Total No. of printed pages = 10

END SEMESTER/RETEST EXAMINATION, J/F 2023

Semester : 1st

Subject Code : Sc-102

MATHEMATICS-I

Full Marks – 70

Time – Three hours

The figures in the margin indicate full marks for the questions.

Instruction :

- *All* questions of PART-A and PART-B are compulsory.

PART – A

Marks – 25

1. Choose the correct answer : 1×10=10

(a) i^{-39} in the form of a+ib can be written as

(i) 0 + 0i (ii) 0 +1i

(iii) 0 – 1i (iv) 1 – 0i

[Turn over

(b) The argument of $z = -1 - i\sqrt{3}$ is

(i) $\frac{\pi}{3}$ (ii) $\frac{-\pi}{3}$

(iii) $\frac{\pi}{6}$ (iv) $\frac{-\pi}{6}$

(c) The value of $\frac{\log\sqrt{8}}{\log 8}$ is equal to

(i) $\frac{1}{2}$ (ii) $\frac{-1}{2}$

(iii) -1 (iv) 2

d) The product of the three cube roots of unity is

(i) 1 (ii) 2

(iii) -1 (iv) -3

(e) The modulus of $\frac{1+2i}{1-3i}$ is equal to

(i) $\frac{1}{2}$ (ii) $\frac{1}{\sqrt{3}}$

(iii) $\frac{1}{\sqrt{2}}$ (iv) $\frac{-1}{\sqrt{2}}$

(f) The next term of the arithmetic progression $\sqrt{7}, \sqrt{28}, \sqrt{63}, \ldots$ is

(i) $\sqrt{70}$ (ii) $\sqrt{84}$

(iii) $\sqrt{97}$ (iv) $\sqrt{112}$

(g) The 20th term from the last term of the arithmetic progression 3,8,13, ... , 253 is

(i) 162 (ii) 115

(iii) 158 (iv) 185

(h) How many words, with or without meaning can be formed using all the letters of the word EQUATION using each letter exactly once ?

(i) 40330 (ii) 4500

(iii) 40320 (iv) 4545

(i) The coefficient of x^5 in the expansion of $(x+3)^8$ is

(i) 1500 (ii) 2100

(iii) 1215 (iv) 1512

(j) Which of the given values of x and y make the following pairs of matrices equal ?

$$\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix}, \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$$

(i) $x = \frac{-1}{3}, y = 7$

(ii) Not possible to find

(iii) $y = 7, x = \frac{-2}{3}$

(iv) $x = \frac{-1}{3}, y = \frac{-2}{3}$

2. Write whether true or false : 1×10=10

(a) $\cos^2 A - \sin^2 A = \frac{1+\tan^2 A}{1-\tan^2 A}$.

(b) $\tan 27^\circ + \tan 18^\circ + \tan 27^\circ \tan 18^\circ = 1$.

(c) $\cos^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{3}$.

(d) $\frac{\sin 5A - \sin 3A}{\cos 5A + \cos 3A} = \tan A$

(e) The general solution of $\sin x = 0$ is $x = n\pi$, where n is an integer.

(f) In any triangle, $a^2 = b^2 + c^2 - 2bc \cos A$.

(g) The slope of line parallel to the x-axis is not defined.

(h) The slope of line parallel to the line $Ax + By + C = 0$ is A.

(i) The distance of a point $(\sin\theta, \cos\theta)$ from the origin is equal to 2 units.

(j) Two lines having slopes $\frac{1}{2}$ and -2 are perpendicular to each other.

3. Choose the correct answers : 1×5=5

(a) The dimensions of a godown are 40m×25m ×10m. If it is filled with boxes each of dimensions 2m×1.25m×1 m, then the number of boxes will be

(i) 1800 (ii) 2000

(iii) 4000 (iv) 8000

(b) A 20 m deep well with diameter 7m is dug and the earth from digging is evenly spread out to form a platform 22m by 14m. The height of the platform is

(i) 3m (ii) 2m

(iii) 2.5m (iv) 3.5m

(c) The cost of whitewashing a room of dimensions 6m×4m×4.2m (including the ceiling) at the rate of ₹ 25 per m^2.

