THE HONG KONG UNIVERSITY OF SCIENCE AND TECHNOLOGY
DEPARTMENT OF ELECTRONIC & COMPUTER ENGINEERING

Introduction to Aerial Robotics
Lecture 7

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Outline

- Optical Flow
- Stereo Vision
- Visual Odometry

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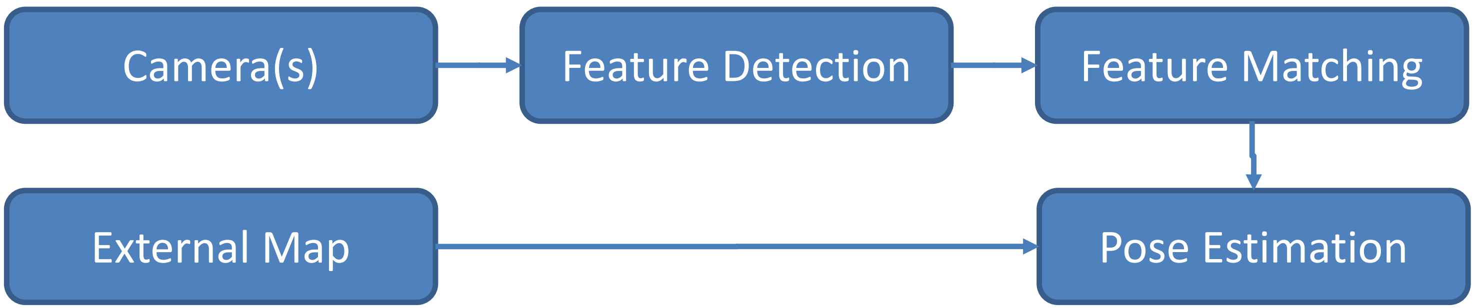
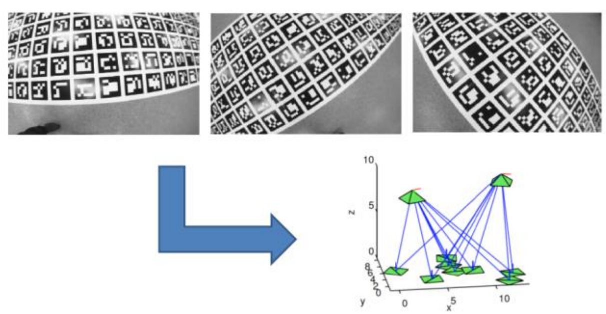
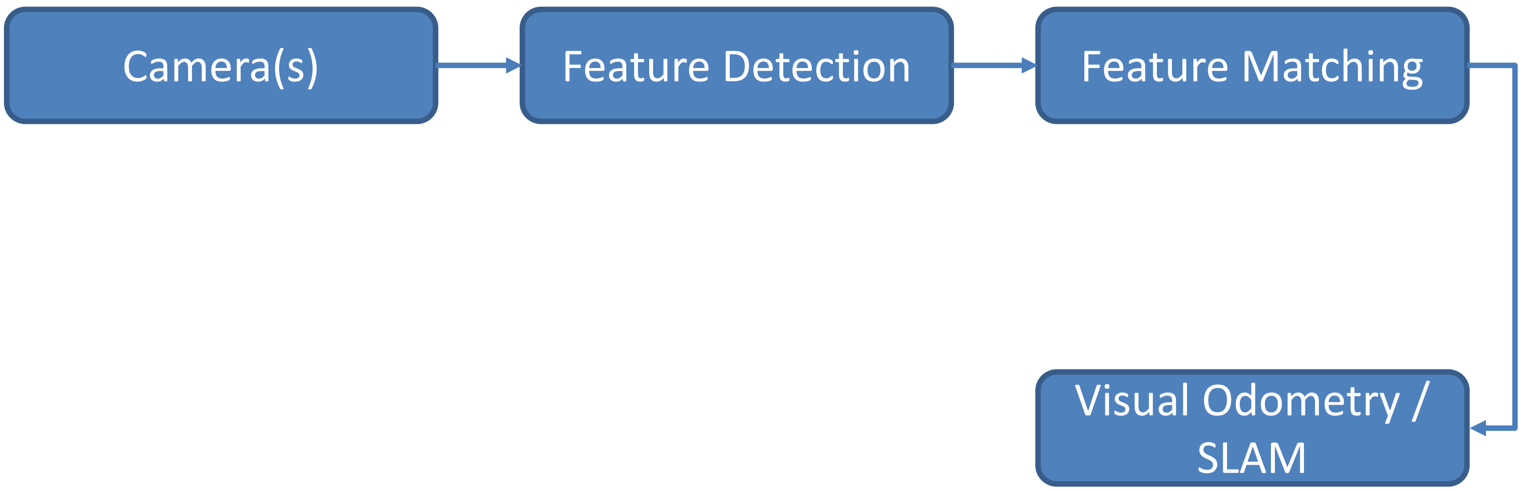
電子及計算機工程學系
DEPARTMENT OF ELECTRONIC & COMPUTER ENGINEERING

