

## Contents

1	Basic	1
1.1	Run	1
1.2	Black Magic	1
1.3	Ternary Search	1
2	Data Structure	1
2.1	BIT RARSQ	1
2.2	zkw RMQ	2
2.3	Segment Tree RARMQ	2
2.4	Treap	2
3	Graph	3
3.1	Topological Sort	3
3.2	Tree Diameter	3
3.3	Directed MST	3
3.4	LCA	4
3.5	Euler Circuit	4
4	Connectivity	6
4.1	Kosaraju SCC	6
4.2	BCC	6
4.3	Articulation Point	7
4.4	Bridges	7
5	Flow & Matching	7
5.1	Relation	7
5.2	Bipartite Matching	8
5.3	KM	8
5.4	Dinic	8
5.5	MCMF	9
5.6	Stable Matching	9
5.7	Max General Graph Matching	10

## 1 Basic

### 1.1 Run

```
1 #use -> sh run.sh {name}
2 g++ -O2 -std=c++14 -Wall -Wextra -Wshadow -o $1 $1.cpp
3 ./ $1 < t.in > t.out
```

### 1.2 Black Magic

```
1 #include <bits/stdc++.h>
2 #include <ext/pb_ds/assoc_container.hpp>
3 #include <ext/pb_ds/tree_policy.hpp>
4 #include <ext/pb_ds/priority_queue.hpp>
5 using namespace std;
6 using namespace __gnu_pbds;
7 using set_t =
8     tree<int, null_type, less<int>, rb_tree_tag,
9         tree_order_statistics_node_update>;
10 using map_t =
11     tree<int, int, less<int>, rb_tree_tag,
12         tree_order_statistics_node_update>;
13 using heap_t =
14     __gnu_pbds::priority_queue<int>;
15 using ht_t =
16     gp_hash_table<int, int>;
17 int main() {
18     //set-----
19     set_t st;
20     st.insert(5); st.insert(6);
21     st.insert(3); st.insert(1);
22
23     // the smallest is (0), biggest is (n-1), kth small
24     // is (k-1)
25     int num = *st.find_by_order(0);
26     cout << num << '\n'; // print 1
27
28     num = *st.find_by_order(st.size() - 1);
29     cout << num << '\n'; // print 6
30
31     // find the index
32     int index = st.order_of_key(6);
```

```
32 cout << index << '\n'; // print 3
33
34 // check if there exists x
35 int x = 5;
36 int check = st.erase(x);
37 if (check == 0) printf("st not contain 5\n");
38 else if (check == 1) printf("st contain 5\n");
39
40 //tree policy like set
41 st.insert(5); st.insert(5);
42 cout << st.size() << '\n'; // print 4
43
44 //map-----
45 map_t mp;
46 mp[1] = 2;
47 cout << mp[1] << '\n';
48 auto tmp = *mp.find_by_order(0); // pair
49 cout << tmp.first << " " << tmp.second << '\n';
50
51 //heap-----
52 heap_t h1, h2;
53 h1.push(1); h1.push(3);
54 h2.push(2); h2.push(4);
55 h1.join(h2);
56 cout << h1.size() << h2.size() << h1.top() << '\n';
57 // 404
58
59 //hash-table-----
60 ht_t ht;
61 ht[85] = 5;
62 ht[89975] = 234;
63 for (auto i : ht) {
64     cout << i.first << " " << i.second << '\n';
65 }
66 }
```

### 1.3 Ternary Search

```
1 const double EPS = 1e-6;
2 // target function
3 double f(double x) { return x * x; }
4 double ternarySearch() {
5     double L = -1e5, R = 1e5;
6     while (R - L > EPS) {
7         double mr = (L + R) / 2.0;
8         double ml = (L + mr) / 2.0;
9         if (f(ml) < f(mr)) {
10             R = mr;
11         } else {
12             L = ml;
13         }
14     }
15     return L;
16 }
```

## 2 Data Structure

### 2.1 BIT RARSQ

```
1 // 1-base
2 #define lowbit(k) (k & -k)
3
4 int n;
5 vector<int> B1, B2;
6
7 void add(vector<int> &tr, int id, int val) {
8     for (; id <= n; id += lowbit(id)) {
9         tr[id] += val;
10     }
11 }
12 void range_add(int l, int r, int val) {
13     add(B1, l, val);
```

```

14 add(B1, r + 1, -val);
15 add(B2, l, val * (1 - 1));
16 add(B2, r + 1, -val * r);
17 }
18 int sum(vector<int> &tr, int id) {
19     int ret = 0;
20     for (; id >= 1; id -= lowbit(id)) {
21         ret += tr[id];
22     }
23     return ret;
24 }
25 int prefix_sum(int id) {
26     return sum(B1, id) * id - sum(B2, id);
27 }
28 int range_sum(int l, int r) {
29     return prefix_sum(r) - prefix_sum(l - 1);
30 }

