1.3

Black Magic

//最左邊 > k 的位置

2 upper\_bound(a, a + n, k);

3 | upper\_bound(a, a + n, k) - 1; //最右邊 ≤ k 的位置

## Contents

11

13

14 }

```
1 #include <bits/stdc++.h>
 1 Basic
                                      #include <ext/pb_ds/assoc_container.hpp>
  1.1 Run .
                                      #include <ext/pb_ds/tree_policy.hpp>
  #include <ext/pb_ds/priority_queue.hpp>
  using namespace std;
  using namespace __gnu_pbds;
 2 Data Structure
                                     7
                                      using set_t =
   tree<int, null_type, less<int>, rb_tree_tag,
   tree_order_statistics_node_update>;
  using map_t =
                                     11
                                        tree<int, int, less<int>, rb_tree_tag,
  3.1 Dijkstra
  12
                                         tree_order_statistics_node_update>;
                                    3
                                     13
  using heap t =
                                        __gnu_pbds::priority_queue<int>;
                                     14
                                    3
   using ht_t =
   16
                                        gp_hash_table<int, int>;
            3.7 Directed MST
                                     17
                                      int main() {
   18
                                        //set-----
   19
                                        set t st:
   st.insert(5); st.insert(6);
 4 Flow & Matching
                                     21
                                        st.insert(3); st.insert(1);
   22
   5
                                        // the smallest is (0), biggest is (n-1), kth small
   is (k-1)
   int num = *st.find_by_order(0);
                                        cout << num << '\n'; // print 1
                                     25
  String
                                     26
   num = *st.find_by_order(st.size() - 1);
                                     27
  28
                                        cout << num << '\n'; // print 6
                                    8
                                     29
                                     30
                                        // find the index
  int index = st.order_of_key(6);
                                     31
  cout << index << '\n'; // print 3</pre>
                                     32
 7 Math
                                     33
   7.1 Extended GCD
           34
                                        // check if there exists x
   int x = 5;
                                     35
   36
                                        int check = st.erase(x);
   37
                                        if (check == 0) printf("st not contain 5\n");
  7.5 Chinese Remainder Thm \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots
  else if (check == 1) printf("st contain 5\n");
                                     39
                                     40
                                        //tree policy like set
 8 Geometry
                                        st.insert(5); st.insert(5);
                                     41
  cout << st.size() << '\n'; // print 4</pre>
                                     42
  //map-----
                                     44
                                     45
                                        map_t mp;
                                     46
                                        mp[1] = 2;
    Basic
                                     47
                                        cout << mp[1] << '\n';
                                     48
                                        auto tmp = *mp.find_by_order(0); // pair
                                        cout << tmp.first << " " << tmp.second << ' \setminus n';
                                     49
 1.1 Run
                                     50
                                        //heap------
                                     51
                                        heap_t h1, h2;
                                     52
1 | #use -> sh run.sh {name}
                                        h1.push(1); h1.push(3);
2 g++ -02 -std=c++14 -Wall -Wextra -Wshadow -o $1 $1.cpp
                                        h2.push(2); h2.push(4);
                                     54
3 | ./$1 < t.in > t.out
                                     55
                                        h1.join(h2);
                                        cout << h1.size() << h2.size() << h1.top() << '\n';</pre>
                                     56
                                     57
 1.2 Default
                                     58
                                        //hash-table-----
                                     59
                                        ht_t ht;
                                     60
1 #include <bits/stdc++.h>
                                        ht[85] = 5:
                                     61
2 using namespace std;
                                     62
                                        ht[89975] = 234;
3 using LL = long long;
                                        for (auto i : ht) {
4 #define IOS ios_base::sync_with_stdio(0); cin.tie(0);
                                         cout << i.first << " " << i.second << '\n';</pre>
                                     64
5 #define pb push_back
                                     65
6 #define eb emplace_back
                                     66 }
7 const int INF = 1e9;
8 const int MOD = 1e9 + 7;
9 const double EPS = 1e-6;
                                      1.4 Binary Search
10 const int MAXN = 0;
12 int main() {
                                     1 lower_bound(a, a + n, k);
                                                          //最左邊 ≥ k 的位置
```

```
4 | lower_bound(a, a + n, k) - 1; //最右邊 < k 的位置</td>5 | [lower_bound, upper_bound)//等於 k 的範圍6 | equal_range(a, a + n, k);
```

## 2 Data Structure

# 2.1 Disjoint Set

```
1 // 0-base
2 const int MAXN = 1000;
3 int boss[MAXN];
4 void init(int n) {
    for (int i = 0; i < n; i++) {</pre>
       boss[i] = -1;
    }
7
8 }
9 int find(int x) {
10
   if (boss[x] < 0) {
11
      return x;
12
13
    return boss[x] = find(boss[x]);
14 }
15 bool uni(int a, int b) {
16
    a = find(a);
17
    b = find(b);
18
    if (a == b) {
       return false;
19
20
21
     if (boss[a] > boss[b]) {
22
       swap(a, b);
23
24
     boss[a] += boss[b];
25
     boss[b] = a;
26
     return true;
27 }
```

