

$$1000 \text{ mbar} = 1 \times 10^5 \text{ Pa}.$$

The millibar is a convenient unit for meteorologists because the average atmospheric pressure at sea level on Earth is  $1.013 \times 10^5 \text{ Pa} = 1013 \text{ mbar} = 1 \text{ atm}$ . Using the equations derived when considering pressure at a depth in a fluid, pressure can also be measured as millimeters or inches of mercury. The pressure at the bottom of a 760-mm column of mercury at  $0^\circ\text{C}$  in a container where the top part is evacuated is equal to the atmospheric pressure. Thus, 760 mm Hg is also used in place of 1 atmosphere of pressure. In vacuum physics labs, scientists often use another unit called the torr, named after Torricelli, who, as we have just seen, invented the mercury manometer for measuring pressure. One torr is equal to a pressure of 1 mm Hg.

Unit	Definition
SI unit: the Pascal	$1 \text{ Pa} = 1 \text{ N/m}^2$
English unit: pounds per square inch ( $\text{lb/in.}^2$ or psi)	$1 \text{ psi} = 6.895 \times 10^3 \text{ Pa}$
Other units of pressure	$1 \text{ atm} = 760 \text{ mmHg}$ $= 1.013 \times 10^5 \text{ Pa}$ $= 14.7 \text{ psi}$ $= 29.9 \text{ inches of Hg}$ $= 1013 \text{ mbar}$  $1 \text{ bar} = 10^5 \text{ Pa}$ $1 \text{ torr} = 1 \text{ mm Hg} = 133.3 \text{ Pa}$

**Table 14.3 Summary of the Units of Pressure**

## 14.3 | Pascal's Principle and Hydraulics

### Learning Objectives

By the end of this section, you will be able to:

- State Pascal's principle
- Describe applications of Pascal's principle
- Derive relationships between forces in a hydraulic system

In 1653, the French philosopher and scientist Blaise Pascal published his *Treatise on the Equilibrium of Liquids*, in which he discussed principles of static fluids. A static fluid is a fluid that is not in motion. When a fluid is not flowing, we say that the fluid is in static equilibrium. If the fluid is water, we say it is in **hydrostatic equilibrium**. For a fluid in static equilibrium, the net force on any part of the fluid must be zero; otherwise the fluid will start to flow.

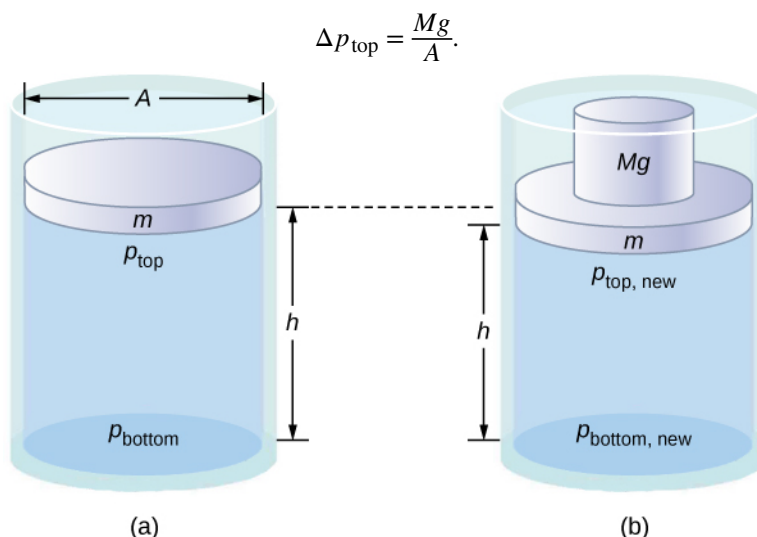
Pascal's observations—since proven experimentally—provide the foundation for hydraulics, one of the most important developments in modern mechanical technology. Pascal observed that a change in pressure applied to an enclosed fluid is transmitted undiminished throughout the fluid and to the walls of its container. Because of this, we often know more about pressure than other physical quantities in fluids. Moreover, Pascal's principle implies that the total pressure in a fluid is the sum of the pressures from different sources. A good example is the fluid at a depth depends on the depth of the fluid and the pressure of the atmosphere.

### Pascal's Principle

**Pascal's principle** (also known as Pascal's law) states that when a change in pressure is applied to an enclosed fluid, it is transmitted undiminished to all portions of the fluid and to the walls of its container. In an enclosed fluid, since atoms of the fluid are free to move about, they transmit pressure to all parts of the fluid *and* to the walls of the container. Any change in pressure is transmitted undiminished.

