1.3 Black Magic

# Contents

11

13

14 }

12 int main() {

```
1 #include <bits/stdc++.h>
 1 Basic
                                          #include <ext/pb_ds/assoc_container.hpp>
   3
                                          #include <ext/pb_ds/tree_policy.hpp>
   1.2 Default . . . . . . . . . . . . . . . . . .
                                          #include <ext/pb_ds/priority_queue.hpp>
   1.3 Black Magic . . . . . . . . . . . . . . . . .
                                          using namespace std;
   1.4 Binary Search . . . . . . . . . . . . .
   using namespace __gnu_pbds;
                                          using set_t =
 2 Data Structure
                                           tree<int, null_type, less<int>, rb_tree_tag,
                                        8
   tree_order_statistics_node_update>;
   using map_t =
                                           tree<int, int, less<int>, rb_tree_tag,
 3 Graph
                                        12
                                            tree_order_statistics_node_update>;
   13
                                          using heap t =
                                      3
                                           __gnu_pbds::priority_queue<int>;
                                        14
                                      3
                                          using ht_t =
                                      3
   4
                                        16
                                           gp_hash_table<int, int>;
   3.6 Tree Diameter
                                        17
                                          int main() {
   18
                                           //set-----
                                        19
                                           set t st:
                                           st.insert(5); st.insert(6);
 4 Flow & Matching
   st.insert(3); st.insert(1);
                                        21
   22
   23
                                           // the smallest is (0), biggest is (n-1), kth small
   is (k-1)
                                           int num = *st.find_by_order(0);
                                           cout << num << '\n'; // print 1
 5 String
                                        25
   26
   num = *st.find_by_order(st.size() - 1);
                                        27
                                        28
                                           cout << num << '\n'; // print 6
                                        29
   30
                                           // find the index
   int index = st.order_of_key(6);
                                           cout << index << '\n'; // print 3</pre>
                                        32
                                        33
   34
                                           // check if there exists x
   int x = 5;
                                        35
   36
                                           int check = st.erase(x);
   if (check == 0) printf("st not contain 5\n");
                                        37
                                           else if (check == 1) printf("st contain 5\n");
   39
 8 Geometry
                                        40
                                           //tree policy like set
                                        41
                                           st.insert(5); st.insert(5);
   cout << st.size() << '\n'; // print 4</pre>
                                        42
          //map-----
                                        44
                                        45
                                           map_t mp;
                                        46
                                           mp[1] = 2;
    Basic
                                        47
                                           cout << mp[1] << '\n';
                                        48
                                           auto tmp = *mp.find_by_order(0); // pair
                                           cout << tmp.first << " " << tmp.second << ' \setminus n';
                                        49
 1.1 Run
                                        50
                                           //heap------
                                        51
                                           heap_t h1, h2;
                                        52
1 #use -> sh run.sh {name}
2 g++ -02 -std=c++14 -Wall -Wextra -Wshadow -o $1 $1.cpp
                                           h1.push(1); h1.push(3);
                                           h2.push(2); h2.push(4);
3 | ./$1 < t.in > t.out
                                        54
                                        55
                                           h1.join(h2);
                                           cout << h1.size() << h2.size() << h1.top() << '\n';</pre>
                                        56
                                        57
 1.2 Default
                                        58
                                           //hash-table-----
                                        59
                                           ht_t ht;
                                        60
1 #include <bits/stdc++.h>
                                           ht[85] = 5:
                                        61
2 using namespace std;
                                           ht[89975] = 234;
3 using LL = long long;
                                           for (auto i : ht) {
4 #define IOS ios_base::sync_with_stdio(0); cin.tie(0);
                                            cout << i.first << " " << i.second << '\n';</pre>
                                        64
5 #define pb push_back
                                        65
6 #define eb emplace_back
                                        66 }
7 const int INF = 1e9;
8 const int MOD = 1e9 + 7;
9 const double EPS = 1e-6;
10 const int MAXN = 0;
```

### 1.4 Binary Search

```
1 lower_bound(a, a + n, k);
                            //最左邊 ≥ k 的位置
2 upper_bound(a, a + n, k);
                            //最左邊 > k 的位置
3 | upper_bound(a, a + n, k) - 1; //最右邊 ≤ k 的位置
```

```
4 | lower_bound(a, a + n, k) - 1; //最右邊 < k 的位置</td>5 | [lower_bound, upper_bound)//等於 k 的範圍6 | equal_range(a, a + n, k);
```

## 1.5 Python

```
1 ### EOF
2 while True:
3
    try:
      pass
    except EOFError:
5
6
      break
7 ###math
8 import math
10 math.ceil(x)#上高斯
11 math.floor(x)#下高斯
12 math.factorial(x)#接乘
13 math.fabs(x)#絕對值
14 math.fsum(arr)#跟sum一樣但更精確(小數點問題)
15 math.gcd(x, y)#bj4
16 math.exp(x)#e^x
17 math.log(x, base)
18 math.log2(x)#2為底
19 math.log10(x)#10為底
20 math.sqrt(x)
21 | math.pow(x, y, mod)#精確些(float型態) MOD!!!
22 math.sin(x)# cos tan asin acos atan atan2(弧度) sinh
      cosh tanh acosh asinh atanh
23 math.hypot(x, y)#歐幾里德範數
24 math.degrees(x)#x從弧度轉角度
25 | math.radians(x)#x從角度轉弧度
26 math.gamma(x)#x的gamma函數
27 math.pi#常數
28 math.e#常數
29 math.inf
30
31 ### ascii
32 ord(x)#char to asc
33 chr(x)#asc to char
34
35 x.encode().hex()#string to hex
36 ### reverse string
37 string = "abc"
```

