Secure Multiparty Computation (SMC or MPC)

Presenter: Yi LIU

Cryptography

- Conventional Usages
  - Confidentiality
    - E.g. Encryption
  - Integrity
    - E.g. MAC
  - Authentication
    - E.g. Signature

- Secure Computing
  - 1 party (e.g. FHE)
  - 2 parties (e.g. Yao's GC)
  - 3+ parties (e.g. secret sharing based MPC)

What is Security?

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- Why your password is secure?

  Hard to guess

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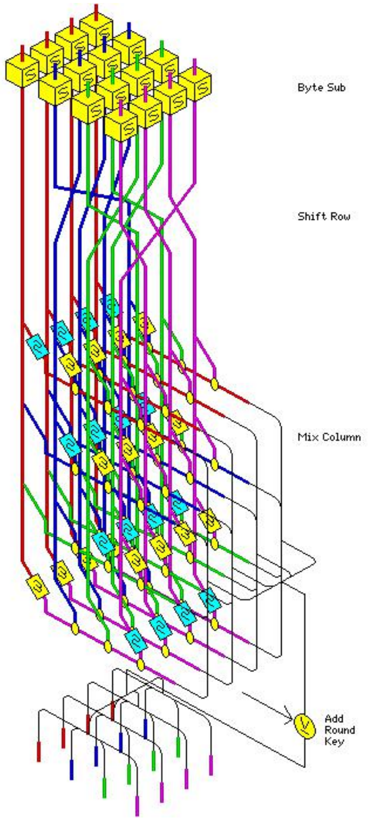
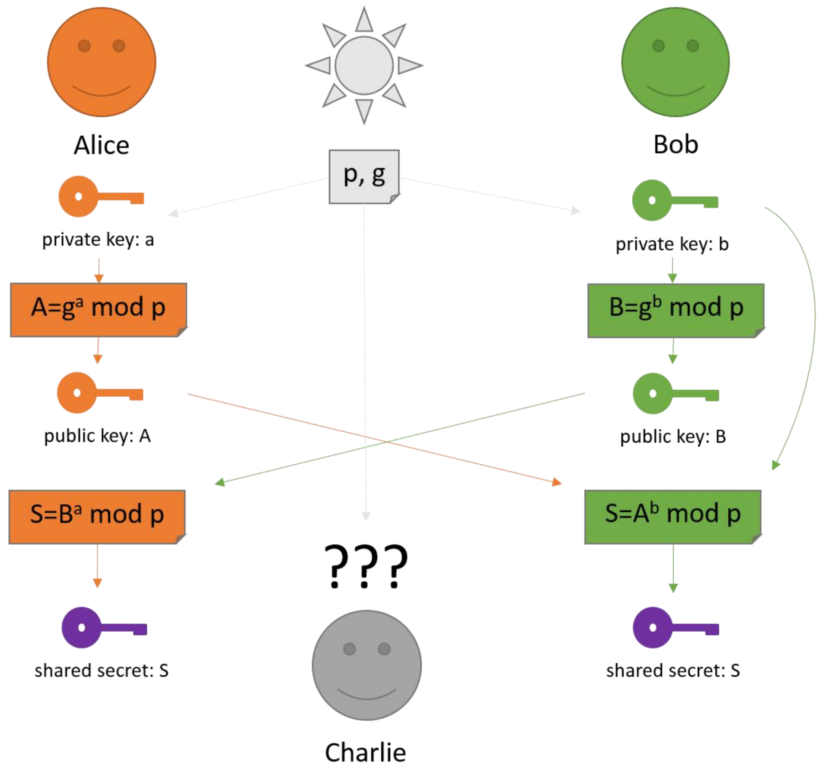
- Why AES (Advanced Encryption Standard) is secure?
  Assumption: tautology



What is Security?

- Why Diffie-Hellman key exchange is secure?
  - DDH (Decisional Diffie–Hellman) Assumption
  - DDH stronger than DLP (Discrete Logarithm Problem)
    - i.e., we can break DDH (specific) WHILE DLP (general) is secure.



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- RSA?
  - RSA Assumption
  - RSA Assumption stronger than/or equal to integer factorization problem
    - i.e., we may break RSA
      WHILE integer factorization is secure. *(Boneh, Venkatesan 98)*

What is Security?

- We say SOMETHING is (τ,ε)-secure if and only if no adversary with running time less than τ can break it with probability more than ε.