## 2.2 BIT RARSQ

```
1 // 1-base
2 #define lowbit(k) (k & -k)
3
4 int n;
5 vector<int> B1, B2;
7 void add(vector<int> &tr, int id, int val) {
    for (; id <= n; id += lowbit(id)) {</pre>
9
      tr[id] += val;
    }
10
11 | }
12 void range_add(int 1, int r, int val) {
13
    add(B1, 1, val);
    add(B1, r + 1, -val);
14
    add(B2, 1, val * (1 - 1));
15
16
    add(B2, r + 1, -val * r);
17 }
18 int sum(vector<int> &tr, int id) {
19
    int ret = 0:
    for (; id >= 1; id -= lowbit(id)) {
20
21
      ret += tr[id];
    }
22
23
    return ret;
24 }
25 int prefix_sum(int id) {
    return sum(B1, id) * id - sum(B2, id);
27 }
28 int range_sum(int 1, int r) {
    return prefix_sum(r) - prefix_sum(l - 1);
29
```

# 2.3 zkw RMQ

```
1 // 0-base
  const int INF = 1e9;
  const int MAXN = ;
  int n;
 6 int a[MAXN], tr[MAXN << 1];</pre>
  // !!! remember to call this function
9 void build() {
10
    for (int i = 0; i < n; i++) {
11
      tr[i + n] = a[i];
12
13
     for (int i = n - 1; i > 0; i--) {
14
       tr[i] = max(tr[i << 1], tr[i << 1 | 1]);
15
16 }
17 void update(int id, int val) {
18
    for (tr[id += n] = val; id > 1; id >>= 1) {
      tr[id >> 1] = max(tr[id], tr[id ^ 1]);
19
20
21 }
22
  int query(int 1, int r) { // [1, r)
    int ret = -INF;
23
     for (1 += n, r += n; 1 < r; 1 >>= 1, r >>= 1) {
24
25
       if (1 & 1) {
26
        ret = max(ret, tr[1++]);
27
       if (r & 1) {
28
29
         ret = max(ret, tr[--r]);
30
31
    }
32
     return ret;
33 }
```

# 3 Graph

#### 3.1 Dijkstra

```
1 // 0-base
  const LL INF = 1e18;
  const int MAXN = ;
3
  struct Edge {
    int to:
    LL cost;
     Edge(int v, LL c) : to(v), cost(c) {}
 7
     bool operator < (const Edge &other) const {</pre>
9
      return cost > other.cost;
10
11 };
12
13 int n;
14 LL dis[MAXN];
15 vector < Edge > G[MAXN];
16
17 void init() {
18
    for (int i = 0; i < n; i++) {
19
      G[i].clear();
20
       dis[i] = INF;
21
    }
22 }
23
  void Dijkstra(int st, int ed = -1) {
    priority_queue < Edge > pq;
24
     pq.emplace(st, 0);
26
     dis[st] = 0;
27
     while (!pq.empty()) {
       auto now = pq.top();
28
       pq.pop();
29
       if (now.to == ed) {
30
31
         return;
32
33
       if (now.cost > dis[now.to]) {
34
         continue:
```

# 3.2 SPFA(negative cycle)

```
1 // 0-base
2 const LL INF = 1e18;
3 const int MAXN = ;
4 struct Edge {
    int to;
    LL cost:
     Edge(int v, LL c) : to(v), cost(c) {}
8 };
9
10 int n;
11 LL dis[MAXN];
12 vector < Edge > G[MAXN];
13
14 void init() {
     for (int i = 0; i < n; i++) {</pre>
15
16
       G[i].clear();
17
       dis[i] = INF;
18
     }
19 }
20 bool SPFA(int st) {
     vector < int > cnt(n, 0);
21
22
     vector<bool> inq(n, false);
     queue<int> q;
23
24
     q.push(st);
25
     dis[st] = 0;
26
27
     inq[st] = true;
28
     while (!q.empty()) {
29
       int now = q.front();
30
       q.pop();
       inq[now] = false;
31
32
       for (auto &e : G[now]) {
33
         if (dis[e.to] > dis[now] + e.cost) {
34
            dis[e.to] = dis[now] + e.cost;
35
           if (!inq[e.to]) {
36
              cnt[e.to]++;
              if (cnt[e.to] > n) {
37
                // negative cycle
38
39
                return false;
40
41
              inq[e.to] = true;
42
              q.push(e.to);
43
44
45
       }
46
     }
47
     return true;
```

## 3.3 Floyd Warshall

```
11
        for (int j = 0; j < n; j++) {
          G[i][j] = INF;
12
13
        G[i][i] = 0;
14
15
16 }
17
   void floyd() {
18
     for (int k = 0; k < n; k++) {
        for (int i = 0; i < n; i++) {
19
          for (int j = 0; j < n; j++) {
  if (G[i][k] != INF && G[k][j] != INF) {</pre>
20
21
               G[i][j] = min(G[i][j], G[i][k] + G[k][j]);
22
23
24
          }
25
     }
26
27 }
```

# 3.4 Topological Sort

```
1 // 0-base
  // if ret.size < n -> cycle
  int n;
  vector<vector<int>> G;
6
  vector<int> topoSort() {
     vector<int> indeg(n), ret;
7
     for (auto &li : G) {
 9
       for (int x : li) {
10
         ++indeg[x];
11
     }
12
     // use priority queue for lexic. largest ans
13
14
     queue < int > q;
     for (int i = 0; i < n; i++) {</pre>
15
16
       if (!indeg[i]) {
17
         q.push(i);
18
19
     }
20
     while (!q.empty()) {
21
      int u = q.front();
       q.pop();
22
23
       ret.pb(u);
       for (int v : G[u]) {
24
25
         if (--indeg[v] == 0) {
           q.push(v);
26
27
28
29
     }
30
     return ret;
31 }
```

