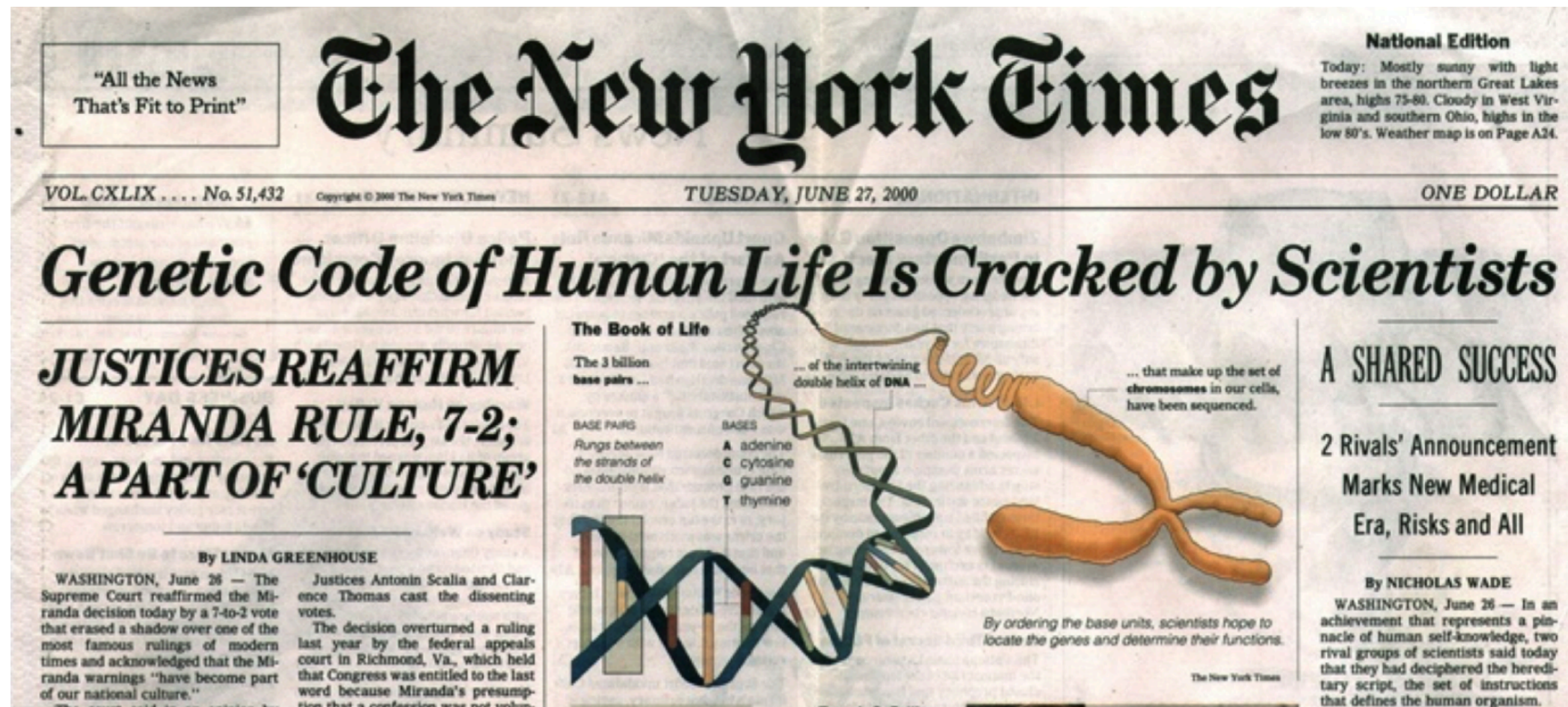


Genomic Analyses from Non-invasive Prenatal Testing Reveal Genetic Associations, Patterns of Viral Infections, and Chinese Population History

**Human genome
project&reference genome**

Human Genome Project



- 70% come from a male donor. The remainders are from one American male donor and two American female donor.
- 3 billion US dollars & 15 years.
- Chinese scientists sequenced 1% of the reference genome.

**How to get our genome
data?**



Reads

GTATGCACGCGATAG
TAGCATTGCGAGACG

TATGTCGCAGTATCT
GGTATGCACGCGATA

CACCCTATGTCGCAG
TGGAGCCGGAGCACC

GAGACGCTGGAGCCG
CGCTGGAGCCGGAGC

Your genome

CGTCTGGGGGGTATGCACGCGATAGCATTGCGAGACGCTGGAGCCGGAGCACCCTATGTCGCAGTATCTGTCTTTGATTCCTG

To cover your
genome, we need
to sequence more

“Genomic
number”

@SQ	SN:1	LN:249250621
@SQ	SN:2	LN:243199373
@SQ	SN:3	LN:198022430
@SQ	SN:4	LN:191154276
@SQ	SN:5	LN:180915260
@SQ	SN:6	LN:171115067
@SQ	SN:7	LN:159138663
@SQ	SN:8	LN:146364022
@SQ	SN:9	LN:141213431
@SQ	SN:10	LN:135534747
@SQ	SN:11	LN:135006516
@SQ	SN:12	LN:133851895
@SQ	SN:13	LN:115169878
@SQ	SN:14	LN:107349540
@SQ	SN:15	LN:102531392
@SQ	SN:16	LN:90354753
@SQ	SN:17	LN:81195210
@SQ	SN:18	LN:78077248
@SQ	SN:19	LN:59128983
@SQ	SN:20	LN:63025520
@SQ	SN:21	LN:48129895
@SQ	SN:22	LN:51304566
@SQ	SN:X	LN:155270560
@SQ	SN:Y	LN:59373566

Allele frequency calculation for a population

Allele Frequency Calculation

- Why not simply count the number of “A,T,C,G”?