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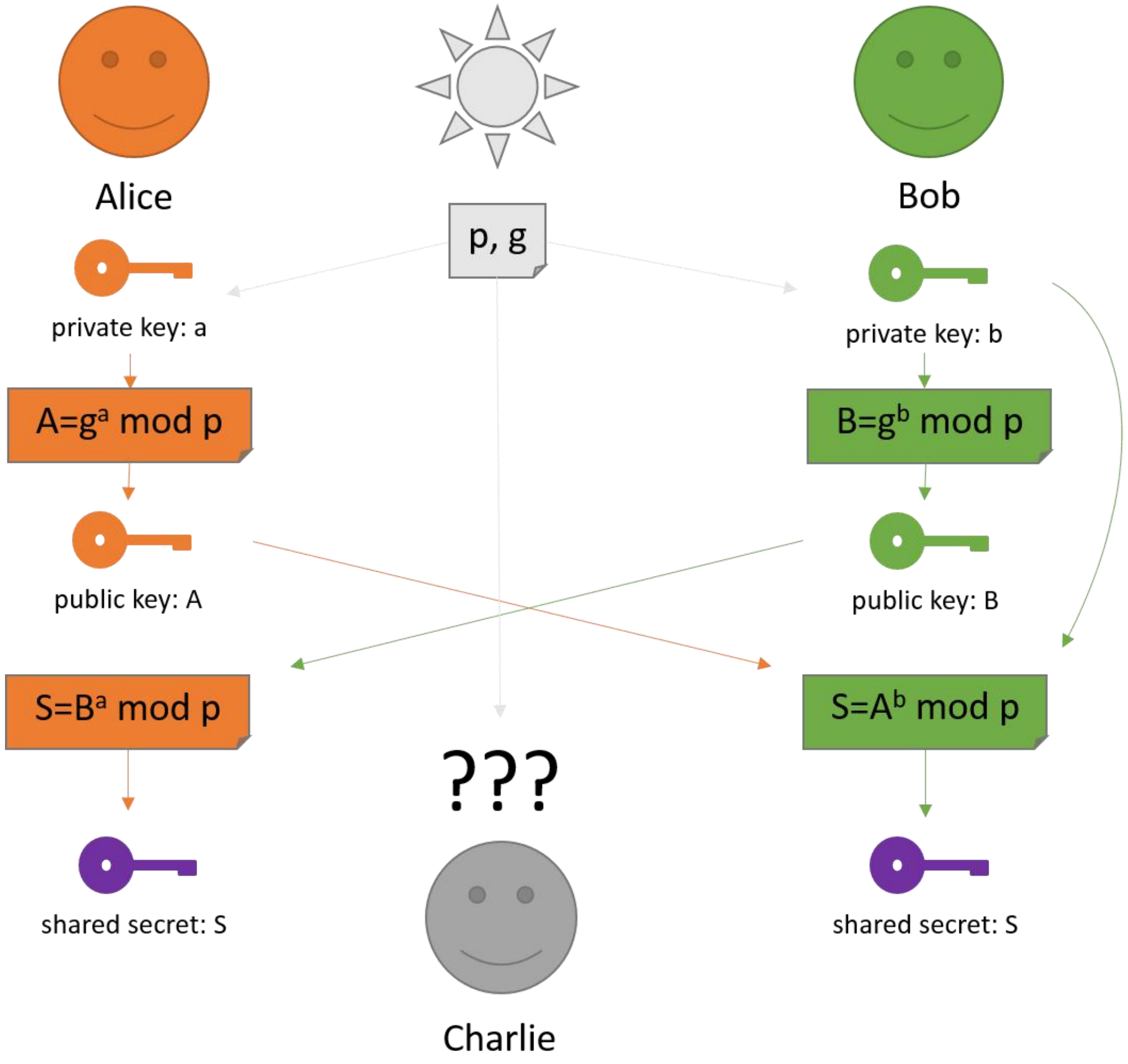
- P protocol is WHAT SECURE under X assumption

Design a Game

- What does “break it” mean?

  Define adversary’s capability via a game

Design a Game



Design a Game

$$(g, p, \mathbb{G}) \leftarrow \mathsf{Gen}(1^\lambda)$$
$$a, b \leftarrow \mathbb{Z}_p$$
$$A := g^a$$
$$B := g^b$$
$$\text{Win iff } S = g^{ab}$$

$$\xrightarrow{g, p, \mathbb{G}, A, B}$$
$$\xleftarrow{S}$$

A

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Running time: τ

Probability of winning: ε

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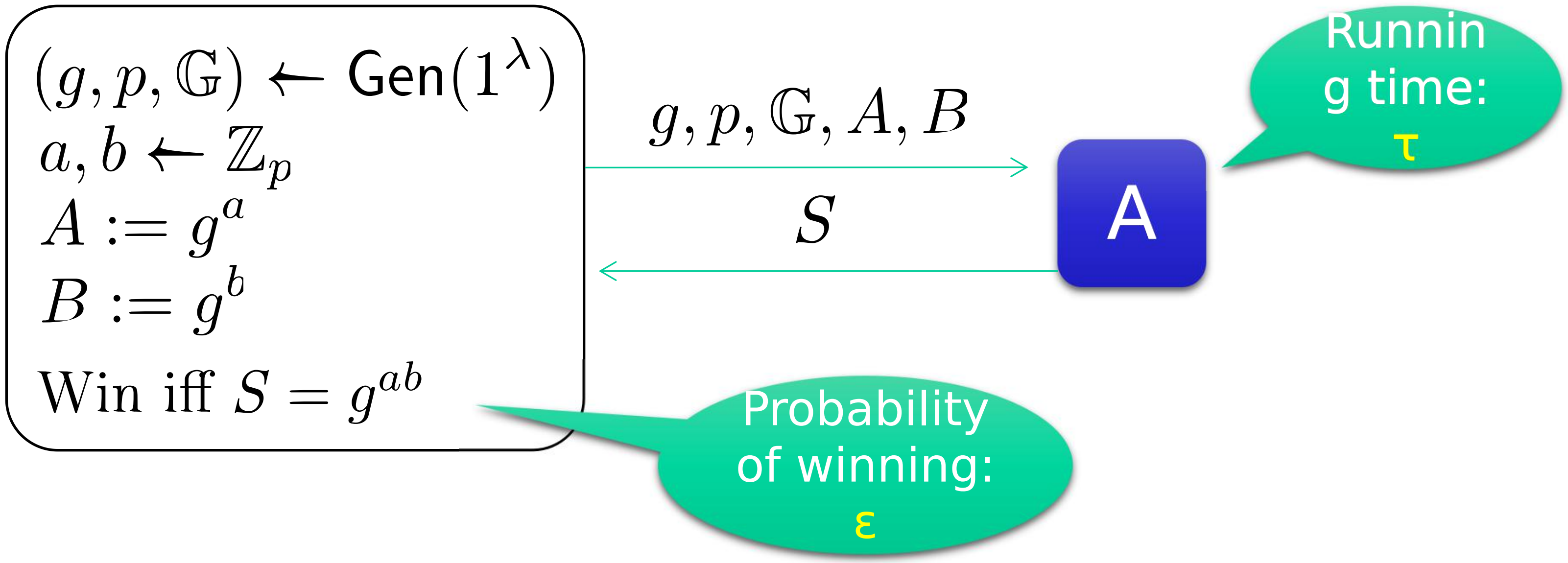
➢What if the adversary can learn the last 10 bits of the key ?
This does not let the adversary win our security game.

