

A Macro-scale Model of Co-evolution for Cities and Transportation Networks

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Systems of Cities and Transportation Networks

Co-evolution

Model : Rationale

- Cities represented by their population follow deterministic growth based on self growth (Gibrat) and interactions with other cities (similar to [?])
- Drivers of network growth are flow demands

Generic Model

Model : Abstract Network

Model : Physical Network

Results

Case studies

Conclusion

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- All code and data available at

<https://github.com/JusteRaimbault/CityNetwork/tree/master/Models/MacroCoevo>

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Model Formalization : Interactions

→ Work under Gibrat independence assumptions, i.e. $\text{Cov}[P_i(t), P_j(t)] = 0$.
If $\vec{P}(t+1) = \mathbf{R} \cdot \vec{P}(t)$ where \mathbf{R} is also independent, then $\mathbb{E}[\vec{P}(t+1)] = \mathbb{E}[\mathbf{R}] \cdot \mathbb{E}[\vec{P}](t)$. Consider expectancies only (higher moments computable similarly)

→ With $\vec{\mu}(t) = \mathbb{E}[\vec{P}(t)]$, we generalize this approach by taking $\vec{\mu}(t+1) = f(\vec{\mu}(t))$

Model Formalization : Interactions

Let $\vec{\mu}(t) = \mathbb{E}[\vec{P}(t)]$ cities population and (d_{ij}) distance matrix

Model specified by

$$f(\vec{\mu}) = r_0 \cdot \text{Id} \cdot \vec{\mu} + \mathbf{G} \cdot \mathbf{1} + \mathbf{N}$$

with

- $G_{ij} = w_G \cdot \frac{V_{ij}}{\langle V_{ij} \rangle}$ and $V_{ij} = \left(\frac{\mu_i \mu_j}{\sum \mu_k^2} \right)^{\gamma_G} \exp(-d_{ij}/d_G)$
- $N_i = w_N \cdot \sum_{kl} \left(\frac{\mu_k \mu_l}{\sum \mu} \right)^{\gamma_N} \exp(-d_{kl,i}/d_N)$ where $d_{kl,i}$ is distance to shortest path between k, l computed with slope impedance ($Z = (1 + \alpha/\alpha_0)^{n_0}$ with $\alpha_0 \simeq 3$)

Model Formalization : Network Growth

Model Formalization : Indicators

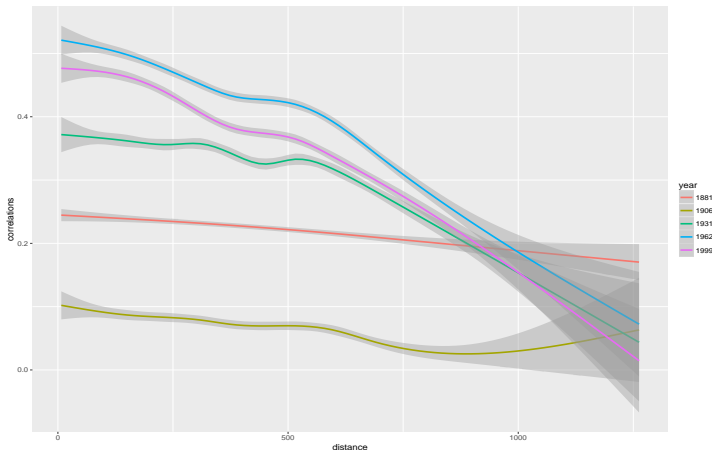
- Initial-final rank correlation (changes in the hierarchy) for variable X : $\rho [X_i(t = 0), X_i(t = t_f)]$
- Trajectory diversity for variable X : with $\tilde{X}_i(t) \in [0; 1]$ rescaled trajectories,

$$\frac{2}{N \cdot (N - 1)} \sum_{i < j} \left(\frac{1}{T} \int_t \left(\tilde{X}_i(t) - \tilde{X}_j(t) \right)^2 \right)^{\frac{1}{2}}$$

Data : stylized facts

Population data for French-cities (Pumain-INED database : 1831-1999)

Non-stationarity of log-returns correlations function of distance



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