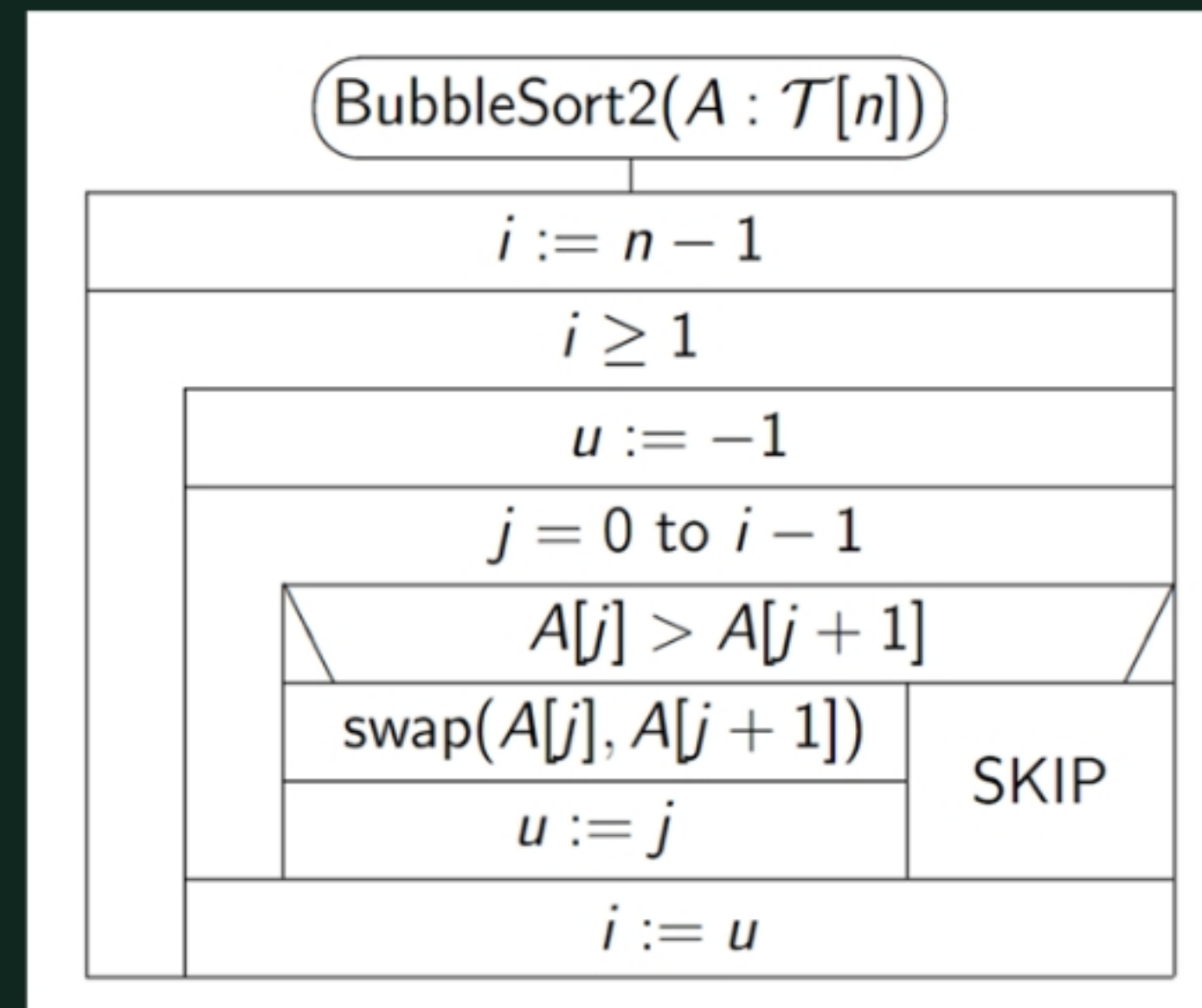


$$T(n) \in \Theta(n^2)$$

$$mT(n) = MT(n)$$

$$Oh(n) \in \Theta(n^2)$$



$$mT(n) \in \Theta(n)$$

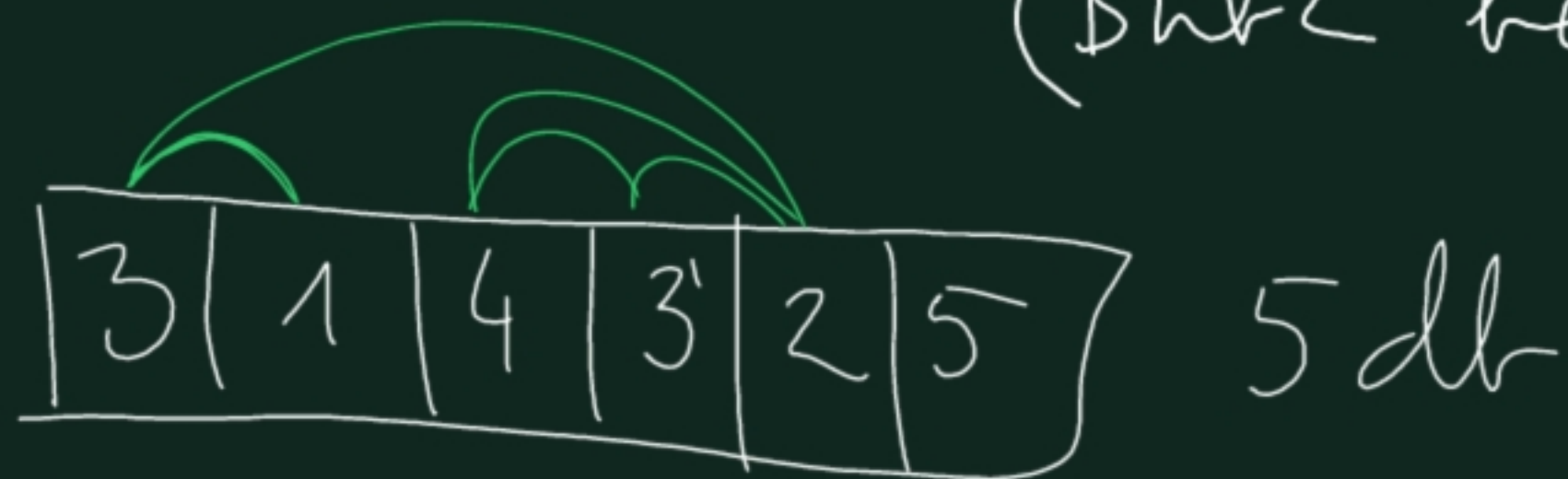
$$MT(n) \in \Theta(n^2)$$

$$Oh(n) \in O(n^2)$$

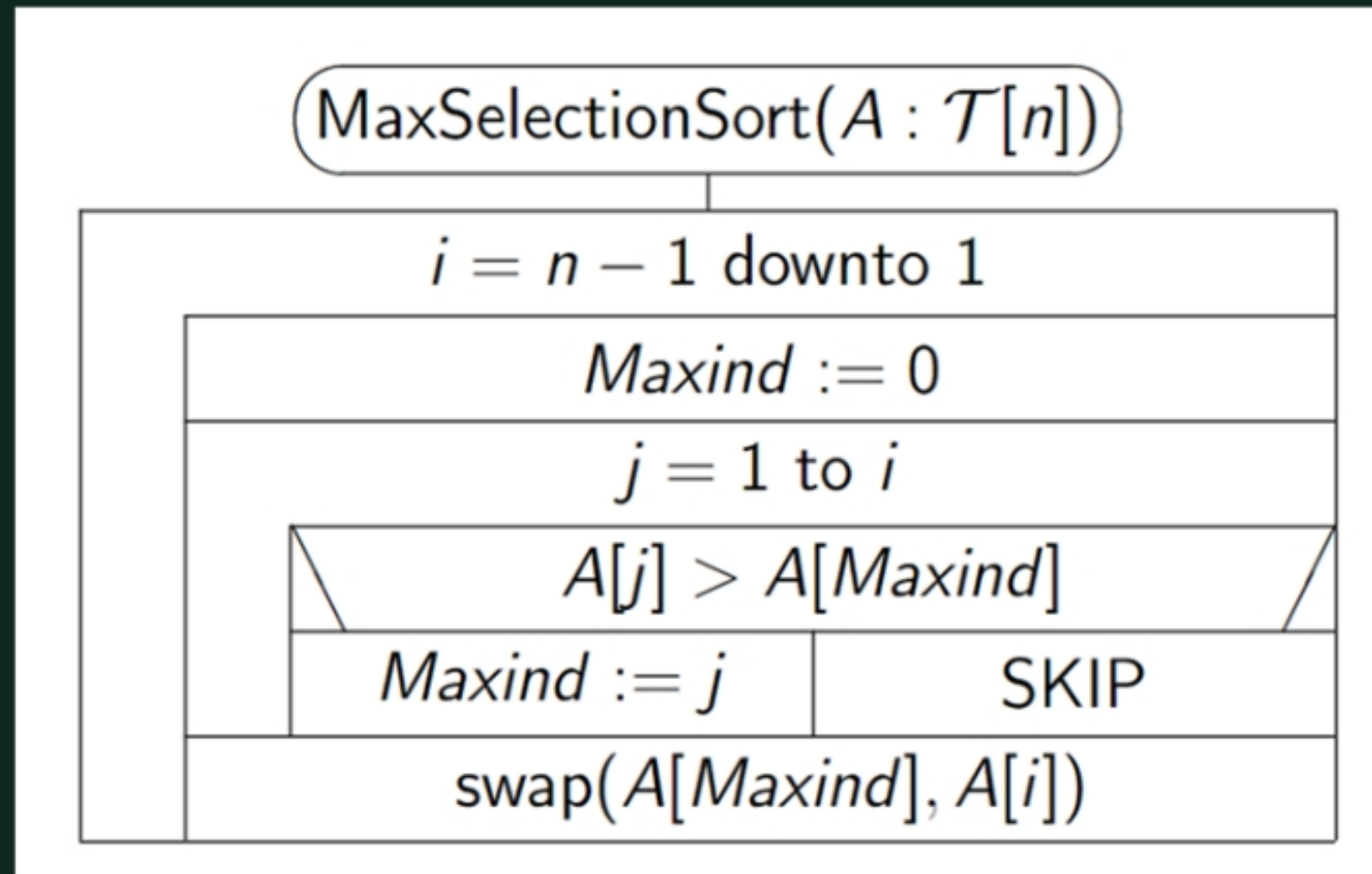
$$\in \Omega(n)$$