Note that this principle does not say that the pressure is the same at all points of a fluid—which is not true, since the pressure

in a fluid near Earth varies with height. Rather, this principle applies to the *change* in pressure. Suppose you place some water in a cylindrical container of height  $H$  and cross-sectional area  $A$  that has a movable piston of mass  $m$  (**Figure 14.15**). Adding weight  $Mg$  at the top of the piston increases the pressure at the top by  $Mg/A$ , since the additional weight also acts over area  $A$  of the lid:




**Figure 14.15** Pressure in a fluid changes when the fluid is compressed. (a) The pressure at the top layer of the fluid is different from pressure at the bottom layer. (b) The increase in pressure by adding weight to the piston is the same everywhere, for example,  $p_{\text{top, new}} - p_{\text{top}} = p_{\text{bottom, new}} - p_{\text{bottom}}$ .

According to Pascal's principle, the pressure at all points in the water changes by the same amount,  $Mg/A$ . Thus, the pressure at the bottom also increases by  $Mg/A$ . The pressure at the bottom of the container is equal to the sum of the atmospheric pressure, the pressure due the fluid, and the pressure supplied by the mass. The change in pressure at the bottom of the container due to the mass is

$$\Delta p_{\text{bottom}} = \frac{Mg}{A}.$$

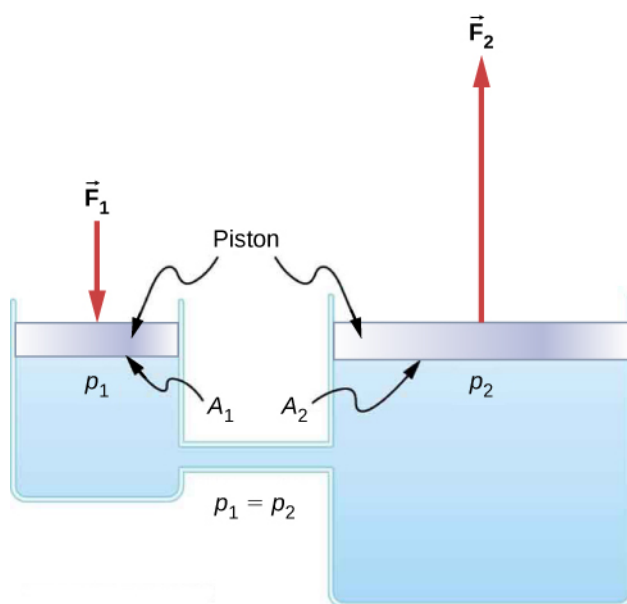
Since the pressure changes are the same everywhere in the fluid, we no longer need subscripts to designate the pressure change for top or bottom:

$$\Delta p = \Delta p_{\text{top}} = \Delta p_{\text{bottom}} = \Delta p_{\text{everywhere}}.$$

 Pascal's Barrel is a great demonstration of Pascal's principle. Watch a **simulation** (<https://openstaxcollege.org//21pascalbarrel>) of Pascal's 1646 experiment, in which he demonstrated the effects of changing pressure in a fluid.

## Applications of Pascal's Principle and Hydraulic Systems

Hydraulic systems are used to operate automotive brakes, hydraulic jacks, and numerous other mechanical systems (**Figure 14.16**).



**Figure 14.16** A typical hydraulic system with two fluid-filled cylinders, capped with pistons and connected by a tube called a hydraulic line. A downward force  $\vec{F}_1$  on the left piston creates a change in pressure that is transmitted undiminished to all parts of the enclosed fluid. This results in an upward force  $\vec{F}_2$  on the right piston that is larger than  $\vec{F}_1$  because the right piston has a larger surface area.

We can derive a relationship between the forces in this simple hydraulic system by applying Pascal's principle. Note first that the two pistons in the system are at the same height, so there is no difference in pressure due to a difference in depth. The pressure due to  $F_1$  acting on area  $A_1$  is simply

