19     ll value, weight;
20 };
21 ll N, W;
22 ll POW[SIZE];
23 ll table_B[1 << 21];
24 Info info[45];
25 int main() {
26     POW[0] = 1;
27     for (int i = 1; i < SIZE; i++) {
28         POW[i] = POW[i - 1] * 2;
29     }
30     scanf("%lld %lld", &N, &W);
31     for (int i = 0; i < N; i++) {
32         scanf("%lld %lld", &info[i].value,
33             &info[i].weight);
34     }
35     if (N == 1) {
36         if (info[0].weight <= W) {
37             printf("%lld\n", info[0].value);
38         } else {
39             printf("0\n");
40         }
41         return 0;
42     }
43     vector<int> A, B;
44     for (int i = 0; i < N / 2; i++) {
45         A.push_back(i);
46     }
47     for (int i = N / 2; i < N; i++) {
48         B.push_back(i);
49     }
50     vector<Info> vec_A, vec_B;
51     for (int state = 0; state < POW[A.size()]; state++) {

```



```

52     ll sum_value = 0;
53     for (int loop = 0; loop < A.size(); loop++) {
54         if (state & POW[loop]) {
55             sum_w += info[A[loop]].weight;
56             sum_value += info[A[loop]].value;
57         }
58     }
59     vec_A.push_back(Info(sum_value, sum_w));
60 }
61 sort(vec_A.begin(), vec_A.end());
62 for (int state = 0; state < POW[B.size()]; state++)
63 {
64     ll sum_w = 0;
65     ll sum_value = 0;
66     for (int loop = 0; loop < B.size(); loop++) {
67         if (state & POW[loop]) {
68             sum_w += info[B[loop]].weight;
69             sum_value += info[B[loop]].value;
70         }
71     }
72     vec_B.push_back(Info(sum_value, sum_w));
73 }
74 sort(vec_B.begin(), vec_B.end());
75 table_B[0] = vec_B[0].value;
76 for (int i = 1; i < vec_B.size(); i++) {
77     //ある重さ以下の最大価値を求める
78     table_B[i] = max(table_B[i - 1], vec_B[i].value);
79 }
80 int tail = vec_B.size() - 1;
81 ll ans = 0;
82 for (int i = 0; i < vec_A.size(); i++) {
83     while (tail >= 0 && vec_A[i].weight +
84            vec_B[tail].weight > W) tail--;
85     if (tail < 0) break;
86     ans = max(ans, vec_A[i].value + table_B[tail]);
87 }
88 printf("%lld\n", ans);
89 return 0;
90 }

```

```

1 // ax + by = c
2 int extgcd(int a, int b, int c, int &x, int &y) {
3     if (b == 0) {
4         x = c / a;
5         y = 0;
6         return a;
7     }
8     int d = extgcd(b, a % b, c, y, x);
9     y -= (a / b) * x;
10    return d;
11 }

```

### 8.3 Gaussian Elimination

