12 13

14

15

16 }

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# 1 Basic

### 1.1 Run

```
1 | #use -> sh run.sh {name}
2 | g++ -02 -std=c++14 -Wall -Wextra -Wshadow -o $1 $1.cpp
3 | ./$1 < t.in > t.out
```

## 1.2 Binary Search

# 1.3 Ternary Search

Data Structure

# 2.1 BIT RARSQ

} else {
 L = ml;

} }

return L;

```
1 // 1-base
2
  #define lowbit(k) (k & -k)
  int n:
3
  vector<int> B1, B2;
  void add(vector<int> &tr, int id, int val) {
     for (; id <= n; id += lowbit(id)) {</pre>
      tr[id] += val;
8
    }
9 }
  void range_add(int 1, int r, int val) {
10
     add(B1, 1, val);
add(B1, r + 1, -val);
11
12
     add(B2, 1, val * (1 - 1));
13
14
     add(B2, r + 1, -val * r);
15 }
16
  int sum(vector<int> &tr, int id) {
17
    int ret = 0;
     for (; id >= 1; id -= lowbit(id)) {
18
19
      ret += tr[id];
20
21
     return ret;
22 }
23
  int prefix_sum(int id) {
    return sum(B1, id) * id - sum(B2, id);
24
25 }
26
  int range_sum(int 1, int r) {
    return prefix_sum(r) - prefix_sum(l - 1);
27
28 }
```

# 2.2 zkw RMQ

```
1 // 0-base
  const int INF = 1e9;
  const int MAXN = ;
  int n;
  int a[MAXN], tr[MAXN << 1];</pre>
  // !!! remember to call this function
6
  void build() {
    for (int i = 0; i < n; i++) {
      tr[i + n] = a[i];
9
10
    for (int i = n - 1; i > 0; i--) {
11
12
       tr[i] = max(tr[i << 1], tr[i << 1 | 1]);
    }
13
14 }
15
  void update(int id, int val) {
    for (tr[id += n] = val; id > 1; id >>= 1) {
16
17
      tr[id >> 1] = max(tr[id], tr[id ^ 1]);
    }
18
  }
19
20
  int query(int 1, int r) { // [1, r)
    int ret = -INF;
21
     for (1 += n, r += n; 1 < r; 1 >>= 1, r >>= 1) {
23
      if (1 & 1) {
24
        ret = max(ret, tr[1++]);
25
26
      if (r & 1) {
27
         ret = max(ret, tr[--r]);
28
29
30
     return ret;
```

### 2.3 Segment Tree RARMQ

```
1 struct Node {
    int val, tag;
     Node *lc, *rc;
     Node() : lc(nullptr), rc(nullptr), tag(0) {}
5
     void pull() {
       if (!lc) {
         val = rc->val;
       } else if (!rc) {
         val = lc->val;
       } else {
10
11
         val = max(lc->val, rc->val);
       }
12
     }
13
     void push() {
14
15
       if (lc) {
16
         lc->tag += tag;
         lc->val += tag;
17
18
       if (rc) {
19
20
         rc->tag += tag;
         rc->val += tag;
21
22
       }
       tag = 0;
23
24
     }
25 };
26
  struct SegmentTree {
     Node *root;
27
     SegmentTree() : root(nullptr) {}
28
     void build(Node* &T, int 1, int r, const
29
         vector<int> &o) {
       T = new Node();
30
31
       if (1 == r) {
32
         T->val = o[1];
33
         return;
34
       int mid = (1 + r) / 2;
35
       build(T->lc, 1, mid, o);
36
       build(T->rc, mid + 1, r, o);
37
       T->pull();
38
39
     void update(Node* &T, int 1, int r, int q1, int qr,
40
         int v) {
41
       if (ql <= 1 && r <= qr) {</pre>
42
         T->val += v;
43
         T->tag += v;
         return:
44
45
       }
       T->push();
46
47
       int mid = (1 + r) / 2;
       if (qr <= mid) {
48
         update(T->lc, 1, mid, ql, qr, v);
49
       } else if (mid < ql) {</pre>
50
51
         update(T->rc, mid + 1, r, ql, qr, v);
52
         update(T->lc, 1, mid, ql, mid, v);
53
         update(T->rc, mid + 1, r, mid + 1, qr, v);
54
55
56
       T->pull();
57
     }
     int query(Node* &T, int 1, int r, int q1, int qr) {
58
59
       if (ql <= 1 && r <= qr) {</pre>
60
         return T->val;
61
       T->push();
62
       int mid = (1 + r) / 2;
63
64
       if (qr <= mid) {
         return query(T->lc, 1, mid, ql, qr);
65
66
       } else if (mid < ql) {</pre>
67
         return query(T->rc, mid + 1, r, ql, qr);
       } else {
68
         return max(query(T->lc, l, mid, ql, mid),
69
70
             query(T->rc, mid + 1, r, mid + 1, qr));
71
72
     }
```

73 };