# 3.5 Kosaraju SCC

```
1 // 0-base
2 int n;
  vector<vector<int>> G, G2; // G2 = G rev
  vector<bool> vis;
  vector<int> s, color;
6 int sccCnt;
7
  void dfs1(int u) {
    vis[u] = true;
    for (int v : G[u]) {
10
      if (!vis[v]) {
         dfs1(v);
11
12
      }
    }
13
    s.pb(u);
14
15 }
16 void dfs2(int u) {
17
    color[u] = sccCnt;
18
    for (int v : G2[u]) {
      if (!color[v]) {
```

19

20

21

22

23

24

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72 73

74 75

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79 80

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91

92

93

94

return ret;

```
20
          dfs2(v);
       }
21
22
     }
23 }
24 void Kosaraju() {
     sccCnt = 0;
25
     for (int i = 0; i < n; i++) {
26
27
       if (!vis[i]) {
         dfs1(i);
28
29
30
     }
     for (int i = n - 1; i \ge 0; i - -) {
31
32
       if (!color[s[i]]) {
33
          ++sccCnt:
34
          dfs2(s[i]);
       }
35
     }
36
37 }
```

#### 3.6 Tree Diameter

```
1 // 0-base;
2 const int MAXN = ;
3
4 struct Edge {
    int to;
6
    int cost;
    Edge(int v, int c) : to(v), cost(c) {}
8 };
9
10 | int n, d = 0;
11 int d1[MAXN], d2[MAXN];
12 vector < Edge > G[MAXN];
13 // dfs(0, -1);
14 void dfs(int u, int from) {
     d1[u] = d2[u] = 0;
15
     for (auto e : G[u]) {
16
       if (e.to == from) {
17
18
         continue;
19
20
       dfs(e.to, u);
       int t = d1[e.to] + e.cost;
21
       if (t > d1[u]) {
22
23
         d2[u] = d1[u];
         d1[u] = t;
24
25
       } else if (t > d2[u]) {
         d2[u] = t;
26
27
    }
28
29
     d = max(d, d1[u] + d2[u]);
30 }
```

## 3.7 Directed MST

```
1 // 0-base
2 const LL INF = 1e18;
  const int MAXN = ;
3
5 struct Edge {
6
    int from;
7
     int to;
8
     LL cost;
     Edge(int u, int v, LL c) : from(u), to(v), cost(c)
9
         {}
10 };
11
12 struct DMST {
13
    int n;
     int vis[MAXN], pre[MAXN], id[MAXN];
14
    LL in[MAXN];
15
16
     vector < Edge > edges;
17
     void init(int _n) {
18
      n = _n;
```

```
edges.clear();
void add_edge(int from, int to, LL cost) {
  edges.eb(from, to, cost);
LL run(int root) {
 LL ret = 0;
  while (true) {
    for (int i = 0; i < n; i++) {</pre>
      in[i] = INF;
    // find in edge
    for (auto &e : edges) {
      if (e.cost < in[e.to] && e.from != e.to) {</pre>
        pre[e.to] = e.from;
        in[e.to] = e.cost;
      }
    }
    // check in edge
    for (int i = 0; i < n; i++) {
      if (i == root) {
        continue;
     if (in[i] == INF) {
        return -1;
      }
    int nodenum = 0;
    memset(id, -1, sizeof(id));
    memset(vis, -1, sizeof(vis));
    in[root] = 0;
    // find cycles
    for (int i = 0; i < n; i++) {
      ret += in[i];
      int v = i;
      while (vis[v] != i && id[v] == -1 && v !=
          root) {
        vis[v] = i;
        v = pre[v];
      if (id[v] == -1 && v != root) {
        for (int j = pre[v]; j != v; j = pre[j]) {
          id[j] = nodenum;
        id[v] = nodenum++;
      }
    }
    // no cycle
    if (nodenum == 0) {
     break;
    for (int i = 0; i < n; i++) {
      if (id[i] == -1) {
        id[i] = nodenum++;
      }
    }
    // grouping the vertices
    for (auto &e : edges) {
      int to = e.to;
      e.from = id[e.from];
      e.to = id[e.to];
      if (e.from != e.to) {
        e.cost -= in[to]; //!!!
      }
    }
    n = nodenum:
    root = id[root];
 }
```

4

```
95 }
96 };
```

#### 3.8 Articulation Point

```
1 int low[MXV], depth[MXV];
2 bool is_cut_vertex[MXV], visit[MXV];
3 vector<int> G[MXV];
5 void dfs(int now, int cur_depth) {
    visit[now] = true;
6
    depth[now] = low[now] = cur_depth;
    int cut_son = 0;
9
    for (auto i : G[now]) {
10
      if (visit[i]) { // ancestor
11
         low[now] = min(low[now], depth[i]);
      } else { // offspring
12
         dfs(i, cur_depth + 1);
13
14
         cut\_son += 1;
         low[now] = min(low[now], low[i]);
15
16
         if (low[i] >= depth[now])
17
           is_cut_vertex[now] = true;
      }
18
19
20
    if (cur_depth == 0)
      is_cut_vertex[now] = (cut_son != 1);
21
22
     return:
23 }
```