Def.: Egy A tömbben 2 elem: $A[i]$ és $A[j]$ inverzióban áll, ha $i < j$ és $A[i] > A[j]$.

ALL.: Bublonekban cserék száma = A -beli inverziók száma
(Bub2-ben is!)



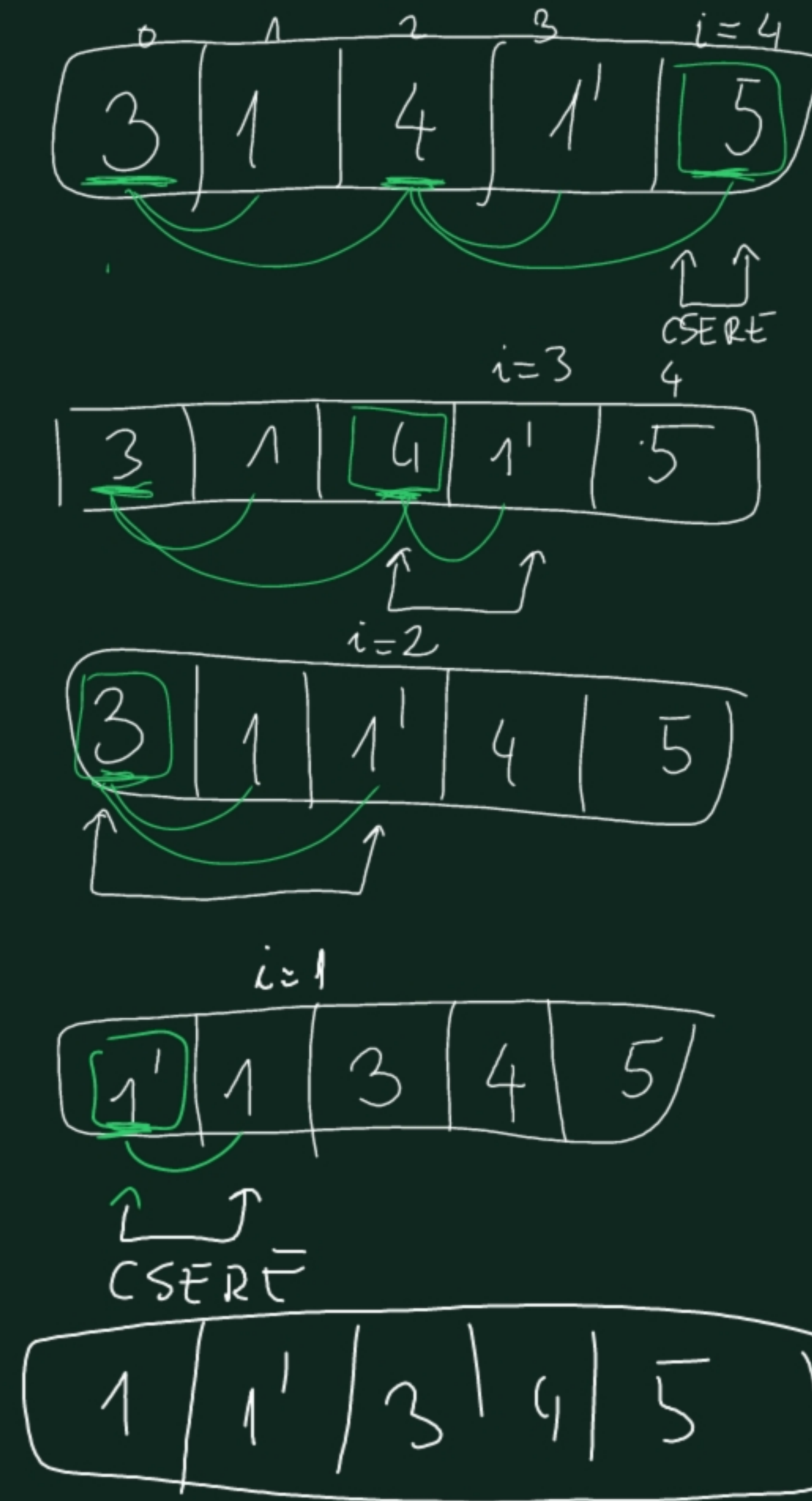
↳ gyakorlatként nézzék meg, hogy a Bublonek
tényleg 5 cserét hajt végre