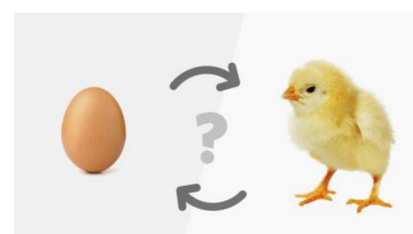
Vision-based Pose Estimation Pipeline (aka. Map-based Localization)





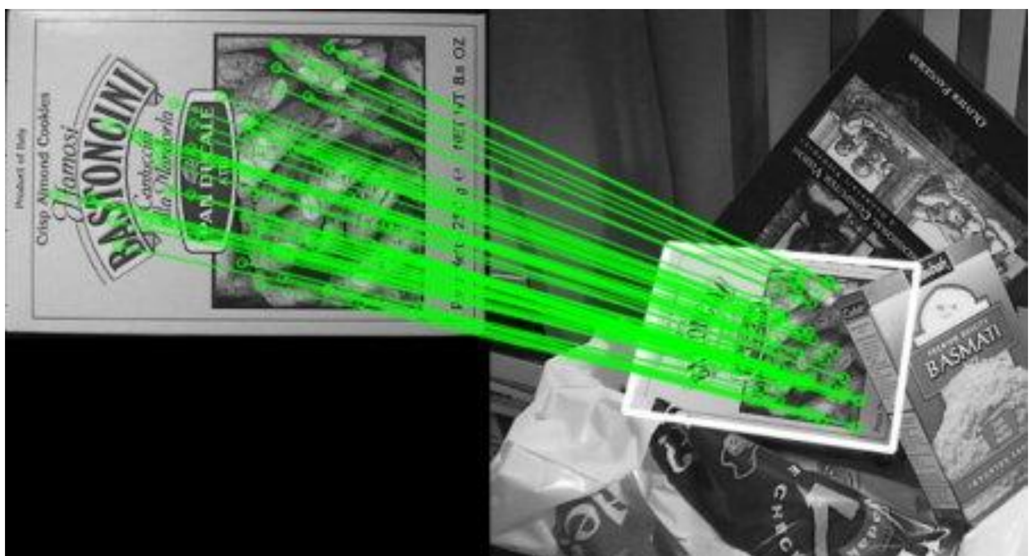
Vision-based Incremental Pose Estimation Pipeline (aka. Visual Odometry)





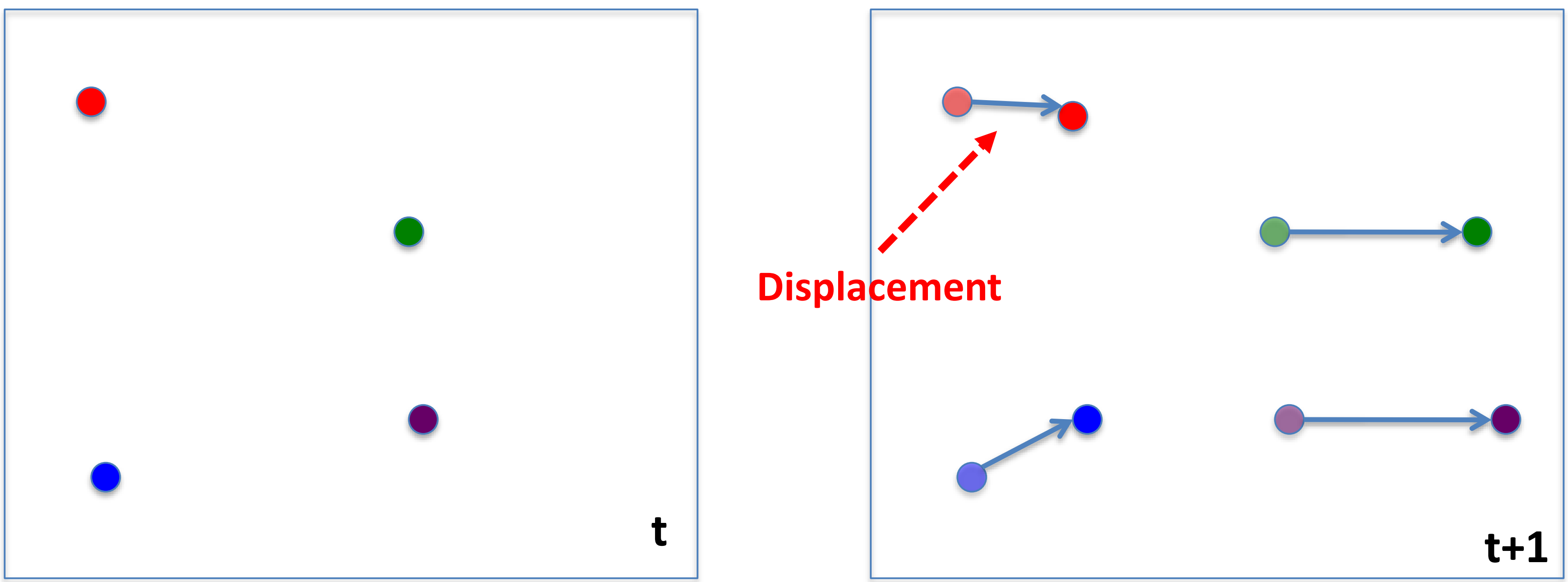
Discrete Feature Matching Approach

- Detect corners features in both images
- Use image patch as feature description
  - Could be extended to color, texture, SIFT/HOG descriptor
- Find correspondences using descriptor matching