# 3.9 Bridge

```
1 int low[MXV], depth[MXV];
2 bool visit[MXV];
3 vector<int> G[MXV];
 4 vector<pair<int, int>> my_cut_edge;
6 void dfs(int now, int cur_depth, int parent) {
    visit[now] = true;
     depth[now] = low[now] = cur_depth;
8
9
     // int cut_son = 0;
     for (auto i : G[now]) {
10
11
       if (i != parent)
12
         continue;
       if (visit[i]) { // ancestor
13
         low[now] = min(low[now], depth[i]);
14
       } else { // offspring
15
16
         dfs(i, cur_depth + 1, now);
17
         // cut_son += 1;
         low[now] = min(low[now], low[i]);
18
19
         if (low[i] > depth[now])
           my_cut_edge.push_bach({now, i});
20
21
    }
22
23
     return:
24 }
25
26 bool is_2_edge_connected(int n) {
27
    int cut_edge = 0;
28
     memset(visit, 0, sizeof(visit));
29
     dfs(1, 0, -1);
30
     return my_cut_edge.size() == 0;
31 }
```

#### 3.10 BCC

```
typedef pair<int, int> PII;
int low[MXV], depth[MXV];
bool is_cut_vertex[MXV], visit[MXV];
vector<int> G[MXV];
vector<PII> BCC[MXV];
int bcc_cnt = 0;
```

```
7 stack<PII> st;
  void dfs(int now, int cur_depth, int f) {
     visit[now] = true;
10
11
     depth[now] = low[now] = cur_depth;
12
     int cut_son = 0;
     for (auto i : G[now]) {
13
14
       if (i == f) continue;
15
       if (visit[i]) { // ancestor
         if (depth[i] < depth[now]) {</pre>
16
17
           low[now] = min(low[now], depth[i]);
18
           st.push({now, i});
19
         }
20
       } else { // offspring
21
         st.push({now, i});
         dfs(i, cur_depth + 1, now);
22
23
         cut_son += 1;
24
         low[now] = min(low[now], low[i]);
25
         if (low[i] >= depth[now]) {
26
           is_cut_vertex[now] = true;
27
           auto t = st.top();
28
           st.pop();
           while (t != make_pair(now, i)) {
29
30
             BCC[bcc_cnt].push_back(t);
31
             t = st.top();
32
             st.pop();
33
34
           BCC[bcc_cnt].push_back(t);
35
           ++bcc cnt:
36
37
38
    if (cur_depth == 0)
39
40
       is_cut_vertex[now] = (cut_son != 1);
41
42 }
```

# 4 Flow & Matching

#### 4.1 Relation

```
1 | 1. 一般圖
2 | |最大匹配| + |最小邊覆蓋| = |V|
3 | |最大獨立集| + |最小點覆蓋| = |V|
4 |最大圖| = |補圖的最大獨立集|
5 | 2. 二分圖
6 | |最大匹配| = |最小點覆蓋|
7 | |最大獨立集| = |艮小邊覆蓋|
8 |最大獨立集| = |V| - |最大匹配|
9 |最大圖| = |補圖的最大獨立集|
```

# 4.2 Bipartite Matching

```
1 // 0-base
  const int MAXN = ;
3 int n;
  vector<int> G[MAXN];
5 int vy[MAXN], my[MAXN];
  bool match(int u) {
7
    for (int v : G[u]) {
8
       if (vy[v]) {
10
         continue;
11
       vy[v] = true;
12
      if (my[v] == -1 || match(my[v])) {
13
         my[v] = u;
14
15
         return true;
16
17
    }
    return false;
18
```

```
19 }
                                                                   61
                                                                                  break;
20 int sol() {
                                                                   62
                                                                               } else {
     int cnt = 0;
                                                                                  update();
21
                                                                   63
                                                                               }
22
     memset(my, -1, sizeof(my));
                                                                   64
     for (int i = 0; i < n; i++) {</pre>
                                                                             }
23
                                                                   65
                                                                           }
24
       memset(vy, 0, sizeof(vy));
                                                                   66
       if (match(i)) {
                                                                           int ans = 0;
25
                                                                   67
26
          cnt++;
                                                                   68
                                                                           for (int i = 1; i <= n; i++) {
                                                                             ans += lx[i] + ly[i];
27
                                                                   69
     }
28
                                                                   70
29
     return cnt;
                                                                   71
                                                                           return ans;
30 }
                                                                   72
                                                                        }
                                                                   73 };
```

#### 4.3 KM

```
1 const int INF = 1e9;
2 const int MAXN = ;
3 struct KM { //1-base
     int n, G[MAXN][MAXN];
    int lx[MAXN], ly[MAXN], my[MAXN];
     bool vx[MAXN], vy[MAXN];
7
     void init(int _n) {
8
       n = n:
9
       for (int i = 1; i <= n; i++) {
         for (int j = 1; j \le n; j++) {
10
           G[i][j] = 0;
11
12
       }
13
14
     }
15
     bool match(int i) {
16
       vx[i] = true;
       for (int j = 1; j <= n; j++) {</pre>
17
         if (lx[i] + ly[j] == G[i][j] && !vy[j]) {
18
19
           vy[j] = true;
           if (!my[j] || match(my[j])) {
20
21
              my[j] = i;
              return true;
22
           }
23
         }
24
25
       }
26
       return false;
27
     void update() {
28
29
       int delta = INF;
       for (int i = 1; i <= n; i++) {
30
31
         if (vx[i]) {
            for (int j = 1; j <= n; j++) {</pre>
32
33
              if (!vy[j]) {
                delta = min(delta, lx[i] + ly[j] -
34
                     G[i][j]);
              }
35
           }
36
37
         }
38
39
       for (int i = 1; i <= n; i++) {
40
         if (vx[i]) {
           lx[i] -= delta;
41
42
         }
         if (vy[i]) {
43
44
           ly[i] += delta;
45
         }
46
       }
47
     }
48
     int run() {
49
       for (int i = 1; i <= n; i++) {</pre>
         lx[i] = ly[i] = my[i] = 0;
50
51
         for (int j = 1; j <= n; j++) {</pre>
52
           lx[i] = max(lx[i], G[i][j]);
53
54
       for (int i = 1; i <= n; i++) {</pre>
55
         while (true) {
56
           for (int i = 1; i <= n; i++) {
57
58
              vx[i] = vy[i] = 0;
59
           if (match(i)) {
60
```