$$T(n) \in \Theta(n^2)$$

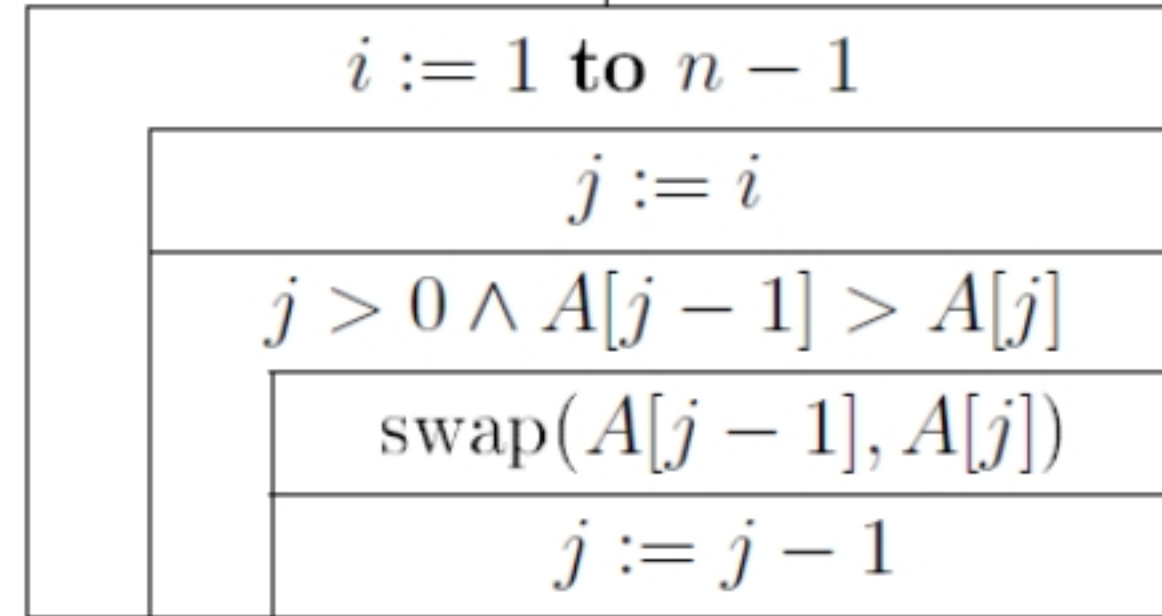
$$Oh(n) \in \Theta(n^2)$$

$$C_{sele}(n) \in \Theta(n)$$