### 2.4 Treap

```
1 struct Treap {
    int val, pri, sz;
     Treap *lc, *rc;
     Treap() {}
    Treap(int _val) {
       val = _val;
       pri = rand();
 7
       sz = 1;
 8
       1c = rc = NULL;
9
   }
10
11 };
12 int getSize(Treap *a) { return (a == NULL ? 0 :
       a->sz); }
  void split(Treap *t, Treap *&a, Treap *&b, int k) {
13
    if (t == NULL) {
15
       a = b = NULL;
16
       return;
17
     if (getSize(t->lc) < k) {</pre>
18
19
       a = t:
       split(t->rc, a->rc, b, k - getSize(t->lc) - 1);
20
21
     } else {
       b = t;
22
       split(t->lc, a, b->lc, k);
23
    }
24
25 }
26
  Treap *merge(Treap *a, Treap *b) {
    if (!a || !b) {
27
28
      return (a ? a : b);
    }
29
30
    if (a->pri > b->pri) {
31
       a->rc = merge(a->rc, b);
32
       return a;
33
    } else {
34
       b \rightarrow lc = merge(a, b \rightarrow lc);
35
       return b:
36
    }
37 }
38 void Insert(Treap *&t, int x, int p) {
39
    Treap *a, *b;
40
    split(t, a, b, x);
41
     t = merge(a, merge(new Treap(p), b));
42 }
43 void Delete(Treap *&t, int x) {
44
    Treap *a, *b, *c;
    split(t, b, c, x);
45
46
     split(b, a, b, x - 1);
47
     t = merge(a, c);
48 }
49
50
51 Usage
52 Treap *root = NULL; // declare
53 root = merge(root, new Treap(val)); // push back
54 Insert(root, x, y); // insert y after x-th element
55
  Delete(root, x); // delete x-th element
```

# 3 Graph

# 3.1 Directed MST

```
1 // 0-base
2 const LL INF = 1e18;
3 const int MAXN = ;
4 
5 struct Edge {
6   int from;
7   int to;
8   LL cost;
9   Edge(int u, int v, LL c) : from(u), to(v), cost(c)
{}
```

```
10 };
11
12 struct DMST {
13
     int n;
14
     int vis[MAXN], pre[MAXN], id[MAXN];
15
     LL in[MAXN];
     vector < Edge > edges;
16
17
     void init(int _n) {
       n = _n;
18
19
       edges.clear();
20
21
     void add_edge(int from, int to, LL cost) {
22
       edges.eb(from, to, cost);
23
24
     LL run(int root) {
       LL ret = 0;
25
       while (true) {
26
         for (int i = 0; i < n; i++) {</pre>
27
           in[i] = INF;
28
29
30
31
          // find in edge
32
         for (auto &e : edges) {
           if (e.cost < in[e.to] && e.from != e.to) {</pre>
33
              pre[e.to] = e.from;
34
              in[e.to] = e.cost;
35
           }
36
         }
37
38
39
          // check in edge
          for (int i = 0; i < n; i++) {</pre>
40
41
            if (i == root) {
42
              continue;
43
           if (in[i] == INF) {
44
45
              return -1;
46
           }
         }
47
48
49
         int nodenum = 0;
50
         memset(id, -1, sizeof(id));
51
         memset(vis, -1, sizeof(vis));
         in[root] = 0;
52
53
          // find cycles
54
          for (int i = 0; i < n; i++) {
55
56
            ret += in[i];
57
            int v = i;
58
            while (vis[v] != i && id[v] == -1 && v !=
                root) {
59
              vis[v] = i;
60
              v = pre[v];
61
            if (id[v] == -1 && v != root) {
62
              for (int j = pre[v]; j != v; j = pre[j]) {
63
                id[j] = nodenum;
64
              }
65
66
              id[v] = nodenum++;
67
           }
68
69
          // no cycle
70
         if (nodenum == 0) {
71
72
           break;
73
74
75
         for (int i = 0; i < n; i++) {
            if (id[i] == -1) {
76
77
              id[i] = nodenum++;
           }
78
79
         }
80
81
          // grouping the vertices
         for (auto &e : edges) {
82
           int to = e.to;
83
84
            e.from = id[e.from];
            e.to = id[e.to];
85
```

```
86
            if (e.from != e.to) {
              e.cost -= in[to]; //!!!
87
88
            }
         }
89
90
91
         n = nodenum;
92
         root = id[root];
93
94
       return ret;
95
     }
96 };
```

#### 3.2 LCA

```
1 const int LOG = 20;
  vector<int> tin(MAXN), tout(MAXN), depth(MAXN);
2
 3 int par[MAXN][LOG];
 4 int timer = 0;
  vector<int> G[MAXN];
 5
  void dfs(int u, int f) {
     tin[u] = ++timer;
     par[u][0] = f;
 9
     for (int v : G[u]) {
       if (v != f) {
10
11
         depth[v] = depth[u] + 1;
12
         dfs(v, u);
13
14
    }
15
     tout[u] = ++timer;
16 }
  void Doubling(int n) {
17
     for (int j = 1; j < LOG; ++j) {</pre>
       for (int i = 1; i <= n; ++i) {
19
20
         par[i][j] = par[par[i][j - 1]][j - 1];
21
    }
22
23 }
24 bool anc(int u, int v) { return tin[u] <= tin[v] &&
       tout[v] <= tout[u]; }</pre>
25
  int LCA(int u, int v) {
    if (depth[u] > depth[v]) {
26
27
       swap(u, v);
28
29
     if (anc(u, v)) {
30
       return u;
31
32
     for (int j = LOG - 1; j >= 0; --j) {
33
      if (!anc(par[u][j], v)) u = par[u][j];
34
35
     return par[u][0];
36 }
37 int dis(int u, int v) {
    int lca = LCA(u, v);
38
39
     return depth[u] + depth[v] - 2 * depth[lca];
40 }
41
42
  /*
43
  dfs(root, root);
44
  Doubling(n);
45 */
```