## 2 Data Structure

38 string\_reverse = string[::-1]

## 2.1 Disjoint Set

```
1 // 0-base
2 const int MAXN = 1000;
3 int boss[MAXN];
4 void init(int n) {
    for (int i = 0; i < n; i++) {</pre>
6
       boss[i] = -1;
7
    }
8 }
9
  int find(int x) {
   if (boss[x] < 0) {
10
11
       return x;
    }
12
13
    return boss[x] = find(boss[x]);
14 }
15 bool uni(int a, int b) {
    a = find(a);
    b = find(b);
17
18
    if (a == b) {
19
       return false;
20
```

```
21    if (boss[a] > boss[b]) {
22        swap(a, b);
23    }
24    boss[a] += boss[b];
25    boss[b] = a;
26    return true;
27 }
```

# 2.2 BIT RARSQ

```
1 // 1-base
  #define lowbit(k) (k & -k)
  vector<int> B1, B2;
5
  void add(vector<int> &tr, int id, int val) {
    for (; id <= n; id += lowbit(id)) {</pre>
8
      tr[id] += val;
10
    }
11 }
12
  void range_add(int 1, int r, int val) {
    add(B1, 1, val);
13
    add(B1, r + 1, -val);
    add(B2, 1, val * (1 - 1));
15
16
    add(B2, r + 1, -val * r);
17 }
18 int sum(vector<int> &tr, int id) {
19
    int ret = 0;
    for (; id >= 1; id -= lowbit(id)) {
20
21
      ret += tr[id];
    }
22
23
    return ret;
24 }
25 int prefix_sum(int id) {
   return sum(B1, id) * id - sum(B2, id);
26
27 }
28 int range_sum(int 1, int r) {
  return prefix_sum(r) - prefix_sum(1 - 1);
```

### 2.3 zkw RMQ

```
1 // 0-base
 2 const int INF = 1e9;
  const int MAXN = ;
 3
 5
  int n;
6 int a[MAXN], tr[MAXN << 1];</pre>
  // !!! remember to call this function
8
 9
  void build() {
    for (int i = 0; i < n; i++) {</pre>
10
      tr[i + n] = a[i];
11
    }
12
    for (int i = n - 1; i > 0; i--) {
13
14
       tr[i] = max(tr[i << 1], tr[i << 1 | 1]);
15
16 }
17
  void update(int id, int val) {
    for (tr[id += n] = val; id > 1; id >>= 1) {
18
19
       tr[id >> 1] = max(tr[id], tr[id ^ 1]);
    }
20
21 }
22 int query(int 1, int r) { // [1, r)
23
    int ret = -INF;
     for (1 += n, r += n; 1 < r; 1 >>= 1, r >>= 1) {
24
25
       if (1 & 1) {
         ret = max(ret, tr[1++]);
26
27
       }
28
       if (r & 1) {
29
         ret = max(ret, tr[--r]);
```

```
31 }
32 return ret;
33 }
```

# 3 Graph

# 3.1 Dijkstra

```
1 // 0-base
2 const LL INF = 1e18;
3 const int MAXN = ;
4 struct Edge {
    int to;
     LL cost;
     Edge(int v, LL c) : to(v), cost(c) {}
8
     bool operator < (const Edge &other) const {</pre>
9
       return cost > other.cost;
10
11 \ \ \ ;
12
13 int n;
14 LL dis[MAXN];
15 vector < Edge > G[MAXN];
16
17 void init() {
18
     for (int i = 0; i < n; i++) {
19
       G[i].clear();
       dis[i] = INF;
20
     }
21
22 }
23
  void Dijkstra(int st, int ed = -1) {
24
     priority_queue < Edge > pq;
     pq.emplace(st, 0);
25
     dis[st] = 0;
26
     while (!pq.empty()) {
27
28
       auto now = pq.top();
29
       pq.pop();
       if (now.to == ed) {
30
31
         return;
32
33
       if (now.cost > dis[now.to]) {
34
         continue;
35
36
       for (auto &e : G[now.to]) {
         if (dis[e.to] > now.cost + e.cost) {
37
           dis[e.to] = now.cost + e.cost;
38
39
            pq.emplace(e.to, dis[e.to]);
40
41
       }
42
     }
43 }
```

# 3.2 SPFA(negative cycle)

```
1 // 0-base
2 const LL INF = 1e18;
3 const int MAXN = ;
 4 struct Edge {
    int to;
6
    LL cost;
7
     Edge(int v, LL c) : to(v), cost(c) {}
8 }:
10 int n;
11 LL dis[MAXN];
12 vector < Edge > G[MAXN];
13
14 void init() {
15
     for (int i = 0; i < n; i++) {
16
       G[i].clear();
17
       dis[i] = INF;
18
```

```
19 }
20 bool SPFA(int st) {
     vector<int> cnt(n, 0);
     vector<bool> inq(n, false);
22
     queue<int> q;
23
24
25
     q.push(st);
26
     dis[st] = 0;
     inq[st] = true;
27
28
     while (!q.empty()) {
29
       int now = q.front();
       q.pop();
30
31
       inq[now] = false;
32
       for (auto &e : G[now]) {
33
         if (dis[e.to] > dis[now] + e.cost) {
           dis[e.to] = dis[now] + e.cost;
34
35
           if (!inq[e.to]) {
36
              cnt[e.to]++;
37
              if (cnt[e.to] > n) {
                // negative cycle
38
                return false;
39
40
41
             inq[e.to] = true;
42
             q.push(e.to);
43
44
         }
45
       }
46
    }
47
     return true:
48 }
```