$$p_1 = \frac{F_1}{A_1}, \text{ as defined by } p = \frac{F}{A}.$$

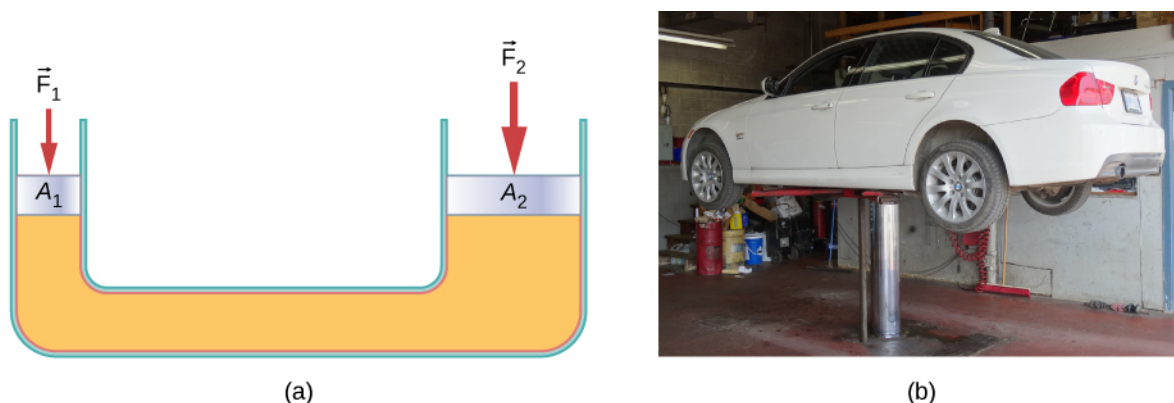
According to Pascal's principle, this pressure is transmitted undiminished throughout the fluid and to all walls of the container. Thus, a pressure  $p_2$  is felt at the other piston that is equal to  $p_1$ . That is,  $p_1 = p_2$ . However, since  $p_2 = F_2/A_2$ , we see that

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}. \quad (14.12)$$

This equation relates the ratios of force to area in any hydraulic system, provided that the pistons are at the same vertical height and that friction in the system is negligible.

Hydraulic systems can increase or decrease the force applied to them. To make the force larger, the pressure is applied to a larger area. For example, if a 100-N force is applied to the left cylinder in **Figure 14.16** and the right cylinder has an area five times greater, then the output force is 500 N. Hydraulic systems are analogous to simple levers, but they have the advantage that pressure can be sent through tortuously curved lines to several places at once.

The **hydraulic jack** is such a hydraulic system. A hydraulic jack is used to lift heavy loads, such as the ones used by auto mechanics to raise an automobile. It consists of an incompressible fluid in a U-tube fitted with a movable piston on each side. One side of the U-tube is narrower than the other. A small force applied over a small area can balance a much larger force on the other side over a larger area (**Figure 14.17**).



**Figure 14.17** (a) A hydraulic jack operates by applying forces  $(F_1, F_2)$  to an incompressible fluid in a U-tube, using a movable piston  $(A_1, A_2)$  on each side of the tube. (b) Hydraulic jacks are commonly used by car mechanics to lift vehicles so that repairs and maintenance can be performed. (credit b: modification of work by Jane Whitney)

From Pascal's principle, it can be shown that the force needed to lift the car is less than the weight of the car:

