END SEMESTER EXAMINATION – 2021

Semester : 1st (New)

Subject Code : Sc-102

MATHEMATICS – I

Full Marks : 70

Time – Three Hours

The figures in the margin indicate full marks for the questions.

Instruction :

All questions of PART–A and PART–B are compulsory.

PART – A

Marks – 25

1. Choose the correct answer of the following questions :   1×10=10

(i) The value of i^{109} is

(a) $-i$ (b) i

(c) -1 (d) 1

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(ii) The modulus of $a + ib$ is

  (a) $a + ib$

  (b) ....

  (c) $\sqrt{a^2 + b^2}$

  (d) $\sqrt{a^2 - b^2}$

(iii) If a polynomial $p(x)$ has no factor common to the factors of $x^3 - 1$, then the partial fraction of the proper fraction $\dfrac{p(x)}{x^3 - 1}$ be of the form

  (a) $\dfrac{Ax^2 + Bx + C}{x^3 - 1}$

  (b) $\dfrac{C}{x-1} + \dfrac{A + Bx}{x^2 + x + 1}$

  (c) $\dfrac{A}{x-1} + \dfrac{Bx^2 + Cx + D}{x^2 + x + 1}$

  (d) $\dfrac{A}{x-1} + \dfrac{B}{(x-1)^2} + \dfrac{C}{(x-1)^3}$

(iv) If $^{n+1}C_3 = 2 \times {}^nC_2$, then the value of n is

  (a) 3

  (b) 4

  (c) 5

  (d) 6

(v) How many arrangements can be made out of the letters of the word 'POLYTECHNIC'?

  (a) 11!

  (b) $\dfrac{11!}{2!}$

  (c) $11 - 2!$

  (d) $^{11}p_2$

(vi) Number of terms in $(2 - x^2)^{14}$ is

  (a) 16

  (b) 15

  (c) 14

  (d) 30

(vii) Coefficient of middle term in expansion of $\left(x - \dfrac{x^3}{5}\right)^5$ is

  (a) $\dfrac{14}{625}$

  (b) $\dfrac{70}{625}$

  (c) $\dfrac{14}{125}$

  (d) $\dfrac{70}{125}$

(viii) The value of $\log_{81} 27$ is

  (a) $\dfrac{1}{4}$

  (b) $\dfrac{1}{2}$

  (c) $\dfrac{5}{4}$

  (d) $\dfrac{3}{4}$

(ix) If A is a matrix of order m×n and B is a matrix of order n×p, then the order of AB is

(a) p×m

(b) p×n

(c) n×p

(d) m×p

(x) If A is a skew symmetric matrix of order 3×3, then the sum of the diagonal elements of A is

(a) 0

(b) 6

(c) 9

(d) 3

2. Choose the correct statement :   1×5=5

(i) (a) $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B$

(b) $\sin(A+B)\sin(A-B) = \cos^2 A - \sin^2 B$

(c) $\sin(A+B)\sin(A-B) = \sin^2 A - \cos^2 B$

(ii) (a) $\cos 2A = \dfrac{1-\tan^2 A}{1+\tan^2 A}$

(b) $\sin 2A = \dfrac{2\tan A}{1-\tan^2 A}$

(c) $\tan 2A = \dfrac{2\tan A}{1+\tan^2 A}$

(iii) (a) $\sin 3\theta = 4\sin^3 \theta - 3\sin \theta$

(b) $\sin 3\theta = 3\sin \theta - 4\sin^3 \theta$

(c) $\sin 3\theta = 3\sin^3 \theta - 4\sin \theta$

(iv) In any $\triangle ABC$

(a) $a^2 = b^2 + c^2 - 2bc \cos B$

(b) $a^2 = b^2 + c^2 + 2bc \cos A$

(c) $a = b\cos C + c\cos B$

(v) (a) $\sin^{-1}(x) + \sin^{-1}(-x) = \dfrac{\pi}{2}, x \in [-1,1]$

(b) $\cos^{-1}(x) + \cos^{-1}(-x) = \pi, x \in [-1,1]$

(c) $\tan^{-1}(x) + \cot^{-1}(-x) = \dfrac{\pi}{2}, x \in \mathbb{R}$

3. Fill in the blanks : $1 \times 5 = 5$

(i) The volume of a sphere of radius 6 unit is _____ .

(ii) The base radius of a cone is 7 units. If the height of the cone is 24 units, its lateral surface is _____ .

(iii) In the Simpson's Rule : $Area = \dfrac{S}{3}[A + 2D + 4E]$, the letter 'S' stands for *Strip* .