Design a Game



- What if the adversary can learn the last 10 bits of the key ?
  This does not let the adversary win our security game.

- We need a better security definition
  No matter how the key will be used latter
  Quantify the leakage of the key

Design a Game

- Idea: use indistinguishability to model "leakage"

  If the adversary cannot even distinguish the real key from a fake one then she knows nothing about the key.

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&K_0 := g^{ab}\\
&K_1 \leftarrow \mathbb{G}\\
&b \leftarrow \{0,1\}\\
&\text{Win iff } b = b^*
\end{aligned}
$$

$$\xrightarrow{g, p, \mathbb{G}, A, B, K_b}$$

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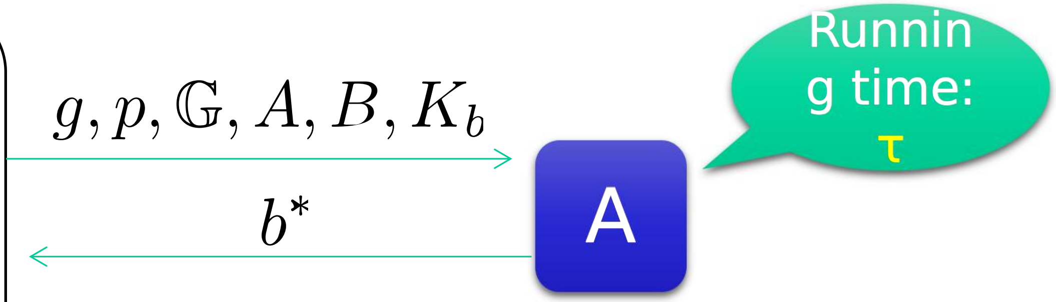

Probability of winning?

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$$

$g, p, \mathbb{G}, A, B, K_b \longrightarrow$ A

$b^* \longleftarrow$ A

Running time: τ

Probability of winning?

$$\mathsf{Adv} := \left|\Pr[win] - \frac{1}{2}\right|$$

Provable Security

- ➢ Rigorous security definition
  - Adversarial model (a.k.a. attacker model)
  - Property based
  - Simulation based

  - Property-based VS Simulation-based
    - https://crypto.stackexchange.com/questions/3814/simulation-based-security