```

## 2.2 zkw RMQ

```

1 // 0-base
2 const int INF = 1e9;
3 const int MAXN = ;
4
5 int n;
6 int a[MAXN], tr[MAXN << 1];
7
8 // !!! remember to call this function
9 void build() {
10     for (int i = 0; i < n; i++) {
11         tr[i + n] = a[i];
12     }
13     for (int i = n - 1; i > 0; i--) {
14         tr[i] = max(tr[i << 1], tr[i << 1 | 1]);
15     }
16 }
17 void update(int id, int val) {
18     for (tr[id += n] = val; id > 1; id >>= 1) {
19         tr[id >> 1] = max(tr[id], tr[id ^ 1]);
20     }
21 }
22 int query(int l, int r) { // [l, r)
23     int ret = -INF;
24     for (l += n, r += n; l < r; l >>= 1, r >>= 1) {
25         if (l & 1) {
26             ret = max(ret, tr[l++]);
27         }
28         if (r & 1) {
29             ret = max(ret, tr[--r]);
30         }
31     }
32     return ret;
33 }

```

## 2.3 Segment Tree RARMQ

```

1 struct Node {
2     int val, tag;
3     Node *lc, *rc;
4     Node() : lc(nullptr), rc(nullptr), tag(0) {}
5     void pull() {
6         if (!lc) {
7             val = rc->val;
8         } else if (!rc) {
9             val = lc->val;
10        } else {
11            val = max(lc->val, rc->val);
12        }
13    }
14    void push() {
15        if (lc) {
16            lc->tag += tag;
17            lc->val += tag;

```

```

18        }
19        if (rc) {
20            rc->tag += tag;
21            rc->val += tag;
22        }
23        tag = 0;
24    }
25 };
26 struct SegmentTree {
27     Node *root;
28     SegmentTree() : root(nullptr) {}
29     void build(Node* &T, int l, int r, const
30         vector<int> &o) {
31         T = new Node();
32         if (l == r) {
33             T->val = o[l];
34             return;
35         }
36         int mid = (l + r) / 2;
37         build(T->lc, l, mid, o);
38         build(T->rc, mid + 1, r, o);
39         T->pull();
40     }
41     void update(Node* &T, int l, int r, int ql, int qr,
42         int v) {
43         if (ql <= l && r <= qr) {
44             T->val += v;
45             T->tag += v;
46             return;
47         }
48         T->push();
49         int mid = (l + r) / 2;
50         if (qr <= mid) {
51             update(T->lc, l, mid, ql, qr, v);
52         } else if (mid < ql) {
53             update(T->rc, mid + 1, r, ql, qr, v);
54         } else {
55             update(T->lc, l, mid, ql, mid, v);
56             update(T->rc, mid + 1, r, mid + 1, qr, v);
57         }
58         T->pull();
59     }
60     int query(Node* &T, int l, int r, int ql, int qr) {
61         if (ql <= l && r <= qr) {
62             return T->val;
63         }
64         T->push();
65         int mid = (l + r) / 2;
66         if (qr <= mid) {
67             return query(T->lc, l, mid, ql, qr);
68         } else if (mid < ql) {
69             return query(T->rc, mid + 1, r, ql, qr);
70         } else {
71             return max(query(T->lc, l, mid, ql, mid),
72                 query(T->rc, mid + 1, r, mid + 1, qr));
73         }
74     }
75 };

```

## 2.4 Treap

```

1 struct Treap {
2     int val, pri, sz;
3     Treap *lc, *rc;
4     Treap() {}
5     Treap(int _val) {
6         val = _val;
7         pri = rand();
8         sz = 1;
9         lc = rc = NULL;
10    }
11 };
12 int getSize(Treap *a) { return (a == NULL ? 0 :
13     a->sz); }
14 void split(Treap *t, Treap *&a, Treap *&b, int k) {
15     if (t == NULL) {