# 3.3 Floyd Warshall

```
1 // 0-base
  // G[i][i] < 0 -> negative cycle
  const LL INF = 1e18;
  const int MAXN = ;
 6
  int n;
7
  LL G[MAXN][MAXN];
  void init() {
9
     for (int i = 0; i < n; i++) {</pre>
11
       for (int j = 0; j < n; j++) {
12
         G[i][j] = INF;
13
14
       G[i][i] = 0;
15
    }
16 }
17
  void floyd() {
18
     for (int k = 0; k < n; k++) {
       for (int i = 0; i < n; i++) {</pre>
19
20
         for (int j = 0; j < n; j++) {
           if (G[i][k] != INF && G[k][j] != INF) {
21
22
             G[i][j] = min(G[i][j], G[i][k] + G[k][j]);
23
24
         }
25
    }
26
27
```

# 3.4 Topological Sort

```
1  // 0-base
2  // if ret.size < n -> cycle
3  int n;
4  vector<vector<int>> G;
5
6  vector<int> topoSort() {
7   vector<int> indeg(n), ret;
8  for (auto &li : G) {
9   for (int x : li) {
10   ++indeg[x];
}
```

```
11
       }
     }
12
     // use priority queue for lexic. largest ans
13
14
     queue<int> q;
     for (int i = 0; i < n; i++) {
15
16
       if (!indeg[i]) {
         q.push(i);
17
18
       }
     }
19
20
     while (!q.empty()) {
21
       int u = q.front();
       q.pop();
22
23
       ret.pb(u);
       for (int v : G[u]) {
24
25
         if (--indeg[v] == 0) {
26
           q.push(v);
27
28
       }
29
     }
30
     return ret;
31 }
```

# 3.5 Kosaraju SCC

```
1 // 0-base
2 int n;
_{3}| vector<vector<int>> G, G2; // G2 = G rev
4 vector < bool > vis;
5 vector<int> s, color;
6 int sccCnt:
7 void dfs1(int u) {
    vis[u] = true;
    for (int v : G[u]) {
10
      if (!vis[v]) {
11
         dfs1(v):
12
    }
13
14
    s.pb(u);
15 }
16 void dfs2(int u) {
17
     color[u] = sccCnt;
     for (int v : G2[u]) {
18
       if (!color[v]) {
19
20
         dfs2(v);
21
       }
22
    }
23 }
24 void Kosaraju() {
25
    sccCnt = 0;
     for (int i = 0; i < n; i++) {
26
27
       if (!vis[i]) {
         dfs1(i);
28
29
       }
30
     for (int i = n - 1; i \ge 0; i - -) {
31
       if (!color[s[i]]) {
32
         ++sccCnt;
33
         dfs2(s[i]);
34
35
       }
36
37 }
```

### 3.6 Tree Diameter

```
1  // 0-base;
2  const int MAXN = ;
3
4  struct Edge {
   int to;
   int cost;
7  Edge(int v, int c) : to(v), cost(c) {}
8  };
9
```

```
10 | int n, d = 0;
11 int d1[MAXN], d2[MAXN];
12 vector < Edge > G[MAXN];
13 // dfs(0, -1);
14 void dfs(int u, int from) {
15
     d1[u] = d2[u] = 0;
     for (auto e : G[u]) {
16
17
       if (e.to == from) {
         continue;
18
19
20
       dfs(e.to, u);
       int t = d1[e.to] + e.cost;
21
22
       if (t > d1[u]) {
         d2[u] = d1[u];
23
24
         d1[u] = t;
       } else if (t > d2[u]) {
25
26
         d2[u] = t;
27
28
29
     d = max(d, d1[u] + d2[u]);
30 }
```

#### 3.7 Directed MST

```
1 // 0-base
  const LL INF = 1e18;
  const int MAXN = ;
 3
  struct Edge {
    int from;
6
     int to;
     LL cost:
 8
     Edge(int u, int v, LL c): from(u), to(v), cost(c)
         {}
10 };
11
12 struct DMST {
13
     int n;
     int vis[MAXN], pre[MAXN], id[MAXN];
14
     LL in[MAXN];
15
16
     vector < Edge > edges;
17
     void init(int _n) {
       n = _n;
18
19
       edges.clear();
20
21
     void add_edge(int from, int to, LL cost) {
22
       edges.eb(from, to, cost);
23
     LL run(int root) {
24
25
       LL ret = 0;
       while (true) {
26
27
         for (int i = 0; i < n; i++) {</pre>
28
           in[i] = INF;
29
30
         // find in edge
31
32
         for (auto &e : edges) {
33
           if (e.cost < in[e.to] && e.from != e.to) {</pre>
             pre[e.to] = e.from;
34
35
              in[e.to] = e.cost;
           }
36
37
         }
38
39
         // check in edge
40
         for (int i = 0; i < n; i++) {</pre>
           if (i == root) {
41
              continue;
           }
43
44
           if (in[i] == INF) {
45
              return -1;
46
47
         }
48
49
         int nodenum = 0;
         memset(id, -1, sizeof(id));
50
         memset(vis, -1, sizeof(vis));
```

```
52
          in[root] = 0;
53
          // find cycles
54
         for (int i = 0; i < n; i++) {</pre>
55
56
            ret += in[i];
57
            int v = i;
            while (vis[v] != i && id[v] == -1 && v !=
58
                root) {
59
              vis[v] = i;
60
              v = pre[v];
61
            if (id[v] == -1 && v != root) {
62
              for (int j = pre[v]; j != v; j = pre[j]) {
63
                id[j] = nodenum;
64
65
              id[v] = nodenum++;
66
67
           }
         }
68
69
70
          // no cycle
         if (nodenum == 0) {
71
72
            break;
73
74
75
         for (int i = 0; i < n; i++) {</pre>
           if (id[i] == -1) {
76
              id[i] = nodenum++;
77
            }
78
79
80
          // grouping the vertices
81
82
          for (auto &e : edges) {
           int to = e.to;
83
            e.from = id[e.from];
84
85
            e.to = id[e.to];
            if (e.from != e.to) {
86
87
              e.cost -= in[to]; //!!!
            }
88
89
90
91
         n = nodenum;
92
         root = id[root];
93
94
       return ret;
95
96 };
```