#### 4.4 Dinic

```
1 #define eb emplace_back
  const LL INF = 1e18;
  const int MAXN = ;
  struct Edge {
     int to;
    LL cap;
    int rev:
    Edge(int v, LL c, int r) : to(v), cap(c), rev(r) {}
9 };
10
  struct Dinic {
11
     int n;
     int level[MAXN], now[MAXN];
12
13
     vector<Edge> G[MAXN];
     void init(int _n) {
14
15
       n = _n;
       for (int i = 0; i <= n; i++) {</pre>
16
17
         G[i].clear();
18
19
     }
20
     void add_edge(int u, int v, LL c) {
21
       G[u].eb(v, c, G[v].size());
       // directed graph
22
       G[v].eb(u, 0, G[u].size() - 1);
23
24
       // undirected graph
25
       // G[v].eb(u, c, G[u].size() - 1);
26
    }
27
     bool bfs(int st, int ed) {
28
       fill(level, level + n + 1, -1);
       queue<int> q;
29
30
       q.push(st);
       level[st] = 0;
31
32
       while (!q.empty()) {
33
         int u = q.front();
34
         q.pop();
35
         for (const auto &e : G[u]) {
           if (e.cap > 0 && level[e.to] == -1) {
36
37
             level[e.to] = level[u] + 1;
             q.push(e.to);
38
39
           }
         }
40
41
42
       return level[ed] != -1;
43
44
     LL dfs(int u, int ed, LL limit) {
       if (u == ed) {
45
46
         return limit;
47
48
       LL ret = 0;
49
       for (int &i = now[u]; i < G[u].size(); i++) {</pre>
         auto &e = G[u][i];
50
         if (e.cap > 0 && level[e.to] == level[u] + 1) {
           LL f = dfs(e.to, ed, min(limit, e.cap));
52
53
           ret += f;
54
           limit -= f;
           e.cap -= f;
55
           G[e.to][e.rev].cap += f;
56
57
           if (!limit) {
58
             return ret;
59
           }
         }
60
```

```
61
                                                                58
                                                                                q.push(v);
       if (!ret) {
62
                                                                59
                                                                               cnt[v]++;
         level[u] = -1;
                                                                                inq[v] = true;
63
                                                                60
64
                                                                61
65
       return ret;
                                                                62
                                                                               if (cnt[v] == n + 2) {
66
                                                                63
                                                                                  negcycle = true;
     LL flow(int st, int ed) {
67
                                                                64
                                                                                  break:
68
       LL ret = 0;
                                                                65
                                                                               }
       while (bfs(st, ed)) {
                                                                66
                                                                             }
69
70
         fill(now, now + n + 1, 0);
                                                                67
71
         ret += dfs(st, ed, INF);
                                                                68
                                                                         }
72
                                                                69
73
                                                                70
       return ret;
    }
                                                                71
                                                                       return dis[ed] != INF;
74
75 };
                                                                72
                                                                     LL sendFlow(int v, LL curFlow) {
                                                                73
                                                                74
                                                                       if (pre[v] == -1) {
                                                                75
                                                                         return curFlow;
  4.5 MCMF
                                                                76
                                                                77
                                                                       int i = pre[v];
1 // 0-base
                                                                       int u = edges[i].u;
                                                                78
2 const LL INF = 1e18;
                                                                79
                                                                       LL cost = edges[i].cost;
  const int MAXN = ;
                                                                80
4 struct Edge {
                                                                81
                                                                       LL f = sendFlow(u, min(curFlow, edges[i].cap));
    int u, v;
                                                                82
    LL cost;
                                                                       ans_cost += f * cost;
                                                                83
7
    LL cap;
                                                                       edges[i].cap -= f;
     Edge(int _u, int _v, LL _c, LL _cap) : u(_u),
                                                                       edges[i ^ 1].cap += f;
                                                                85
         v(_v), cost(_c), cap(_cap) {}
                                                                86
                                                                       return f;
9 };
                                                                87
10 struct MCMF {
                      // inq times
                                                                     pair<LL, LL> run(int st, int ed) {
                                                                88
    int n, pre[MAXN], cnt[MAXN];
11
                                                                89
                                                                       ans_flow = ans_cost = 0;
12
     LL ans_flow, ans_cost, dis[MAXN];
                                                                90
                                                                       while (SPFA(st, ed)) {
13
     bool inq[MAXN];
                                                                91
                                                                         ans_flow += sendFlow(ed, INF);
     vector<int> G[MAXN];
14
                                                                92
15
     vector < Edge > edges;
                                                                93
                                                                       return make_pair(ans_flow, ans_cost);
     void init(int _n) {
16
                                                                94
                                                                    }
17
       n = _n;
                                                                95 };
       edges.clear();
18
       for (int i = 0; i < n; i++) {
19
20
         G[i].clear();
21
       }
                                                                        String
22
     void add_edge(int u, int v, LL c, LL cap) {
23
                                                                         Manacher
24
25
       G[u].pb(edges.size());
       edges.eb(u, v, c, cap);
26
                                                                 1 int p[2 * MAXN];
27
       G[v].pb(edges.size());
                                                                  int Manacher(const string &s) {
28
       edges.eb(v, u, -c, 0);
                                                                     string st = "@#";
29
                                                                    for (char c : s) {
     bool SPFA(int st, int ed) {
30
                                                                       st += c;
31
       for (int i = 0; i < n; i++) {</pre>
                                                                 6
                                                                       st += '#';
         pre[i] = -1;
32
                                                                7
                                                                     }
         dis[i] = INF;
33
                                                                     st += '$';
                                                                 8
         cnt[i] = 0;
34
                                                                     int id = 0, mx = 0, ans = 0;
                                                                9
         inq[i] = false;
35
                                                                10
                                                                     for (int i = 1; i < st.length() - 1; i++) {</pre>
36
                                                                       p[i] = (mx > i ? min(p[2 * id - i], mx - i) : 1);
                                                                11
37
       queue<int> q;
                                                                12
                                                                       for (; st[i - p[i]] == st[i + p[i]]; p[i]++);
       bool negcycle = false;
38
                                                                13
                                                                       if (mx < i + p[i]) {</pre>
39
                                                                         mx = i + p[i];
                                                                14
40
       dis[st] = 0;
                                                                15
                                                                         id = i;
41
       cnt[st] = 1;
                                                                16
       inq[st] = true;
42
                                                                17
                                                                       ans = max(ans, p[i] - 1);
43
       q.push(st);
                                                                18
44
                                                                19
                                                                     return ans;
45
       while (!q.empty() && !negcycle) {
                                                                20 }
46
         int u = q.front();
47
         q.pop();
48
         inq[u] = false;
49
         for (int i : G[u]) {
                                                                  5.2 Trie
50
           int v = edges[i].v;
51
           LL cost = edges[i].cost;
           LL cap = edges[i].cap;
                                                                 1 const int MAXL = ;
52
                                                                 2 const int MAXC = ;
54
           if (dis[v] > dis[u] + cost && cap > 0) {
                                                                 3 struct Trie {
55
             dis[v] = dis[u] + cost;
                                                                    int nex[MAXL][MAXC];
56
             pre[v] = i;
                                                                     int len[MAXL];
```

