A Theory of co-evolutive networked territorial systems : Exemplification of Network Necessity

Working Paper

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Abstract

This paper is the application of a theoretical paper developing a theory of co-evolutive networked territorial systems. We apply simple models of urban growth for systems of cities, which include in particular the role of physical networks.

1 Context and Objective

1.1 Exemplifying Network Necessity

We propose to support our hypothesis that *physical transportation networks are necessary to explain the morphogenesis of territorial systems* (aka *Network Necessity*) by showing on a relatively simple case that the integration of physical networks into some models effectively increase their explanative power (being careful on the precise definition of model improvement to avoid overfitting). We work on simple territorial systems that are country-wide city systems, and more particularly French cities, on a time scale corresponding to that spatial scale, i.e. two last centuries. Taking into account physical networks can improve the understanding of city growth within that system in two ways : a qualitative one, for which the extended model would exhibit qualitative features corresponding to stylized facts empirically observed but that more basic models do not manage to reproduce, and a quantitative way, in the sense that model extension improves explained variance further than the mechanic improvement due to the introduction of supplementary degrees of freedom. If at least one of these is unveiled in our particular case, the evidence will support the theory at these scale and in this context.

1.2 Model context

[Bretagnolle et al., 2000] already propose a spatial extension of the Gibrat model (*detail*)

[Favaro and Pumain, 2011] is a more refined extension with economic cycles

2 Model Description

2.1 From Gibrat to Marius : the dilemma of formulation

2.2 Model description

We choose to work on a deterministic extension of the Gibrat model, what is equivalent to consider only expectancies in time as detailed before. Let $\vec{P}(t) = (P_i(t))_i$ be the population of cities in time. Under Gibrat independence assumptions, we have $\text{Cov}[P_i(t), P_j(t)] = 0$. If $\vec{P}(t+1) = \mathbf{R} \cdot \vec{P}(t)$ where $\mathbf{R}$ is also independent, then $\mathbb{E}\left[\vec{P}(t+1)\right] = \mathbf{R}\cdot\mathbb{E}\left[\vec{P}\right](t)$. With $\vec{\mu}(t) = \mathbb{E}\left[\vec{P}(t)\right]$, we generalize this approach by taking $\vec{\mu}(t+1) = f(\vec{\mu}(t))$. In our case, we take

$$f(\vec{\mu}) = r_0 \cdot \mathbf{Id} \cdot \vec{\mu} + \mathbf{G} \cdot \mathbf{1} + \mathbf{N} \cdot$$

with

- $G_{ij} = w_G \cdot \frac{V_{ij}}{<V_{ij}>}$ and $V_{ij} = \left(\frac{\mu_i \mu_j}{\sum {\mu_k}^2}\right)^{\gamma_G} \exp\left(-d_{ij}/d_G\right)$
- $N_i = w_N \cdot \sum_{kl}\left(\frac{\mu_k \mu_l}{\sum \mu}\right)^{\gamma_N} \exp\left(-d_{kl,i}\right)/d_N$ where $d_{kl,i}$ is distance to shortest path between k,l computed with slope impedance ($Z = (1+\alpha/\alpha_0)^{n_0}$ with $\alpha_0 \simeq 3$)

3 Results

3.1 Data

Population data

Physical flows As stated above, this modeling exercise focuses on exploring the role of physical flows, whatever the effective shape of the network. We do not need for this reason network data which is furthermore not easily available at different time periods, and physical flows are assumed to take the geographical shortest path that include terrain slope (to avoid geographical absurdities such as cities with a difficult access having an overestimated growth rate). Using the 1km resolution Digital Elevation Model openly available from IGN [], we construct an impedance field of the form

$$Z = \left(1 + \frac{\alpha}{\alpha_0}\right)^{n_0}$$

We took fixed parameter values $\alpha_0 = 3$ (corresponding to approximatively a 5% slope).

3.2 Implementation

Data preprocessing, result processing and models profiling are implemented in R. For performances reasons and an easier integration into the OpenMole software for model exploration [Reuillon et al., 2013], a `scala` version was also developed. The typical question of trade-off between implementation performance and interoperability appeared quickly as an issue, as a blind exploration and calibration can difficultly provide useful thematic conclusions for that kind of model. Finding an improvement in model fit among one parameter dimension is significant if the geographical situation is visualized and the improvement is confirmed as reasonable and not an absurdity.

3.3 Model Exploration

3.4 Model Calibration

4 Discussion

5 Supplementary Materials

5.1 Integrating Gibrat

Analytical resolution is possible for some aspects of the Gibrat model. We detail here the computation for some.

Expectancies If working with expectancies, it makes no sense to proceed to Monte Carlo simulation as a direct resolution gives a deterministic recurrence relation on expectancies. Let $\mu_t = \mathbb{E}[P(t)]$

Covariance

Distribution

5.2 A Bayesian iterative approach

Readers familiar with Bayesian Signal Processing techniques will have remarked that similarities exist between the described models and iterative filters such as the Kalman filter, more precisely particle filters.

5.3 Model explanative power : propositions for an empirical AIC

References

[Bretagnolle et al., 2000] Bretagnolle, A., Mathian, H., Pumain, D., and Rozenblat, C. (2000). Long-term dynamics of european towns and cities: towards a spatial model of urban growth. Cybergeo: European Journal of Geography.

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