### 3.3 Euler Circuit

七橋問題根據起點與終點是否相同,分成 Euler path(不同)及 Euler circuit(相同)。

- 判斷法
- · 無向圖部分,將點分成奇點(度數為奇數)和偶點(度數為偶數)。
  - Euler path:奇點數為 0 或 2
  - Euler circuit:沒有奇點
- · 有向圖部分,將點分成出點(出度 入度 = 1)和入點(入度 出度 = 1)還 有平衡點(出度 = 入度)。

76

dfs((pin ? st : root));

```
- Euler path:出點和入點個數同時為 0 或 1。
          - Euler circuit:只有平衡點。
     · 求出一組解
     • 用 DFS 遍歷整張圖,設 S 為離開的順序,無向圖的答案為 S ,有向圖的答案
       為反向的 S 。
     · DFS 起點選定:
          - Euler path:無向圖選擇任意一個奇點,有向圖選擇出點。
          - Euler circuit:任意一點。
1 // Code from Eric
2 #define 11 long long
3 #define PB push_back
4 #define EB emplace_back
5 #define PII pair<int, int>
6 #define MP make_pair
  #define all(x) x.begin(), x.end()
8 #define maxn 50000+5
9 //structure
10 struct Eular {
    vector<PII> adj[maxn];
11
12
    vector<bool> edges;
     vector<PII> path;
13
    int chk[maxn];
14
15
     int n;
    void init(int _n) {
16
17
       n = _n;
       for (int i = 0; i <= n; i++) adj[i].clear();</pre>
18
       edges.clear();
19
20
       path.clear();
       memset(chk, 0, sizeof(chk));
21
22
     void dfs(int v) {
23
       for (auto i : adj[v]) {
24
25
         if (edges[i.first] == true) {
           edges[i.first] = false;
26
           dfs(i.second);
27
           path.EB(MP(i.second, v));
28
29
30
      }
31
     }
32
     void add_Edge(int from, int to) {
33
       edges.PB(true);
       // for bi-directed graph
34
35
       adj[from].PB(MP(edges.size() - 1, to));
       adj[to].PB(MP(edges.size() - 1, from));
36
37
       chk[from]++;
38
       chk[to]++:
39
       // for directed graph
       // adj[from].PB(MP(edges.size()-1, to));
40
41
       // check[from]++;
42
43
     bool eular_path() {
44
       int st = -1;
45
       for (int i = 1; i <= n; i++) {
         if (chk[i] % 2 == 1) {
46
           st = i;
47
48
           break;
49
        }
       }
50
       if (st == -1) {
51
52
         return false;
53
54
       dfs(st);
55
       return true;
56
     void print_path(void) {
57
58
       for (auto i : path) {
         printf("%d %d\n", i.first, i.second);
59
60
61
62 };
```

```
1 // Code from allen(lexicographic order)
 2 #include <bits/stdc++.h>
  using namespace std;
  const int ALP = 30;
  const int MXN = 1005;
  int n;
  int din[ALP], dout[ALP];
 8 int par[ALP];
  vector<string> vs[MXN], ans;
  bitset<MXN> vis, used[ALP];
11
  void djsInit() {
    for (int i = 0; i != ALP; ++i) {
12
13
       par[i] = i;
    }
14
15
  }
16 int Find(int x) { return (x == par[x] ? (x) : (par[x]
       = Find(par[x])); }
17
  void init() {
    djsInit();
18
19
     memset(din, 0, sizeof(din));
     memset(dout, 0, sizeof(dout));
20
     vis.reset();
     for (int i = 0; i != ALP; ++i) {
22
23
       vs[i].clear();
24
       used[i].reset();
    }
25
26
    return;
27 }
28
  void dfs(int u) {
     for (int i = 0; i != (int)vs[u].size(); ++i) {
29
       if (used[u][i]) {
30
31
         continue:
       }
32
33
       used[u][i] = 1;
34
       string s = vs[u][i];
35
       int v = s[s.size() - 1] - 'a';
36
       dfs(v);
37
       ans.push_back(s);
    }
38
39 }
40 bool solve() {
41
     int cnt = 1;
     for (int i = 0; i != n; ++i) {
42
43
       string s:
44
       cin >> s;
45
       int from = s[0] - 'a', to = s.back() - 'a';
46
       ++din[to];
47
       ++dout[from];
48
       vs[from].push_back(s);
49
       vis[from] = vis[to] = true;
50
       if ((from = Find(from)) != (to = Find(to))) {
51
         par[from] = to;
52
         ++cnt;
53
       }
    }
54
55
     if ((int)vis.count() != cnt) {
56
      return false;
57
58
     int root, st, pin = 0, pout = 0;
     for (int i = ALP - 1; i >= 0; --i) {
59
60
       sort(vs[i].begin(), vs[i].end());
       if (vs[i].size()) root = i;
61
62
       int d = dout[i] - din[i];
       if (d == 1) {
63
64
         ++pout;
65
         st = i;
       } else if (d == -1) {
66
67
         ++pin;
       } else if (d != 0) {
68
69
         return false;
70
       }
71
72
     if (pin != pout || pin > 1) {
73
       return false;
74
75
     ans.clear();
```

```
77
     return true;
78 }
79 int main() {
80
    int t;
81
     cin >> t;
     while (t--) {
82
       cin >> n;
83
84
       init();
       if (!solve()) {
85
         cout << "***\n";
86
87
          continue:
88
       for (int i = ans.size() - 1; i >= 0; --i) {
89
         cout << ans[i] << ".\n"[i == 0];</pre>
90
91
92
93 }
```