```

```

15     a = b = NULL;
16     return;
17 }
18 if (getSize(t->lc) < k) {
19     a = t;
20     split(t->rc, a->rc, b, k - getSize(t->lc) - 1);
21 } else {
22     b = t;
23     split(t->lc, a, b->lc, k);
24 }
25 }
26 Treap *merge(Treap *a, Treap *b) {
27     if (!a || !b) {
28         return (a ? a : b);
29     }
30     if (a->pri > b->pri) {
31         a->rc = merge(a->rc, b);
32         return a;
33     } else {
34         b->lc = merge(a, b->lc);
35         return b;
36     }
37 }
38 void Insert(Treap *t, int x, int p) {
39     Treap *a, *b;
40     split(t, a, b, x);
41     t = merge(a, merge(new Treap(p), b));
42 }
43 void Delete(Treap *t, int x) {
44     Treap *a, *b, *c;
45     split(t, b, c, x);
46     split(b, a, b, x - 1);
47     t = merge(a, c);
48 }
49
50 /*
51 Usage
52 Treap *root = NULL; // declare
53 root = merge(root, new Treap(val)); // push back
54 Insert(root, x, y); // insert y after x-th element
55 Delete(root, x); // delete x-th element
56 */

```

## 3 Graph

### 3.1 Topological Sort

```

1 // 0-base
2 // if ret.size < n -> cycle
3 int n;
4 vector<vector<int>> G;
5
6 vector<int> topoSort() {
7     vector<int> indeg(n), ret;
8     for (auto &li : G) {
9         for (int x : li) {
10             ++indeg[x];
11         }
12     }
13     // use priority queue for lexic. largest ans
14     queue<int> q;
15     for (int i = 0; i < n; i++) {
16         if (!indeg[i]) {
17             q.push(i);
18         }
19     }
20     while (!q.empty()) {
21         int u = q.front();
22         q.pop();
23         ret.pb(u);
24         for (int v : G[u]) {
25             if (--indeg[v] == 0) {
26                 q.push(v);
27             }
28         }
29     }
30 }

```

```

28     }
29 }
30 return ret;
31 }

```

### 3.2 Tree Diameter

```

1 // 0-base;
2 const int MAXN = ;
3
4 struct Edge {
5     int to;
6     int cost;
7     Edge(int v, int c) : to(v), cost(c) {}
8 };
9
10 int n, d = 0;
11 int d1[MAXN], d2[MAXN];
12 vector<Edge> G[MAXN];
13 // dfs(0, -1);
14 void dfs(int u, int from) {
15     d1[u] = d2[u] = 0;
16     for (auto e : G[u]) {
17         if (e.to == from) {
18             continue;
19         }
20         dfs(e.to, u);
21         int t = d1[e.to] + e.cost;
22         if (t > d1[u]) {
23             d2[u] = d1[u];
24             d1[u] = t;
25         } else if (t > d2[u]) {
26             d2[u] = t;
27         }
28     }
29     d = max(d, d1[u] + d2[u]);
30 }

```

### 3.3 Directed MST

```

1 // 0-base
2 const LL INF = 1e18;
3 const int MAXN = ;
4
5 struct Edge {
6     int from;
7     int to;
8     LL cost;
9     Edge(int u, int v, LL c) : from(u), to(v), cost(c) {}
10 };
11
12 struct DMST {
13     int n;
14     int vis[MAXN], pre[MAXN], id[MAXN];
15     LL in[MAXN];
16     vector<Edge> edges;
17     void init(int _n) {
18         n = _n;
19         edges.clear();
20     }
21     void add_edge(int from, int to, LL cost) {
22         edges.pb(Edge(from, to, cost));
23     }
24     LL run(int root) {
25         LL ret = 0;
26         while (true) {
27             for (int i = 0; i < n; i++) {
28                 in[i] = INF;
29             }
30             // find in edge
31             for (auto &e : edges) {

```

```

33     if (e.cost < in[e.to] && e.from != e.to) {
34         pre[e.to] = e.from;
35         in[e.to] = e.cost;
36     }
37 }
38
39 // check in edge
40 for (int i = 0; i < n; i++) {
41     if (i == root) {
42         continue;
43     }
44     if (in[i] == INF) {
45         return -1;
46     }
47 }
48
49 int nodenum = 0;
50 memset(id, -1, sizeof(id));
51 memset(vis, -1, sizeof(vis));
52 in[root] = 0;
53
54 // find cycles
55 for (int i = 0; i < n; i++) {
56     ret += in[i];
57     int v = i;
58     while (vis[v] != i && id[v] == -1 && v !=
59         root) {
60         vis[v] = i;
61         v = pre[v];
62     }
63     if (id[v] == -1 && v != root) {
64         for (int j = pre[v]; j != v; j = pre[j]) {
65             id[j] = nodenum;
66         }
67         id[v] = nodenum++;
68     }
69 }
70
71 // no cycle
72 if (nodenum == 0) {
73     break;
74 }
75
76 for (int i = 0; i < n; i++) {
77     if (id[i] == -1) {
78         id[i] = nodenum++;
79     }
80 }
81
82 // grouping the vertices
83 for (auto &e : edges) {
84     int to = e.to;
85     e.from = id[e.from];
86     e.to = id[e.to];
87     if (e.from != e.to) {
88         e.cost -= in[to]; //!!!!
89     }
90 }
91
92 n = nodenum;
93 root = id[root];
94 return ret;
95 }
96 };