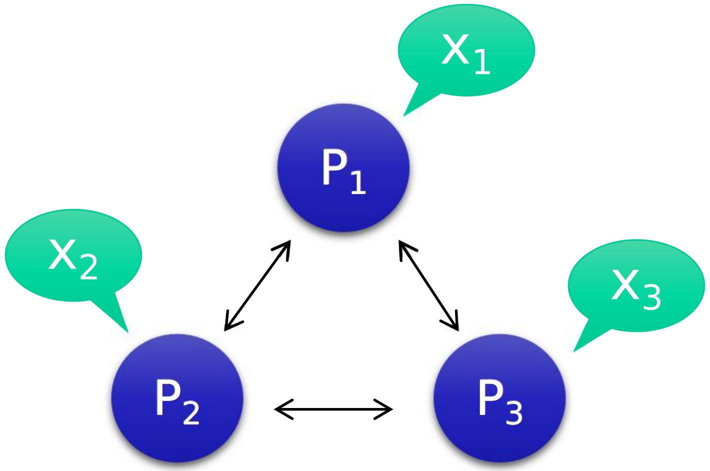
- ➢ Precise assumption

  Hard problems

- ➢ Formal proof

  Usually via reduction

Secure Multiparty Computation



- Input parties
- Computing parties
- Output parties

$F(x_1,x_2,x_3)$

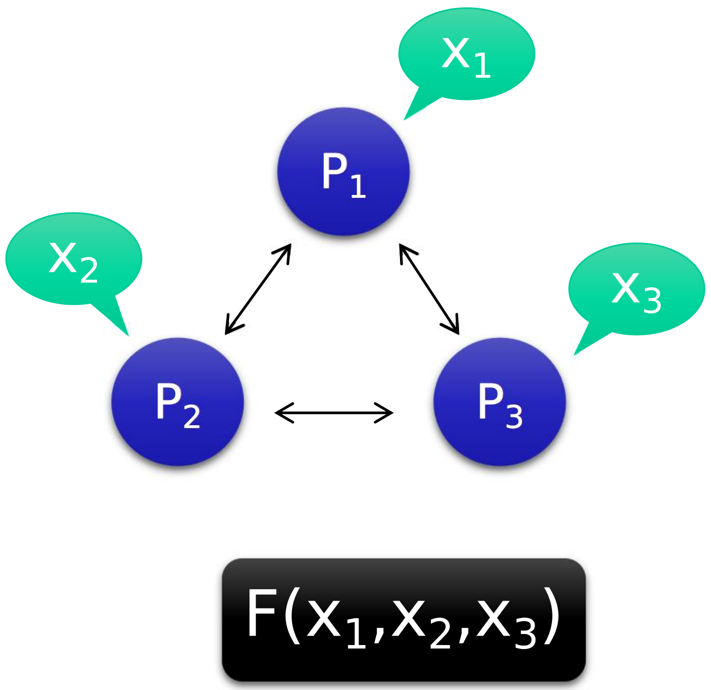
Secure Multiparty Computation

- Secure two-party computation (2PC) FOCS'82
- Yao's Millionaires' Problem

姚期智
Turing Award (2000)

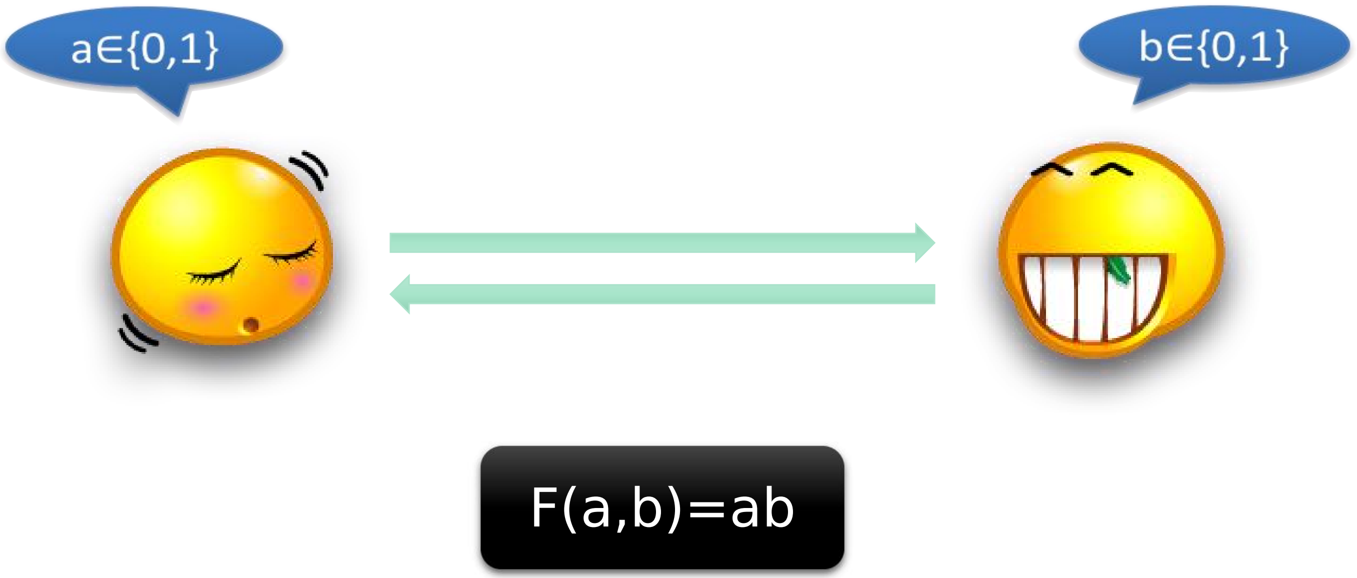
Goal of MPC

- What is the security goal?
  - Input privacy:
    - P_i's input is unknown
  - Output correctness:
    - $F(x_1,x_2,x_3)$ is correct
  - Input independency:
    - One player's input should not depends on the others'.
  - Fairness:
    - Either everyone get the output or none of them gets the output.
  - Guaranteed output delivery (GOD)



Special Case: 2-party Computation

- Example: Private VETO



Oblivious Transfer (OT)

- Sender has x_0,x_1; receiver has b
- Receiver obtains x_b only
- Sender learns nothing

Oblivious Transfer

- **Trapdoor permutation (I, D, F, F^{-1})**
  - I: samples a function f and trapdoor t in the family
  - D(f): uniformly samples a value in the domain of f
  - F(f,x): computes f(x)
  - $F^{-1}(t,y)$: computes $f^{-1}(y)$
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- **Enhanced trapdoor permutations**
  - Hard to invert y, even given the random coins used to sample y (using D)

Oblivious Transfer

- **Hard-core predicate B**
  - Given y=f(x), can guess B(x) with probability only negligibly greater than ½
  - Equivalently, given y=f(x), the bit B(x) is pseudorandom

Oblivious Transfer Protocol

- Sender's input: (z_0, z_1); receiver's input b

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  - Sender chooses (f,t) using sampling algorithm I
  - Sender sends f to receiver

Oblivious Transfer Protocol

- **Sender's input: (z_0,z_1); receiver's input b**
- **Sender's first message:**
  - Sender chooses **(f,t)** using sampling algorithm **I**
  - Sender sends **f** to receiver
- **Receiver's first message:**
  - Receiver chooses x_b and computes $y_b=f(x_b)$
  - Receiver chooses random y_{1-b}
  - Receiver sends (y_0,y_1) to sender