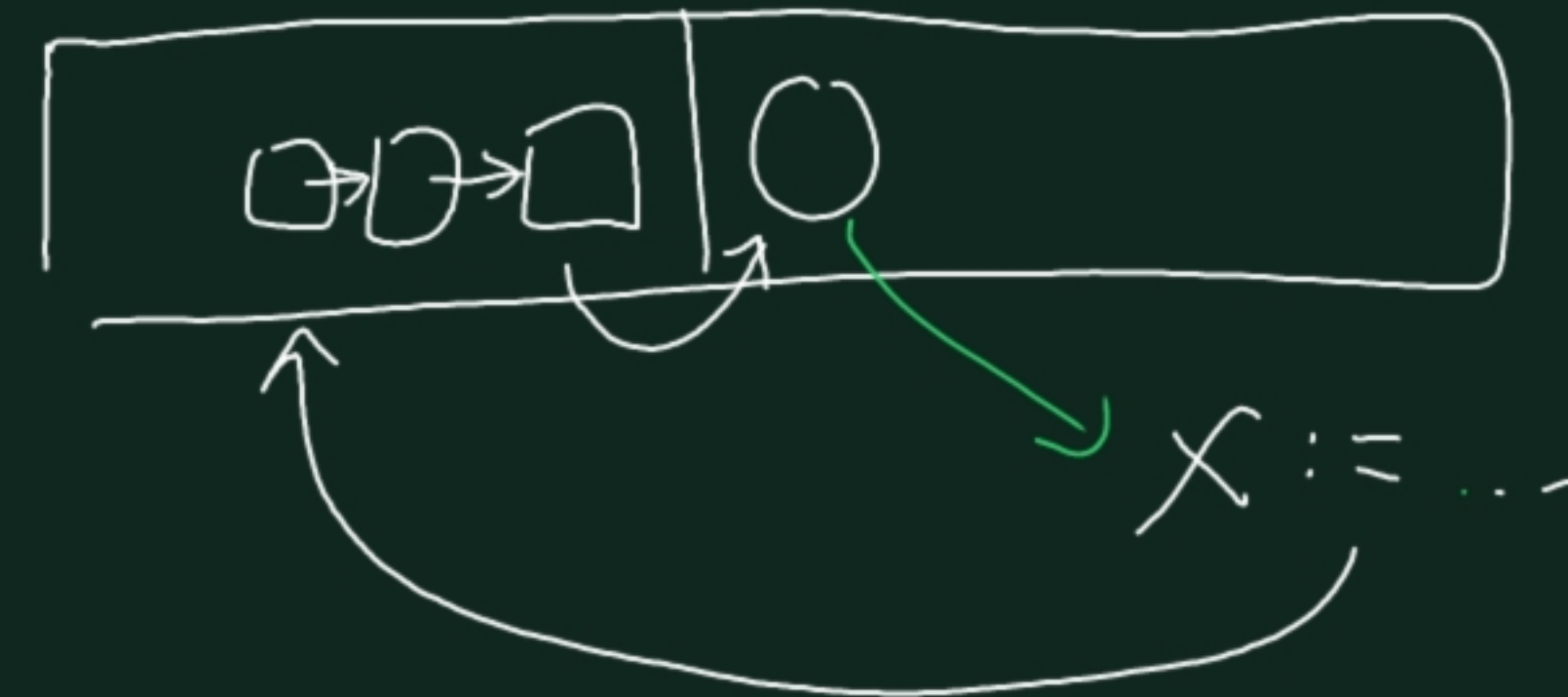
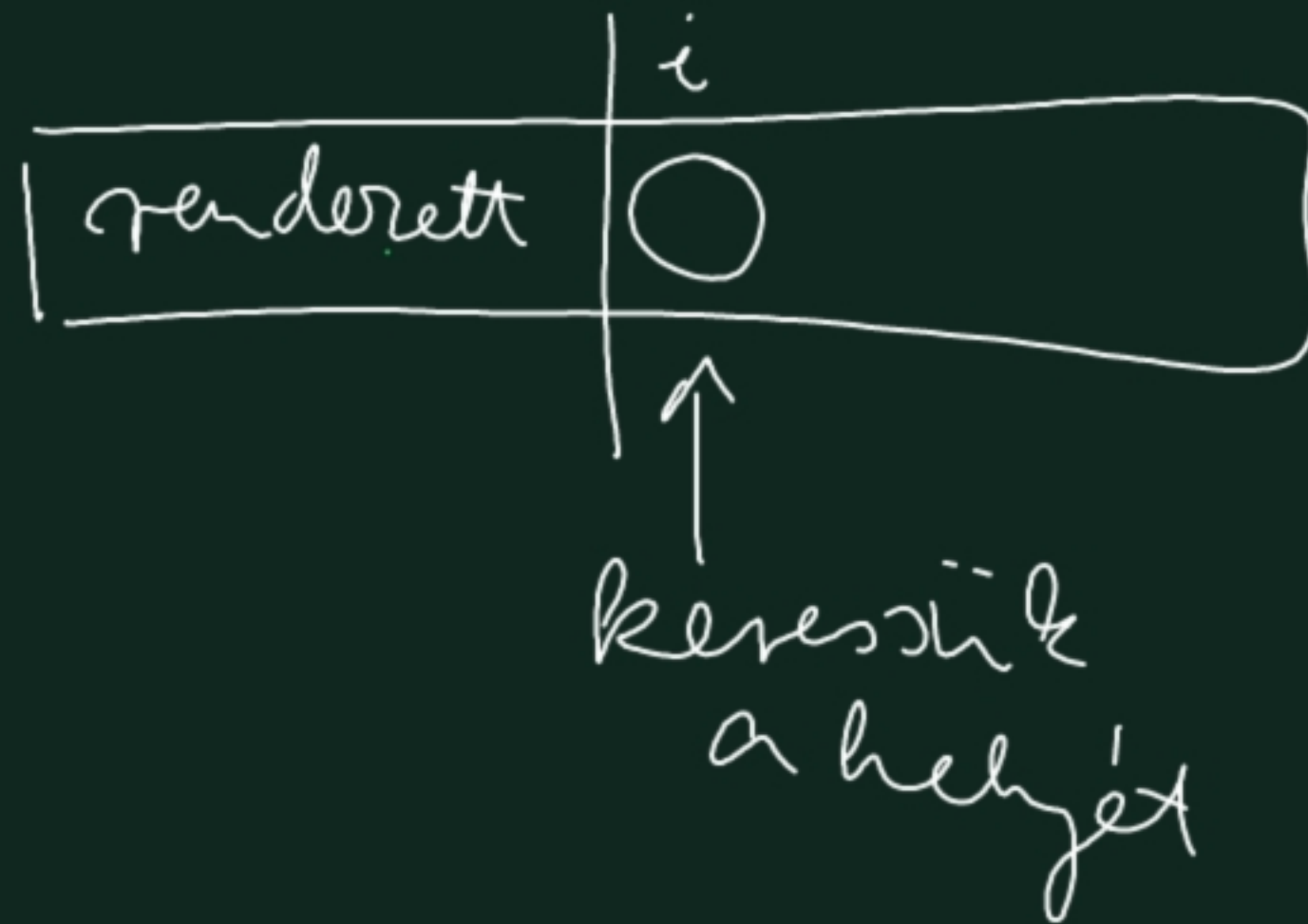
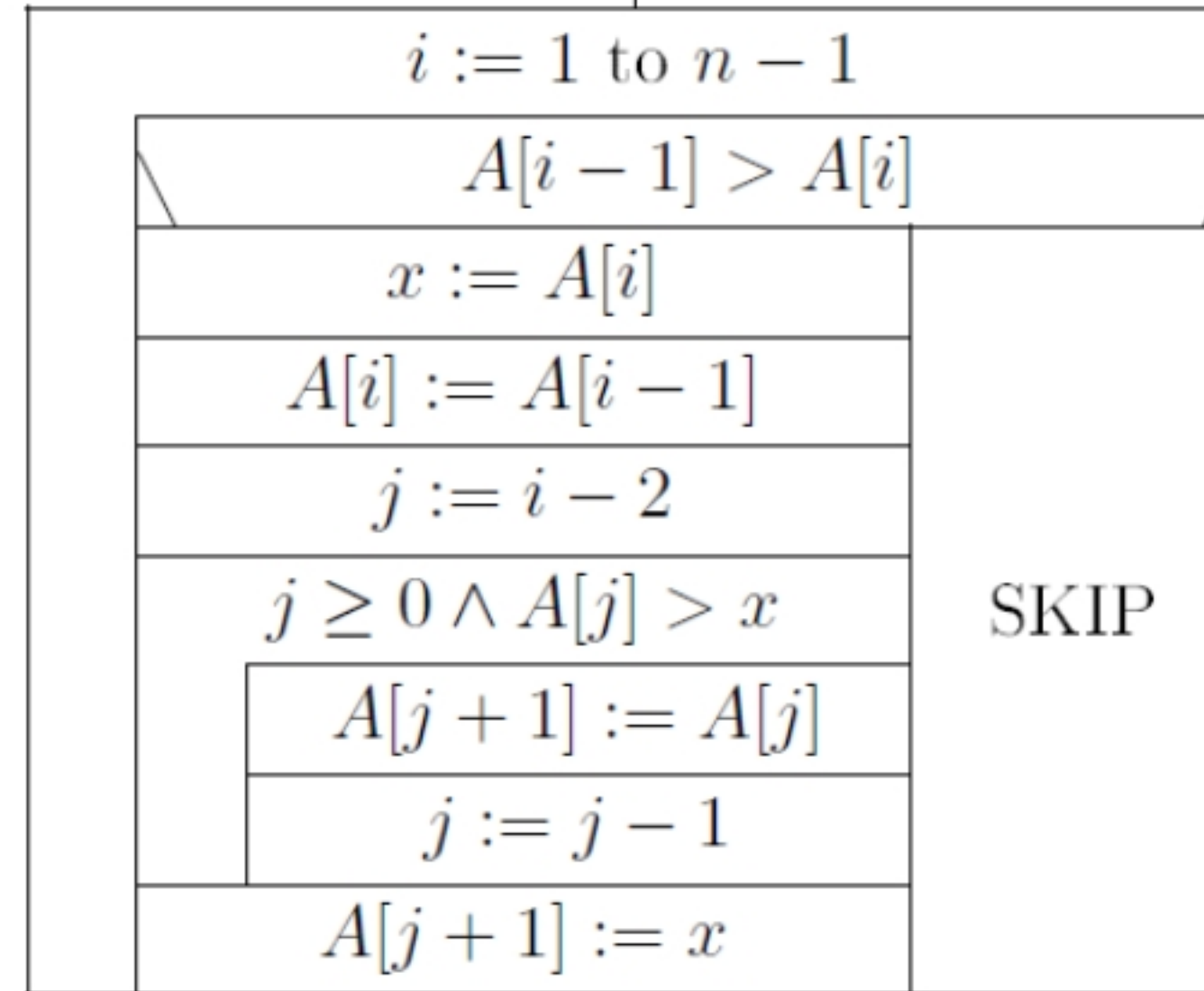