# 3.8 BCC

```
1 typedef pair<int, int> PII;
2 int low[MXV], depth[MXV];
3 bool is_cut_vertex[MXV], visit[MXV];
4 vector<int> G[MXV];
5 vector < PII > BCC[MXV];
6 int bcc_cnt = 0;
7 stack<PII> st;
9 vector<pair<int, int>> my_cut_edge;
10
11 void dfs(int now, int cur_depth, int f) {
12
    visit[now] = true;
    depth[now] = low[now] = cur_depth;
13
14
    int cut_son = 0;
    for (auto i : G[now]) {
15
16
       if (i == f) continue;
       if (visit[i]) { // ancestor
17
18
         if (depth[i] < depth[now]) { // #</pre>
           low[now] = min(low[now], depth[i]);
19
20
           st.push({now, i});
21
         }
      } else { // offspring
22
         st.push({now, i});
23
24
         dfs(i, cur_depth + 1, now);
25
         cut_son += 1;
         low[now] = min(low[now], low[i]);
26
         if (low[i] >= depth[now]) {
27
```

```
28
           is_cut_vertex[now] = true;
           auto t = st.top();
29
30
           st.pop();
           while (t != make_pair(now, i)) {
31
32
             BCC[bcc_cnt].push_back(t);
33
             t = st.top();
             st.pop();
34
35
           BCC[bcc_cnt].push_back(t);
36
37
           ++bcc_cnt;
38
         }
         // ###
39
         if (low[i] > depth[now])
40
41
           my_cut_edge.push_bach({now, i});
42
    }
43
44
    if (cur_depth == 0)
45
       is_cut_vertex[now] = (cut_son != 1);
46
    return:
47 }
48
49
  bool is_2_edge_connected(int n) {
50
    memset(visit, 0, sizeof(visit));
51
    dfs(1, 0, -1);
52
     return my_cut_edge.size() == 0;
53 }
```

# 4 Flow & Matching

#### 4.1 Relation

```
1 | 1. 一般圖
2 | 1最大匹配 | + 1最小邊覆蓋 | = |V|
3 | 1最大獨立集 | + 1最小點覆蓋 | = |V|
4 | 1最大圖 | = |補圖的最大獨立集 |
5 | 2. 二分圖
6 | 1最大匹配 | = |最小點覆蓋 |
7 | 1最大獨立集 | = |最小邊覆蓋 |
8 | 1最大獨立集 | = |V| - |最大匹配 |
9 | 1最大圖 | = |補圖的最大獨立集 |
```

### 4.2 Bipartite Matching

```
1 // 0-base
  const int MAXN = ;
 3
  int n;
  vector<int> G[MAXN];
 5 int vy[MAXN], my[MAXN];
 7
  bool match(int u) {
 8
     for (int v : G[u]) {
       if (vy[v]) {
10
         continue;
       }
11
12
       vy[v] = true;
       if (my[v] == -1 || match(my[v])) {
13
14
         my[v] = u;
15
         return true;
16
17
     }
18
     return false;
19 }
20 int sol() {
21
    int cnt = 0;
22
     memset(my, -1, sizeof(my));
     for (int i = 0; i < n; i++) {</pre>
23
       memset(vy, 0, sizeof(vy));
24
25
       if (match(i)) {
26
         cnt++;
27
28
     }
```

72 } 73 };

```
4.3 KM
```

return cnt;