int sz:

if (!inq[v]) {

57

```
void init() {
       memset(nex, 0, sizeof(nex));
8
       memset(len, 0, sizeof(len));
9
10
11
12
     void insert(const string &str) {
       int p = 0;
13
14
       for (char c : str) {
         int id = c - 'a';
15
         if (!nex[p][id]) {
16
17
           nex[p][id] = ++sz;
18
19
         p = nex[p][id];
20
21
       len[p] = str.length();
     }
22
     vector<int> find(const string &str, int i) {
23
24
       int p = 0;
       vector<int> ans;
25
26
       for (; i < str.length(); i++) {</pre>
         int id = str[i] - 'a';
27
28
         if (!nex[p][id]) {
29
           return ans;
         }
30
         p = nex[p][id];
31
         if (len[p]) {
32
           ans.pb(len[p]);
33
34
35
       }
36
       return ans;
37
38 };
```

#### 5.3 Z-value

```
1 // 0-base
2 // 對於個長度為 n 的字串 s
3 // 定義函數 z[i] 表示 s 和 s[i, n - 1]
4|//(即以 s[i] 開頭的後綴)的最長公共前綴(LCP)的長度
5 // z[0] = 0 \circ
6 vector<int> z_function(string s) {
    int n = (int)s.length();
    vector<int> z(n);
    for (int i = 1, l = 0, r = 0; i < n; ++i) {
10
      if (i <= r && z[i - l] < r - i + 1) {</pre>
11
        z[i] = z[i - 1];
12
      } else {
13
        z[i] = max(0, r - i + 1);
        while (i + z[i] < n \&\& s[z[i]] == s[i + z[i]])
14
            ++z[i];
15
      if (i + z[i] - 1 > r) l = i, r = i + z[i] - 1;
16
17
    }
18
    return z;
19 }
```

## 6 DP

# 6.1 LIS

```
1 int LIS(vector<int> &a) {
2
    vector<int> s;
3
    for (int i = 0; i < a.size(); i++) {</pre>
       if (s.empty() || s.back() < a[i]) {</pre>
         s.push_back(a[i]);
         *lower_bound(s.begin(), s.end(), a[i],
8
           [](int x, int y) {return x < y;}) = a[i];
9
      }
    }
10
11
    return s.size();
```

#### 6.2 LCS

```
1 int LCS(string s1, string s2) {
    int n1 = s1.size(), n2 = s2.size();
    vector<vector<int>> dp(n1 + 1, vector<int>(n2 + 1,
         0)):
    for (int i = 1; i <= n1; i++) {
5
      for (int j = 1; j <= n2; j++) {
        if (s1[i - 1] == s2[j - 1]) {
6
7
           dp[i][j] = dp[i - 1][j - 1] + 1;
        } else {
8
           dp[i][j] = max(dp[i - 1][j], dp[i][j - 1]);
10
        }
11
12
    }
13
    return dp[n1][n2];
```