| Oh | Csele |
|----|-------|
| 4 | 1 |
| 3 | 1 |
| 2 | 1 |
| 1 | 1 |

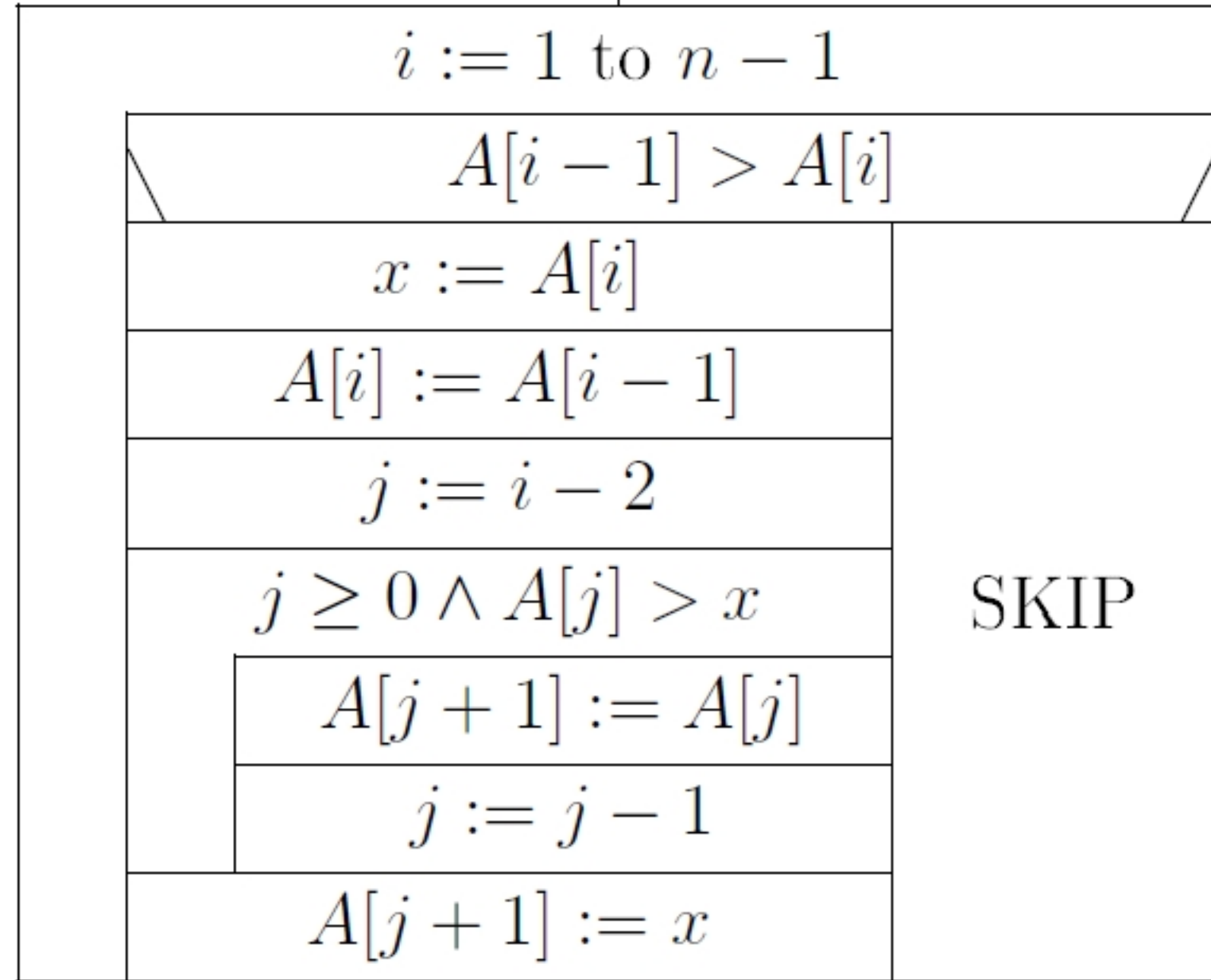
naiveInsertionSort($A : \mathcal{T}[n]$)



insertionSort($A : \mathcal{T}[n]$)



insertionSort($A : \mathcal{T}[n]$)