Frame-to-Frame Feature Matching Problem Definition

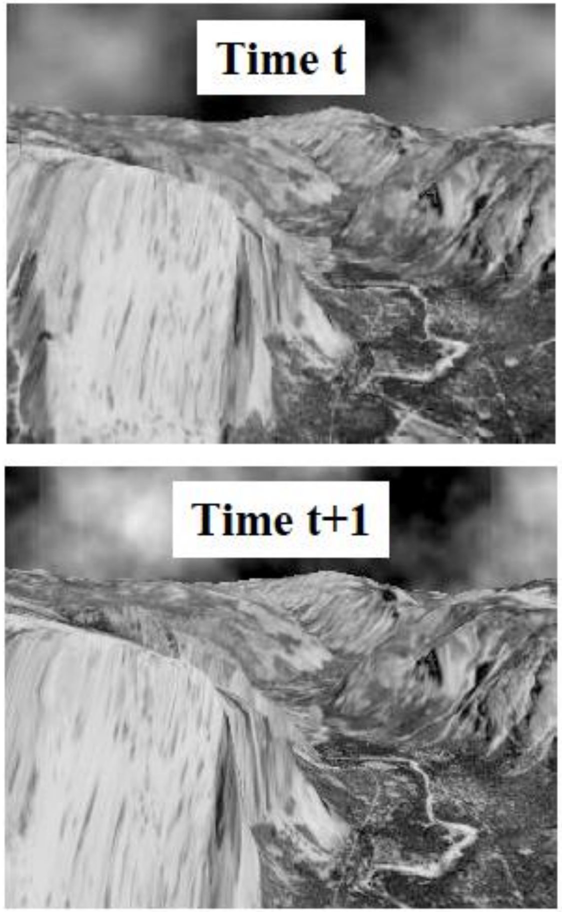
- Define regions of interests, or points of interests in the first image at time t
- Search for correspondences in the second image at time $t + 1$