# 4 Connectivity

### 4.1 Articulation Point

```
1 // from aizu
  typedef long long int 11;
3 typedef unsigned long long int ull;
4 #define BIG_SIZE 2000000000
5 #define MOD 1000000007
6 #define EPS 0.000000001
  using namespace std;
8 #define SIZE 100000
9 vector<int> G[SIZE];
10 int N;
11 bool visited[SIZE];
12 int visited_order[SIZE], parent[SIZE], lowest[SIZE],
      number:
13 void dfs(int cur, int pre_node) {
14
    visited_order[cur] = lowest[cur] = number;
15
    number++:
16
    visited[cur] = true;
17
    int next:
     for (int i = 0; i < G[cur].size(); i++) {</pre>
18
19
      next = G[cur][i];
20
       if (!visited[next]) {
21
         parent[next] = cur;
22
         dfs(next. cur):
23
         lowest[cur] = min(lowest[cur], lowest[next]);
24
       } else if (visited[next] == true && next !=
           pre_node) {
25
         lowest[cur] = min(lowest[cur],
             visited_order[next]);
26
27
    }
28 }
29
  void art_points() {
    for (int i = 0; i < N; i++) visited[i] = false;</pre>
30
31
    number = 1;
32
    dfs(0, -1);
     int tmp_parent, root_num = 0;
33
34
    vector<int> V;
35
     for (int i = 1; i < N; i++) {
36
       tmp_parent = parent[i];
37
       if (tmp_parent == 0) {
38
         root_num++;
39
       } else if (visited_order[tmp_parent] <=</pre>
           lowest[i]) {
40
         V.push_back(tmp_parent);
41
      }
42
    if (root_num >= 2) {
43
44
      V.push_back(0);
45
46
    sort(V.begin(), V.end());
    V.erase(unique(V.begin(), V.end());
47
    for (int i = 0; i < V.size(); i++) {</pre>
```

```
49
       printf("%d\n", V[i]);
    }
50
51 }
52 int main() {
53
    int E;
     scanf("%d %d", &N, &E);
    int from, to;
55
    for (int i = 0; i < E; i++) {
       scanf("%d %d", &from, &to);
57
58
       G[from].push_back(to);
59
       G[to].push_back(from);
    }
60
61
    art_points();
62 }
```

# 4.2 Bridges

```
1 // from aizu
2 typedef long long int 11;
  typedef unsigned long long int ull;
  #define BIG_NUM 2000000000
  #define MOD 1000000007
6 #define EPS 0.000000001
7
  using namespace std;
8
  struct Edge {
    bool operator<(const struct Edge &arg) const {</pre>
      if (s != arg.s) {
10
         return s < arg.s;</pre>
11
12
      } else {
13
         return t < arg.t;</pre>
14
    }
15
16
    int s, t;
17 };
18 struct Info {
19
    Info(int arg_to, int arg_edge_id) {
20
      to = arg_to;
       edge_id = arg_edge_id;
    }
22
23
    int to, edge_id;
24
25
  int V, E, number;
  int order[100000], lowlink[100000];
27
  bool visited[100000];
28
  Edge edge[100000];
29
  vector < Info > G[100000];
  void recursive(int cur) {
30
    order[cur] = number++;
    lowlink[cur] = order[cur];
32
33
     int next;
     for (int i = 0; i < G[cur].size(); i++) {</pre>
34
      next = G[cur][i].to;
35
36
       if (order[next] == -1) {
37
         visited[G[cur][i].edge_id] = true;
38
         recursive(next);
39
         lowlink[cur] = min(lowlink[cur], lowlink[next]);
       } else if (visited[G[cur][i].edge_id] == false) {
40
41
         lowlink[cur] = min(lowlink[cur], order[next]);
42
43
    }
44 }
45
  int main() {
46
    scanf("%d %d", &V, &E);
47
     for (int i = 0; i < E; i++) {</pre>
48
       scanf("%d %d", &edge[i].s, &edge[i].t);
      if (edge[i].s > edge[i].t) {
49
         swap(edge[i].s, edge[i].t);
      }
51
52
      G[edge[i].s].push_back(Info(edge[i].t, i));
53
      G[edge[i].t].push_back(Info(edge[i].s, i));
54
55
     sort(edge, edge + E);
56
     number = 0;
57
     for (int i = 0; i < V; i++) {
58
      order[i] = -1;
      lowlink[i] = -1;
```

```
60
     for (int i = 0; i < E; i++) {</pre>
61
       visited[i] = false;
62
     }
63
64
     recursive(0);
65
     int from, to;
     for (int i = 0; i < E; i++) {</pre>
66
67
       from = edge[i].s;
       to = edge[i].t;
68
       if (order[edge[i].s] > order[edge[i].t]) {
69
70
          swap(from, to);
71
72
       if (order[from] < lowlink[to]) {</pre>
          printf("%d %d\n", edge[i].s, edge[i].t);
73
74
     }
75
76
     return 0;
```

# 5 Flow & Matching

### 5.1 Relation

```
1 | 1. 一般圖
2 | |最大匹配| + |最小邊覆蓋| = |V|
3 | |最大匹配| + |最小點覆蓋| = |V|
4 | |最大圖| = |補圖的最大獨立集|
5 | 2. 二分圖
6 | |最大匹配| = |最小點覆蓋|
7 | |最大獨立集| = |最小邊覆蓋|
8 | |最大獨立集| = |V| - |最大匹配|
9 | |最大圖| = |補圖的最大獨立集|
```