Because of sequencing errors, systemic bias, etc, the probability should be incorporated.

Allele frequency calculation

- **Maximum Likelihood estimation**

idea: given observations (sequenced reads from different individuals), MLE attempts to find the parameters of probabilistic model to maximize the likelihood $L(p)$

For N unrelated individuals with a single read covering the position, the likelihood function for the read data D_i , for a single variant candidate site in individual i , of the allele frequency $p = (p_A, p_C, p_G, p_T)$, is defined as:

$$L(p) = \prod_{i=1}^N P(D_i | p) = \prod_{i=1}^N \sum_{b \in \{A, C, G, T\}} p(b | p) p(D_i | b) \quad (1)$$

where $p(b | p) = p_b$ and the genotype likelihood assuming a haploid model is $p(D_i | b) = \{1 - \varepsilon_i \text{ if } D_i = b \text{ and } \varepsilon_i/3, \text{ if } D_i \neq b\}$. ε_i corresponds to the GATK corresponds to the GATK-recalibrated error rate converted from the PHRED-scale base quality.

Allele frequency calculation

- **EM algorithm**

EM is an iterative method to find maximum likelihood estimates of parameters in statistical model, where the model depends on unobserved latent variables.

We obtain the maximum likelihood estimate $\hat{p} = \operatorname{argmax}_p L(p)$ using the EM algorithm with starting value computed by the observed allele frequency:

$$p_b = \frac{\sum D_i = b}{N} \quad (2)$$

In the E step, we compute the posterior probability of allele b for individual i at a site j as one of the four A/C/G/T bases:

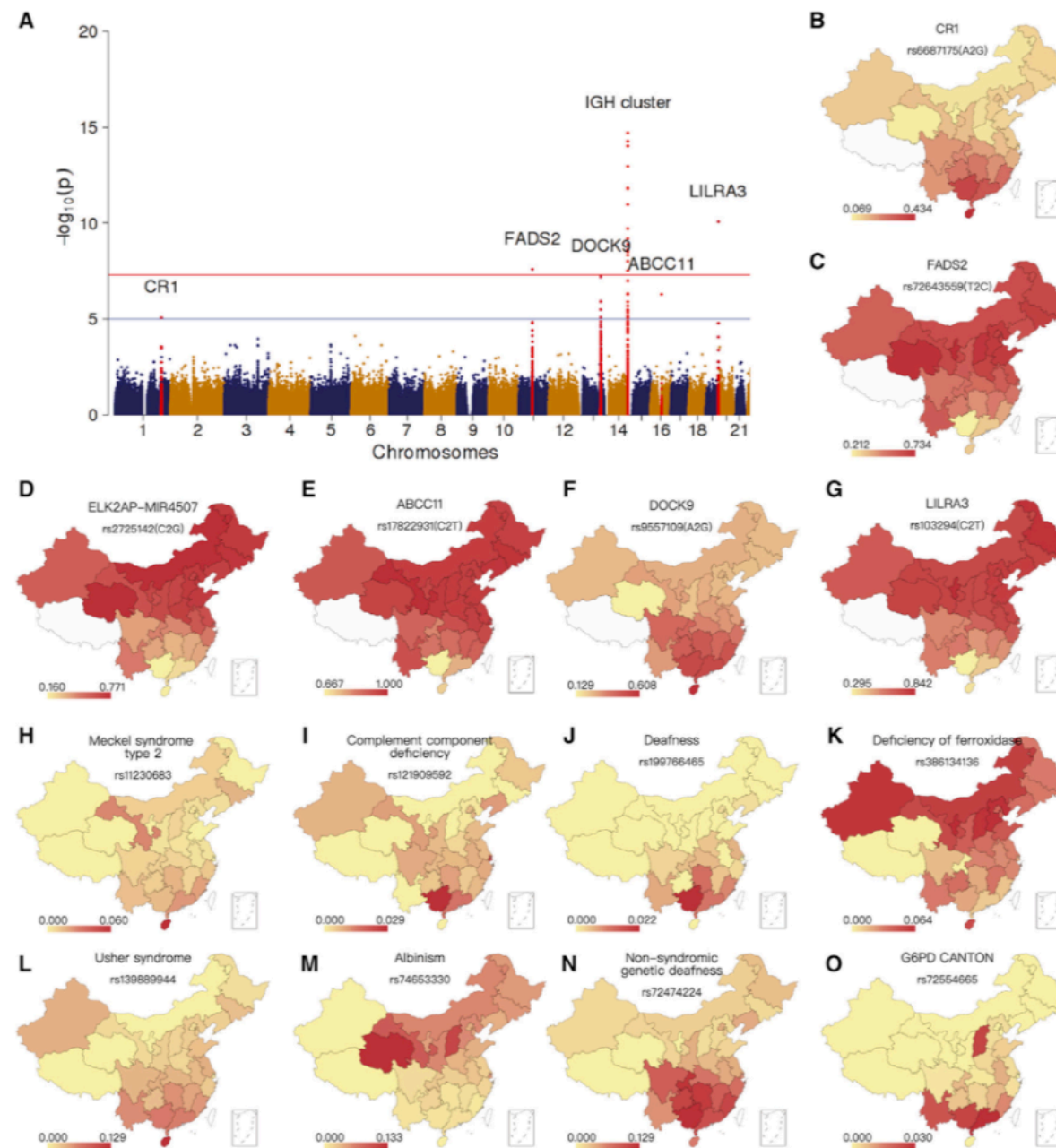
$$P(b | D_i) = \frac{p(b | p)p(D_i | b)}{\sum_{b' \in \{A, C, G, T\}} p(b' | p)p(D_i | b')} \quad (3)$$