```

1 const int MAXN = 300;
2 const double EPS = 1e-8;
3 int n;
4 double A[MAXN][MAXN];
5 void Gauss() {
6     for (int i = 0; i < n; i++) {
7         bool ok = 0;
8         for (int j = i; j < n; j++) {
9             if (fabs(A[j][i]) > EPS) {
10                swap(A[j], A[i]);
11                ok = 1;
12                break;
13            }
14        }
15        if (!ok) continue;
16        double fs = A[i][i];
17        for (int j = i + 1; j < n; j++) {
18            double r = A[j][i] / fs;
19            for (int k = i; k < n; k++) {
20                A[j][k] -= A[i][k] * r;
21            }
22        }
23    }
24 }

```

## 8 Math

### 8.1 Number Theory

- Inversion:  
 $aa^{-1} \equiv 1 \pmod{m}$ .  $a^{-1}$  exists iff  $\gcd(a, m) = 1$ .
- Linear inversion:  
 $a^{-1} \equiv (m - \lfloor \frac{m}{a} \rfloor) \times (m \bmod a)^{-1} \pmod{m}$
- Fermat's little theorem:  
 $a^p \equiv a \pmod{p}$  if  $p$  is prime.
- Euler function:  
 $\phi(n) = n \prod_{p|n} \frac{p-1}{p}$
- Euler theorem:  
 $a^{\phi(n)} \equiv 1 \pmod{n}$  if  $\gcd(a, n) = 1$ .
- Extended Euclidean algorithm:  
 $ax + by = \gcd(a, b) = \gcd(b, a \bmod b) = \gcd(b, a - \lfloor \frac{a}{b} \rfloor b) = bx_1 + (a - \lfloor \frac{a}{b} \rfloor b)y_1 = ay_1 + b(x_1 - \lfloor \frac{a}{b} \rfloor y_1)$
- Divisor function:  
 $\sigma_x(n) = \sum_{d|n} d^x$ .  $n = \prod_{i=1}^r p_i^{a_i}$ .  
 $\sigma_x(n) = \prod_{i=1}^r \frac{p_i^{(a_i+1)x} - 1}{p_i^x - 1}$  if  $x \neq 0$ .  $\sigma_0(n) = \prod_{i=1}^r (a_i + 1)$ .
- Chinese remainder theorem:  
 $x \equiv a_i \pmod{m_i}$ .  
 $M = \prod m_i$ .  $M_i = M / m_i$ .  $t_i = M_i^{-1}$ .  
 $x = kM + \sum a_i t_i M_i$ ,  $k \in \mathbb{Z}$ .

### 8.2 Extended GCD

### 8.4 Phi

- 歐拉函數計算對於一個整數  $N$ ，小於等於  $N$  的正整數中，有幾個和  $N$  互質
- 如果  $\gcd(p, q) = 1$ ,  $\Phi(p) \cdot \Phi(q) = \Phi(p \cdot q)$
- $\Phi(p^k) = p^{k-1} \times (p - 1)$

```

1 void phi_table(int n) {
2     phi[1] = 1;
3     for (int i = 2; i <= n; i++) {
4         if (phi[i]) {
5             continue;
6         }
7         for (int j = i; j < n; j += i) {
8             if (!phi[j]) {
9                 phi[j] = j;
10            }
11            phi[j] = phi[j] / i * (i - 1);
12        }
13    }
14 }

```

## 9 Geometry

### 9.1 Point

```

1 // notice point type!!!
2 using dvt = int;
3 const double EPS = 1e-6;
4 const double PI = acos(-1);
5

```

```

6 struct Pt {
7     dvt x;
8     dvt y;
9 };
10 bool operator < (const Pt &a, const Pt &b) {
11     return a.x == b.x ? a.y < b.y : a.x < b.x;
12 }
13 bool operator == (const Pt &a, const Pt &b) {
14     return a.x == b.x && a.y == b.y;
15 }
16 Pt operator + (const Pt &a, const Pt &b) {
17     return {a.x + b.x, a.y + b.y};
18 }
19 Pt operator - (const Pt &a, const Pt &b) {
20     return {a.x - b.x, a.y - b.y};
21 }
22 // multiply constant
23 Pt operator * (const Pt &a, const dvt c) {
24     return {a.x * c, a.y * c};
25 }
26 Pt operator / (const Pt &a, const dvt c) {
27     return {a.x / c, a.y / c};
28 }
29 // |a| x |b| x cos(x)
30 dvt iproduct(const Pt &a, const Pt &b) {
31     return a.x * b.x + a.y * b.y;
32 }
33 // |a| x |b| x sin(x)
34 dvt cross(const Pt &a, const Pt &b) {
35     return a.x * b.y - a.y * b.x;
36 }
37 dvt dis_pp(const Pt &a, const Pt &b) {
38     dvt dx = a.x - b.x;
39     dvt dy = a.y - b.y;
40     return sqrt(dx * dx + dy * dy);
41 }

```

## 9.2 Line

$$d(P, L) = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$$

