CS 61A Midterm 1 HKN Review

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Agenda

Hosted by HKN (hkn.eecs.berkeley.edu)

Office hours from 11AM to 5PM in 290 Cory, 345 Soda

Check our website for exam archive, course guide, course surveys, tutoring schedule.

- What Would Python Print
- Environment Diagrams
- Higher Order Functions
- Lambda Functions
- Newton's Method
- Recursion

Follow along at bit.ly/1owv0DT!

What will Python print?

```
def while_loop(n):
    i, j = 0, 1
    while i < n:
        j += 1
        while j < n:
            i += 1
            if j % i == 1:
                print(i, j)
            j += 1
        i += 1

while_loop(5)
```

```
>>> while_loop(5)
```

What will Python print?

```
def best(n):
    def pikachu():
        print('pika!')
        return n
    bulbasaur = lambda: 2
    charmander = (3, 'pika!')
    if pikachu() < bulbasaur():
        n += charmander[0]
    elif pikachu() == charmander[1]:
        print('pikachu!')
    else:
        if pikachu() % 2 == 1:
            return 'squirtle'
```

```
>>> best(4)
________
>>> ashy_boy = best(3)
________
>>> ashy_boy
________
>>> best(1)
________
```

Controlling your printing

```
def print_moar(stuff):
    i = 0
    while stuff and i < 2:
        stuff = print(stuff, print('soumya'))
        i += 1
    return stuff

>>> brian = print_moar('cs61a')
_________
>>> brian
_________
```

Boolean Expressions

BONUS!

For reference, look at lab 2 titled "Control"

- A *boolean expression* is one that evaluates to either `True`, `False`, or sometimes an `Error`.
- When evaluating boolean expressions, we follow the same rules as those used for evaluating other statements and function calls.
- The order of operations for booleans (from highest priority to lowest) is: **not, and, or**

The following will evaluate to `True`:

```
True and not False or not True and False
```

You can rewrite it using parentheses to make it more clear:

```
(True and (not False)) or ((not True) and False)
```

More Boolean Expressions

BONUS!

Short-circuiting

- Expressions are evaluated from left to right in Python.
- Expressions with **and** will evaluate to `True` only if *all* the operands are `True`. For multiple **and** expressions, Python will go left to right until it runs into the first `False` value -- then the expression will immediately evaluate to `False`.
- Expressions with **or** will evaluate to `True` if *at least one* of the operands is `True`. For multiple **or** expressions, Python will go left to right until it runs into the first `True` value -- then the expression will immediately evaluate to `True`. For example:

  ```
  5 > 6 or 4 == 2*2 or 1/0
  ```

  This evaluates to `True` because of short-circuiting.

Environment Diagrams

```
lamps = lambda you, me: you + me
def brian(mark):
    def flour(based):
        return based(mark)
    mark = 'hi'
    return flour
lamps, brian = brian, lamps
answer = lamps(brian(3, 2))
answer(print)
```

Environment Diagrams

```
fruit = lambda: apples
apples = 5
def domo(eats, apples):
        def rawr():
                return fruit()
        print(eats())
        rawr()
domo(fruit, 42)
```

Number Fun

Implement `digit_span`, a function that takes as input a positive integer and returns the difference between its largest digit and its smallest digit.

```
def digit_span(n):
    """Return the difference between the largest
    and smallest digits.

    >>> digit_span(2013)
    3
    >>> digit_span(7)
    0
    """
```

Higher-Order Functions

A function that takes in a function as an argument and/or returns a function.

```
def sum_if_even(n):
    """
    Returns the sum of the even natural numbers from 1
    to n inclusive.
    >>> sum_if_even(10)
    30
    >>> sum_if_even(2)
    2
    >>> sum_if_even(15)
    56
    """
```

Higher-Order Functions

```
def product_if_prime(n):
    """
    Returns the product of the prime natural numbers
    from 1 to n inclusive.
    >>> product_if_prime(10)
    210
    >>> product_if_prime(2)
    2
    >>> product_if_prime(6)
    30
    """
```

Higher-Order Functions

```
def sum_if_even(n):
    i = 0
    total = 0
    while i <= n:
        i += 1
        if i % 2 == 0:
            total += i
    return total
```

```
def product_if_prime(n):
    i = 0
    total = 1
    while i <= n:
        i += 1
        if is_prime(i):
            total *= i
    return total
```

```
def accumulate_if_pred(n, start, pred, combiner):
```