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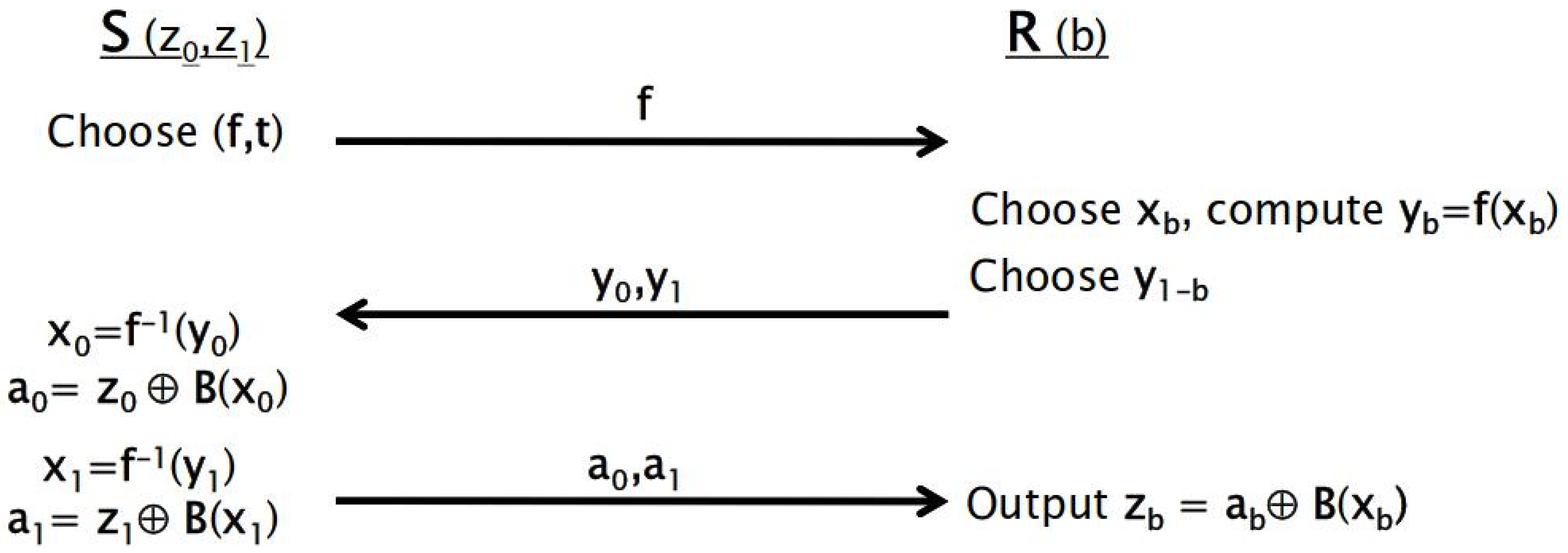
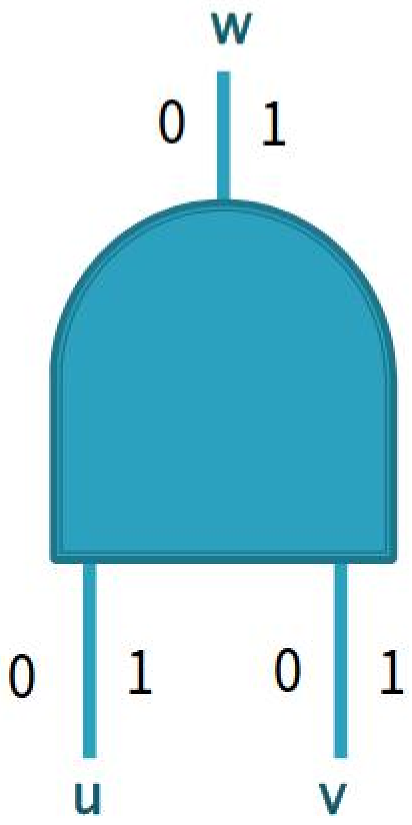
  - Receiver sends (y_0,y_1) to sender
- **Sender's second message:**
  - Sender computes (x_0,x_1) by inverting
  - Sender computes $a_i = z_i \oplus B(x_i)$
  - Sender sends (a_0,a_1) to receiver
- **Receiver outputs $z_b = a_b \oplus x_b$**

