
Indirect Evidence of Network Effects in a System of Cities

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Abstract

This paper is the application of a theoretical paper developing a theory of co-evolutive networked territorial systems. We apply simple models of urban growth for systems of cities, which include in particular the role of physical networks.

Keywords

Urban Systems, Urban Growth, Spatial Interactions

1 Introduction

1.1 Modeling Urban Growth

Understanding processes driving urban growth is more crucial than ever, as urban population recently crossed the symbolic proportion of half world population .Future of world economies and sustainability of future societies seem to be deeply interlinked with the dynamics of urban systems.A better knowledge of how cities differentiate, interact and grow is thus a relevant topic both theoretically and for application. Many disciplines have studied models of urban growth with different objectives and taking different aspects into account. Economics still have a difficulty to consider spatial interactions in their models **Krugman (1998)**, whereas geography fails to embrace a certain level of complexity. The example of this two disciplines shows how it is difficult to make bridges, as it needed exceptional minds to translate from one to the other (as P. Hall did for Von Thunen work **Taylor (2016)**)

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1.2 Urban Growth and Spatial Interaction

Bretagnolle et al. (2000) already proposed a spatial extension of the Gibrat model. The gravity-based interaction model that Sanders (1992) use to apply concept of Synergetics to cities (getting indeed out of the scope of synergetics by taking expectancies and getting rid of master equation and the probabilistic formulation of trajectories) is also close to this idea of interdependent urban growth, contained physically in the phenomenon of migration between cities. A more refined extension with economic cycles and innovation waves was developed by Favaro and Pumain (2011), yielding a system dynamics version of the core of Simpop models.

1.3 Urban Growth and Networks

We work on simple territorial systems that are country-wide city systems, and more particularly French cities, on a time scale corresponding to that spatial scale, i.e. two last centuries. Taking into account physical networks can improve the understanding of city growth within that system in two ways : a qualitative one, for which the extended model would exhibit qualitative features corresponding to stylized facts empirically observed but that more basic models do not manage to reproduce, and a quantitative way, in the sense that model extension improves explained variance further than the mechanic improvement due to the introduction of supplementary degrees of freedom. If at least one of these is unveiled in our particular case, the evidence will support the theory at these scale and in this context.

The rest of this paper is organized as follows : our model is introduced and formally described in next section ; we then describe results obtained through exploration and calibration of our model on data for french cities, in particular the unveiling of network effects significantly influencing growth processes. We finally discuss interpretations of these results and implications for planning.

2 Model Description

2.1 From Gibrat to Marius : the dilemma of formulation

Some confusion may arise when surveying at stochastic and deterministic models of urban growth. To what extent is a proposed model “complex” and is the simulation of stochasticity necessary ?

2.2 Model description

We choose to work on a deterministic extension of the Gibrat model, what is equivalent to consider only expectancies in time as explained in the previous subsection. Let $\vec{P}(t) = (P_i(t))_i$ be the population of cities in time. Under Gibrat independence assumptions, we have

$$\text{Cov}[P_i(t), P_j(t)] = 0$$

A linear extended version would write $\vec{P}(t+1) = \mathbf{R} \cdot \vec{P}(t)$ where \mathbf{R} is an independent random matrix of growth rates (identity in the initial case). It yields directly thanks to the independence assumptions that $\mathbb{E}[\vec{P}(t+1)] = \mathbb{E}[\mathbf{R}] \cdot \mathbb{E}[\vec{P}](t)$. We generalize this linear relation to a non-linear relation that allows to be more consistent regarding some thematic considerations, by taking with denoting $\vec{\mu}(t) = \mathbb{E}[\vec{P}(t)]$, the relation $\vec{\mu}(t+1) = f(\vec{\mu}(t))$ (note that in that case, stochastic and deterministic versions are not equivalent anymore*). In our case, we take

