$$p_i = \frac{J}{\Delta X_{\bar{i}} Z^{\star}_C - \Delta X_{\bar{i}} Z^{\star}_{\bar{i}}}$$

$$U_i(C) - U_i(NC) = p_{\bar{i}} \left(\Delta X_i Z^{\star}_C - \Delta X_i Z^{\star}_i\right) - J$$

$$p_i = \frac{1}{1 + \exp\left(-\beta_{DC} \cdot \left(\frac{\Delta X_i Z^{\star}_C - \Delta X_i Z^{\star}_i}{1+\exp\left(-\beta_{DC}\left(p_i \cdot \left(\Delta X_{\bar{i}} Z^{\star}_C - \Delta X_{\bar{i}} Z^{\star}_{\bar{i}}\right) - J\right)\right)} - J\right)\right)}$$

$A_{max} = E_{max} = 500; r_A = 1; r_E = 0.8; \gamma_E = 0.9; \gamma_A = 0.65; \beta_l = 1.8; \lambda = 0.005; r_0 = 2$
$N_{expl} = 25; I = 0.001; J = 0.0001; \nu = 5; E_{ext}(t_0) = 3E_{max}; t_f = 4$

Initial distribution of Actives and Employments around governance centers at positions $\vec{x}_i$ by

$$A(\vec{x}) = A_{max} \cdot \exp\left(\frac{\|\vec{x} - \vec{x}_i\|}{r_A}\right); E(\vec{x}) = E_{max} \cdot \exp\left(\frac{\|\vec{x} - \vec{x}_i\|}{r_E}\right)$$

For facility patches, employments are added by $E(\vec{x}) = E(\vec{x}) + \frac{k_{ext} \cdot E_{max}}{n_{ext}}$.

Transportation module : computation of flows ϕ_{ij} by solving on p_i, q_j by a fixed point method (Furness algorithm), the system of gravital flows

$$\begin{cases} \phi_{ij} = p_i q_j A_i E_j \exp\left(-\lambda_{tr} d_{ij}\right) \\ \sum_k \phi_{kj} = E_j; \sum_k \phi_{ik} = A_i \\ p_i = \frac{1}{\sum_k q_k E_k \exp\left(-\lambda_{tr} d_{ik}\right)}; q_j = \frac{1}{\sum_k p_k A_k \exp\left(-\lambda_{tr} d_{kj}\right)} \end{cases}$$

Trajectories then attributed by effective shortest path, and corresponding congestion c obtained (no Wardrop equilibrium).
Speed of network given by BPR function $v(c) = v_0 \left(1 - \frac{c}{\kappa}\right)^{\gamma_c}$. Congestion not used in current studies (infinite capacity κ).

Land-Use module : we assume that residential/employments relocations are at equilibrium at the time scale of a tick, that corresponds to transportation infrastructure evolution time scale which is much larger (Bretagnolle, 2009).
We take a Cobb-douglas function for utilities of actives/employments at a given cell

$$U_i(A) = X_i(A)^{\gamma_A} \cdot F_i(A)^{1-\gamma_A}; F_i(A) = \frac{1}{A_i E_i}$$

$$U_j(E) = X_j(E)^{\gamma_E} \cdot F_j(E)^{1-\gamma_E}; F_j(E) = 1$$

where $X_i(A) = A_i \cdot \sum_j E_j \exp(-\lambda \cdot d_{ij})$ and $X_j(E) = E_j \cdot \sum_i A_i \exp(-\lambda \cdot d_{ij})$.
Relocations are then done deterministically following a discrete choice model :

$$A_i(t+1) = \sum_i A_i(t) \cdot \frac{\exp(\beta U_i(A))}{\sum_i \exp(\beta U_i(A))}$$

$$E_j(t+1) = \sum_j E_j(t) \cdot \frac{\exp(\beta U_j(E))}{\sum_j \exp(\beta U_j(E))}$$

Effective distances computation

- Euclidian distance matrix $d(i,j)$ computed analytically
- Network shortest paths between network intersections (rasterized network) updated in a dynamic way (addition of new paths and update/change of old paths if needed when a link is added), correspondance between network patches and closest intersection also updated dynamically ; $O(N_{inters}^3)$
- Weak component clusters and distance between clusters updated ; $O(N_{nw}^2)$
- Network distances between network patches updated, through the heuristic of only minimal connexions between clusters ; $O(N_{nw}^2)$
- Effective distances (taking paces/congestion into account) updated as minimum between euclidian time and $\min_{C,C'} d(i,C) + d_{nw}(p_C(i), p'_C(j)) + d(C',j)$; $O(N_{clusters}^2 \cdot N^2)$ [Approximed with $\min_C$ only in the implementation, consistent within the interaction ranges ~ 5 patches taken in the model].