```

### 3.4 LCA

```

1 const int LOG = 20;
2 vector<int> tin(MAXN), tout(MAXN), depth(MAXN);
3 int par[MAXN][LOG];
4 int timer = 0;
5 vector<int> G[MAXN];
6
7 void dfs(int u, int f) {
8     tin[u] = ++timer;

```

```

9     par[u][0] = f;
10    for (int v : G[u]) {
11        if (v != f) {
12            depth[v] = depth[u] + 1;
13            dfs(v, u);
14        }
15    }
16    tout[u] = ++timer;
17 }
18
19 void Doubling(int n) {
20     for (int j = 1; j < LOG; ++j) {
21         for (int i = 1; i <= n; ++i) {
22             par[i][j] = par[par[i][j - 1]][j - 1];
23         }
24     }
25 }
26
27 bool anc(int u, int v) { return tin[u] <= tin[v] &&
28     tout[v] <= tout[u]; }
29
30 int LCA(int u, int v) {
31     if (depth[u] > depth[v]) {
32         swap(u, v);
33     }
34     if (anc(u, v)) {
35         return u;
36     }
37     for (int j = LOG - 1; j >= 0; --j) {
38         if (!anc(par[u][j], v)) u = par[u][j];
39     }
40     return par[u][0];
41 }
42
43 int dis(int u, int v) {
44     int lca = LCA(u, v);
45     return depth[u] + depth[v] - 2 * depth[lca];
46 }
47
48 /*
49 dfs(root, root);
50 Doubling(n);
51 */

```

### 3.5 Euler Circuit

七橋問題根據起點與終點是否相同，分成 Euler path (不同) 及 Euler circuit (相同)。

- 判斷法
  - 無向圖部分，將點分成奇點（度數為奇數）和偶點（度數為偶數）。
    - Euler path：奇點數為 0 或 2
    - Euler circuit：沒有奇點
  - 有向圖部分，將點分成出點（出度 - 入度 = 1）和入點（入度 - 出度 = 1）還有平衡點（出度 = 入度）。
    - Euler path：出點和入點個數同時為 0 或 1。
    - Euler circuit：只有平衡點。
- 求出一組解
- 用 DFS 遍歷整張圖，設  $s$  為離開的順序，無向圖的答案為  $s$ ，有向圖的答案為反向的  $s$ 。
- DFS 起點選定：
  - Euler path：無向圖選擇任意一個奇點，有向圖選擇出點。
  - Euler circuit：任意一點。

```

1 // Code from Eric
2 #define ll long long
3 #define PB push_back
4 #define EB emplace_back
5 #define PII pair<int, int>
6 #define MP make_pair