### 5.2 Bipartite Matching

```
1 // 0-base
2 const int MAXN = ;
3 int n;
4 vector<int> G[MAXN];
5 int vy[MAXN], my[MAXN];
6 bool match(int u) {
     for (int v : G[u]) {
       if (vy[v]) {
8
9
         continue;
10
11
       vy[v] = true;
       if (my[v] == -1 || match(my[v])) {
12
13
         my[v] = u;
14
         return true;
15
       }
     }
16
17
     return false;
18 }
19 int sol() {
    int cnt = 0;
21
     memset(my, -1, sizeof(my));
     for (int i = 0; i < n; i++) {</pre>
22
23
       memset(vy, 0, sizeof(vy));
       if (match(i)) {
24
25
         cnt++;
26
       }
27
     }
28
     return cnt;
29 }
```

### 5.3 KM

```
1 const int INF = 1e9;
2 const int MAXN = ;
```

```
struct KM { //1-base
     int n, G[MAXN][MAXN];
     int lx[MAXN], ly[MAXN], my[MAXN];
     bool vx[MAXN], vy[MAXN];
 7
     void init(int _n) {
       n = _n;
 8
       for (int i = 1; i <= n; i++) {</pre>
10
         for (int j = 1; j <= n; j++) {</pre>
            G[i][j] = 0;
11
12
13
       }
14
15
     bool match(int i) {
       vx[i] = true;
16
17
       for (int j = 1; j <= n; j++) {
         if (lx[i] + ly[j] == G[i][j] && !vy[j]) {
18
19
            vy[j] = true;
20
            if (!my[j] || match(my[j])) {
21
              mv[j] = i;
22
              return true;
            }
23
24
         }
       }
25
26
       return false;
27
     void update() {
28
29
       int delta = INF;
30
       for (int i = 1; i <= n; i++) {
31
         if (vx[i]) {
32
            for (int j = 1; j <= n; j++) {
              if (!vy[j]) {
33
                delta = min(delta, lx[i] + ly[j] -
                     G[i][j]);
35
           }
36
37
         }
38
       for (int i = 1; i <= n; i++) {
39
40
         if (vx[i]) {
41
            lx[i] -= delta;
42
43
         if (vy[i]) {
            ly[i] += delta;
44
45
       }
46
47
     }
48
     int run() {
49
       for (int i = 1; i <= n; i++) {</pre>
50
         lx[i] = ly[i] = my[i] = 0;
         for (int j = 1; j <= n; j++) {</pre>
51
52
            lx[i] = max(lx[i], G[i][j]);
         }
53
54
55
       for (int i = 1; i <= n; i++) {</pre>
         while (true) {
56
            for (int i = 1; i <= n; i++) {
57
             vx[i] = vy[i] = 0;
58
59
60
            if (match(i)) {
61
              break:
            } else {
62
63
              update();
64
         }
65
66
67
       int ans = 0;
       for (int i = 1; i <= n; i++) {</pre>
68
69
         ans += lx[i] + ly[i];
70
71
       return ans;
72
     }
73 };
```

# 5.4 Dinic

1 #define eb emplace\_back

```
2 const LL INF = 1e18;
3 const int MAXN = ;
4 struct Edge {
    int to;
6
    LL cap;
7
    int rev;
    Edge(int v, LL c, int r) : to(v), cap(c), rev(r) {}
8
9 };
10 struct Dinic {
11
    int n;
12
    int level[MAXN], now[MAXN];
     vector < Edge > G[MAXN];
13
     void init(int _n) {
14
       n = n;
15
16
       for (int i = 0; i <= n; i++) {</pre>
17
         G[i].clear();
       }
18
19
    }
     void add_edge(int u, int v, LL c) {
20
21
       G[u].eb(v, c, G[v].size());
       // directed graph
22
23
       G[v].eb(u, 0, G[u].size() - 1);
24
       // undirected graph
25
       // G[v].eb(u, c, G[u].size() - 1);
26
27
     bool bfs(int st, int ed) {
       fill(level, level + n + 1, -1);
28
29
       queue<int> q;
       q.push(st);
30
31
       level[st] = 0;
       while (!q.empty()) {
32
33
         int u = q.front();
34
         q.pop();
35
         for (const auto &e : G[u]) {
           if (e.cap > 0 && level[e.to] == -1) {
36
37
             level[e.to] = level[u] + 1;
             q.push(e.to);
38
           }
39
         }
40
41
       }
42
       return level[ed] != -1;
43
     LL dfs(int u, int ed, LL limit) {
44
45
       if (u == ed) {
         return limit;
46
47
48
       LL ret = 0:
       for (int &i = now[u]; i < G[u].size(); i++) {</pre>
49
50
         auto &e = G[u][i];
         if (e.cap > 0 && level[e.to] == level[u] + 1) {
51
52
           LL f = dfs(e.to, ed, min(limit, e.cap));
53
           ret += f;
54
           limit -= f;
55
           e.cap -= f;
           G[e.to][e.rev].cap += f;
56
57
           if (!limit) {
58
             return ret;
59
60
        }
61
       if (!ret) {
62
         level[u] = -1;
63
64
65
       return ret;
66
67
    LL flow(int st, int ed) {
       LL ret = 0;
68
69
       while (bfs(st, ed)) {
         fill(now, now + n + 1, 0);
70
         ret += dfs(st, ed, INF);
71
72
       }
73
       return ret;
    }
74
75 };
```