Higher-Order Functions

Start with a specific implementation and highlight the parts that are different.

```
def accumulate_if_pred(n, start, pred, combiner):
    i = 0
    total = 1
    while i <= n:
        i += 1
        if is_prime(i):
            total *= i
    return total
```

Higher-Order Functions

Generalize carefully and you're done!

```
def accumulate_if_pred(n, start, pred, combiner):
    i = 0
    while i <= n:
        i += 1
        if is_prime(i):
            total *= i
    return start
```

Higher-Order Functions

Generalize carefully and you’re done!

```
def accumulate_if_pred(n, start, pred, combiner):
    i = 0
    while i <= n:
        i += 1
        if pred(i):
            total *= i
    return start
```

Higher-Order Functions

Generalize carefully and you're done!

```
def accumulate_if_pred(n, start, pred, combiner):
    i = 0
    while i <= n:
        i += 1
        if pred(i):
            start = combiner(start, i)
    return start
```

Lambda Functions

- Unnamed function, no assignments

```
    "lambda <arguments>: <return value>"
>>> g = lambda y: y % 2
>>> g(4)
________
>>> g(7)
________
>>> h = lambda x: lambda y: z
>>> h(1)
________
>>> h(1000)(1)
________
```

Lambda Functions

And, we can call a lambda expression without ever giving it a name!

```
>>> f = lambda x: x + 1
>>> f(4)
```

____

```
>>> (lambda x: x + 1)(4)
```

____

```
>>> (lambda y: y(3))(lambda x: x + 4)
```

____

Lambda Functions

Try on your own!

```
>>> x = lambda x: lambda: lambda y: 2 * x + 3 * y
>>> x
______
>>> x(3)
______
>>> x(3)(4)
______
>>> x(3)()
______
>>> x(3)()(7)
______
```

Lambda Functions

Try on your own!

```
>>> x = lambda x: lambda: lambda y: 2 * x + 3 * y
```

We can rewrite the lambda expressions using HOFs:

```
def L1(x):
# The line above could be "def x(x):" Why?
    def L2():
        def L3(y):
            return 2 * x + 3 * y
        return L3
    return L2
```

Applications of Higher-Order Functions: Newton's Method

Newton's method is used to find (approximately) the roots (or *zeros*) of a function f, which are the input values for which the function evaluates to zero.

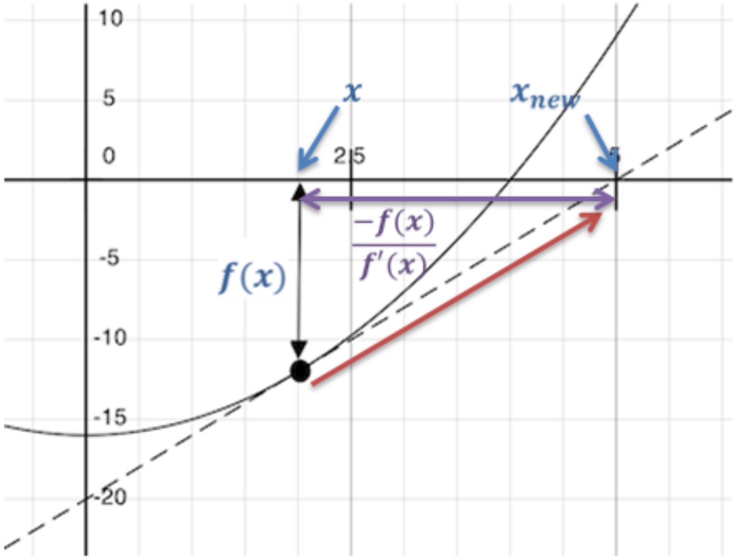
Many mathematical problems are equivalent to finding roots of specific functions.

"Square root of 2 is x, $x^2 = 2$" is equivalent to "Find a root of $x^2 - 2$."

"(Any) Angle x whose sine is 0.5" is equivalent to "Find a root of sin(x) - 0.5."

Applications of Higher-Order Functions: Newton's Method

1. Start with a function `f` and a guess x.
2. Compute the value of the function `f` at x.
3. If zero, we are done; else, compute the derivative of `f` at x, `f'(x)`.
4. Update guess to be `x - (f(x)/f'(x))`.