```

```

7 #define all(x) x.begin(), x.end()
8 #define maxn 50000+5
9
10 //structure
11 struct Euler {
12     vector<PII> adj[maxn];
13     vector<bool> edges;
14     vector<PII> path;
15     int chk[maxn];
16     int n;
17
18     void init(int _n) {
19         n = _n;
20         for (int i = 0; i <= n; i++) adj[i].clear();
21         edges.clear();
22         path.clear();
23         memset(chk, 0, sizeof(chk));
24     }
25
26     void dfs(int v) {
27         for (auto i : adj[v]) {
28             if (edges[i.first] == true) {
29                 edges[i.first] = false;
30                 dfs(i.second);
31                 path.EB(MP(i.second, v));
32             }
33         }
34     }
35
36     void add_Edge(int from, int to) {
37         edges.PB(true);
38
39         // for bi-directed graph
40         adj[from].PB(MP(edges.size() - 1, to));
41         adj[to].PB(MP(edges.size() - 1, from));
42         chk[from]++;
43         chk[to]++;
44
45         // for directed graph
46         // adj[from].PB(MP(edges.size()-1, to));
47         // check[from]++;
48     }
49
50     bool eular_path() {
51         int st = -1;
52         for (int i = 1; i <= n; i++) {
53             if (chk[i] % 2 == 1) {
54                 st = i;
55                 break;
56             }
57         }
58         if (st == -1) {
59             return false;
60         }
61         dfs(st);
62         return true;
63     }
64
65     void print_path(void) {
66         for (auto i : path) {
67             printf("%d %d\n", i.first, i.second);
68         }
69     }
70 };
71
72 // Code from allen(lexicographic order)
73 #include <bits/stdc++.h>
74 using namespace std;
75 const int ALP = 30;
76 const int MXN = 1005;
77 int n;
78 int din[ALP], dout[ALP];
79 int par[ALP];
80 vector<string> vs[MXN], ans;
81 bitset<MXN> vis, used[ALP];
82
83 void djsInit() {
84     for (int i = 0; i != ALP; ++i) {
14         par[i] = i;
15     }
16 }
17
18 int Find(int x) { return (x == par[x] ? (x) : (par[x]
19     = Find(par[x]))); }
20
21 void init() {
22     djsInit();
23     memset(din, 0, sizeof(din));
24     memset(dout, 0, sizeof(dout));
25     vis.reset();
26     for (int i = 0; i != ALP; ++i) {
27         vs[i].clear();
28         used[i].reset();
29     }
30     return;
31 }
32
33 void dfs(int u) {
34     for (int i = 0; i != (int)vs[u].size(); ++i) {
35         if (used[u][i]) {
36             continue;
37         }
38         used[u][i] = 1;
39         string s = vs[u][i];
40         int v = s[s.size() - 1] - 'a';
41         dfs(v);
42         ans.push_back(s);
43     }
44 }
45
46 bool solve() {
47     int cnt = 1;
48     for (int i = 0; i != n; ++i) {
49         string s;
50         cin >> s;
51         int from = s[0] - 'a', to = s.back() - 'a';
52         ++din[to];
53         ++dout[from];
54         vs[from].push_back(s);
55         vis[from] = vis[to] = true;
56         if ((from = Find(from)) != (to = Find(to))) {
57             par[from] = to;
58             ++cnt;
59         }
60     }
61     if ((int)vis.count() != cnt) {
62         return false;
63     }
64     int root, st, pin = 0, pout = 0;
65     for (int i = ALP - 1; i >= 0; --i) {
66         sort(vs[i].begin(), vs[i].end());
67         if (vs[i].size()) root = i;
68         int d = dout[i] - din[i];
69         if (d == 1) {
70             ++pout;
71             st = i;
72         } else if (d == -1) {
73             ++pin;
74         } else if (d != 0) {
75             return false;
76         }
77     }
78     if (pin != pout || pin > 1) {
79         return false;
80     }
81     ans.clear();
82     dfs((pin ? st : root));
83     return true;
84 }
85
86 int main() {
87     int t;
88     cin >> t;
89     while (t--) {

```

```

90     init();
91     if (!solve()) {
92         cout << "***\n";
93         continue;
94     }
95     for (int i = ans.size() - 1; i >= 0; --i) {
96         cout << ans[i] << ".\n"[i == 0];
97     }
98 }
99 }

```

## 4 Connectivity

### 4.1 Kosaraju SCC

```

1 // 0-base
2 int n;
3 vector<vector<int>> G, G2; // G2 = G rev
4 vector<bool> vis;
5 vector<int> s, color;
6 int sccCnt;
7 void dfs1(int u) {
8     vis[u] = true;
9     for (int v : G[u]) {
10         if (!vis[v]) {
11             dfs1(v);
12         }
13     }
14     s.pb(u);
15 }
16 void dfs2(int u) {
17     color[u] = sccCnt;
18     for (int v : G2[u]) {
19         if (!color[v]) {
20             dfs2(v);
21         }
22     }
23 }
24 void Kosaraju() {
25     sccCnt = 0;
26     for (int i = 0; i < n; i++) {
27         if (!vis[i]) {
28             dfs1(i);
29         }
30     }
31     for (int i = n - 1; i >= 0; i--) {
32         if (!color[s[i]]) {
33             ++sccCnt;
34             dfs2(s[i]);
35         }
36     }
37 }

```

### 4.2 BCC

```

1 typedef pair<int, int> PII;
2 int low[MXV], depth[MXV];
3 bool is_cut_vertex[MXV], visit[MXV];
4 vector<int> G[MXV];
5 vector<PII> BCC[MXV];
6 int bcc_cnt = 0;
7 stack<PII> st;
8
9 vector<pair<int, int>> my_cut_edge;
10
11 void dfs(int now, int cur_depth, int f) {
12     visit[now] = true;
13     depth[now] = low[now] = cur_depth;
14     int cut_son = 0;
15     for (auto i : G[now]) {
16         if (i == f) continue;
17         if (visit[i]) { // ancestor