(i) ₹ 2265.50 (ii) ₹ 2250

(iii) ₹ 2700.50 (iv) ₹ 2700

(d) The total surface area of a solid cylinder is 462 cm^2. If its curved surface area is $\frac{1}{3}$ of that of its total surface area, then the volume of the cylinder in cm^3 is

(i) 593 (ii) 539

(iii) 590 (iv) 597

(e) If the total surface area of a hemisphere is 27π cm^2, then its diameter in cm is

(i) 3 (ii) 6

(iii) 9 (iv) 12

PART – B

Marks – 45

4 (a) If $\log m + \log n = \log(m+n)$, then show that $n = \frac{m}{m-1}$. 2

(b) For any two complex numbers z_1 and z_2, prove that

$\mathrm{Re}(z_1 z_2) = \mathrm{Re}(z_1)\,\mathrm{Re}(z_2) - \mathrm{Im}(z_1)\,\mathrm{Im}(z_2)$. 2

(c) Prove that $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ is independent of θ. 3

5. (a) If a,b,c are in arithmetic progression, then prove that

$a\left(\frac{1}{b}+\frac{1}{c}\right)$, $b\left(\frac{1}{c}+\frac{1}{a}\right)$ and $c\left(\frac{1}{a}+\frac{1}{b}\right)$ are in arithmetic progression. 2

(b) Find the middle term in the expansion of $\left(3-\frac{x^3}{6}\right)^8$. 2

(c) Find the value of n if ${}^{n-1}P_4 : {}^nP_4 = 3 : 5$, where $n > 4$. 2

6. (a) In how many of the distinct permutations of the letters in the word MISSISSIPPI do the four I's not come together? 2

(b) The sum of the first three terms of a GP is $\frac{39}{10}$ and their product is 1. Find the common ratio of the GP. 2

(c) Resolve into simple fractions :

$$\frac{27x^2+32x+16}{(3x+2)^2(1-x)} \text{ where } |x|<\frac{2}{3}. \quad 3$$

7 Show that (any *four*) : 4×2=8

(a) $\text{cosec}^2\theta + \sec^2\theta = \frac{\sec^2\theta}{\sin^2\theta}$

(b) $\frac{2}{\sqrt{2+\sqrt{2+2\cos 4x}}} = \sec x$.

(c) $\frac{\cos 8^\circ - \sin 8^\circ}{\cos 8^\circ + \sin 8^\circ} = \tan 37^\circ$.

(d) $\frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)} = \left(\frac{1+\tan x}{1-\tan x}\right)^2$.

(e) $\cos 6x = 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$.

(f) $\cos(420^\circ)\sin(390^\circ) + \sin(-300^\circ)\cos(-330^\circ) = 1$.

(g) $\frac{a^2-c^2}{b^2} = \frac{\sin(A-C)}{\sin(A+C)}$.

8. Solve any *two* questions : 3×2=6

(a) Prove that in any triangle ABC,

$$\tan\frac{B-C}{2} = \frac{b-c}{b+c}\cot\frac{A}{2}$$

(b) Prove that $\cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ = \frac{1}{16}$.

(c) Prove that

$$\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right) = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x \text{ where } \frac{-1}{\sqrt{2}} \le x \le 1.$$

9. Solve any *two* questions : $3 \times 2 = 6$

(a) Find the equation of the line which passes through the point (2,2) and which cuts of intercepts on the coordinate axes whose sum is equal to 9.

(b) The perpendicular from the origin to a line meets it at the point (–2, 9). Find the equation of the line.

(c) Find the coordinates of the foot of the perpendicular drawn from the point (2, 3) to the straight line whose equation is $y = 3x + 4$.

10. (a) Calculate the total area of a field with a base of 50m and ordinates 4, 6, 5, 7, 7, 6, 6.5, 7.5, 8, 8 and 9m respectively. 3

(b) Calculate the lateral surface area and total surface area of the frustum of a square pyramid whose larger base edge is 24 cm, smaller base edge is 14 cm and whose lateral edge is 13 cm. 2