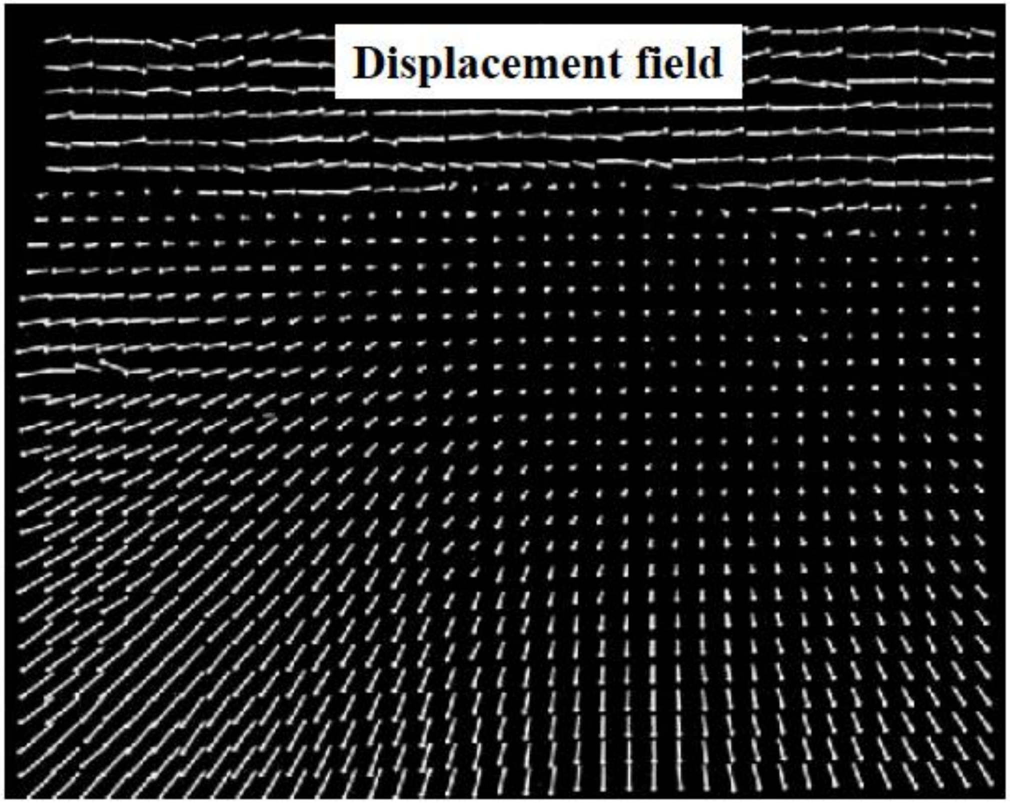


Slide adapted from Kostas Daniilidis

Optical Flow

Differential Approach: Optical Flow





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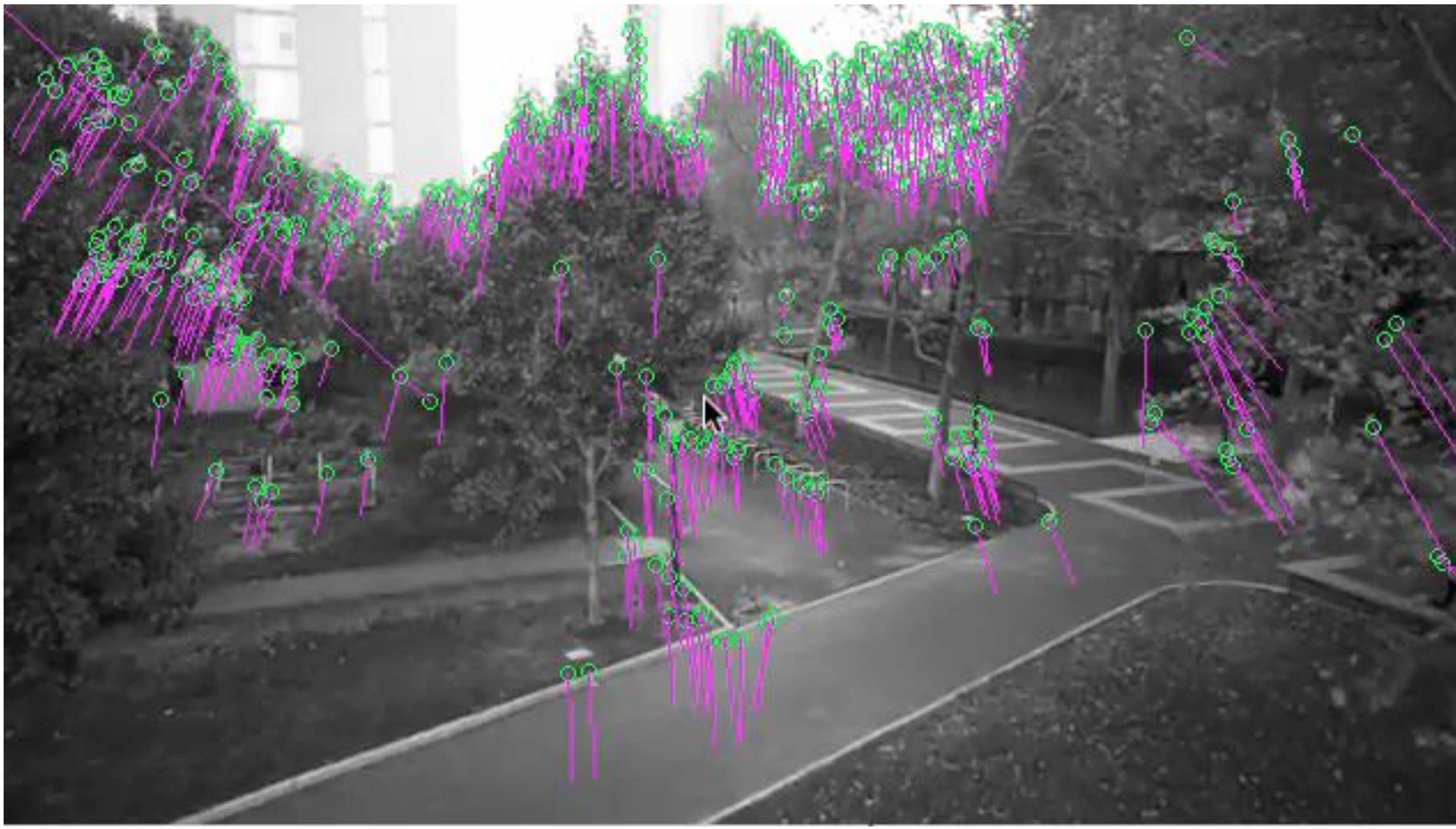
Differential Approach: Optical Flow



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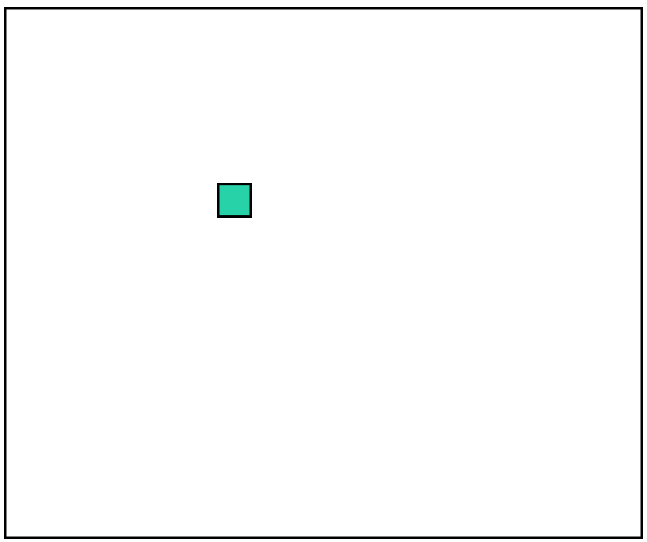
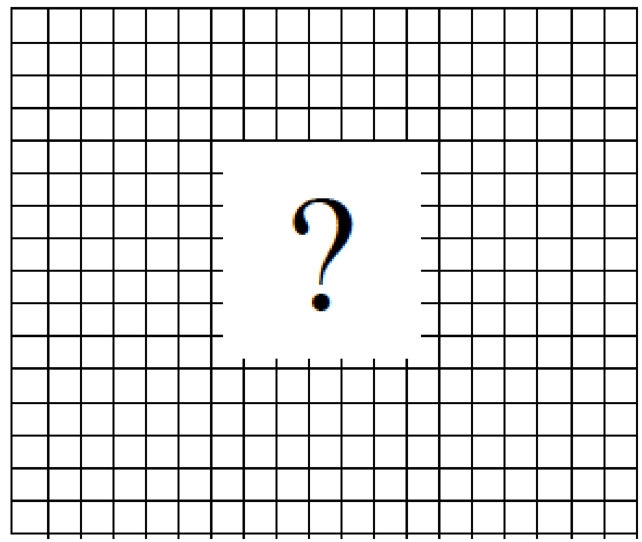
Differential Approach: Optical Flow