29

30 }

```
1 const int INF = 1e9;
   const int MAXN = ;
3 struct KM { //1-base
     int n, G[MAXN][MAXN];
     int lx[MAXN], ly[MAXN], my[MAXN];
6
     bool vx[MAXN], vy[MAXN];
7
     void init(int _n) {
       n = _n;
8
       for (int i = 1; i <= n; i++) {</pre>
9
         for (int j = 1; j <= n; j++) {</pre>
10
11
            G[i][j] = 0;
12
         }
       }
13
14
15
     bool match(int i) {
16
       vx[i] = true;
       for (int j = 1; j <= n; j++) {</pre>
17
18
         if (lx[i] + ly[j] == G[i][j] && !vy[j]) {
19
            vy[j] = true;
20
            if (!my[j] || match(my[j])) {
21
              my[j] = i;
22
              return true;
            }
23
24
         }
       }
25
26
       return false;
27
     void update() {
28
29
       int delta = INF;
       for (int i = 1; i <= n; i++) {</pre>
30
31
         if (vx[i]) {
            for (int j = 1; j <= n; j++) {</pre>
32
33
              if (!vy[j]) {
                delta = min(delta, lx[i] + ly[j] -
34
                     G[i][j]);
35
           }
36
         }
37
38
39
       for (int i = 1; i <= n; i++) {
40
          if (vx[i]) {
           lx[i] -= delta;
41
42
         if (vy[i]) {
43
44
            ly[i] += delta;
45
       }
46
47
     }
     int run() {
48
49
       for (int i = 1; i <= n; i++) {
50
         lx[i] = ly[i] = my[i] = 0;
51
          for (int j = 1; j <= n; j++) {
52
            lx[i] = max(lx[i], G[i][j]);
53
         }
54
       for (int i = 1; i <= n; i++) {</pre>
55
56
         while (true) {
57
            for (int i = 1; i <= n; i++) {
              vx[i] = vy[i] = 0;
58
59
           if (match(i)) {
60
61
              break;
            } else {
62
63
              update();
64
         }
65
66
       int ans = 0;
67
       for (int i = 1; i <= n; i++) {</pre>
68
69
         ans += lx[i] + ly[i];
70
```

# 4.4 Dinic

return ans;

```
1 #define eb emplace_back
  const LL INF = 1e18;
  const int MAXN = ;
  struct Edge {
    int to;
     LL cap;
7
    int rev:
     Edge(int v, LL c, int r) : to(v), cap(c), rev(r) {}
9 };
10 struct Dinic {
11
     int n;
     int level[MAXN], now[MAXN];
12
13
     vector < Edge > G[MAXN];
     void init(int _n) {
14
15
       n = _n;
       for (int i = 0; i <= n; i++) {</pre>
16
17
         G[i].clear();
18
       }
19
20
     void add_edge(int u, int v, LL c) {
       G[u].eb(v, c, G[v].size());
21
       // directed graph
22
23
       G[v].eb(u, 0, G[u].size() - 1);
       // undirected graph
24
25
       // G[v].eb(u, c, G[u].size() - 1);
26
27
     bool bfs(int st, int ed) {
28
       fill(level, level + n + 1, -1);
       queue<int> q;
29
       q.push(st);
30
       level[st] = 0;
31
       while (!q.empty()) {
32
33
         int u = q.front();
34
         a.pop():
35
         for (const auto &e : G[u]) {
           if (e.cap > 0 && level[e.to] == -1) {
36
37
             level[e.to] = level[u] + 1;
38
             q.push(e.to);
39
40
         }
41
42
       return level[ed] != -1;
43
44
     LL dfs(int u, int ed, LL limit) {
       if (u == ed) {
45
         return limit;
46
47
       }
48
       LL ret = 0;
49
       for (int &i = now[u]; i < G[u].size(); i++) {</pre>
         auto &e = G[u][i];
50
51
         if (e.cap > 0 && level[e.to] == level[u] + 1) {
52
           LL f = dfs(e.to, ed, min(limit, e.cap));
53
           ret += f;
54
           limit -= f;
           e.cap -= f;
55
56
           G[e.to][e.rev].cap += f;
57
           if (!limit) {
58
             return ret;
59
         }
60
61
       }
       if (!ret) {
62
63
         level[u] = -1;
64
65
       return ret;
66
     LL flow(int st, int ed) {
67
68
       LL ret = 0;
69
       while (bfs(st, ed)) {
         fill(now, now + n + 1, 0);
```

