PANINIpy: Package of Algorithms for Nonparametric Inference with Networks In Python

18 September 2024

Summary

Complex networks provide a highly flexible representation of the relational structure within a variety of real-world systems, from city streets to the Internet [Barabási 2016]. The topology and dynamics of real network data are often too complex to summarize or visualize using traditional data analysis methods, which has triggered a substantial research movement within multiple fields—including physics, computer science, sociology, mathematics, and economics among others—to develop new tools for statistical inference and machine learning specifically suited for networks.

Research on complex network inference has the goal of learning meaningful structural and dynamical regularities in network data in a manner that is often independent of the particular application of interest but relies on fundamental principles that govern a wide range of networked systems, such as transitivity, degree heterogeneity, and assortativity (Newman 2018). A substantial amount of research within complex network inference over the last two decades has focused on the construction and evaluation of algorithms for *community detection*—the task of inferring groups of nodes that exhibit particularly strong connectivity or that have shared roles or features (Fortunato 2010). Another major area of interest is *network reconstruction* (Peel, Peixoto, and De Domenico 2022), which aims to infer statistically significant functional connections from time series or other activity patterns as well as identify spurious correlations and missing edges in partially observed noisy network data. A third notable focus area within complex network inference is the clustering of network populations or multilayer networks arising in longitudinal and cross-sectional studies (Young, Kirkley, and Newman 2022).

Although community detection, network reconstruction, and network population clustering are some of the most widely researched areas in complex network inference, there are a broad range of tasks for which there is active development

of new methods. For example, there is a large new body of work aimed at inferring statistically significant structure in higher order networks (Battiston et al. 2021) and networks with different types of metadata on the nodes and/or edges (Fajardo-Fontiveros, Guimerà, and Sales-Pardo 2022).

Statement of Need

Due to their discrete, relational, and heterogeneous nature, complex networks present new obstacles for statistical inference. Many inference objectives on networks are intrinsically combinatorial and produce complex summaries in the form of sets or partitions. These factors make scalability and interpretability of critical importance for practical algorithms, which are not often easily accommodated within learning frameworks that focus on continuous ordered data. There are also a number of ways uncertainty can be introduced in the collection of a network dataset, whether through measurement error, sampling bias, or fluctuations across experimental settings in longitudinal or cross-sectional studies. These factors underscore the importance of developing new principled and flexible methods for extracting structural and dynamical regularities in networks that do not rely on ad hoc parameter choices or heuristics, allowing them to be robust in the presence of noise.

PANINIpy is a flexible and easy-to-use collection of nonparametric statistical inference methods for unsupervised learning with network data. These methods are unified in their motivation from fundamental principles–currently, the Minimum Description Length (MDL) principle underlies all the methods in PANINIpy—and their lack of dependence on arbitrary parameter choices that can impose unwanted biases in inference results. PANINIpy is highly accessible for practitioners as its methods do not require the user to manually tune any input parameters and everything is written from scratch in pure Python to be optimized for each task of interest without reliance on existing packages. PANINIpy therefore provides an important complement to existing network analysis packages in Python such as NetworkX (Hagberg, Swart, and Schult 2008) that focus primarily on network metrics, network visualization, and community detection.

Related Software Packages

There are number of existing Python packages containing individual methods that perform nonparametric inference with networks, but none that are unified under this scope with the ease-to-use pure Python implementation of PANINIpy. The Graph-Tool (Peixoto 2014) package includes a number of flexible, principled Bayesian methods for complex network inference, many of which are nonparametric. As its core functionalities are implemented in C++, Graph-tool is highly efficient given the computational demand of the inference problems it considers. Graph-tool relies on unique data structures and Markov chain Monte Carlo

methods for greater speed and flexibility, but these features are often challenging for new users to navigate. Other popular packages such as NetworkX (Hagberg, Swart, and Schult 2008) and iGraph (Csardi and Nepusz 2006) also have methods for complex network inference—largely for the task of community detection—but are much broader in scope, being used primarily for network summary statistics and visualization. PANINIpy fills an important gap in the software space for network inference methods with very simple dependencies in pure Python.

Current Modules

Modules can be flexibly added to the package as needed. All modules take as input a standard representation of a network (either as an edgelist or an adjacency list in Python). The existing modules at the time of this publication are:

- **hypergraph_binning**: Methods for identifying MDL-optimal temporally contiguous partitions of event data between distinct node sets (e.g. users and products). Utilizes method of (A. Kirkley 2024b).
- **population_clustering**: Methods for performing clustering of observed network populations, multilayer network layers, or temporal networks. Utilizes method of (A. Kirkley et al. 2023). Also includes method for generating synthetic network populations using the method of (Young, Kirkley, and Newman 2022).
- **distributional_regionalization**: Methods for performing MDL-based regionalization on distributional (e.g. census) data over space. Utilizes method of (A. Kirkley 2022).
- **hub_identification**: Methods for inferring hub nodes in a network using different information theoretic criteria. Utilizes method of (A. Kirkley 2024a).
- **community_regionalization**: Perform contiguous regionalization of spatial network data, applicable to a wide class of community detection objectives. Utilizes method of (Morel-Balbi and Kirkley 2024).
- **network_backbones**: Perform global and local network backboning for a weighted network. Utilizes method of (Alec Kirkley 2024).