- Detect corners features in first image
- Use image patch as feature description
  - Could be extended to color and texture descriptors
- Use Lucas-Kanade algorithm to compute displacement of the pixels in the patch
  - Motion model could be translation (2-DoF), affine (6-DoF), or more general 3D models
- Subpixel accuracy
- Do not need repeated detection

Given image patch in one image

We don't want to search everywhere in the second image for a match





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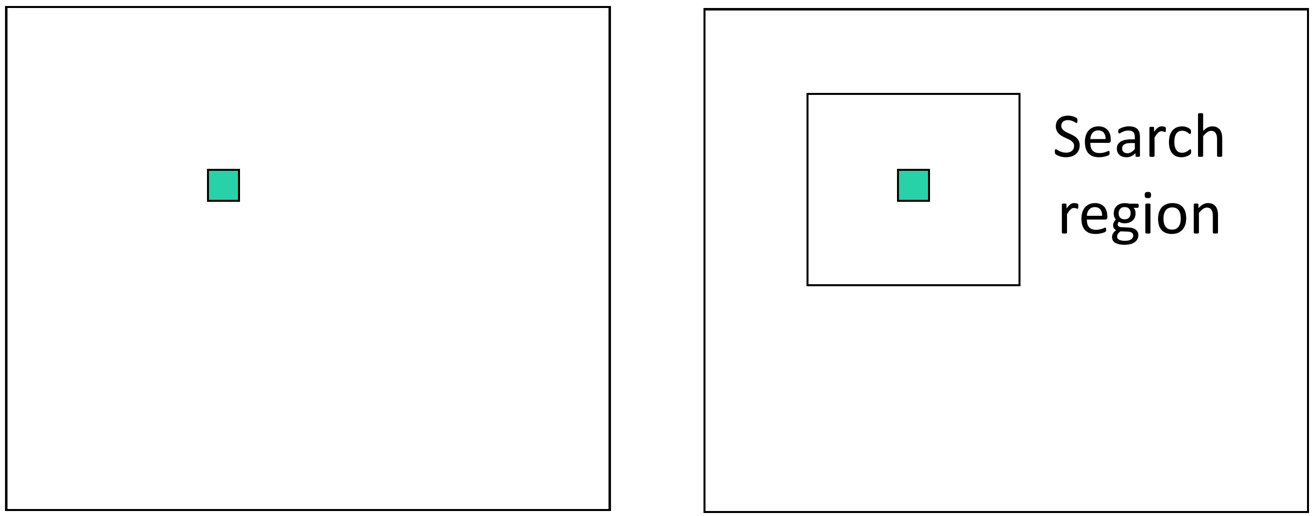
Given image patch in one image