```

```

18         if (depth[i] < depth[now]) { // #
19             low[now] = min(low[now], depth[i]);
20             st.push({now, i});
21         }
22     } else { // offspring
23         st.push({now, i});
24         dfs(i, cur_depth + 1, now);
25         cut_son += 1;
26         low[now] = min(low[now], low[i]);
27         if (low[i] >= depth[now]) {
28             is_cut_vertex[now] = true;
29             auto t = st.top();
30             st.pop();
31             while (t != make_pair(now, i)) {
32                 BCC[bcc_cnt].push_back(t);
33                 t = st.top();
34                 st.pop();
35             }
36             BCC[bcc_cnt].push_back(t);
37             ++bcc_cnt;
38         }
39         // ###
40         if (low[i] > depth[now])
41             my_cut_edge.push_back({now, i});
42     }
43 }
44 if (cur_depth == 0)
45     is_cut_vertex[now] = (cut_son != 1);
46 return;
47 }
48
49 bool is_2_edge_connected(int n) {
50     memset(visit, 0, sizeof(visit));
51     dfs(1, 0, -1);
52     return my_cut_edge.size() == 0;
53 }

```

### 4.3 Articulation Point

```

1 // from aizu
2 typedef long long int ll;
3 typedef unsigned long long int ull;
4 #define BIG_SIZE 2000000000
5 #define MOD 1000000007
6 #define EPS 0.000000001
7 using namespace std;
8
9 #define SIZE 100000
10
11 vector<int> G[SIZE];
12 int N;
13 bool visited[SIZE];
14 int visited_order[SIZE], parent[SIZE], lowest[SIZE],
    number;
15
16 void dfs(int cur, int pre_node) {
17     visited_order[cur] = lowest[cur] = number;
18     number++;
19
20     visited[cur] = true;
21
22     int next;
23
24     for (int i = 0; i < G[cur].size(); i++) {
25         next = G[cur][i];
26         if (!visited[next]) {
27             parent[next] = cur;
28             dfs(next, cur);
29             lowest[cur] = min(lowest[cur], lowest[next]);
30         } else if (visited[next] == true && next !=
            pre_node) {
31             lowest[cur] = min(lowest[cur],
                visited_order[next]);
32         }
33     }
34 }

```

```

35
36 void art_points() {
37     for (int i = 0; i < N; i++) visited[i] = false;
38
39     number = 1;
40     dfs(0, -1);
41
42     int tmp_parent, root_num = 0;
43
44     vector<int> V;
45
46     for (int i = 1; i < N; i++) {
47         tmp_parent = parent[i];
48         if (tmp_parent == 0) {
49             root_num++;
50         } else if (visited_order[tmp_parent] <=
51             lowest[i]) {
52             V.push_back(tmp_parent);
53         }
54     }
55     if (root_num >= 2) {
56         V.push_back(0);
57     }
58     sort(V.begin(), V.end());
59     V.erase(unique(V.begin(), V.end()), V.end());
60
61     for (int i = 0; i < V.size(); i++) {
62         printf("%d\n", V[i]);
63     }
64 }
65
66 int main() {
67     int E;
68     scanf("%d %d", &N, &E);
69     int from, to;
70     for (int i = 0; i < E; i++) {
71         scanf("%d %d", &from, &to);
72         G[from].push_back(to);
73         G[to].push_back(from);
74     }
75     art_points();
76 }

```

## 4.4 Bridges

```

1 // from aizu
2 typedef long long int ll;
3 typedef unsigned long long int ull;
4 #define BIG_NUM 2000000000
5 #define MOD 1000000007
6 #define EPS 0.000000001
7 using namespace std;
8
9 struct Edge {
10     bool operator<(const struct Edge &arg) const {
11         if (s != arg.s) {
12             return s < arg.s;
13         } else {
14             return t < arg.t;
15         }
16     }
17     int s, t;
18 };
19 struct Info {
20     Info(int arg_to, int arg_edge_id) {
21         to = arg_to;
22         edge_id = arg_edge_id;
23     }
24     int to, edge_id;
25 };
26
27 int V, E, number;
28 int order[100000], lowlink[100000];
29 bool visited[100000];
30 Edge edge[100000];
31 vector<Info> G[100000];

```

```

32
33 void recursive(int cur) {
34     order[cur] = number++;
35     lowlink[cur] = order[cur];
36
37     int next;
38
39     for (int i = 0; i < G[cur].size(); i++) {
40         next = G[cur][i].to;
41
42         if (order[next] == -1) {
43             visited[G[cur][i].edge_id] = true;
44             recursive(next);
45             lowlink[cur] = min(lowlink[cur], lowlink[next]);
46         } else if (visited[G[cur][i].edge_id] == false) {
47             lowlink[cur] = min(lowlink[cur], order[next]);
48         }
49     }
50 }
51
52
53 int main() {
54     scanf("%d %d", &V, &E);
55     for (int i = 0; i < E; i++) {
56         scanf("%d %d", &edge[i].s, &edge[i].t);
57         if (edge[i].s > edge[i].t) {
58             swap(edge[i].s, edge[i].t);
59         }
60         G[edge[i].s].push_back(Info(edge[i].t, i));
61         G[edge[i].t].push_back(Info(edge[i].s, i));
62     }
63
64     sort(edge, edge + E);
65
66     number = 0;
67     for (int i = 0; i < V; i++) {
68         order[i] = -1;
69         lowlink[i] = -1;
70     }
71     for (int i = 0; i < E; i++) {
72         visited[i] = false;
73     }
74
75     recursive(0);
76
77     int from, to;
78     for (int i = 0; i < E; i++) {
79         from = edge[i].s;
80         to = edge[i].t;
81         if (order[edge[i].s] > order[edge[i].t]) {
82             swap(from, to);
83         }
84         if (order[from] < lowlink[to]) {
85             printf("%d %d\n", edge[i].s, edge[i].t);
86         }
87     }
88     return 0;
89 }