We compute the updated allele frequency p' in the M step as

$$p'_b = \frac{\sum_{i=1}^N P(b | D_i)}{N} \quad (4)$$

When the change in the maximum likelihood is less than 0.001, we terminate the algorithm.

Allele frequency calculation



Fitness test

Decision of allelic type

- Method: log-likelihood ratio test

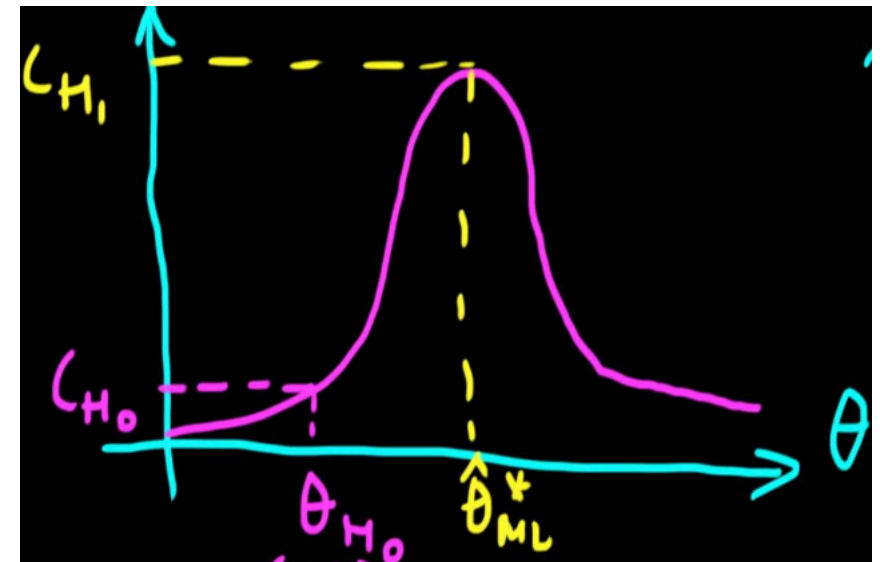
Fitness test

$$H_0 : \theta = \theta_0$$

$$H_1 : \theta = \theta_{ML}$$

$$LR = 2(\log L(O | \theta_{ML}) - \log L(O | \theta))$$

$$LR \sim \chi^2(1)$$



Decision of allelic type

1. iteratively set the allele frequency of one of the four nucleotides to zero to obtain models of tri-allelic loci .

$$LRT_{4vs3} = -2\log\left(\frac{\hat{f}_3(p_x=0)}{\hat{f}_4}\right)$$

where x is one of the 4 bases, f is likelihood function $L(p) = \prod_{i=1}^N P(D_i | p) = \prod_{i=1}^N \sum_{b \in \{A,C,G,T\}} p(b | p) p(D_i | b)$

2. If the p values of LRT_{4vs3} test are significant, the variant will be classified as a tetra-allelic loci (H_0 is rejected). If not, we further to testify:

1.
$$LRT_{3vs2} = -2\log\left(\frac{\hat{f}_2(p_x=0, p_y=0)}{\hat{f}_3(p_x=0)}\right)$$

where x is the base which makes the p value of LRT_{4vs3} maximum, y is one of the rest 3 bases.

Decision of allelic type

$$LRT_{3vs2} = -2\log\left(\frac{\hat{f}_2(p_x=0, p_y=0)}{\hat{f}_3(p_x=0)}\right)$$

3. Similarly, if p value of LRT_{3vs2} is significant, this loci is classified as tri-allelic loci. Otherwise, we choose the base y which makes p value of LRT_{3vs2} maximum, to continue test the bi-allelic versus mono-allelic assumption:

$$LRT_{2vs1} = -2\log\left(\frac{\hat{f}_1(p_x=0, p_y=0, p_z=0)}{\hat{f}_2(p_x=0, p_y=0)}\right)$$