#### 5.5 MCMF

```
1 // 0-base
  const LL INF = 1e18;
2
3 const int MAXN = ;
4 struct Edge {
    int u, v;
    LL cost;
    LL cap:
7
    Edge(int _u, int _v, LL _c, LL _cap) : u(_u),
         v(_v), cost(_c), cap(_cap) {}
9 };
10
  struct MCMF {
                     // inq times
    int n, pre[MAXN], cnt[MAXN];
11
    LL ans_flow, ans_cost, dis[MAXN];
12
13
    bool inq[MAXN];
14
    vector<int> G[MAXN];
15
    vector<Edge> edges;
16
     void init(int _n) {
17
       n = _n;
       edges.clear();
18
       for (int i = 0; i < n; i++) {</pre>
19
20
         G[i].clear();
21
22
23
     void add_edge(int u, int v, LL c, LL cap) {
24
25
       G[u].pb(edges.size());
26
       edges.eb(u, v, c, cap);
27
       G[v].pb(edges.size());
       edges.eb(v, u, -c, 0);
28
29
    bool SPFA(int st, int ed) {
30
31
       for (int i = 0; i < n; i++) {
32
         pre[i] = -1;
33
         dis[i] = INF;
         cnt[i] = 0;
34
35
         inq[i] = false;
       }
36
37
       queue<int> q;
38
       bool negcycle = false;
39
       dis[st] = 0;
       cnt[st] = 1;
40
       inq[st] = true;
41
42
       q.push(st);
43
       while (!q.empty() && !negcycle) {
44
         int u = q.front();
         q.pop();
45
46
         inq[u] = false;
         for (int i : G[u]) {
47
48
           int v = edges[i].v;
49
           LL cost = edges[i].cost;
           LL cap = edges[i].cap;
50
51
           if (dis[v] > dis[u] + cost && cap > 0) {
52
             dis[v] = dis[u] + cost;
53
             pre[v] = i;
             if (!inq[v]) {
54
55
               q.push(v);
56
               cnt[v]++;
57
               inq[v] = true;
58
               if (cnt[v] == n + 2) {
                 negcycle = true;
59
60
                 break;
61
               }
62
             }
63
         }
64
65
       }
       return dis[ed] != INF;
66
67
68
     LL sendFlow(int v, LL curFlow) {
       if (pre[v] == -1) {
69
70
         return curFlow;
71
       }
       int i = pre[v];
72
73
       int u = edges[i].u;
       LL cost = edges[i].cost;
```

```
75
       LL f = sendFlow(u, min(curFlow, edges[i].cap));
       ans_cost += f * cost;
76
77
       edges[i].cap -= f;
       edges[i ^ 1].cap += f;
78
79
       return f;
80
    pair<LL, LL> run(int st, int ed) {
81
82
       ans_flow = ans_cost = 0;
       while (SPFA(st, ed)) {
83
84
         ans_flow += sendFlow(ed, INF);
85
       return make_pair(ans_flow, ans_cost);
86
87
    }
88 };
```

# 6 String

#### 6.1 Manacher

```
1 int p[2 * MAXN];
2 int Manacher(const string &s) {
    string st = "@#";
    for (char c : s) {
       st += c;
       st += '#';
6
7
    }
    st += '$';
8
    int id = 0, mx = 0, ans = 0;
    for (int i = 1; i < st.length() - 1; i++) {</pre>
11
       p[i] = (mx > i ? min(p[2 * id - i], mx - i) : 1);
12
       for (; st[i - p[i]] == st[i + p[i]]; p[i]++);
13
       if (mx < i + p[i]) {</pre>
         mx = i + p[i];
14
15
         id = i;
       }
16
17
       ans = max(ans, p[i] - 1);
18
    }
19
    return ans;
20 }
```

### 6.2 Trie

```
1 const int MAXL = ;
2 const int MAXC = ;
3 struct Trie {
    int nex[MAXL][MAXC];
    int len[MAXL];
     int sz;
6
7
     void init() {
       memset(nex, 0, sizeof(nex));
8
9
       memset(len, 0, sizeof(len));
10
       sz = 0;
11
12
     void insert(const string &str) {
13
       int p = 0;
       for (char c : str) {
14
         int id = c - 'a'
15
         if (!nex[p][id]) {
16
17
           nex[p][id] = ++sz;
18
19
         p = nex[p][id];
20
21
       len[p] = str.length();
     }
22
23
     vector<int> find(const string &str, int i) {
24
       int p = 0;
       vector<int> ans;
25
       for (; i < str.length(); i++) {</pre>
26
         int id = str[i] - 'a';
27
28
         if (!nex[p][id]) {
29
           return ans;
30
```

