CS 70 Midterm 2 Review

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(with some questions by Chris Gioia)

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Poisson's Paradise

On an island called Poisson's Paradise, the government is trying to create a social security system that will help uniquely identify the different citizens that live on the island. Their outdated database only allows them to store SSN's that contain exactly 2 different numerical digits (where each social security number is 11 digits long). As more and more people move to this island, the government officials are wondering whether there are enough unique SSN's for all of the people.

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If I choose the two numbers to be 3 and 5:

An example valid SSN is:
$\underline{3}\ \underline{3}\ \underline{5}\ \underline{5}\ \underline{3}\ \underline{3}\ \underline{5}\ \underline{3}\ \underline{3}\ \underline{5}\ \underline{3}$

An example of an invalid SSN is:
$\underline{3}\ \underline{3}\ \underline{5}\ \underline{5}\ \underline{3}\ \underline{3}\ \underline{5}\ \underline{3}\ \underline{3}\ \underline{5}\ \underline{7}$ (invalid because the SSN contains more than two types of digits)

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How many different social security numbers can they create that contain exactly 2 different numerical digits? (You may leave it as an unsimplified expression).
$\binom{10}{2} \times (2^{11} - 2)$

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(b) What is the probability that at least one state has two members in the committee? $1 - \frac{\binom{50}{20}\cdot 2^{20}}{\binom{100}{20}}$

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(a) How many distinguishable ways are there to organize these three files?

(b) Suppose the boss asks James to organize the files so that all types of files are contiguous to each other. For instance, if $m = 3$, the sequence *bbbaaaccc* is contiguous, but the sequence *bbaabaccc* is not contiguous. What is the probability that a random permutation of the files will be contiguous?

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(b) Let $S = (X + Y) \mod p$ and $T = XY \mod p$. What are the distributions of S and T?

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(c) What are the expectations $\mathbb{E}[S]$ and $\mathbb{E}[T]$? $\mathbb{E}[S] = \frac{(p-1)}{2}; \mathbb{E}[T] = \frac{(p-1)^2}{2p}$

Coin Flips

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Am I Crazy?

Chris wants to know if he has bipolar disorder. Let C be the event that Chris has bipolar disorder. The incidence of bipolar disorder in the general population is 1%. He is really poor, so he asks his older brother Alex to make his diagnosis. If someone has bipolar disorder, Alex correctly identifies it 80% of the time, but if someone does not have bipolar disorder, Alex makes a false accusation of bipolar 30% of the time. Let A be the event that Alex diagnoses Chris of having bipolar disorder. Assume $Pr[C] = 0.01$.

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(a) Find $Pr[A|C]$

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Am I Crazy? (cont.)

(e) Let's generalize the answer to (d). Let D be the event that a psychologist diagnoses Chris as having bipolar disorder. $Pr[D|C] = p$ and $Pr[D|\neg C] = q$. Let $Pr[C] = s$. Let B' be the event that Chris receives a correct diagnosis from the psychologist.

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   i Find $Pr[B']$.

   ii What is the effect of raising p? (Does this make intuitive sense?)

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i Find $Pr[B']$. $sp + (1-s)(1-q)$

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It depends...

Search and Rescue

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(a) After how many days of searching the city and not finding the boy will the probability that the boy is in the forest be greater than 90%?

(b) Suppose the local fire department has searched the city for **at most** 2 days.

  i What is the probability that they found the boy?

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(a) Every Starcraft game you play, you have a 55% chance of winning. If you play 10 games today, what is the probability that you will win more than 8 of them?

(b) On your daily drive to school, you pass by an average of 10 cars. What is the probability that you will pass by 50 cars tomorrow?

(c) If you are trapped on an island where all the fresh water is contaminated and you have a 2% chance of getting water poisoning each day, how many days can you expect to survive without getting poisoned?

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Binomial distribution: $\binom{10}{9} \cdot 0.55^9 \cdot 0.45^1 + \binom{10}{10} \cdot 0.55^{10} \cdot 0.45^0$

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Poisson distribution: $\lambda = 10 \implies \frac{10^{50}}{50!} \cdot e^{-10}$

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Geometric distribution: $E(X) = \frac{1}{p} \implies$ You will live for $\frac{1}{0.02} = 50$ days.

Distributions (cont.)

When you solve homework problems, you can solve an average of 10 per day. However, because you are often busy on Reddit and/or Facebook, you get easily distracted and you usually get 80% of them correct. What is the probability that you will solve 30 questions **and** get exactly 20 of them correct on any given day?

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$$
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& \Pr[\text{You answer 30 questions} \cap \text{You answer 20 questions correctly}] \\
= \; & \Pr[\text{You answer 30 questions}] \\
& \times \Pr[\text{You get 20 questions correct} \mid \text{You answer 30 questions}] \\
= \; & \frac{10^{30}}{30!} \cdot e^{-10} \cdot \binom{30}{20} \cdot 0.8^{20} \cdot 0.2^{10}
\end{aligned}
$$

The evolution of a social network (Midterm 2, Fall 2011)

Say one person in a class of n people knows a secret, perhaps where the midterm is. Occasionally, a randomly chosen person A **who doesn't know the secret** calls a randomly chosen person $B (B \neq A)$ and learns the secret if B knows it. Let X_2 be a random variable that represents the number of calls (no two calls are simultaneous) until two people know the secret.

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(c) Let X_i be the number of calls needed to go from $i-1$ people knowing the secret to i people. What is $E[X_i]$?

(d) What is the expected time for everyone to know the secret?

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$$\sum_{i \geq 2}^{n} \frac{n-1}{i-1} = (n-1)(\ln(n-1) + \gamma)$$

Graphs

Wikipedia: "A **complete** graph is an undirected graph in which every pair of distinct vertices is connected by a unique edge."

(a) How many edges are in a complete graph with 100 nodes?

(b) Does an Eulerian path exist such a graph?

(c) Does an Eulerian cycle exist in such a graph?

(d) Does a Hamiltonian path exist in such a graph?

(e) Does a Hamiltonian cycle exist in such a graph?

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Graphs

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(a) How many edges are in a complete graph with 100 nodes? $\frac{100 \cdot 99}{2}$

(b) Does an Eulerian path exist such a graph? No

(c) Does an Eulerian cycle exist in such a graph? No

(d) Does a Hamiltonian path exist in such a graph?

(e) Does a Hamiltonian cycle exist in such a graph?

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(d) Does a Hamiltonian path exist in such a graph? Yes

(e) Does a Hamiltonian cycle exist in such a graph? Yes

Graphs (cont.)

Now we generate a complete graph with 101 nodes. Then we remove 1 randomly chosen edge.

(a) Does an Eulerian path exist in this graph?

(b) Does an Eulerian cycle exist in this graph?

Graphs (cont.)

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Hamiltonian Paths and Cycles

How do you determine whether a graph has a Hamiltonian path/cycle?

Hamiltonian Paths and Cycles

How do you determine whether a graph has a Hamiltonian path/cycle?
You take CS170. This is an NP-complete problem.

Hypercubes

Hypercubes

...?