$i = 3$

$i = 4$

$i = 5$

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|----|----|---|---|----|---|
| A | 3 | 8 | 11 | 14 | 5 | 2 | 16 | 1 |

i

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|----|----|---|---|----|---|
| A | 3 | 8 | 11 | 14 | 5 | 2 | 16 | 1 |

i

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|----|----|---|----|---|
| A | 3 | | 8 | 11 | 14 | 2 | 16 | 1 |

i

x=5

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|----|----|---|----|---|
| A | 3 | 5 | 8 | 11 | 14 | 2 | 16 | 1 |

i

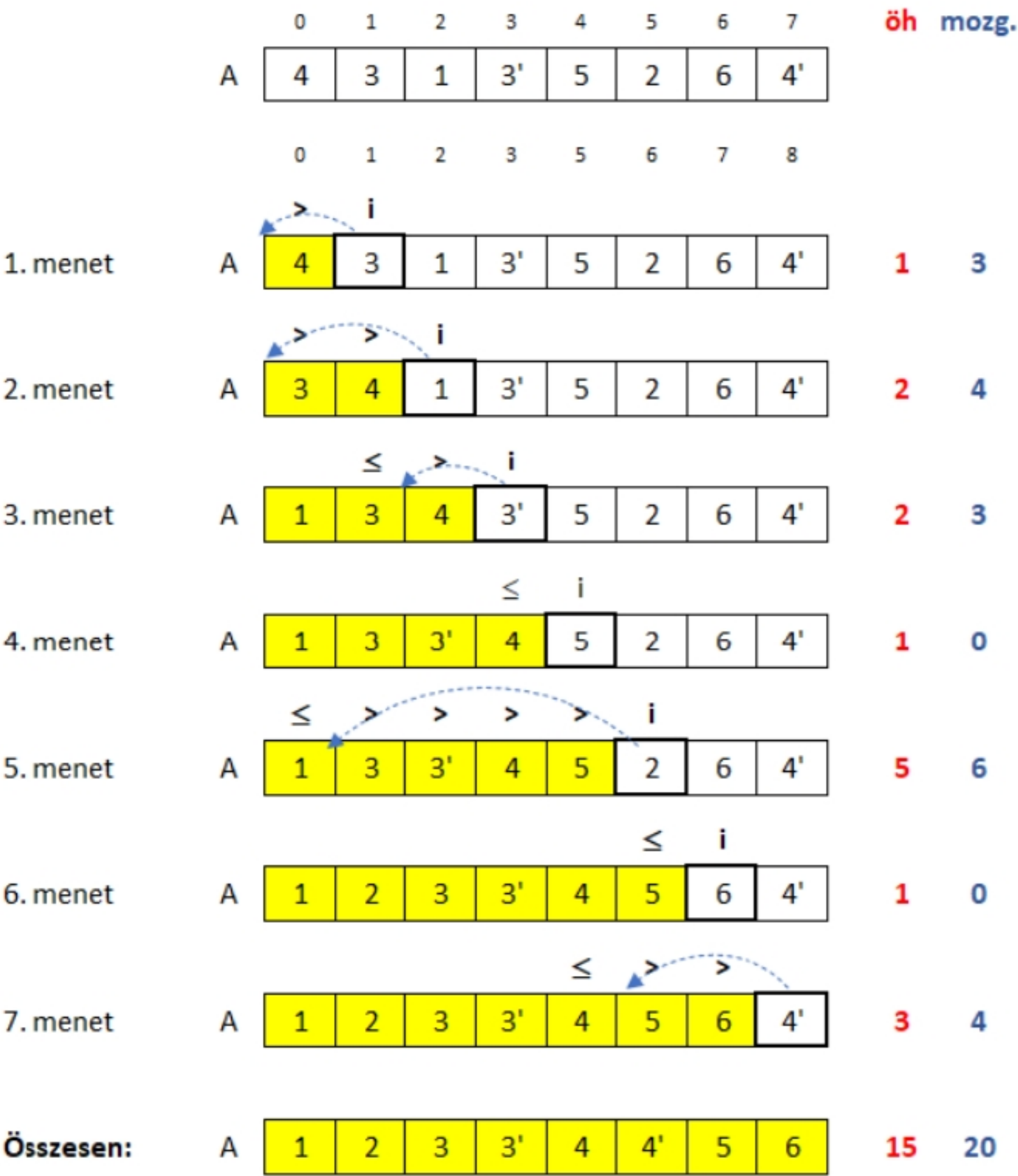
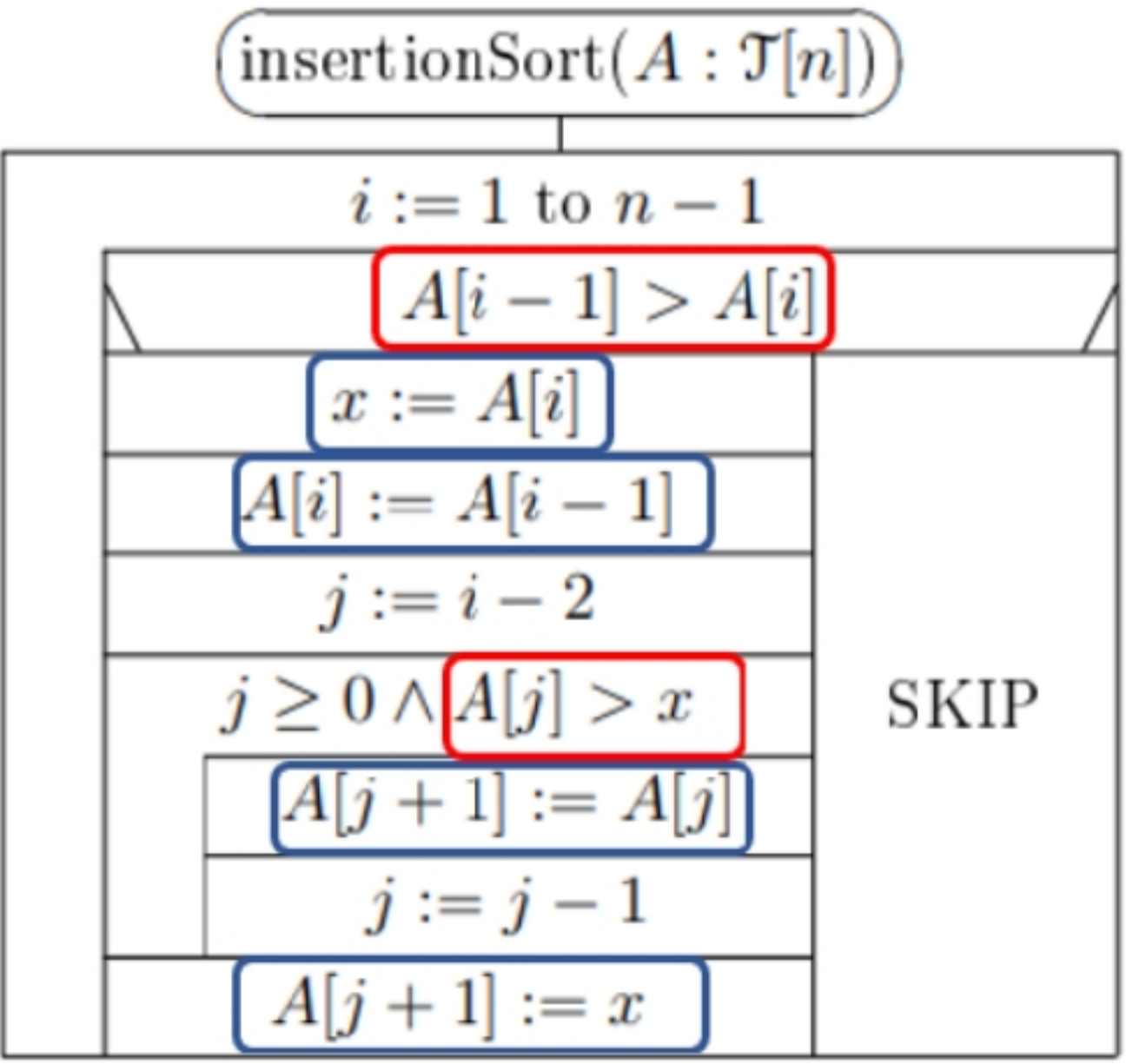
x=5

$$MT(n) \in \Theta(n)$$

$$MT(n) \in \Theta(n^2)$$

Piros keret jelzi a kulcs-összehasonlításokat, kék a kulcs-mozgatásokat.