71

67

ret += dfs(st, ed, INF);

```
72
                                                                69
73
                                                                70
       return ret;
    }
                                                                       return dis[ed] != INF;
74
                                                                71
75 };
                                                                72
                                                               73
                                                                     LL sendFlow(int v, LL curFlow) {
                                                                74
                                                                       if (pre[v] == -1) {
                                                                75
                                                                         return curFlow;
  4.5 MCMF
                                                                76
                                                                77
                                                                       int i = pre[v];
1 // 0-base
                                                                78
                                                                       int u = edges[i].u;
2 const LL INF = 1e18;
                                                                       LL cost = edges[i].cost;
                                                                79
3 const int MAXN = ;
                                                                80
  struct Edge {
                                                                       LL f = sendFlow(u, min(curFlow, edges[i].cap));
                                                                81
    int u, v;
                                                                82
    LL cost;
                                                                       ans_cost += f * cost;
                                                                83
7
    LL cap;
                                                                       edges[i].cap -= f;
                                                                84
8
    Edge(int _u, int _v, LL _c, LL _cap) : u(_u),
                                                                       edges[i ^ 1].cap += f;
                                                                85
         v(_v), cost(_c), cap(_cap) {}
                                                                86
                                                                       return f;
9 };
                                                                87
10 struct MCMF {
                      // inq times
                                                                     pair<LL, LL> run(int st, int ed) {
                                                                88
    int n, pre[MAXN], cnt[MAXN];
11
                                                                89
                                                                       ans_flow = ans_cost = 0;
12
    LL ans_flow, ans_cost, dis[MAXN];
                                                                       while (SPFA(st, ed)) {
                                                                90
13
    bool inq[MAXN];
                                                                91
                                                                         ans_flow += sendFlow(ed, INF);
    vector < int > G[MAXN];
14
                                                                92
15
     vector < Edge > edges;
                                                                93
                                                                       return make_pair(ans_flow, ans_cost);
16
    void init(int _n) {
                                                                94
17
       n = _n;
                                                                95 };
18
       edges.clear();
       for (int i = 0; i < n; i++) {</pre>
19
20
         G[i].clear();
21
       }
                                                                        String
22
    void add_edge(int u, int v, LL c, LL cap) {
23
24
       // directed
                                                                  5.1
                                                                         Manacher
25
       G[u].pb(edges.size());
       edges.eb(u, v, c, cap);
26
                                                                 1 int p[2 * MAXN];
27
       G[v].pb(edges.size());
                                                                2 int Manacher(const string &s) {
       edges.eb(v, u, -c, 0);
28
                                                                     string st = "@#";
29
                                                                     for (char c : s) {
    bool SPFA(int st, int ed) {
30
                                                                       st += c;
31
       for (int i = 0; i < n; i++) {</pre>
                                                                 6
                                                                       st += '#';
32
         pre[i] = -1;
                                                                 7
                                                                     }
         dis[i] = INF;
33
                                                                     st += '$';
                                                                8
         cnt[i] = 0;
34
                                                                     int id = 0, mx = 0, ans = 0;
                                                                9
35
         inq[i] = false;
                                                                     for (int i = 1; i < st.length() - 1; i++) {</pre>
                                                                10
36
                                                                11
                                                                       p[i] = (mx > i ? min(p[2 * id - i], mx - i) : 1);
37
       queue<int> q;
                                                                12
                                                                       for (; st[i - p[i]] == st[i + p[i]]; p[i]++);
       bool negcycle = false;
38
                                                                       if (mx < i + p[i]) {</pre>
                                                                13
39
                                                                         mx = i + p[i];
                                                                14
       dis[st] = 0;
40
                                                                         id = i;
                                                                15
41
       cnt[st] = 1;
                                                                16
                                                                       }
       inq[st] = true;
42
                                                                17
                                                                       ans = max(ans, p[i] - 1);
       q.push(st);
43
                                                                18
44
                                                                19
       while (!q.empty() && !negcycle) {
                                                                     return ans;
45
                                                                20 }
46
         int u = q.front();
47
         q.pop();
         inq[u] = false;
48
49
         for (int i : G[u]) {
                                                                  5.2 Trie
50
           int v = edges[i].v;
51
           LL cost = edges[i].cost;
52
           LL cap = edges[i].cap;
                                                                 1 const int MAXL = ;
                                                                 2 const int MAXC = ;
53
           if (dis[v] > dis[u] + cost && cap > 0) {
54
                                                                 3 struct Trie {
                                                                    int nex[MAXL][MAXC];
             dis[v] = dis[u] + cost;
55
56
             pre[v] = i;
                                                                     int len[MAXL];
             if (!inq[v]) {
57
                                                                6
                                                                     int sz:
58
               q.push(v);
                                                                     void init() {
                                                                       memset(nex, 0, sizeof(nex));
59
               cnt[v]++;
                                                                 8
60
               inq[v] = true;
                                                                9
                                                                       memset(len, 0, sizeof(len));
61
                                                                10
                                                                       sz = 0;
               if (cnt[v] == n + 2) {
                                                                11
62
                  negcycle = true;
                                                                12
                                                                     void insert(const string &str) {
63
64
                  break;
                                                                13
                                                                       int p = 0;
65
                                                                14
                                                                       for (char c : str) {
             }
                                                                         int id = c - 'a';
66
                                                                15
```

if (!nex[p][id]) {

16

```
17
            nex[p][id] = ++sz;
18
19
         p = nex[p][id];
20
21
       len[p] = str.length();
22
     }
     vector<int> find(const string &str, int i) {
23
24
       int p = 0;
       vector<int> ans;
25
       for (; i < str.length(); i++) {</pre>
26
27
          int id = str[i] - 'a';
          if (!nex[p][id]) {
28
29
            return ans;
         }
30
31
          p = nex[p][id];
          if (len[p]) {
32
            ans.pb(len[p]);
33
34
35
       }
36
       return ans;
     }
37
38 };
```

#### 5.3 Z-value

```
1 // 0-base
2 // 對於個長度為 n 的字串 s
3 // 定義函數 z[i] 表示 s 和 s[i, n - 1]
4 | // (即以 s[i] 開頭的後綴)的最長公共前綴 (LCP)的長度
5 | 1/ z [0] = 0
6 vector<int> z_function(string s) {
    int n = (int)s.length();
    vector<int> z(n);
9
    for (int i = 1, l = 0, r = 0; i < n; ++i) {
      if (i <= r && z[i - l] < r - i + 1) {</pre>
10
        z[i] = z[i - 1];
11
12
      } else {
        z[i] = max(0, r - i + 1);
13
        while (i + z[i] < n \&\& s[z[i]] == s[i + z[i]])
14
            ++z[i]:
15
      if (i + z[i] - 1 > r) l = i, r = i + z[i] - 1;
16
    }
17
18
    return z;
19 }
```

# 6 DP

## 6.1 LIS

```
int LIS(vector<int> &a) {
  vector<int> s;
  for (int i = 0; i < a.size(); i++) {
    if (s.empty() || s.back() < a[i]) {
        s.push_back(a[i]);
    } else {
        *lower_bound(s.begin(), s.end(), a[i],
        [](int x, int y) {return x < y;}) = a[i];
    }
  }
  return s.size();
}</pre>
```