We don’t want to search everywhere in the second image for a match

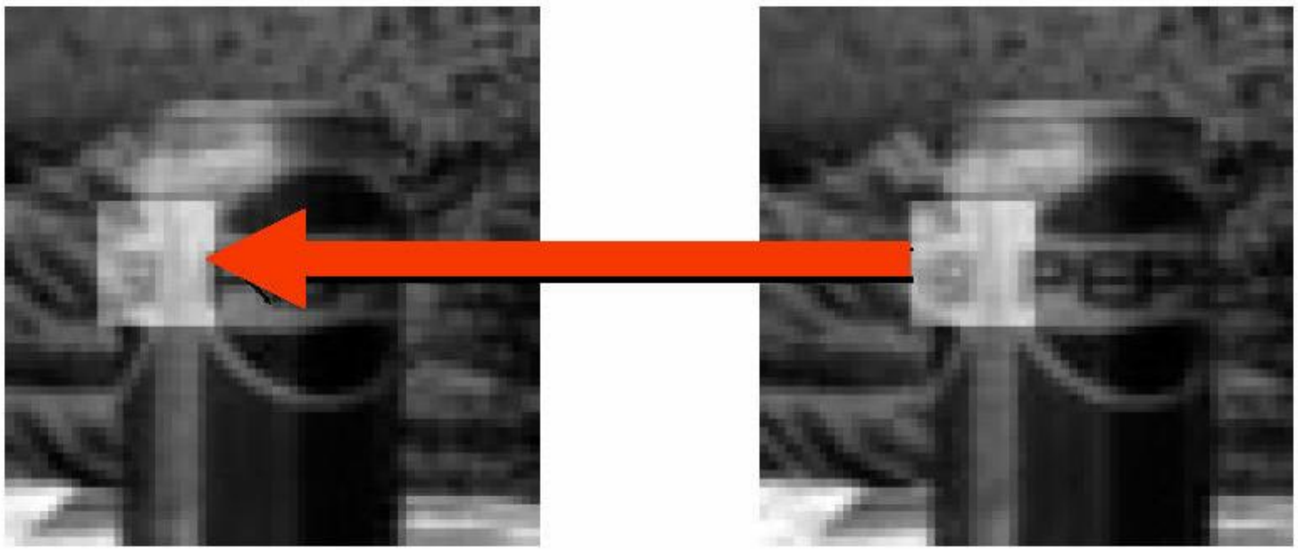


- The motion is known to be “small”, we can bound the search region.

Slide adapted from Kostas Daniilidis

Optical Flow Assumption: Brightness Constancy

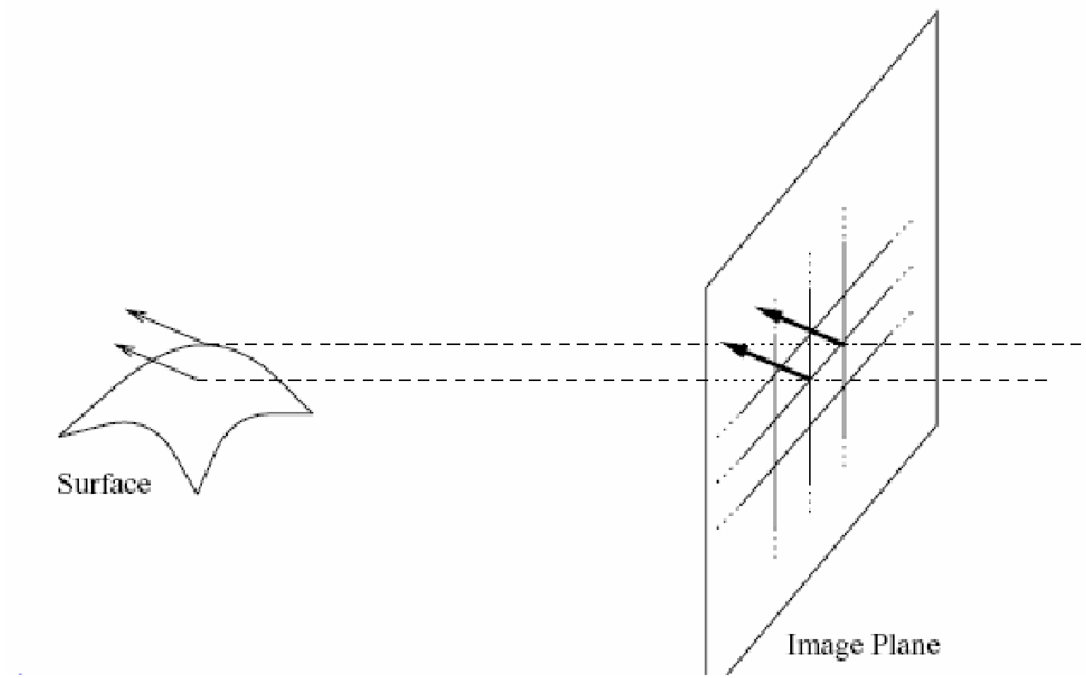
- Image measurements (brightness) in a small region remains the same even though their location may change