### 7 DP

### 7.1 Huge Knapsack

```
1 // from aizu
  #include <bits/stdc++.h>
  typedef long long int 11;
  typedef unsigned long long int ull;
  #define BIG_NUM 200000000
  #define MOD 1000000007
8 #define EPS 0.000000001
  using namespace std;
10 #define SIZE 25
11
  struct Info {
    Info() { value = 0, weight = 0; }
12
    Info(ll arg_value, ll arg_weight) {
13
14
       value = arg_value;
      weight = arg_weight;
15
16
    bool operator<(const struct Info &arg) const {</pre>
17
         return weight < arg.weight; }</pre>
18
    ll value, weight;
19
20
  };
21 11 N, W;
22 11 POW[SIZE];
23 | 11 table_B[1 << 21];
  Info info[45];
24
25
  int main() {
    POW[0] = 1;
26
     for (int i = 1; i < SIZE; i++) {</pre>
27
28
      POW[i] = POW[i - 1] * 2;
29
30
     scanf("%11d %11d", &N, &W);
     for (int i = 0; i < N; i++) {</pre>
31
32
       scanf("%11d %11d", &info[i].value,
           &info[i].weight);
33
    if (N == 1) {
34
      if (info[0].weight <= W) {</pre>
35
36
         printf("%11d\n", info[0].value);
37
      } else {
38
         printf("0 \ n");
39
40
      return 0;
    }
41
42
     vector<int> A, B;
43
     for (int i = 0; i < N / 2; i++) {
44
      A.push_back(i);
45
46
    for (int i = N / 2; i < N; i++) {
      B.push_back(i);
47
48
     vector<Info> vec_A, vec_B;
49
     for (int state = 0; state < POW[A.size()]; state++)</pre>
         {
51
       11 \text{ sum\_w} = 0;
52
       11 sum_value = 0;
       for (int loop = 0; loop < A.size(); loop++) {</pre>
53
         if (state & POW[loop]) {
54
55
           sum_w += info[A[loop]].weight;
56
           sum_value += info[A[loop]].value;
57
      }
58
```

```
59
       vec_A.push_back(Info(sum_value, sum_w));
60
     sort(vec_A.begin(), vec_A.end());
61
     for (int state = 0; state < POW[B.size()]; state++)</pre>
62
63
       11 \text{ sum\_w} = 0;
       11 sum_value = 0;
64
65
       for (int loop = 0; loop < B.size(); loop++) {</pre>
         if (state & POW[loop]) {
66
           sum_w += info[B[loop]].weight;
67
68
            sum_value += info[B[loop]].value;
         }
69
70
       }
       vec_B.push_back(Info(sum_value, sum_w));
71
72
     sort(vec_B.begin(), vec_B.end());
73
     table_B[0] = vec_B[0].value;
74
75
     for (int i = 1; i < vec_B.size(); i++) {</pre>
       //ある重さ以下の最大価値を求める
76
       table_B[i] = max(table_B[i - 1], vec_B[i].value);
77
78
     int tail = vec_B.size() - 1;
79
80
     11 \text{ ans} = 0;
     for (int i = 0; i < vec_A.size(); i++) {</pre>
81
       while (tail >= 0 && vec_A[i].weight +
82
            vec_B[tail].weight > W) tail--;
       if (tail < 0) break;</pre>
83
84
       ans = max(ans, vec_A[i].value + table_B[tail]);
85
86
87
     printf("%11d\n", ans);
88
     return 0;
89 }
```

# 8 Math

#### 8.1 Number Theory

```
· Inversion:
   aa^{-1} \equiv 1 \pmod{m}. a^{-1} exists iff gcd(a, m) = 1.
· Linear inversion:
   a^{-1} \equiv (m - \lfloor \frac{m}{a} \rfloor) \times (m \mod a)^{-1} \pmod m
• Fermat's little theorem:
   a^p \equiv a \pmod{p} if p is prime.
\cdot Euler function:
   \phi(n) = n \prod_{p \mid n} \frac{p-1}{p}
· Euler theorem:
   a^{\phi(n)} \equiv 1 \pmod{n} if \gcd(a, n) = 1.
• Extended Euclidean algorithm:
   ax+by=\gcd(a,b)=\gcd(b,a\bmod b)=\gcd(b,a-\lfloor\frac{a}{b}\rfloor b)=bx_1+(a+b)
   \lfloor \frac{a}{b} \rfloor b) y_1 = a y_1 + b (x_1 - \lfloor \frac{a}{b} \rfloor y_1)
· Divisor function:
   \sigma_x(n) = \sum_{d|n} d^x. n = \prod_{i=1}^r p_i^{a_i}.
   \sigma_x(n) = \prod_{i=1}^r \frac{p_i^{(a_i+1)x} - 1}{p_i^x - 1} \text{ if } x \neq 0. \ \sigma_0(n) = \prod_{i=1}^r (a_i + 1).
```

## 8.2 Extended GCD

 $x \equiv a_i \pmod{m_i}$ .

· Chinese remainder theorem:

 $M = \prod_{i=1}^{n} m_{i}$ .  $M_{i} = M/m_{i}$ .  $t_{i} = M_{i}^{-1}$ .  $x = kM + \sum_{i=1}^{n} a_{i}t_{i}M_{i}$ ,  $k \in \mathbb{Z}$ .

```
1  // ax + by = c
2  int extgcd(int a, int b, int c, int &x, int &y) {
3   if (b == 0) {
4     x = c / a;
5     y = 0;
6     return a;
7  }
8  int d = extgcd(b, a % b, c, y, x);
```

```
9 y -= (a / b) * x;
10 return d;
11 }
```

### 8.3 Gaussian Elimination

```
1 const int MAXN = 300;
  const double EPS = 1e-8;
  int n;
  double A[MAXN][MAXN];
  void Gauss() {
    for (int i = 0; i < n; i++) {
       bool ok = 0;
       for (int j = i; j < n; j++) {</pre>
         if (fabs(A[j][i]) > EPS) {
9
10
           swap(A[j], A[i]);
11
           ok = 1;
12
           break;
         }
13
14
       if (!ok) continue;
15
       double fs = A[i][i];
16
17
       for (int j = i + 1; j < n; j++) {
18
         double r = A[j][i] / fs;
         for (int k = i; k < n; k++) {
19
20
           A[j][k] -= A[i][k] * r;
21
22
    }
23
24 }
```