$$f(\vec{\mu}) = r_0 \cdot \mathbf{Id} \cdot \vec{\mu} + \mathbf{G}(\vec{\mu}) \cdot \mathbf{1} + \mathbf{N}(\vec{\mu}) \quad (1)$$

with

- $G_{ij} = w_G \cdot \frac{V_{ij}}{\langle V_{ij} \rangle}$ and $V_{ij} = \left(\frac{\mu_i \mu_j}{\sum \mu_k^2} \right)^{\gamma_G} \cdot \exp(-d_{ij}/d_G)$
- $N_i = w_N \cdot \sum_{kl} \left(\frac{\mu_k \mu_l}{\sum \mu} \right)^{\gamma_N} \exp(-d_{kl,i}/d_N)$ where $d_{kl,i}$ is distance to shortest path between k, l computed with slope impedance ($Z = (1 + \alpha/\alpha_0)^{n_0}$ with $\alpha_0 \simeq 3$)

The first component is the pure Gibrat model, that we obtain by setting the weights $w_G = w_N = 0$. The second component captures direct interdependencies between cities, under the form of a separable gravity potential such as the one used in Sanders (1992).

Model Parameter Space

2.3 Data

Population data We work with the Pumain-INED historical database for French Cities, which give populations of Urban Areas (INSEE definition) at time intervals of mostly 5 years, from 1830 to 1995. The latest version of the database, described in integrates the definition of Urban Areas, allowing to follow them on long time-period, according to Bretagnole's long time cities ontology Bretagnolle (2009) (that constructs a definition of cities as evolving entities which boundaries are not fixed in time).

Physical flows As stated above, this modeling exercise focuses on exploring the role of physical flows, whatever the effective shape of the network. We do not need for this reason network data which is furthermore not easily available at different time periods, and physical flows are assumed to take the geographical shortest path that include terrain slope (to avoid geographical absurdities such as cities with a difficult access having an overestimated growth rate). Using the 1km resolution Digital Elevation Model openly available from IGN, we construct an impedance field of the form

$$Z = \left(1 + \frac{\alpha}{\alpha_0} \right)^{n_0}$$

*precisely because of the non-linearity

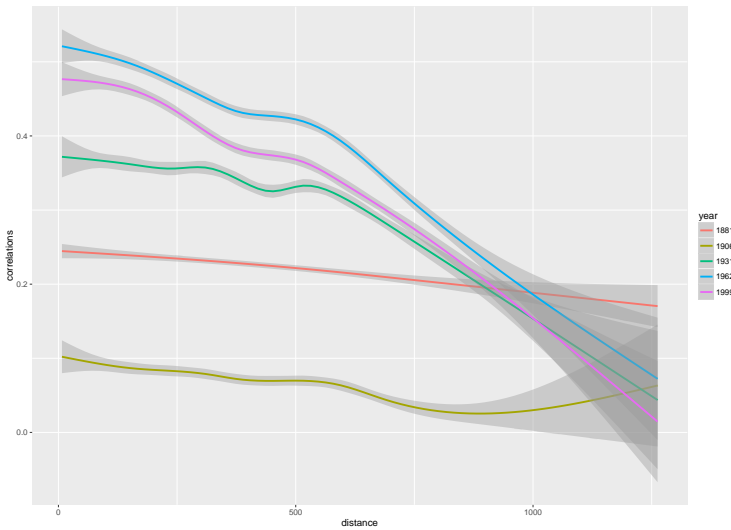


Figure 1. Time-series correlations as a function of distance. Solid line correspond to smoothed correlations, computed between each normalized population time-series, on successive periods.

We took fixed parameter values $\alpha_0 = 3$ (corresponding to approximatively a 5% slope) and $n_0 = 3$. Supplementary Material S1 justifies this choice by investigating the sensibility of paths to these parameters.

