MIDTERM 3 REVIEW (FALL 2011)

Collected by HKN

1 Discrete-Time Fourier Series

(Quiz 3, Fall 2009) The signal $x(n)$ is defined as $x(-1) = x(1) = 1, x(0) = -2$. x is also periodic, $p = 100$.

1. How many DFS coefficients does x have? Find these coefficients.
2. Determine the following quantities:

$$\sum_{k \in \langle p \rangle} X_k, \qquad \sum_{k \in \langle p \rangle} (-1)^k X_k.$$

2 Continuous-Time Fourier Series

(EE120 Midterm 1, Fall 2011) The continuous-time signal $x(t)$ is defined as:

$$x(t) = \sum_{n=-\infty}^{\infty} (-1)^n \delta(t - nT).$$

Find the continuous-time Fourier series coefficients X_k of x.

3 DTFT, CTFT

1. *Time-shifting property of the DTFT.* Let $X(\omega)$ be the discrete-time Fourier transform of the discrete-time signal $x(n)$. We shift $x(n)$ in time by N to obtain the new signal $\hat{x}(n) = x(n - N)$ with a Fourier transform $\hat{X}(\omega)$. Establish a relationship between $X(\omega)$ and $\hat{X}(\omega)$.
2. *Differentiation property of the CTFT.* Let $X(\omega)$ be the continuous-time Fourier transform of the continuous-time signal $x(t)$. We construct the new signal $\hat{x}(t)$ as $\hat{x}(t) = \dot{x}(t) = \frac{d}{dt}x(t)$ with a Fourier transform $\hat{X}(\omega)$. Establish a relationship between $X(\omega)$ and $\hat{X}(\omega)$. Feel free to switch the differentiation and integration operators if needed.
3. Determine the signal $x(n)$ with DTFT

$$X(\omega) = \begin{cases} A, & |\omega| < \Omega, \\ 0, & \text{otherwise.} \end{cases}$$

   for $|\omega| < \pi$. Remember that the DTFT of any discrete-time signal is 2π-periodic (why?), so we only need to specify the function for any frequency range of size 2π.

4. *Convolution property of the DTFT.* If x_1 has DTFT X_1 and x_2 has DTFT X_2, then $x_1 * x_2$ has DTFT $X_1 \cdot X_2$. Informally, 'convolution in time is multiplication in frequency'. Using this property, find $(x_1 * x_2)$, if

$$x_1(t) = \frac{\sin\left(\frac{\pi}{3}n\right)}{\pi n}, \qquad x_2(t) = \frac{\sin\left(\frac{\pi}{4}(n-2)\right)}{\pi(n-2)}.$$