(iv) If m_1 and m_2 be the slopes of two perpendicular lines, then the relation between m_1 and m_2 is _____ .

(v) The equation of a straight line passing through $(0, -1)$ and parallel to x-axis is _____ .

4. Write true or false : $1 \times 5 = 5$

(i) If two columns of a determinant are same (identical), then the value of the determinant will be zero.

(ii) The value of $\sin(-1230°)$ is $\dfrac{1}{2}$.

(iii) The principal value of $\cos^{-1}(\cos\dfrac{5\pi}{4})$ is $\dfrac{\pi}{4}$.

(iv) In a triangle ABC, if the sides are $a=7, b=5$ and $c=8$, then the value of A will be 60°.

(v) If a base of field 55m and number of ordinates are 11, then the breadth of each strip is equal to 5m.

PART – B

Marks – 45

5. Answer any *five* of the following questions : $2 \times 5 = 10$

(i) If $x = 1 - i$, then prove that $x^2 - 2x + 2 = 0$.

(ii) Find the argument of $2 + 2\sqrt{-3}$.

(iii) In how many different ways can the letters of the word PANDEMIC be arranged without changing the order of the vowels in the word ?

(iv) Write the general term in the expansion of $\left(9x - \dfrac{1}{3\sqrt{x}}\right)^{18}$ and find its 13th term.

(v) Prove that

$$\log 2 + 16 \log \frac{16}{15} + 12 \log \frac{25}{24} + 7 \log \frac{81}{80} = 1.$$

(vi) Insert three G.M.s between 1 and $\frac{1}{16}$.

(vii) If $A = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B^T = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$, then find $(A+2B)^T$.

6. Resolve into partial fractions (any *one*) :

$$3 \times 1 = 3$$

(i) $\dfrac{x}{(x+1)(x-2)^2}$

(ii) $\dfrac{2x-3}{x^2+6x+8}$.

7. Answer any *two* of the following questions :

$$3 \times 2 = 6$$

(i) If 3rd and 7th terms of an Arithmetic Progression are 18 and 30 respectively, then find the progression.

(ii) If a,b,c in A.P. and x,y,z are in G.P., then prove that $x^{b-c} y^{c-a} z^{a-b} = 1$.

(iii) By using properties of determinant find the value of

$$\begin{vmatrix} 1+x & y & z \\ x & 1+y & z \\ x & y & 1+z \end{vmatrix}.$$

8. Prove any *three* of the following : $2 \times 3 = 6$

(i) $\dfrac{\cos 5^0 + \sin 5^0}{\cos 5^0 - \sin 5^0} = \tan 50^0$

(ii) $\tan\left(\dfrac{\pi}{4} + \dfrac{\theta}{2}\right) = \sec\theta + \tan\theta$

(iii) $\tan^{-1}\dfrac{1}{2} + \tan^{-1}\dfrac{1}{3} = \dfrac{\pi}{4}$.

(iv) In any triangle ABC, $\dfrac{a+b}{c} = \dfrac{\cos\left(\dfrac{A-B}{2}\right)}{\sin\dfrac{C}{2}}$.

9. Answer any *two* of the following questions :

$$3 \times 2 = 6$$

(i) If $A+B+C = \dfrac{\pi}{2}$, then prove that

$\tan A \tan B + \tan B \tan C + \tan C \tan A = 1$.

(ii) Solve : $2\cos^2 x + 3\cos x - 2 = 0$, $0 \le x \le 2\pi$.

(iii) Show that $\cos^{-1}\left(\dfrac{12}{13}\right) + \sin^{-1}\left(\dfrac{3}{5}\right) = \sin^{-1}\left(\dfrac{56}{65}\right)$.

10. Answer any *two* of the following questions :

$3 \times 2 = 6$

(i) An irregular plot has the following offsets measured from one end at equal distance.

X	0	12	24	36	48	60	72	84	96	108	120
d	53	52	47	49	53	63	58	61	52	49	48

Find the area of the plot.

(ii) Find the whole surface area of a right prism whose height is 75 cm and whose base is a regular octagon of side 12 cm.

(iii) A regular pyramid has a base area of 56 cm² and a volume of 224 cm³. What is the height of the pyramid ?

11. Answer the following questions : $2+3+3=8$

(i) If the three points $(3, 0)$, $(a, -2)$ and $(8, 2)$ are collinear, then find the value of a.

(ii) Find the equation of a straight line passing through $(-2, 4)$ and perpendicular to the line $4x - 5y + 2 = 0$.

(iii) Find the equations of two lines parallel to $5x - 12y + 26 = 0$ at a distance of 4 units from it.