MIDTERM 1 REVIEW (FALL 2012)

Conducted by HKN

1 Signals

1. Assume the signal $x(n)$ defined as

$$x(n) = \sum_{k=-2}^{2} k\delta(n-k).$$

Plot the following signals:

(a) $x(n)$
(b) $x(3-n)$
(c) $x(2n)$
(d) $x(n-2)\delta(n-2)$
(e) $\frac{1}{2}x(n) + \frac{1}{2}(-1)^n x(n)$

2 Dirac Delta

1. What do the following expressions evaluate to?

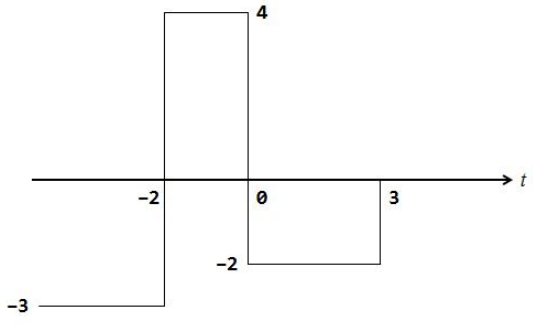
(a)

$$\int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} \delta(\tau) f(t-\tau) d\tau \right] \delta(t)\, dt$$

(b)

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \delta(\tau)\, \delta(t)\, d\tau\, dt$$

2. Assume a signal $f(t)$ whose plot is given by:



Plot $\dot{f}(t) = \frac{df}{dt}(t)$.

3. Let the signal $x(t)$ be defined as $x(t) = \delta(t+2) - \delta(t-2)$. Let $y(t)$ be another signal defined by

$$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau.$$

Plot the signal $y(t)$.

3 Complex Numbers

1. Simplify and plot $(-1 + i\sqrt{3})^8$.
2. Find the five roots z_1, z_2, z_3, z_4, z_5 of the polynomial $z^5 - z^3 + z$. Plot the five roots on the complex plane. Then, find the sum

$$\left(\sum_{k=1}^{5} z_k \right)^{e^{e^{20}}}.$$

3. Plot the magnitude $|F(\omega)|$ and phase $\angle F(\omega)$ of the function

$$F(\omega) = \cos(\omega) e^{i\omega}.$$

4. Plot the magnitude $|G(\omega)|$ and phase $\angle G(\omega)$ of the function

$$G(\omega) = \frac{1}{1 + i\omega}.$$

5. Use the relationship (as derived from homework) that

$$\prod_{k=1}^{N} D_k = N,$$

where $D_k = |z_k - z_0|$, to prove that

$$\prod_{k=1}^{N} \left| \sin\left(k \frac{\pi}{N}\right) \right| = \frac{N}{2^{N-1}}.$$