```

## 5 Flow & Matching

### 5.1 Relation

- 1 一般圖
- 2  $|最大匹配| + |最小邊覆蓋| = |V|$
- 3  $|最大獨立集| + |最小點覆蓋| = |V|$
- 4  $|最大圖| = |補圖的最大獨立集|$
- 5 二分圖
- 6  $|最大匹配| = |最小點覆蓋|$
- 7  $|最大獨立集| = |最小邊覆蓋|$
- 8  $|最大獨立集| = |V| - |最大匹配|$
- 9  $|最大圖| = |補圖的最大獨立集|$





```

40     }
41     }
42     return level[ed] != -1;
43 }
44 LL dfs(int u, int ed, LL limit) {
45     if (u == ed) {
46         return limit;
47     }
48     LL ret = 0;
49     for (int &i = now[u]; i < G[u].size(); i++) {
50         auto &e = G[u][i];
51         if (e.cap > 0 && level[e.to] == level[u] + 1) {
52             LL f = dfs(e.to, ed, min(limit, e.cap));
53             ret += f;
54             limit -= f;
55             e.cap -= f;
56             G[e.to][e.rev].cap += f;
57             if (!limit) {
58                 return ret;
59             }
60         }
61     }
62     if (!ret) {
63         level[u] = -1;
64     }
65     return ret;
66 }
67 LL flow(int st, int ed) {
68     LL ret = 0;
69     while (bfs(st, ed)) {
70         fill(now, now + n + 1, 0);
71         ret += dfs(st, ed, INF);
72     }
73     return ret;
74 }
75 };

```

## 5.5 MCMF

```

1 // 0-base
2 const LL INF = 1e18;
3 const int MAXN = ;
4 struct Edge {
5     int u, v;
6     LL cost;
7     LL cap;
8     Edge(int _u, int _v, LL _c, LL _cap) : u(_u),
9         v(_v), cost(_c), cap(_cap) {}
10 };
11 struct MCMF { // inq times
12     int n, pre[MAXN], cnt[MAXN];
13     LL ans_flow, ans_cost, dis[MAXN];
14     bool inq[MAXN];
15     vector<int> G[MAXN];
16     vector<Edge> edges;
17     void init(int _n) {
18         n = _n;
19         edges.clear();
20         for (int i = 0; i < n; i++) {
21             G[i].clear();
22         }
23     }
24     void add_edge(int u, int v, LL c, LL cap) {
25         // directed
26         G[u].pb(edges.size());
27         edges.eb(u, v, c, cap);
28         G[v].pb(edges.size());
29         edges.eb(v, u, -c, 0);
30     }
31     bool SPFA(int st, int ed) {
32         for (int i = 0; i < n; i++) {
33             pre[i] = -1;
34             dis[i] = INF;
35             cnt[i] = 0;
36             inq[i] = false;
37         }
38     }

```

```

37     queue<int> q;
38     bool negcycle = false;
39
40     dis[st] = 0;
41     cnt[st] = 1;
42     inq[st] = true;
43     q.push(st);
44
45     while (!q.empty() && !negcycle) {
46         int u = q.front();
47         q.pop();
48         inq[u] = false;
49         for (int i : G[u]) {
50             int v = edges[i].v;
51             LL cost = edges[i].cost;
52             LL cap = edges[i].cap;
53
54             if (dis[v] > dis[u] + cost && cap > 0) {
55                 dis[v] = dis[u] + cost;
56                 pre[v] = i;
57                 if (!inq[v]) {
58                     q.push(v);
59                     cnt[v]++;
60                     inq[v] = true;
61
62                     if (cnt[v] == n + 2) {
63                         negcycle = true;
64                         break;
65                     }
66                 }
67             }
68         }
69     }
70
71     return dis[ed] != INF;
72 }
73 LL sendFlow(int v, LL curFlow) {
74     if (pre[v] == -1) {
75         return curFlow;
76     }
77     int i = pre[v];
78     int u = edges[i].u;
79     LL cost = edges[i].cost;
80
81     LL f = sendFlow(u, min(curFlow, edges[i].cap));
82
83     ans_cost += f * cost;
84     edges[i].cap -= f;
85     edges[i ^ 1].cap += f;
86     return f;
87 }
88 pair<LL, LL> run(int st, int ed) {
89     ans_flow = ans_cost = 0;
90     while (SPFA(st, ed)) {
91         ans_flow += sendFlow(ed, INF);
92     }
93     return make_pair(ans_flow, ans_cost);
94 }
95 };

