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Indirect Evidence of Network Effects in a System of Cities

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Abstract

This paper is the application of a theoretical paper developing a theory of co-evolutive networked territorial systems. We apply simple models of urban growth for systems of cities, which include in particular the role of physical networks.

Keywords

Urban Systems, Urban Growth, Spatial Interactions

Introduction

We propose to support our hypothesis that *physical transportation networks are necessary to explain the morphogenesis of territorial systems* (aka *Network Necessity*) by showing on a relatively simple case that the integration of physical networks into some models effectively increase their explanative power (being careful on the precise definition of model improvement to avoid overfitting). We work on simple territorial systems that are country-wide city systems, and more particularly French cities, on a time scale corresponding to that spatial scale, i.e. two last centuries. Taking into account physical networks can improve the understanding of city growth within that system in two ways : a qualitative one, for which the extended model would exhibit qualitative features corresponding to stylized facts empirically observed but that more basic models do not manage to reproduce, and a quantitative way, in the sense that model extension improves explained variance further than the mechanic improvement due to the introduction of supplementary degrees of freedom. If at least one of these is unveiled in our particular case, the evidence will support the theory at these scale and in this context.

? already proposed a spatial extension of the Gibrat model. Later, **?** developed a more refined extension with economic cycles and innovation waves, yielding a system dynamics version of the core of Simpop models.

Model Description

From Gibrat to Marius : the dilemma of formulation

Model description

We choose to work on a deterministic extension of the Gibrat model, what is equivalent to consider only expectancies in time as detailed before. Let $\vec{P}(t) = (P_i(t))_i$ be the population of cities in time. Under Gibrat independence assumptions, we have $\text{Cov}[P_i(t), P_j(t)] = 0$. If $\vec{P}(t+1) = \mathbf{R} \cdot \vec{P}(t)$ where $\mathbf{R}$ is also independent, then $\mathbb{E}\left[\vec{P}(t+1)\right] = \mathbf{R} \cdot \mathbb{E}\left[\vec{P}\right](t)$. With $\vec{\mu}(t) = \mathbb{E}\left[\vec{P}(t)\right]$, we generalize this approach by taking $\vec{\mu}(t+1) = f(\vec{\mu}(t))$. In our case, we take

$$f(\vec{\mu}) = r_0 \cdot \mathbf{Id} \cdot \vec{\mu} + \mathbf{G} \cdot \mathbf{1} + \mathbf{N} \cdot$$

with

- $G_{ij} = w_G \cdot \frac{V_{ij}}{<V_{ij}>}$ and $V_{ij} = \left(\frac{\mu_i \mu_j}{\sum \mu_k{}^2}\right)^{\gamma_G} \exp\left(-d_{ij}/d_G\right)$
- $N_i = w_N \cdot \sum_{kl} \left(\frac{\mu_k \mu_l}{\sum \mu}\right)^{\gamma_N} \exp\left(-d_{kl,i}\right)/d_N$ where $d_{kl,i}$ is distance to shortest path between k, l computed with slope impedance ($Z = (1 + \alpha/\alpha_0)^{n_0}$ with $\alpha_0 \simeq 3$)

The first component is the pure Gibrat model, that we obtain by setting the weights $w_G = w_N = 0$. The second component captures interdependencies between

Results

Data

Population data

Physical flows As stated above, this modeling exercise focuses on exploring the role of physical flows, whatever the effective shape of the network. We do not need for this reason network data which is furthermore not easily available at different time periods, and physical flows are assumed to take the geographical shortest path that include terrain slope (to avoid geographical absurdities such as cities with a difficult access having an overestimated growth rate). Using the 1km resolution Digital Elevation Model openly available from IGN , we construct an impedance field of the form

$$Z = \left(1 + \frac{\alpha}{\alpha_0}\right)^{n_0}$$

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We took fixed parameter values $\alpha_0 = 3$ (corresponding to approximatively a 5% slope).

Implementation

Data preprocessing, result processing and models profiling are implemented in R. For performances reasons and an easier integration into the OpenMole software for model exploration **?**, a `scala` version was also developed. The typical question of trade-off between implementation performance and interoperability appeared quickly as an issue, as a blind exploration and calibration can difficultly provide useful thematic conclusions for that kind of model. Finding an improvement in model fit among one parameter dimension is significant if the geographical situation is visualized and the improvement is confirmed as reasonable and not an absurdity.

Model Exploration

Model Calibration

Remarks

(i) In `\runninghead` use '*et al.*' if there are three or more authors.
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The body of the article

Mathematics

sagej.cls makes the full functionality of $\mathcal{AMS}$TeX available. We encourage the use of the `align`, `gather` and `multline` environments for displayed mathematics. amsthm is used for setting theorem-like and proof environments. The usual `\newtheorem` command needs to be used to set up the environments for your particular document.

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sagej.cls includes the graphicx package for handling figures.

Figures are called in as follows:

```
\begin{figure}
\centering
\includegraphics{<figure name>}
\caption{<Figure caption>}
\end{figure}
```

```
\begin{table}
\small\sf\centering
\caption{<Table caption.>}
\begin{tabular}{<table alignment>}
\toprule
<column headings>\\
\midrule
<table entries
(separated by & as usual)>\\
<table entries>\\
.
.
.\\
\bottomrule
\end{tabular}
\end{table}
```

Figure 1. Example table layout.

For further details on how to size figures, etc., with the graphicx package see, for example, Kopka and Daly (2003) or Mittelbach and Goossens (2004).

The standard coding for a table is shown in Figure 1.

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The commands available are:

```
\begin{acks}
To typeset an
  "Acknowledgements" section.
\end{acks}

\begin{biog}
To typeset an
  "Author biography" section.
\end{biog}

\begin{biogs}
To typeset an
  "Author Biographies" section.
\end{biogs}

\begin{dci}
To typeset a "Declaration of
  conflicting interests" section.
\end{dci}

\begin{funding}
To typeset a "Funding" section.
\end{funding}

\begin{sm}
To typeset a
  "Supplemental material" section.
\end{sm}
```

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To avoid any confusion for papers that use Vancouver style references, footnotes/endnotes should be edited into the text.

References

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Acknowledgements

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http://www.sunrise-setting.co.uk

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