#### 7 Math

#### 7.1 Extended GCD

```
1 // ax + by = c
  int extgcd(int a, int b, int c, int &x, int &y) {
    if (b == 0) {
      x = c / a;
5
      y = 0;
6
      return a;
7
    }
    int d = extgcd(b, a % b, c, y, x);
8
    y -= (a / b) * x;
10
    return d;
11 }
```

#### 7.2 Gaussian Elimination

```
1 const double EPS = 1e-6;
  double Gauss(vector<vector<double>> &d) {
     int n = d.size(), m = d[0].size();
     double det = 1;
5
     for (int i = 0; i < m; i++) {
6
       int p = -1;
       for (int j = i; j < n; j++) {
         if (fabs(d[j][i]) < EPS) {</pre>
9
           continue;
10
         if (p == -1 || fabs(d[j][i]) > fabs(d[p][i])) {
11
           p = j;
12
         }
13
14
15
       if (p == -1) {
16
         continue;
17
18
       if (p != i) {
         det *= -1;
19
20
       for (int j = 0; j < m; j++) {
21
22
         swap(d[p][j], d[i][j]);
23
24
       for (int j = 0; j < n; j++) {
25
         if (i == j) {
26
           continue;
27
         double z = d[j][i] / d[i][i];
28
29
         for (int k = 0; k < m; k++) {
30
           d[j][k] -= z * d[i][k];
31
32
33
     }
34
     for (int i = 0; i < n; i++) {</pre>
35
       det *= d[i][i];
36
```

```
7.3 Prime Table
```

return det;

37

38 }

```
1 vector<int> p;
2 bitset < MAXN > is_notp;
3 void PrimeTable(int n) {
     is_notp.reset();
     is_notp[0] = is_notp[1] = 1;
     for (int i = 2; i <= n; ++i) {</pre>
6
       if (!is_notp[i]) {
         p.push_back(i);
9
10
       for (int j = 0; j < (int)p.size(); ++j) {</pre>
11
         if (i * p[j] > n) {
12
           break;
13
         is_notp[i * p[j]] = 1;
14
15
         if (i % p[j] == 0) {
16
           break;
17
18
       }
19
     }
20 }
```

#### 7.4 Phi Table

• 歐拉函數計算對於一個整數 N,小於等於 N 的正整數中,有幾個和 N 互質

```
・ 如果 \gcd(p,q)=1,\Phi(p)\cdot\Phi(q)=\Phi(p\cdot q)
```

```
• \Phi(p^k) = p^{k-1} \times (p-1)
```

```
1 void phi_table(int n) {
    phi[1] = 1;
     for (int i = 2; i <= n; i++) {
       if (phi[i]) {
         continue;
6
7
       for (int j = i; j < n; j += i) {
8
         if (!phi[j]) {
9
          phi[j] = j;
10
11
         phi[j] = phi[j] / i * (i - 1);
12
13
    }
14 }
```

#### 7.5 Chinese Remainder Thm

```
1 / / 参数可为负数的扩展欧几里德定理
void exOJLD(int a, int b, int& x, int& y) {
    //根据欧几里德定理
    if (b == 0) { //任意数与0的最大公约数为其本身。
5
     x = 1;
     y = 0;
6
    } else {
8
      int x1, y1;
      exOJLD(b, a % b, x1, y1);
9
      if (a * b < 0) { //异号取反
10
       x = -y1;
11
12
       y = a / b * y1 - x1;
      } else { //同号
13
       x = y1;
14
15
       y = x1 - a / b * y1;
16
17
   }
18 }
19 //剩余定理
20 int calSYDL(int a[], int m[], int k) {
```

```
//这个可以刪除
21
    int N[k];
    int mm = 1; //最小公倍数
22
    int result = 0;
23
    for (int i = 0; i < k; i++) {
25
      mm *= m[i];
26
    for (int j = 0; j < k; j++) {
27
      int L, J;
28
29
      exOJLD(mm / m[j], -m[j], L, J);
      N[j] = m[j] * J + 1; // 1

N[j] = mm / m[j] * L; // 2
30
31
          1和2这两个值应该是相等的。
32
      result += N[j] * a[j];
33
34
    return (result % mm + mm) % mm;
35
    //落在(0,
        mm)之间,这么写是为了防止result初始为负数,本例中不可能为
36
    //写成: return result%mm;即可。
37 }
38
39 int main() {
   int a[3] = {2, 3, 6}; // a[i]=n%m[i]
40
41
    int m[3] = \{3, 5, 7\};
    cout << calSYDL(a, m, 3) << endl;
42
    //輸出為滿足兩條陣列的最小n,第3參數為陣列長度
43
    //所有滿足答案的數字集合為n+gcd(m0, m1, m2...)*k,
44
        k為正數
45
    return 0;
46 }
```

## 7.6 Josephus

#### 7.7 Catalan

```
C_0 = 1 and C_{n+1} = \frac{2(2n+1)}{n+2}C_n
1 long long f[N] = {1}, i, t, p;
  int main() {
    for (int i = 1; i \le 100; i++) {
3
       f[i] = f[i - 1] * (4 * i - 2) % mod;
       for (t = i + 1, p = mod - 2; p; t = (t * t) %
           mod, p >>= 1LL) {
         if (p & 1) {
7
           f[i] *= t;
8
           f[i] %= mod;
9
         }
10
11
    }
12
```