```

1 struct Line {
2     Pt st;
3     Pt ed;
4 };
5 // return point side
6 // left, on line, right -> 1, 0, -1
7 int side(Line l, Pt a) {
8     dvt cross_val = cross(a - l.st, l.ed - l.st);
9     if (cross_val > EPS) {
10         return 1;
11     } else if (cross_val < -EPS) {
12         return -1;
13     } else {
14         return 0;
15     }
16 }
17 // AB infinity, CD segment
18 bool has_intersection(Line AB, Line CD) {
19     int c = side(AB, CD.st);
20     int d = side(AB, CD.ed);
21     if (c == 0 || d == 0) {
22         return true;
23     } else {
24         // different side
25         return c == -d;
26     }
27 }
28 // find intersection point, two line, not seg
29 pair<int, Pt> intersection(Line a, Line b) {
30     Pt A = a.ed - a.st;
31     Pt B = b.ed - b.st;
32     Pt C = b.st - a.st;
33     dvt mom = cross(A, B);
34     dvt son = cross(C, B);

```

```

35     if (std::abs(mom) <= EPS) {
36         if (std::abs(son) <= EPS) {
37             return {1, {}}; // same line
38         } else {
39             return {2, {}}; // parallel
40         }
41     } else { // ok
42         return {0, a.st + A * (son / mom)};
43     }
44 }
45 // line to point distance
46 dvt dis_lp(Line l, Pt a) {
47     return area3x2(l.st, l.ed, a) / dis_pp(l.st, l.ed);
48 }

```

## 9.3 Area

```

1 // triangle
2 dvt area3(Pt a, Pt b, Pt c) {
3     return std::abs(cross(b - a, c - a) / 2);
4 }
5 dvt area3x2(Pt a, Pt b, Pt c) { // for integer
6     return std::abs(cross(b - a, c - a));
7 }
8 // simple convex area(can in)
9 dvt area(vector<Pt> &a) {
10     dvt ret = 0;
11     for (int i = 0, sz = a.size(); i < sz; i++) {
12         ret += cross(a[i], a[(i + 1) % sz]);
13     }
14     return std::abs(ret) / 2;
15 }
16 // check point in/out a convex
17 int io_convex(vector<Pt> convex, Pt q) {
18     // convex is Counterclockwise
19     for (int i = 0, sz = convex.size(); i < sz; i++) {
20         Pt cur = convex[i] - q;
21         Pt nex = convex[(i + 1) % sz] - q;
22         dvt cross_val = cross(cur, nex);
23         if (std::abs(cross_val) <= EPS) {
24             return 0; // on edge
25         }
26         if (cross_val < 0) {
27             return -1; // outside
28         }
29     }
30     return 1; // inside
31 }

```

## 9.4 Convex Hull

```

1 vector<Pt> convex_hull(vector<Pt> &a) {
2     sort(a.begin(), a.end());
3     a.erase(unique(a.begin(), a.end()), a.end());
4     int sz = a.size(), m = 0;
5     vector<Pt> ret(sz + 5); // safe 1 up
6     for (int i = 0; i < sz; i++) {
7         while (m > 1 &&
8             cross(ret[m - 1] - ret[m - 2], a[i] - ret[m - 2]) <= EPS) {
9             m--;
10        }
11        ret[m++] = a[i];
12    }
13    int k = m;
14    for (int i = sz - 2; i >= 0; i--) {
15        while (m > k &&
16            cross(ret[m - 1] - ret[m - 2], a[i] - ret[m - 2]) <= EPS) {
17            m--;
18        }
19        ret[m++] = a[i];
20    }

```

---

```
21 |   if (sz > 1) {  
22 |       m--;  
23 |   }  
24 |   ret.resize(m);  
25 |   return ret;  
26 | }
```