Please refer to the documentation for details on the methodology, implementation, and usage for each module.
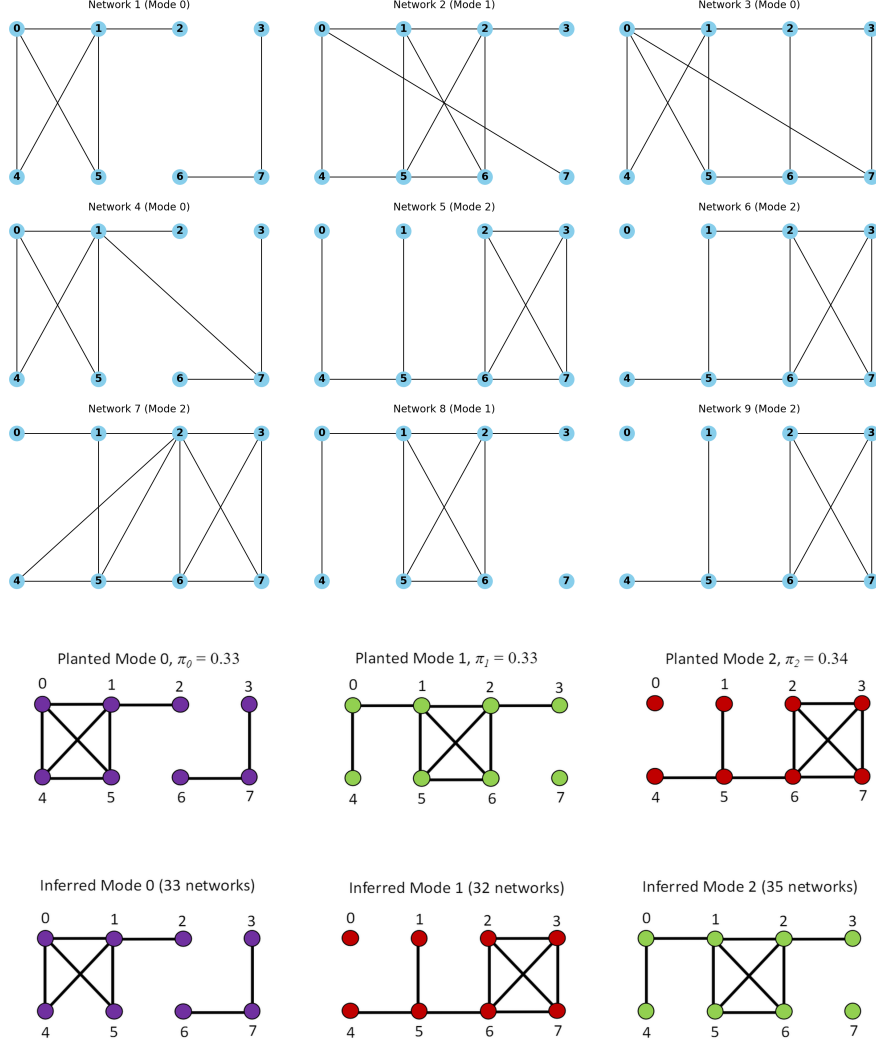
Example Figures

Acknowledgments

Figure 1: (Top) Sample networks from a synthetic network population generated with the `generate_synthetic` method, along with the mode (cluster) from which each sample was generated. (Bottom) Planted and inferred cluster modes using the discontiguous MDL clustering algorithm of A. Kirkley et al. (2023), implemented as an `MDL_populations` class method. Colors indicate the correspondence between the true and inferred modes. The mixture probability π_k of generating from each planted mode, as well as the number of networks within each inferred mode's cluster, are indicated.
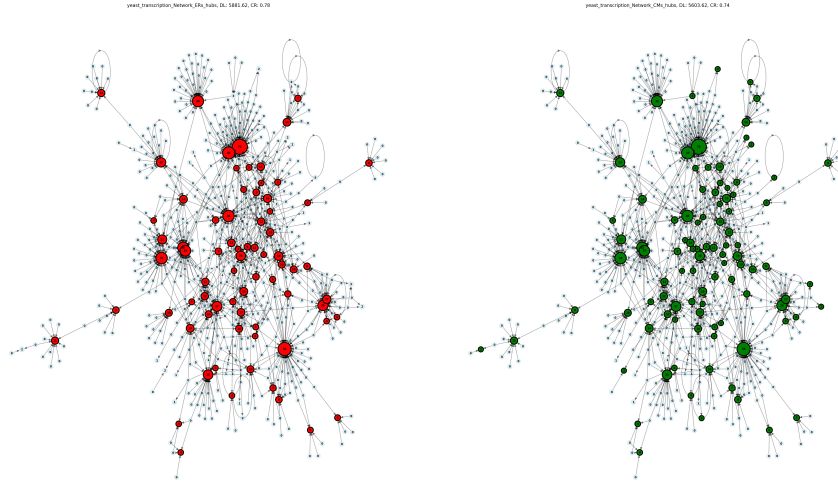
Figure 2: (Left) Yeast transcription network (Milo et al. 2002) with red nodes indicating the hubs inferred using the ER encoding of A. Kirkley (2024a), implemented as an 'Network_hubs' class method. (Right) Same network, with green nodes indicating hubs inferred using the CM encoding. Nodes are scaled proportionally to their degrees.

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