Oblivious Transfer Protocol

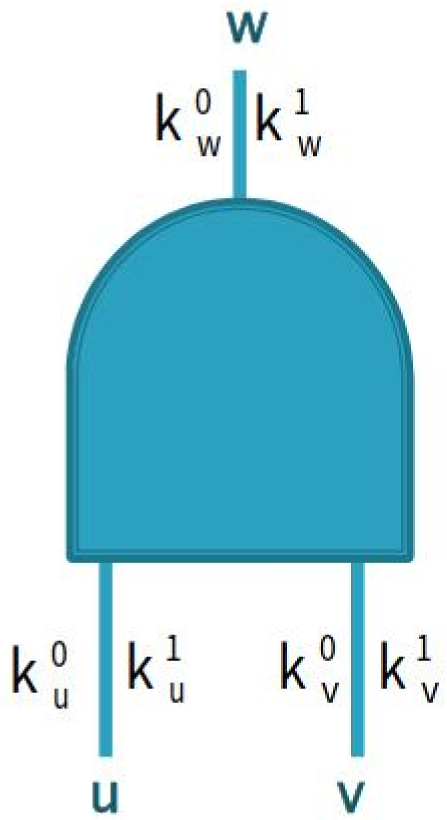


Yao's Garbled Circuit



u	v	w
0	0	0
0	1	0
1	0	0
1	1	1

Yao's Garbled Circuit



u	v	w
k_u^0	k_v^0	k_w^0
k_u^0	k_v^1	k_w^0
k_u^1	k_v^0	k_w^0
k_u^1	k_v^1	k_w^1

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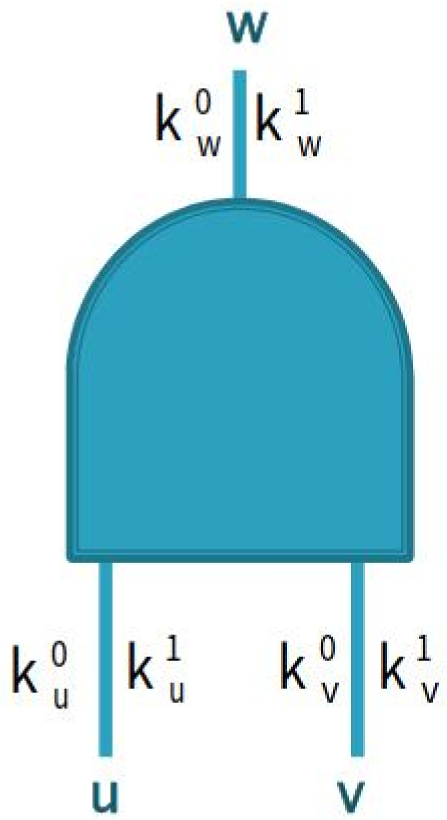


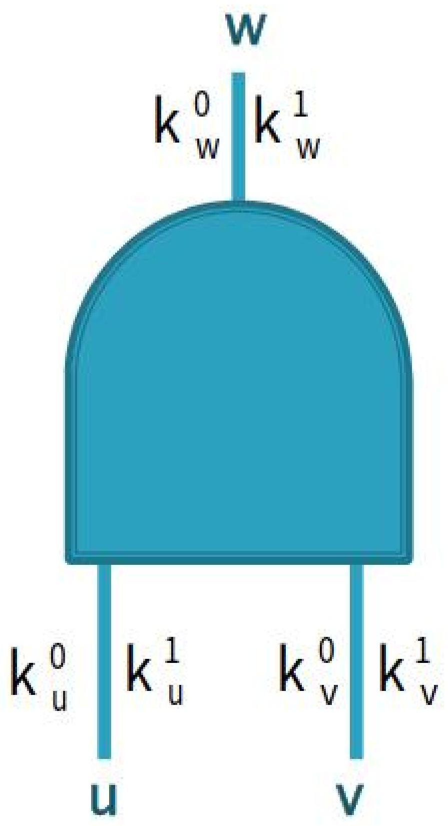
u	v	w
k_u^0	k_v^0	$E_{k_u^0}(E_{k_v^0}(k_w^0))$
k_u^0	k_v^1	$E_{k_u^0}(E_{k_v^1}(k_w^0))$
k_u^1	k_v^0	$E_{k_u^1}(E_{k_v^0}(k_w^0))$
k_u^1	k_v^1	$E_{k_u^1}(E_{k_v^1}(k_w^1))$

Yao's Garbled Circuit

- **The actual garbled gate**

$$E_{k_u^1}(E_{k_v^0}(k_w^0))$$
$$E_{k_u^0}(E_{k_v^1}(k_w^0))$$
$$E_{k_u^1}(E_{k_v^1}(k_w^1))$$
$$E_{k_u^0}(E_{k_v^0}(k_w^0))$$

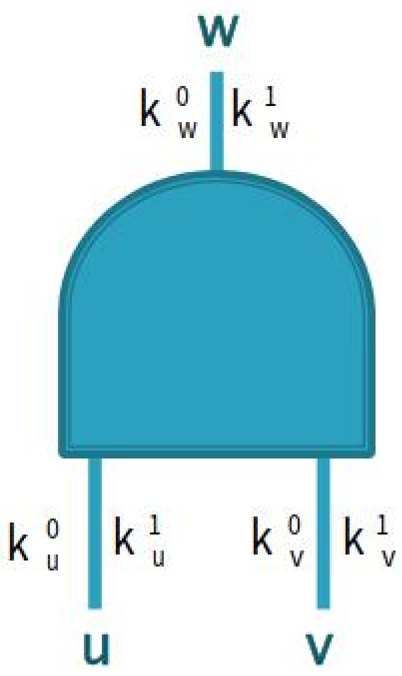
- Given k_u^0 and k_v^1 can obtain k_w^0 only
- Furthermore, since the table is permuted, the party has no idea if it obtained the 0 or 1 key



Yao's Garbled Circuit

- If the gate is an output gate, need to provide the "decryption" of the output wire
- Output translation table

$$[(0,\ k_w^0),(1,\ k_w^1)]$$



Yao's Protocol

- Input: x and y of length n

Yao's Protocol

- **Input**: **x** and **y** of length **n**
- P_1 generates a garbled circuit **G(C)**
  - k_L^0, k_L^1 are the keys on wire w_L
  - Let $w_1,\ldots,w_n$ be the input wires of P_1 and $w_{n+1},\ldots,w_{2n}$ be the input wires of P_2

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- P_1 and P_2 run n OTs in parallel
  - P_1 inputs k_{n+i}^0, k_{n+i}^1
  - P_2 inputs y_i