Jelöljük, mely kulcsokkal történt összehasonlítás!



Összefoglaló rendezés / Merge Sort

$\text{mergeSort}(A : \mathcal{T}[n])$

| |
|--|
| $B : \mathcal{T}[n] ; B[0..n) := A[0..n)$ |
| // Sort $B[0..n)$ into $A[0..n)$ non-decreasingly: |
| $\text{ms}(B, A)$ |

$\text{ms}(B, A : \mathcal{T}[n])$

| | |
|--|------|
| // Initially $B[0..n) = A[0..n)$. | |
| // Sort $B[0..n)$ into $A[0..n)$ non-decreasingly: | |
| \ $n > 1$ / | |
| $m := \lfloor \frac{n}{2} \rfloor$ | SKIP |
| $\text{ms}(A[0..m), B[0..m))$ // Sort $A[0..m)$ into $B[0..m)$ | |
| $\text{ms}(A[m..n), B[m..n))$ // Sort $A[m..n)$ into $B[m..n)$ | |
| $\text{merge}(B[0..m), B[m..n), A[0..n))$ // sorted merge | |

$\text{merge}(A : \mathcal{T}[l] ; B : \mathcal{T}[m] ; C : \mathcal{T}[n])$

| | |
|---|----------------------|
| // sorted merge of A and B into C where $l + m = n$ | |
| $k := 0$ // in loop, copy into $C[k]$ | |
| $i := 0 ; j := 0$ // from $A[i]$ or $B[j]$ | |
| $i < l \wedge j < m$ | |
| \ $A[i] \leq B[j]$ / | |
| $C[k] := A[i]$ | $C[k] := B[j]$ |
| $i := i + 1$ | $j := j + 1$ |
| $k := k + 1$ | |
| \ $i < l$ / | |
| $C[k..n) := A[i..l)$ | $C[k..n) := B[j..m)$ |

$\text{merge}(A : \mathcal{T}[l] ; B : \mathcal{T}[m] ; C : \mathcal{T}[n])$

// sorted merge of A and B into C where $l + m = n$

$k := 0$ // in loop, copy into $C[k]$

$i := 0 ; j := 0$ // from $A[i]$ or $B[j]$

$i < l \wedge j < m$

$A[i] \leq B[j]$

$C[k] := A[i]$

$C[k] := B[j]$

$i := i + 1$

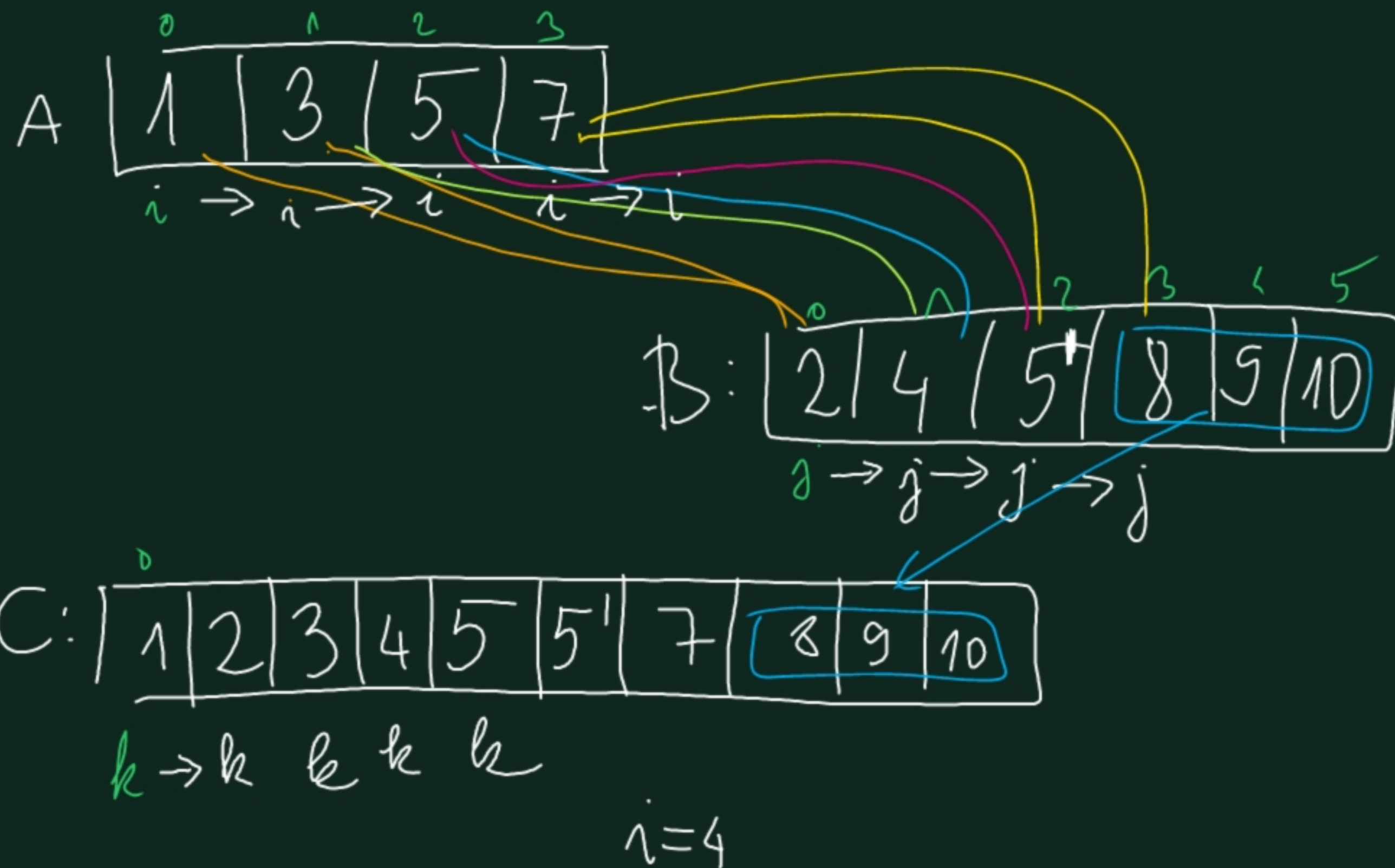
$j := j + 1$

$k := k + 1$

$i < l$

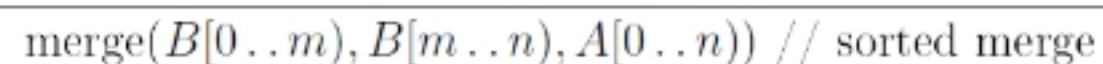
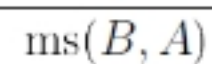
$C[k..n) := A[i..l)$