```

## 5.6 Stable Matching

```

1 int t, n, b[N][N], bi[N], g[N][N], bg[N], gb[N];
2
3 void sol() {
4     deque<int> dq;
5     memset(gb, 0, sizeof(gb));
6     memset(bi, 0, sizeof(bi));
7     for (int i = 1; i <= n; i++) dq.push_back(i);
8     while (!dq.empty()) {
9         int x = dq.front();
10        dq.pop_front();
11        int y = b[x][++bi[x]];
12        if (!gb[y]) {
13            gb[y] = x;
14            bg[x] = y;

```

```

15     } else if (g[y][x] < g[y][gb[y]]) {
16         dq.push_back(gb[y]);
17         gb[y] = x;
18         bg[x] = y;
19     } else {
20         dq.push_back(x);
21     }
22 }
23 for (int i = 1; i <= n; i++) {
24     cout << bg[i] << '\n';
25 }
26 }
27
28 int main() {
29     int x;
30     cin >> t;
31     for (int i = 0; i < t; i++) {
32         cin >> n;
33         for (int i = 1; i <= n; i++) {
34             for (int j = 1; j <= n; j++) {
35                 cin >> b[i][j];
36             }
37         }
38         for (int i = 1; i <= n; i++) {
39             for (int j = 1; j <= n; j++) {
40                 cin >> x;
41                 g[i][x] = j;
42             }
43         }
44         if (i) cout << '\n';
45         sol();
46     }
47 }

```

```

39     for (int i = 0; i <= n; ++i) fa[i] = i, s[i] = -1;
40     while (!q.empty()) q.pop();
41     q.push(r);
42     s[r] = 0;
43     while (!q.empty()) {
44         int x = q.front();
45         q.pop();
46         for (int u : g[x]) {
47             if (s[u] == -1) {
48                 pre[u] = x, s[u] = 1;
49                 if (match[u] == n) {
50                     for (int a = u, b = x, last; b != n; a =
                        last, b = pre[a])
51                         last = match[b], match[b] = a, match[a] =
                            b;
52                     return true;
53                 }
54                 q.push(match[u]);
55                 s[match[u]] = 0;
56             } else if (!s[u] && Find(u) != Find(x)) {
57                 int l = LCA(u, x, n);
58                 Blossom(x, u, l);
59                 Blossom(u, x, l);
60             }
61         }
62     }
63     return false;
64 }
65
66 int Solve(int n) {
67     int res = 0;
68     for (int x = 0; x < n; ++x) {
69         if (match[x] == n) res += Bfs(x, n);
70     }
71     return res;
72 }

```

## 5.7 Max General Graph Matching

```

1 #define maxn
2
3 int fa[maxn], pre[maxn], match[maxn], s[maxn],
    v[maxn];
4 vector<int> g[maxn];
5 queue<int> q;
6 void Init(int n) {
7     for (int i = 0; i <= n; ++i) match[i] = pre[i] = n;
8     for (int i = 0; i < n; ++i) g[i].clear();
9 }
10 void AddEdge(int u, int v) {
11     g[u].push_back(v);
12     g[v].push_back(u);
13 }
14 int Find(int u) { return u == fa[u] ? u : fa[u] =
    Find(fa[u]); }
15 int LCA(int x, int y, int n) {
16     static int tk = 0;
17     tk++;
18     x = Find(x), y = Find(y);
19     for (;;) swap(x, y) {
20         if (x != n) {
21             if (v[x] == tk) return x;
22             v[x] = tk;
23             x = Find(pre[match[x]]);
24         }
25     }
26 }
27
28 void Blossom(int x, int y, int l) {
29     while (Find(x) != l) {
30         pre[x] = y, y = match[x];
31         if (s[y] == 1) q.push(y), s[y] = 0;
32         if (fa[x] == x) fa[x] = l;
33         if (fa[y] == y) fa[y] = l;
34         x = pre[y];
35     }
36 }
37
38 bool Bfs(int r, int n) {

```