# 8 Geometry

#### 8.1 Point

```
1 // notice point type!!!
2 using dvt = int;
3 const double EPS = 1e-6;
4 const double PI = acos(-1);
5
6 struct Pt {
7 dvt x;
8 dvt y;
9 };
```

```
10 bool operator < (const Pt &a, const Pt &b) {
   return a.x == b.x ? a.y < b.y : a.x < b.x;</pre>
11
12 }
13 bool operator == (const Pt &a, const Pt &b) {
14
    return a.x == b.x && a.y == b.y;
15 }
16 Pt operator + (const Pt &a, const Pt &b) {
17 return {a.x + b.x, a.y + b.y};
18 | }
19 Pt operator - (const Pt &a, const Pt &b) {
20
   return {a.x - b.x, a.y - b.y};
21 }
22 // multiply constant
23 Pt operator * (const Pt &a, const dvt c) {
    return {a.x * c, a.y * c};
25 }
26 Pt operator / (const Pt &a, const dvt c) {
27 return {a.x / c, a.y / c};
28 | }
29 // |a| x |b| x <math>cos(x)
30 dvt iproduct(const Pt &a, const Pt &b) {
    return a.x * b.x + a.y * b.y;
31
32 }
33 // |a| \times |b| \times \sin(x)
34 dvt cross(const Pt &a, const Pt &b) {
35
   return a.x * b.y - a.y * b.x;
36 }
37 dvt dis_pp(const Pt &a, const Pt, &b) {
38
   dvt dx = a.x - b.x;
    dvt dy = a.y - b.y;
39
    return sqrt(dx * dx, dy * dy);
40 l
41 }
```

# 8.2 Line

```
1 struct Line {
   Pt st:
2
    Pt ed;
4 };
5 // return point side
6 // left, on line, right -> 1, 0, -1
7 int side(Line 1, Pt a) {
    dvt cross_val = cross(a - 1.st, 1.ed - 1.st);
9
    if (cross_val > EPS) {
10
      return 1;
11
     } else if (cross_val < -EPS) {</pre>
      return -1;
12
13
    } else {
14
       return 0;
15
16 }
17 // AB infinity, CD segment
18 bool has_intersection(Line AB, Line CD) {
    int c = side(AB, CD.st);
19
20
     int d = side(AB, CD.ed);
    if (c == 0 || d == 0) {
21
      return true;
22
23
    } else {
      // different side
24
25
       return c == -d;
26
    }
27 }
28 // find intersection point, two line, not seg
29 pair<int, Pt> intersection(Line a, Line b) {
30
    Pt A = a.ed - a.st;
     Pt B = b.ed - b.st;
31
     Pt C = b.st - a.st;
32
33
    dvt mom = cross(A, B);
     dvt son = cross(C, B);
34
35
     if (std::abs(mom) <= EPS) {</pre>
      if (std::abs(son) <= EPS) {</pre>
36
        return {1, {}}; // same line
37
38
       } else {
39
         return {2, {}}; // parallel
       }
40
     } else {
                          // ok
```

#### 8.3 Area

```
1 // triangle
2 dvt area3(Pt a, Pt b, Pt c) {
3
  return std::abs(cross(b - a, c - a) / 2);
4 }
5 dvt area3x2(Pt a, Pt b, Pt c) { // for integer
   return std::abs(cross(b - a, c - a));
7 }
8 // simple convex area(can in)
9 dvt area(vector<Pt> &a) {
    dvt ret = 0;
10
11
    for (int i = 0, sz = a.size(); i < sz; i++) {
12
      ret += cross(a[i], a[(i + 1) % sz]);
    }
13
14
    return std::abs(ret) / 2;
15 }
16 // check point in/out a convex
17 int io_convex(vector<Pt> convex, Pt q) {
    // convex is Counterclockwise
18
    for (int i = 0, sz = convex.size(); i < sz; i++) {</pre>
19
      Pt cur = convex[i] - q;
20
21
      Pt nex = convex[(i + 1) \% sz] - q;
22
      dvt cross_val = cross(cur, nex);
      if (std::abs(cross_val) <= EPS) {</pre>
23
24
         return 0; // on edge
25
26
      if (cross_val < 0) {</pre>
27
         return -1; // outside
28
29
    }
                    // inside
30
    return 1;
```

#### 8.4 Convex Hull

```
1 vector <Pt> convex_hull(vector <Pt> &a) {
     sort(a.begin(), a.end());
 3
     a.erase(unique(a.begin(), a.end()), a.end());
     int sz = a.size(), m = 0;
     vector<Pt> ret(sz + 5); // safe 1 up
     for (int i = 0; i < sz; i++) {</pre>
6
 7
       while (m > 1 &&
         cross(ret[m - 1] - ret[m - 2], a[i] - ret[m -
 8
            2]) <= EPS) {
a
         m - - :
       }
10
11
       ret[m++] = a[i];
    }
12
13
     int k = m;
14
     for (int i = sz - 2; i >= 0; i--) {
       while (m > k \&\&
15
16
         cross(ret[m - 1] - ret[m - 2], a[i] - ret[m -
            2]) <= EPS) {
17
       }
18
19
       ret[m++] = a[i];
20
     if (sz > 1) {
21
22
23
     }
24
     ret.resize(m);
25
     return ret;
26 }
```