$C[k..n) := B[j..m)$

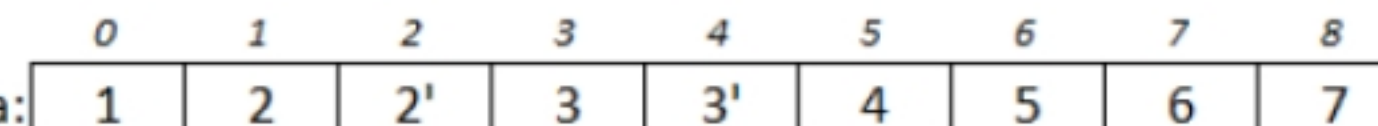


all it $\neq n$

$Oh = 7$



SKIP



Merge futások szemléltetése:

| | | | | | | | | | | | | | | | | | | | | | | |
|---|-------|---|---|---|---|----|---|---|---|----|---------------------------------|-----|---|---|---|---|----|---|---|----|----|----|
| | START | | | | | | | | | | | END | | | | | | | | | | öh |
| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | | | |
| 1 | A | 4 | 2 | 6 | 3 | 2' | 5 | 1 | 7 | 3' | merge(B[0,1), B[1, 2),A[0,2)) | A | 2 | 4 | 6 | 3 | 2' | 5 | 1 | 7 | 3' | 1 |
| | B | 4 | 2 | 6 | 3 | 2' | 5 | 1 | 7 | 3' | (B-ből A-ba) | B | 4 | 2 | 6 | 3 | 2' | 5 | 1 | 7 | 3' | |
| | START | | | | | | | | | | | END | | | | | | | | | | |
| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | | | |
| 2 | A | 2 | 4 | 6 | 3 | 2' | 5 | 1 | 7 | 3' | merge(B[2, 3), B[3, 4), A[2,4)) | A | 2 | 4 | 3 | 6 | 2' | 5 | 1 | 7 | 3' | 1 |
| | B | 4 | 2 | 6 | 3 | 2' | 5 | 1 | 7 | 3' | (B-ből A-ba) | B | 4 | 2 | 6 | 3 | 2' | 5 | 1 | 7 | 3' | |
| | START | | | | | | | | | | | END | | | | | | | | | | |
| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | | | |
| 3 | A | 2 | 4 | 3 | 6 | 2' | 5 | 1 | 7 | 3' | merge(A[0, 2), A[2, 4),B[2,4)) | A | 2 | 4 | 3 | 6 | 2' | 5 | 1 | 7 | 3' | 3 |
| | B | 4 | 2 | 6 | 3 | 2' | 5 | 1 | 7 | 3' | (A-ból B-be) | B | 2 | 3 | 4 | 6 | 2' | 5 | 1 | 7 | 3' | |
| | START | | | | | | | | | | | END | | | | | | | | | | |
| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | | | |
| 4 | A | 2 | 4 | 3 | 6 | 2' | 5 | 1 | 7 | 3' | merge(B[4, 5), B[5, 6),A[4,6)) | A | 2 | 4 | 3 | 6 | 2' | 5 | 1 | 7 | 3' | 1 |
| | B | 2 | 3 | 4 | 6 | 2' | 5 | 1 | 7 | 3' | (B-ből A-ba) | B | 2 | 3 | 4 | 6 | 2' | 5 | 1 | 7 | 3' | |
| | START | | | | | | | | | | | END | | | | | | | | | | |
| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | | | |
| 5 | A | 2 | 4 | 3 | 6 | 2' | 5 | 1 | 7 | 3' | merge(A[7, 8), A[8, 9),B[7,9)) | A | 2 | 4 | 3 | 6 | 2' | 5 | 1 | 7 | 3' | |
| | B | 2 | 3 | 4 | 6 | 2' | 5 | 1 | 7 | 3' | (A-ból B-be) | B | 2 | 3 | 4 | 6 | 2' | 5 | 1 | 3' | 7 | 1 |

6

START

| | | | | | | | | | |
|---|---|---|---|---|----|---|---|----|----|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | |
| A | 2 | 4 | 3 | 6 | 2' | 5 | 1 | 7 | 3' |
| B | 2 | 3 | 4 | 6 | 2' | 5 | 1 | 3' | 7 |

merge(B[6, 7), B[7, 9),A[6,9))
(B-ből A-ba)

END

| | | | | | | | | | |
|---|---|---|---|---|----|---|---|----|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | |
| A | 2 | 4 | 3 | 6 | 2' | 5 | 1 | 3' | 7 |
| B | 2 | 3 | 4 | 6 | 2' | 5 | 1 | 3' | 7 |

7

START

| | | | | | | | | | |
|---|---|---|---|---|----|---|---|----|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | |
| A | 2 | 4 | 3 | 6 | 2' | 5 | 1 | 3' | 7 |
| B | 2 | 3 | 4 | 6 | 2' | 5 | 1 | 3' | 7 |

merge(A[4,6), A[6, 9),B[4,9))
(A-ból B-be)

END

| | | | | | | | | | |
|---|---|---|---|---|----|----|----|----|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | |
| A | 2 | 4 | 3 | 6 | 2' | 5 | 1 | 3' | 7 |
| B | 2 | 3 | 4 | 6 | 1 | 2' | 3' | 5 | 7 |

8

START

| | | | | | | | | | |
|---|---|---|---|---|----|----|----|----|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | |
| A | 2 | 4 | 3 | 6 | 2' | 5 | 1 | 3' | 7 |
| B | 2 | 3 | 4 | 6 | 1 | 2' | 3' | 5 | 7 |

merge(B[0, 4),B[4, 9), A[0,9))
(B-ből A-ba)

END

| | | | | | | | | | |
|---|---|---|----|---|----|----|----|---|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | |
| A | 1 | 2 | 2' | 3 | 3' | 4 | 5 | 6 | 7 |
| B | 2 | 3 | 4 | 6 | 1 | 2' | 3' | 5 | 7 |

Verem → feltöltött pdf