A semi-parametrized model Our model is assumed as hybrid as it relies on semi-parametrization on real data. It could be possible to be a full toy-model, initial configuration and physical environment being constructed as synthetic data. As [Raimbault \(2016\)](#) points out, it should even be a step in an extensive study of model, using synthetic data to unveil sensibility of dynamics to meta-parameters defining setup. This enterprise is however out of the scope of this paper, as we aim here to extract advanced stylized facts from a dataset, and we focus therefore on the semi-parametrized version of the model.

2.4 Model Evaluation

We work on an explanatory rather than an exploratory model, and indicators to evaluate model outputs are therefore not linked to a performance of trajectories or obtained final states, but to a distance to phenomenon we want to explain, i.e. the data. We use therefore the following complementary indicators :

- Logarithms of mean-square error
- Mean-square error on logarithms

Figure 2. Example of output of the model. The graphical interface allows to explore interactively on which cities changes operate after a parameter change, what is necessary to interpret raw calibration results.

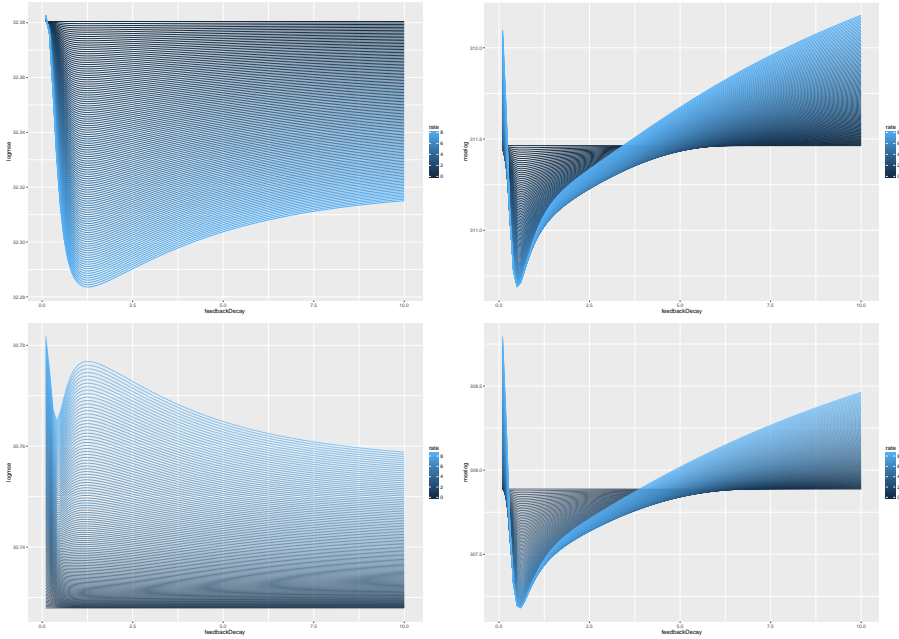


Figure 3. Evidence of network effects revealed by model exploration. Feedback with fixed gravity : first evidences of network effects ; confirmed with effect of α_0

3 Results

3.1 Implementation

Data preprocessing, result processing and models profiling are implemented in R. For performances reasons and an easier integration into the OpenMole software for model exploration described by [Reuillon et al. \(2013\)](#), a scala version was also developed. The typical question of trade-off between implementation performance and interoperability appeared quickly as an issue, as a blind exploration and calibration can difficultly provide useful thematic conclusions for that kind of model. Finding an improvement in model fit among one parameter dimension is significant if the geographical situation is visualized and the improvement is confirmed as reasonable and not an absurdity.

3.2 Model Exploration

3.3 Model Calibration

4 Discussion

We propose to support our hypothesis that *physical transportation networks are necessary to explain the morphogenesis of territorial systems* (aka *Network Necessity*) by showing on a relatively simple case that the integration of physical networks into some models effectively increase their explanative power (being careful on the precise definition of model improvement to avoid overfitting).

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Physical Flows Parametrization

Reproducibility