### 6.2 LCS

```
int LCS(string s1, string s2) {
  int n1 = s1.size(), n2 = s2.size();
  vector<vector<int>> dp(n1 + 1, vector<int>(n2 + 1,
      0));
  for (int i = 1; i <= n1; i++) {</pre>
```

```
5
       for (int j = 1; j \le n2; j++) {
         if (s1[i - 1] == s2[j - 1]) {
6
7
           dp[i][j] = dp[i - 1][j - 1] + 1;
8
         } else {
9
           dp[i][j] = max(dp[i - 1][j], dp[i][j - 1]);
10
11
12
    }
    return dp[n1][n2];
13
```

# 7 Math

# 7.1 Extended GCD

#### 7.2 Gaussian Elimination

```
1 const double EPS = 1e-6;
  double Gauss(vector<vector<double>> &d) {
    int n = d.size(), m = d[0].size();
     double det = 1;
     for (int i = 0; i < m; i++) {
       int p = -1;
7
       for (int j = i; j < n; j++) {
         if (fabs(d[j][i]) < EPS) {</pre>
8
9
           continue;
10
         if (p == -1 || fabs(d[j][i]) > fabs(d[p][i])) {
11
12
           p = j;
13
14
       if (p == -1) {
15
16
         continue;
17
18
       if (p != i) {
19
         det *= -1;
20
       for (int j = 0; j < m; j++) {
21
         swap(d[p][j], d[i][j]);
22
23
24
       for (int j = 0; j < n; j++) {
         if (i == j) {
25
26
           continue;
27
         double z = d[j][i] / d[i][i];
28
         for (int k = 0; k < m; k++) {
29
           d[j][k] -= z * d[i][k];
30
31
         }
32
33
34
     for (int i = 0; i < n; i++) {
35
       det *= d[i][i];
    }
36
37
    return det;
38 }
```

#### 7.3 Prime Table

```
1 vector<int> p;
2 bitset < MAXN > is_notp;
3 void PrimeTable(int n) {
    is_notp.reset();
     is_notp[0] = is_notp[1] = 1;
     for (int i = 2; i <= n; ++i) {
6
       if (!is_notp[i]) {
7
8
         p.push_back(i);
9
       for (int j = 0; j < (int)p.size(); ++j) {</pre>
10
11
         if (i * p[j] > n) {
12
           break:
13
         }
         is_notp[i * p[j]] = 1;
14
15
         if (i % p[j] == 0) {
16
           break;
17
18
       }
19
     }
20 }
```

### 7.4 Phi

- · 歐拉函數計算對於一個整數 N,小於等於 N 的正整數中,有幾個和 N 互質
- ・ 如果  $gcd(p,q) = 1, \Phi(p) \cdot \Phi(q) = \Phi(p \cdot q)$
- $\Phi(p^k) = p^{k-1} \times (p-1)$

```
1 void phi_table(int n) {
    phi[1] = 1;
2
     for (int i = 2; i <= n; i++) {
      if (phi[i]) {
         continue;
6
7
      for (int j = i; j < n; j += i) {
8
         if (!phi[j]) {
9
           phi[j] = j;
10
         phi[j] = phi[j] / i * (i - 1);
11
12
13
    }
14 }
```

### 7.5 Chinese Remainder Thm

```
1 / / 参数可为负数的扩展欧几里德定理
void exOJLD(int a, int b, int& x, int& y) {
3
   //根据欧几里德定理
    if (b == 0) { //任意数与0的最大公约数为其本身。
      x = 1;
6
      y = 0;
7
    } else {
8
      int x1, y1;
      exOJLD(b, a % b, x1, y1);
9
      if (a * b < 0) { //异号取反
10
11
       x = -y1;
12
       y = a / b * y1 - x1;
      } else { //同号
13
       x = y1;
14
       y = x1 - a / b * y1;
15
16
17
    }
18 }
19 //剩余定理
20 int calSYDL(int a[], int m[], int k) {
    int N[k]; //这个可以删除
21
    int mm = 1; //最小公倍数
22
    int result = 0;
23
    for (int i = 0; i < k; i++) {
24
25
      mm *= m[i];
26
    for (int j = 0; j < k; j++) {
```

```
28
      int L, J;
      exOJLD(mm / m[j], -m[j], L, J);
29
     N[j] = m[j] * J + 1; // 1
30
     N[j] = mm / m[j] * L; // 2
31
         1和2这两个值应该是相等的。
32
     result += N[j] * a[j];
33
    return (result % mm + mm) % mm;
34
35
        mm)之间,这么写是为了防止result初始为负数,本例中不可能为
    //写成:return result%mm;即可。
36
37 }
38
39
  int main() {
   int a[3] = {2, 3, 6}; // a[i]=n%m[i]
40
   int m[3] = \{3, 5, 7\};
    cout << calSYDL(a, m, 3) << endl;</pre>
42
43
    //輸出為滿足兩條陣列的最小n,第3參數為陣列長度
    //所有滿足答案的數字集合為n+gcd(m0,m1,m2...)*k,
       k為正數
    return 0;
45
46 }
```

### 7.6 Josephus

#### 7.7 Catalan

```
C_0 = 1 and C_{n+1} = \frac{2(2n+1)}{n+2}C_n
1 long long f[N] = {1}, i, t, p;
2 int main() {
     for (int i = 1; i <= 100; i++) {</pre>
       f[i] = f[i - 1] * (4 * i - 2) % mod;
       for (t = i + 1, p = mod - 2; p; t = (t * t) %
5
            mod, p >>= 1LL) {
         if (p & 1) {
6
            f[i] *= t;
8
            f[i] %= mod;
9
10
11
     }
12 }
```