t+1 **t**

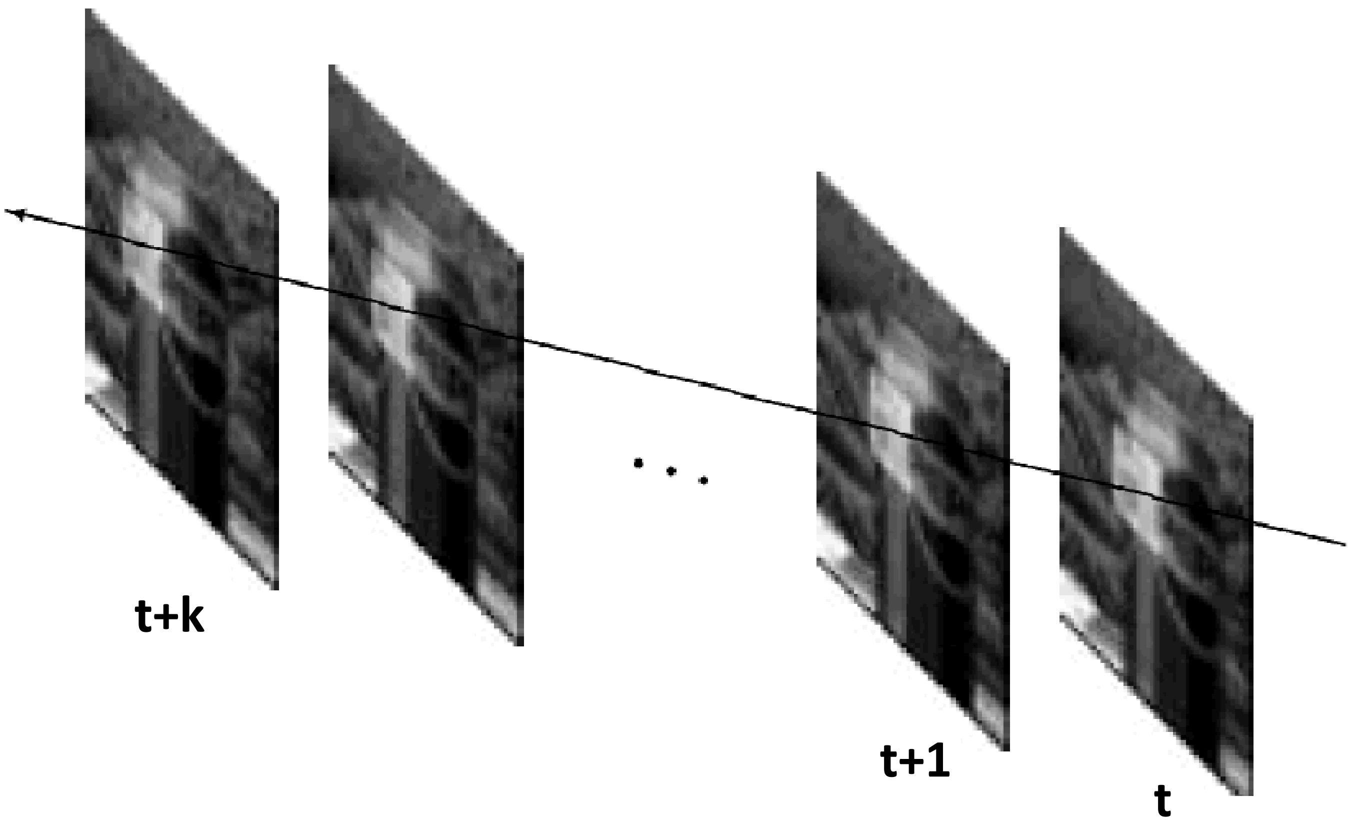
Optical Flow Assumption: Spatial Coherence

- Neighboring points in the scene typically belong to the same surface and have similar motions



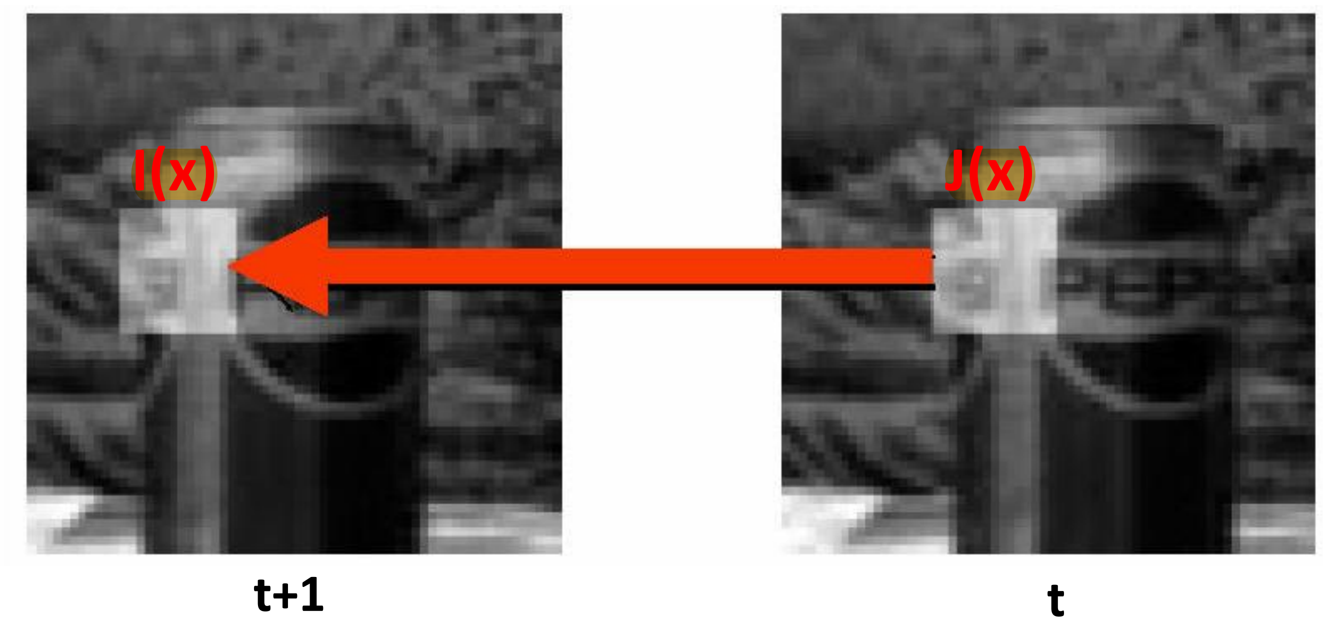
Optical Flow Assumption: Temporal Persistence