### 8.4 Phi

- · 歐拉函數計算對於一個整數 N,小於等於 N 的正整數中,有幾個和 N 互質
- ・ 如果  $\gcd(p,q)=1,\Phi(p)\cdot\Phi(q)=\Phi(p\cdot q)$
- $\Phi(p^k) = p^{k-1} \times (p-1)$

```
1 void phi_table(int n) {
     phi[1] = 1;
     for (int i = 2; i <= n; i++) {</pre>
       if (phi[i]) {
5
         continue;
 6
7
       for (int j = i; j < n; j += i) {
 8
         if (!phi[j]) {
 9
           phi[j] = j;
10
11
         phi[j] = phi[j] / i * (i - 1);
12
     }
13
14 }
```

## 9 Geometry

#### 9.1 Point

```
14
    return a.x == b.x && a.y == b.y;
15 }
16 Pt operator + (const Pt &a, const Pt &b) {
17
   return {a.x + b.x, a.y + b.y};
18 }
19 Pt operator - (const Pt &a, const Pt &b) {
   return {a.x - b.x, a.y - b.y};
20
21 }
22 // multiply constant
23 Pt operator * (const Pt &a, const dvt c) {
24
   return {a.x * c, a.y * c};
25 }
26 Pt operator / (const Pt &a, const dvt c) {
   return {a.x / c, a.y / c};
27
28 }
29 // |a| \times |b| \times cos(x)
30 dvt iproduct(const Pt &a, const Pt &b) {
31 return a.x * b.x + a.y * b.y;
32 }
33 // |a| \times |b| \times \sin(x)
34 dvt cross(const Pt &a, const Pt &b) {
   return a.x * b.y - a.y * b.x;
35
36 }
37 dvt dis_pp(const Pt &a, const Pt, &b) {
    dvt dx = a.x - b.x;
   dvt dy = a.y - b.y;
39
    return sqrt(dx * dx, dy * dy);
41 | }
```

#### 9.2 Line

```
d(P, L) = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}
```

```
1 struct Line {
    Pt st;
2
    Pt ed;
3
4 };
5 // return point side
6 // left, on line, right -> 1, 0, -1
7 int side(Line 1, Pt a) {
    dvt cross_val = cross(a - 1.st, 1.ed - 1.st);
    if (cross_val > EPS) {
10
      return 1;
11
    } else if (cross_val < -EPS) {</pre>
12
       return -1;
    } else {
13
14
       return 0:
    }
15
16 }
17 // AB infinity, CD segment
18 bool has_intersection(Line AB, Line CD) {
    int c = side(AB, CD.st);
19
    int d = side(AB, CD.ed);
20
21
     if (c == 0 || d == 0) {
22
      return true;
    } else {
23
24
      // different side
       return c == -d;
25
26
    }
27 }
28 // find intersection point, two line, not seg
29 pair<int, Pt> intersection(Line a, Line b) {
    Pt A = a.ed - a.st;
30
31
     Pt B = b.ed - b.st;
    Pt C = b.st - a.st;
32
33
     dvt mom = cross(A, B);
34
     dvt son = cross(C, B);
35
     if (std::abs(mom) <= EPS) {</pre>
      if (std::abs(son) <= EPS) {</pre>
36
        return {1, {}}; // same line
37
      } else {
        return {2, {}}; // parallel
39
40
41
     } else {
                          // ok
       return {0, a.st + A * (son / mom)};
42
```

```
43    }
44 }
45    // line to point distance
46    dvt dis_lp(Line 1, Pt a) {
47        return area3x2(1.st, 1.ed, a) / dis_pp(1.st, 1.ed);
48 }
```

#### 9.3 Area

```
1 // triangle
2 dvt area3(Pt a, Pt b, Pt c) {
  return std::abs(cross(b - a, c - a) / 2);
4 }
5
  dvt area3x2(Pt a, Pt b, Pt c) { // for integer
6
   return std::abs(cross(b - a, c - a));
7 }
8 // simple convex area(can in)
9 dvt area(vector<Pt> &a) {
    dvt ret = 0;
10
    for (int i = 0, sz = a.size(); i < sz; i++) {</pre>
11
12
      ret += cross(a[i], a[(i + 1) % sz]);
    }
13
    return std::abs(ret) / 2;
14
15 }
  // check point in/out a convex
16
int io_convex(vector<Pt> convex, Pt q) {
18
    // convex is Counterclockwise
19
    for (int i = 0, sz = convex.size(); i < sz; i++) {</pre>
20
      Pt cur = convex[i] - q;
      Pt nex = convex[(i + 1) \% sz] - q;
21
       dvt cross_val = cross(cur, nex);
22
23
      if (std::abs(cross_val) <= EPS) {</pre>
        return 0; // on edge
24
25
      if (cross_val < 0) {</pre>
26
27
         return -1; // outside
28
29
    }
30
    return 1;
                   // inside
31 }
```

### 9.4 Convex Hull

```
1 | vector < Pt > convex_hull(vector < Pt > &a) {
    sort(a.begin(), a.end());
     a.erase(unique(a.begin(), a.end()), a.end());
 3
     int sz = a.size(), m = 0;
     vector<Pt> ret(sz + 5); // safe 1 up
     for (int i = 0; i < sz; i++) {</pre>
 7
       while (m > 1 &&
         cross(ret[m - 1] - ret[m - 2], a[i] - ret[m -
 8
             2]) <= EPS) {
9
10
      }
11
       ret[m++] = a[i];
12
     }
13
     int k = m;
     for (int i = sz - 2; i >= 0; i--) {
14
15
       while (m > k \&\&
         cross(ret[m - 1] - ret[m - 2], a[i] - ret[m -
16
            2]) <= EPS) {
17
18
19
       ret[m++] = a[i];
     }
20
21
     if (sz > 1) {
22
23
     ret.resize(m);
24
25
     return ret;
```