# 8 Geometry

#### 8.1 Point

```
1 // notice point type!!!
2 using dvt = int;
3 const double EPS = 1e-6;
4 const double PI = acos(-1);
6 struct Pt {
    dvt x;
8
    dvt y;
9 };
10 bool operator < (const Pt &a, const Pt &b) {
  return a.x == b.x ? a.y < b.y : a.x < b.x;
11
12 }
13 bool operator == (const Pt &a, const Pt &b) {
14
   return a.x == b.x && a.y == b.y;
15 }
16 Pt operator + (const Pt &a, const Pt &b) {
```

```
17
    return {a.x + b.x, a.y + b.y};
18 }
19 Pt operator - (const Pt &a, const Pt &b) {
20
    return {a.x - b.x, a.y - b.y};
21 }
22 // multiply constant
23 Pt operator * (const Pt &a, const dvt c) {
24 return {a.x * c, a.y * c};
25 | }
26 Pt operator / (const Pt &a, const dvt c) {
27
    return {a.x / c, a.y / c};
28 }
29 // |a| x |b| x <math>cos(x)
30 dvt iproduct(const Pt &a, const Pt &b) {
    return a.x * b.x + a.y * b.y;
32 | }
33 // |a| \times |b| \times \sin(x)
34 dvt cross(const Pt &a, const Pt &b) {
    return a.x * b.y - a.y * b.x;
35
36 }
37 dvt dis_pp(const Pt &a, const Pt, &b) {
    dvt dx = a.x - b.x;
38
39
    dvt dy = a.y - b.y;
    return sqrt(dx * dx, dy * dy);
40
41 }
```

#### 8.2 Line

```
d(P, L) = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}
```

```
1 struct Line {
2
   Pt st;
3
    Pt ed;
4 };
  // return point side
6 // left, on line, right -> 1, 0, -1
7 int side(Line 1, Pt a) {
    dvt cross_val = cross(a - 1.st, 1.ed - 1.st);
    if (cross_val > EPS) {
9
10
       return 1;
11
    } else if (cross_val < -EPS) {</pre>
12
       return -1;
13
    } else {
14
       return 0;
15
    }
16 }
17 // AB infinity, CD segment
18 bool has_intersection(Line AB, Line CD) {
19
    int c = side(AB, CD.st);
    int d = side(AB, CD.ed);
20
    if (c == 0 || d == 0) {
21
22
       return true;
23
     } else {
24
       // different side
25
       return c == -d;
    }
26
27 }
28 // find intersection point, two line, not seg
29 pair<int, Pt> intersection(Line a, Line b) {
30
    Pt A = a.ed - a.st;
     Pt B = b.ed - b.st;
31
32
     Pt C = b.st - a.st;
     dvt mom = cross(A, B);
33
34
     dvt son = cross(C, B);
    if (std::abs(mom) <= EPS) {</pre>
35
36
       if (std::abs(son) <= EPS) {</pre>
37
         return {1, {}}; // same line
38
      } else {
39
         return {2, {}}; // parallel
       }
40
    } else {
41
42
       return {0, a.st + A * (son / mom)};
43
44 }
45 // line to point distance
```

```
46 dvt dis_lp(Line 1, Pt a) {
47    return area3x2(l.st, l.ed, a) / dis_pp(l.st, l.ed);
48 }
```

#### 8.3 Area

```
1 // triangle
2 dvt area3(Pt a, Pt b, Pt c) {
    return std::abs(cross(b - a, c - a) / 2);
3
4 }
  dvt area3x2(Pt a, Pt b, Pt c) { // for integer
    return std::abs(cross(b - a, c - a));
7 }
8
  // simple convex area(can in)
9 dvt area(vector<Pt> &a) {
10
    dvt ret = 0;
    for (int i = 0, sz = a.size(); i < sz; i++) {</pre>
      ret += cross(a[i], a[(i + 1) % sz]);
12
13
14
     return std::abs(ret) / 2;
15 }
16 // check point in/out a convex
17 int io_convex(vector<Pt> convex, Pt q) {
     // convex is Counterclockwise
19
     for (int i = 0, sz = convex.size(); i < sz; i++) {</pre>
20
       Pt cur = convex[i] - q;
       Pt nex = convex[(i + 1) % sz] - q;
21
22
       dvt cross_val = cross(cur, nex);
       if (std::abs(cross_val) <= EPS) {</pre>
23
24
         return 0; // on edge
       }
25
       if (cross_val < 0) {</pre>
26
27
         return -1; // outside
28
29
    }
                    // inside
30
    return 1;
31 }
```

## 8.4 Convex Hull

```
1 vector < Pt > convex_hull(vector < Pt > &a) {
    sort(a.begin(), a.end());
     a.erase(unique(a.begin(), a.end()), a.end());
     int sz = a.size(), m = 0;
     vector<Pt> ret(sz + 5); // safe 1 up
     for (int i = 0; i < sz; i++) {
 6
 7
       while (m > 1 &&
         cross(ret[m - 1] - ret[m - 2], a[i] - ret[m -
 8
             2]) <= EPS) {
 9
      }
10
11
       ret[m++] = a[i];
    }
12
     int k = m;
13
14
     for (int i = sz - 2; i >= 0; i--) {
15
       while (m > k &&
16
         cross(ret[m - 1] - ret[m - 2], a[i] - ret[m -
             2]) <= EPS) {
17
         m - - :
       }
18
19
       ret[m++] = a[i];
     }
20
21
     if (sz > 1) {
22
       m - -;
23
24
     ret.resize(m);
25
     return ret;
26 }
```