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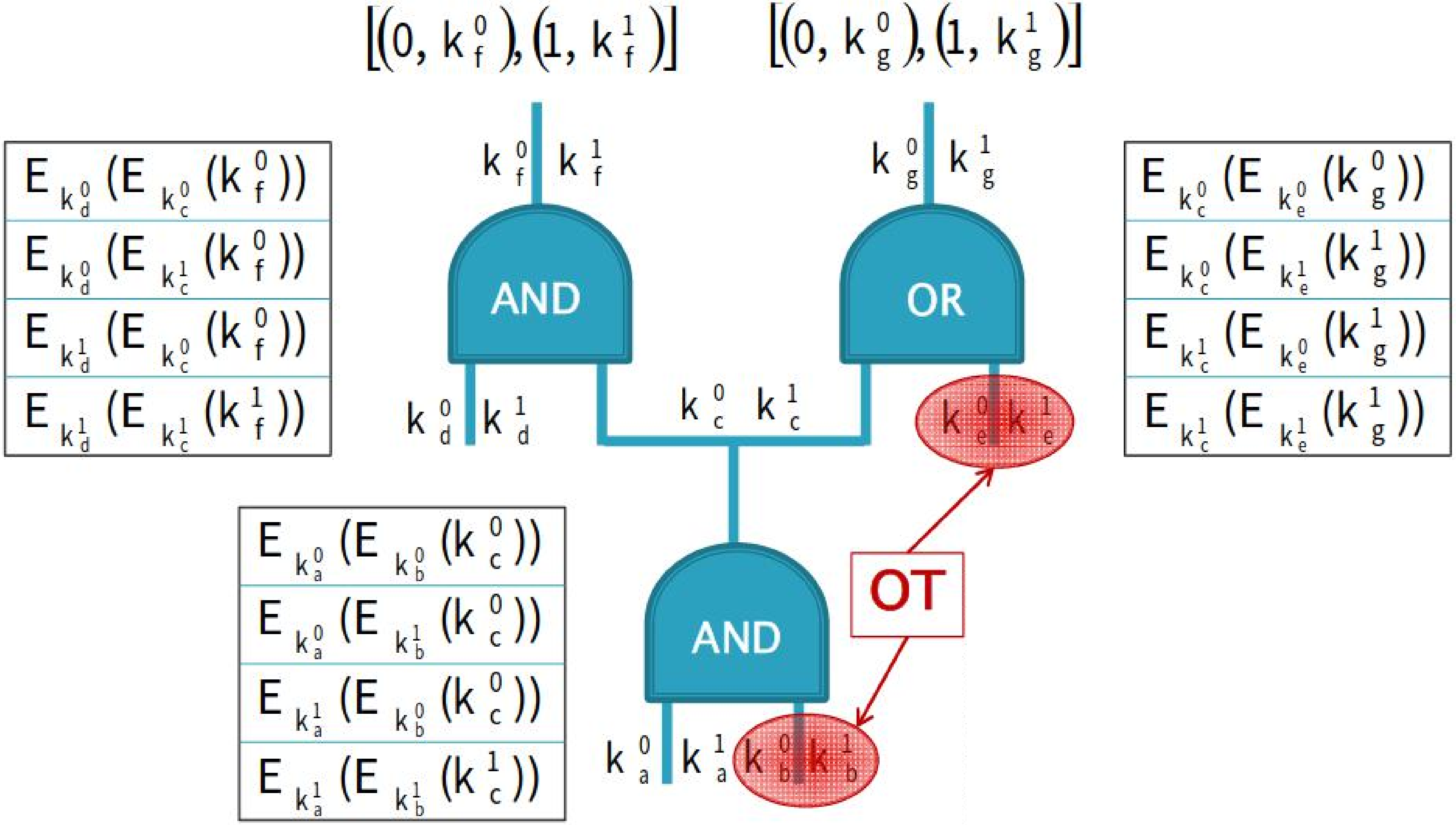
- P_1 and P_2 run n OTs in parallel
  - P_1 inputs k_{n+i}^0, k_{n+i}^1
  - P_2 inputs y_i
- Given all keys, P_2 computes **G(C)** and obtains **C(x,y)**
  - P_2 sends result to P_1

The Example Circuit

(input wires $P_1 = d,a$; $P_2 = b,e$)



How to gambling over Wechat?

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- What is a commitment?

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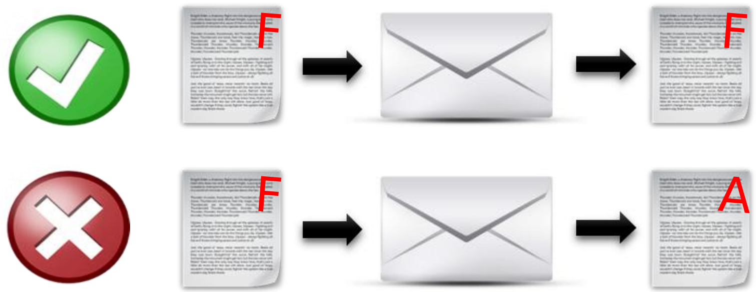
- Binding Property:



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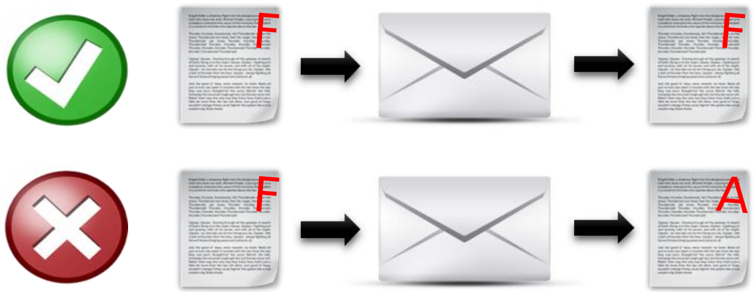
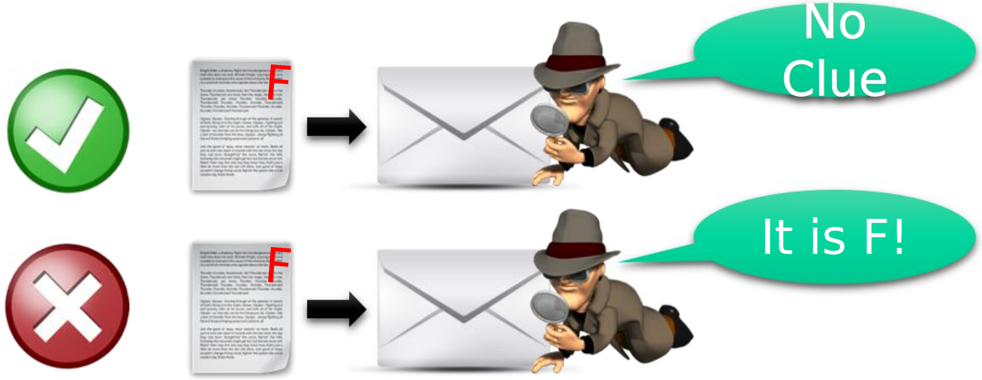


- Hiding Property:



Design a Commitment

- First attempt
  - H(m) or g^m

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- Pederson commitment
  - Commitment key: $\mathsf{ck} := (g, h)$
  - Committing: $\mathsf{Com}_{\mathsf{ck}}(m; r) := g^m h^r$
  - Opening: reveal (m, r) and checking

Pederson Commitment

- Perfect Hiding

Pederson Commitment

- Perfect Hiding

$$\Pr[C=c] = \sum \Pr[C=c|M=m]\cdot\Pr[M=m]$$

$$= \sum \Pr[R=r \text{ s.t. } c=\mathsf{Com}_{\mathsf{ck}}(m;r)]\cdot\Pr[M=m]$$

$$= \sum \frac{1}{|\mathbb{G}|}\cdot\Pr[M=m] = \frac{1}{|\mathbb{G}|}\sum \Pr[M=m] = \frac{1}{|\mathbb{G}|}$$

Pederson Commitment

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$$\Pr[M=m|C=c] = \frac{\Pr[C=c|M=m] \cdot \Pr[M=m]}{\Pr[C=c]}$$

$$= \frac{\Pr[R=r \text{ s.t. } c = \mathsf{Com}_{\mathsf{ck}}(m;r)] \cdot \Pr[M=m]}{1/|\mathbb{G}|}$$

$$= \frac{1/|\mathbb{G}| \cdot \Pr[M=m]}{1/|\mathbb{G}|} = \Pr[M=m]$$

Pederson Commitment

$$\mathsf{Com}_{\mathsf{ck}}(m;r) := g^m h^r$$

- Computational Binding

  If the adversary does not know the discrete logarithm of h, then she could not double open the commitment.

  Assuming discrete logarithm problem is hard, the Pederson commitment is binding.

Given (m_1, r_1) and (m_2, r_2) such that

$$\mathsf{Com}_{\mathsf{ck}}(m_1; r_1) = c = \mathsf{Com}_{\mathsf{ck}}(m_2; r_2)$$

we can compute $\mathsf{DLog}_g(h) = \frac{m_1 - m_2}{r_2 - r_1}$

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More: https://cs.nyu.edu/courses/spring12/CSCI-GA.3210-001/lect/lecture14.pdf

Toy Protocol

- Suppose a man claims that he has a machine (e.g. quantum computer) that can efficiently solve any discrete logarithm problem.

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- Suppose a man claims that he has a machine (e.g. quantum computer) that can efficiently solve any discrete logarithm problem.
  - The protocol to verify his claim:

$\mathbb{G}, g, S$

$\text{Com}(x)$

s

$\text{Open } x$

C

Check if $g^s = S$

Check if $g^x = S$

Plug-in Pederson Commitment

$\mathbb{G}, g, S$

$\mathsf{Com}(x)$

s

Open x

C

Check if $g^s = S$

Check if $g^x = S$

Plug-in Pederson Commitment

$\xleftarrow{\mathbb{G}, g, S}$

$\xrightarrow{\mathsf{Com}(x)}$

$\xleftarrow{s}$

$\xrightarrow{\text{Open } x}$

C

Check if $g^s = S$

Check if $g^x = S$

Is this protocol secure?

Compute the commitment as $c = S \cdot h^r$,
and later send $x = s$ and r to open it.

Plug-in Pederson Commitment

$$\xleftarrow{\mathbb{G}, g, S}$$

$$\xrightarrow{c = g^x h^r}$$

$$\xleftarrow{s}$$

$$\xrightarrow{x, r}$$

C

Check if $g^s = S$

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Plug-in Pederson Commitment

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Plug-in Pederson Commitment

$\mathbb{G}, g, S$

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x, r

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Plug-in Pederson Commitment

$\mathbb{G}, g, S$

$c = g^x h^r$

Check $= S$

Composing two secure protocols together is always secure!
(See Universal Composability)

Is this protocol secure?

Compute the commitment as $c = S \cdot h^r$,
and later send $x = s$ and r to open it.

Recommended Reading

So You're Starting a PhD?

Mike Rosulek

http://web.engr.oregonstate.edu/~rosulekm/advising.html

Acknowledge

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