Animation: http://en.wikipedia.org/wiki/File:NewtonIteration_Ani.gif

Applications of Higher-Order Functions: Newton's Method

Newton's method is an instance of *iterative improvement*.

Step 1: ***Guess*** an answer to the problem.

Step 2: If the guess is ***(approximately) correct***, it is the solution; otherwise, ***update*** the guess and repeat this step.

Applications of Higher-Order Functions: Newton's Method

```
def improve(update, close, guess=1):
    while not close(guess):
        guess = update(guess)
    return guess
```

Applications of Higher-Order Functions: Newton's Method

```
def improve(update, close, guess=1,
            max_updates=100):
    k = 1
    while not close(guess) and k < max_updates:
        guess = update(guess)
        k = k + 1
    return guess
```

Applications of Higher-Order Functions: Newton's Method

```
def approx_eq(x, y, tolerance=1e-15):
    return abs(x - y) < tolerance


def newton_update(f, df):
    def update(guess):
        return guess - (f(guess) / df(guess))
    return update


def find_zero(f, df):
    # Uses Newton’s method to find a zero of a function
    # f, whose derivative is the function df.
    def near_zero(guess):
        return approx_eq(f(guess), 0)
    return improve(newton_update(f, df), near_zero)
```

Applications of Higher-Order Functions: Newton's Method

"Square root of 2 is x, $x^2 = 2$."

```
find_zero(lambda x: x**2 - 2, lambda x: 2*x)
```

"(Any) Angle x whose sine is 0.5."

```
find_zero(lambda x: sin(x) - 0.5, lambda x: cos(x))
```

"Number x that is one less than its square, or $x = x^2 - 1$."

```
find_zero(______________________________________)
```

Applications of Higher-Order Functions: Newton's Method

Not every function has a clean formula for its derivative. Write a function `derivative` that takes in a function and returns another function that approximates its derivative. Recall that

$$f'(x) \approx \frac{f(x+dx)-f(x)}{dx}$$

```
>>> derivative(lambda x: x**2)(3)    # Example usage
6.00010000012054


def derivative(f, dx=1e-4):
```

Applications of Higher-Order Functions: Newton's Method

Write a function `find_extremum` that takes a function, and finds a local extremum of that argument function. A *local extremum* is a point at which the derivative of the function is zero. (You may assume that the argument function will never have any saddle points.)

An example call is provided:

```
>>> f = lambda x: (x-1)**2 # parabola with minimum at x=1
>>> find_extremum(f)
0.999995 # very close to 1
```

Recursion

BONUS!

A function is ***recursive*** if the body calls the function itself, either directly or indirectly.

```
def factorial(n):
    if n == 1 or n == 0:
        return 1
    return n * factorial(n - 1)
```

Recursion

BONUS!

A recursive function has two important components:

1. A *base case*, where the function does not recursively call itself, and instead returns a direct answer. This is reserved for the simplest inputs.
2. A *recursive case*, where the function calls itself. The call must be made with an argument that drives the function towards the base case.

Recursion

A recursive function has two important components:

1. A *base case*.
2. A *recursive case*.

```
def factorial(n):
    if n == 1 or n == 0:
        return 1
    return n * factorial(n - 1)
```

Visualization: http://goo.gl/ux5MuQ

Recursion

Write a recursive function `add_until` that adds all of the integers from `1` to a positive integer `n`.

```
def add_until(n):
```

Recursion

Write a recursive function `eat_chocolate` that takes in a number of chocolate pieces and returns a string as follows:

```
>>> eat_chocolate(5)
"nom nom nom nom nom"
>>> eat_chocolate(2)
"nom nom"
>>> eat_chocolate(1)
"nom"
>>> eat_chocolate(0)
"No chocolate :("
```

Recursion

Write a function `count_occurrences` that takes in a number and a digit and counts the number of times the digit appears in the number.

```
def count_occurrences(num, digit):
```

Recursion

Write the function `call_until_one` recursively. Recall that it takes a function we are interested in as an argument. It returns another function that, when called on a number, will tell you how many times you can call that original function on the number until it will return a value less than or equal to 1.

```
>>> f = call_until_one(lambda x: x - 1)
>>> f(100)
99

>>> g = call_until_one(lambda x: x / 2)
>>> g(128)
7
```