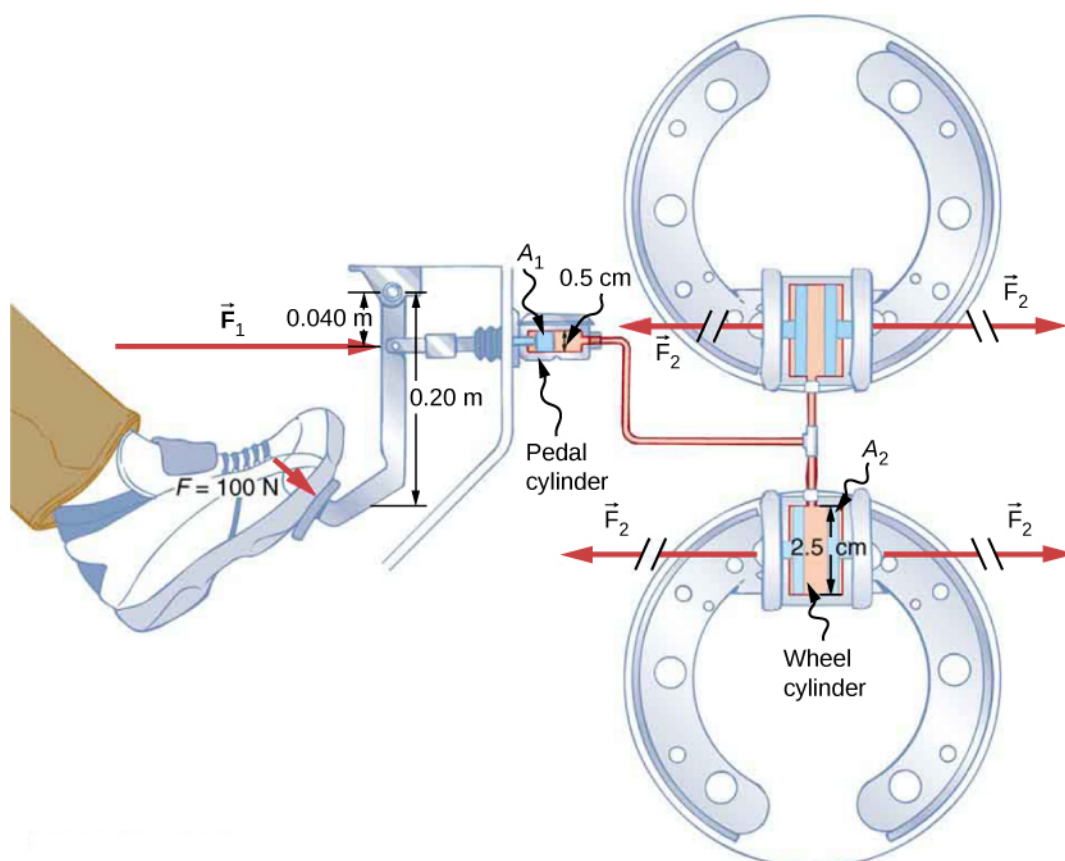
$$F_1 = \frac{A_1}{A_2} F_2,$$

where  $F_1$  is the force applied to lift the car,  $A_1$  is the cross-sectional area of the smaller piston,  $A_2$  is the cross sectional area of the larger piston, and  $F_2$  is the weight of the car.

### Example 14.3

#### Calculating Force on Wheel Cylinders: Pascal Puts on the Brakes

Consider the automobile hydraulic system shown in **Figure 14.18**. Suppose a force of 100 N is applied to the brake pedal, which acts on the pedal cylinder (acting as a “master” cylinder) through a lever. A force of 500 N is exerted on the pedal cylinder. Pressure created in the pedal cylinder is transmitted to the four wheel cylinders. The pedal cylinder has a diameter of 0.500 cm and each wheel cylinder has a diameter of 2.50 cm. Calculate the magnitude of the force  $F_2$  created at each of the wheel cylinders.



**Figure 14.18** Hydraulic brakes use Pascal's principle. The driver pushes the brake pedal, exerting a force that is increased by the simple lever and again by the hydraulic system. Each of the identical wheel cylinders receives the same pressure and, therefore, creates the same force output  $F_2$ . The circular cross-sectional areas of the pedal and wheel cylinders are represented by  $A_1$  and  $A_2$ , respectively.

### Strategy

We are given the force  $F_1$  applied to the pedal cylinder. The cross-sectional areas  $A_1$  and  $A_2$  can be calculated from their given diameters. Then we can use the following relationship to find the force  $F_2$ :

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}.$$

Manipulate this algebraically to get  $F_2$  on one side and substitute known values.

### Solution

Pascal's principle applied to hydraulic systems is given by  $\frac{F_1}{A_1} = \frac{F_2}{A_2}$ :

$$\begin{aligned} F_2 &= \frac{A_2}{A_1} F_1 = \frac{\pi r_2^2}{\pi r_1^2} F_1 \\ &= \frac{(1.25 \text{ cm})^2}{(0.250 \text{ cm})^2} \times 500 \text{ N} = 1.25 \times 10^4 \text{ N}. \end{aligned}$$

### Significance

This value is the force exerted by each of the four wheel cylinders. Note that we can add as many wheel cylinders as we wish. If each has a 2.50-cm diameter, each will exert  $1.25 \times 10^4$  N. A simple hydraulic system, as an example of a simple machine, can increase force but cannot do more work than is done on it. Work is force times distance moved, and the wheel cylinder moves through a smaller distance than the pedal cylinder. Furthermore, the more wheels added, the smaller the distance each one moves. Many hydraulic systems—such as power brakes and those in bulldozers—have a motorized pump that actually does most of the work in the system.



**14.3 Check Your Understanding** Would a hydraulic press still operate properly if a gas is used instead of a liquid?

## 14.4 | Archimedes' Principle and Buoyancy

### Learning Objectives

By the end of this section, you will be able to:

- Define buoyant force
- State Archimedes' principle
- Describe the relationship between density and Archimedes' principle

When placed in a fluid, some objects float due to a buoyant force. Where does this buoyant force come from? Why is it that some things float and others do not? Do objects that sink get any support at all from the fluid? Is your body buoyed by the atmosphere, or are only helium balloons affected (**Figure 14.19**)?



(a)



(b)



(c)

**Figure 14.19** (a) Even objects that sink, like this anchor, are partly supported by water when submerged. (b) Submarines have adjustable density (ballast tanks) so that they may float or sink as desired. (c) Helium-filled balloons tug upward on their strings, demonstrating air's buoyant effect. (credit b: modification of work by Allied Navy; credit c: modification of work by "Crystl"/Flickr)

Answers to all these questions, and many others, are based on the fact that pressure increases with depth in a fluid. This means that the upward force on the bottom of an object in a fluid is greater than the downward force on top of the object. There is an upward force, or **buoyant force**, on any object in any fluid (**Figure 14.20**). If the buoyant force is greater than the object's weight, the object rises to the surface and floats. If the buoyant force is less than the object's weight, the object sinks. If the buoyant force equals the object's weight, the object can remain suspended at its present depth. The buoyant force is always present, whether the object floats, sinks, or is suspended in a fluid.