- Image motion of a surface patch changes smoothly over time



Lucas-Kanade (KLT) Tracking

- Intensity constancy constraint: $J(x+d) = I(x)$
  - $J(x) = I(x, t+1)$
  - $I(x) = I(x, t)$



Lucas-Kanade (KLT) Tracking

- Define Sum of Squared Difference (SSD) error as:
  - $\epsilon = \int_{W} [\, J(x+d) - I(x) \,]^{2} \omega(x) dx$
  - $\omega(x)$ is the smoothing term
  - Minimize ϵ with respect to $d \in R^{2\times 1}$

- 4 steps for solving this problem:
  - Set $\frac{\partial \epsilon}{\partial d}$ to 0
  - Linearization by Taylor expansion on $J(x+d)$ with respect to d
  - Solve the resulting linearized system
  - Iterative refinement

Step 1: Set Derivative to 0

- Differentiate SSD with respect to d and set to 0:

$$\frac{1}{2}\frac{\partial_{\in}}{\partial_{d}} = \int_{w} [J(x+d) - I(x)]\, \boxed{g}\, w\, dx = 0$$

$$g = \left(\frac{\partial J}{\partial x}, \frac{\partial J}{\partial y}\right)^{T}$$

Slide adapted from Kostas Daniilidis

Step 2: Linearization

- Combining previous equations:

$$\frac{1}{2}\frac{\partial_\epsilon}{\partial_d} = \int_w [J(x+d) - I(x)]\, g\, w\, dx = 0$$

$$J(x+d) = J(x) + g^T d$$

$$\downarrow$$

$$\int_W g\,(g^T d) w\, dx = \int_W [I(x) - J(x)] g\, w\, dx$$

Slide adapted from Kostas Daniilidis

Step 3: Solve Linear System

$$\int_W g\,(g^T d) w\, dx = \int_W [I(x) - J(x)] g\, w\, dx$$

$$\sum_{i,j} \begin{bmatrix} g_x(i,j) g_x(i,j) & g_y(i,j) g_x(i,j) \\ g_x(i,j) g_y(i,j) & g_y(i,j) g_y(i,j) \end{bmatrix}$$

A: second moment matrix

Slide adapted from Kostas Daniilidis

Step 3: Solve Linear System

$$\int_W g\,(g^T d) w\,dx = \int_W [I(x) - J(x)] g\,w\,dx$$

$$\underline{\sum_{i,j} \begin{bmatrix} g_x(i,j)[I(i,j) - J(i,j)] \\ g_y(i,j)[I(i,j) - J(i,j)] \end{bmatrix}}$$

Error vector b

Slide adapted from Kostas Daniilidis

Step 3: Solve Linear System

$$\int_W g(g^T d) w \, dx = \int_W [I(x) - J(x)] g \, w \, dx$$

$$A = \sum_{i,j} \begin{bmatrix} g_x(i,j) g_x(i,j) & g_y(i,j) g_x(i,j) \\ g_x(i,j) g_y(i,j) & g_y(i,j) g_y(i,j) \end{bmatrix}$$

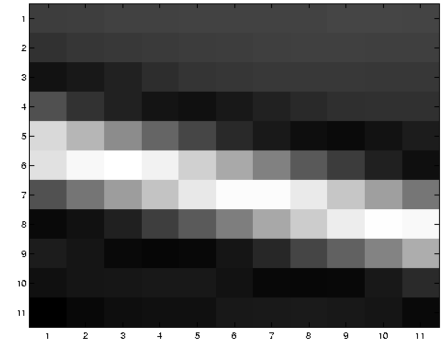
$$b = \sum_{i,j} \begin{bmatrix} g_x(i,j)[I(i,j) - J(i,j)] \\ g_y(i,j)[I(i,j) - J(i,j)] \end{bmatrix}$$

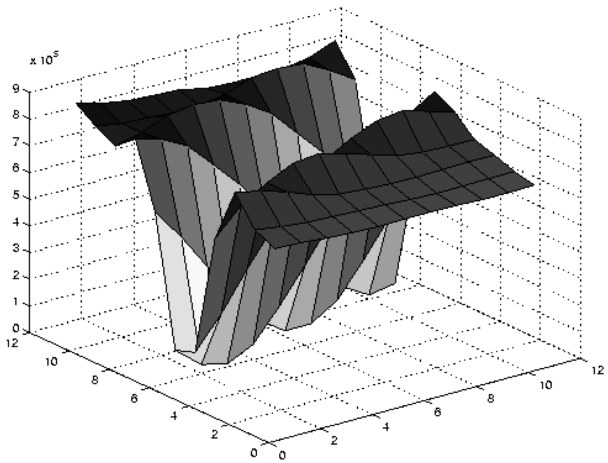
$$A\,d = b$$
$$d = A^{-1} b$$

- What if A is not full rank? Recall the structure of A:
  - Same as the one for corner detection
  - Eigenvalues and eigenvectors of A tells whether we are tracking a corner

Slide adapted from Kostas Daniilidis

Edge



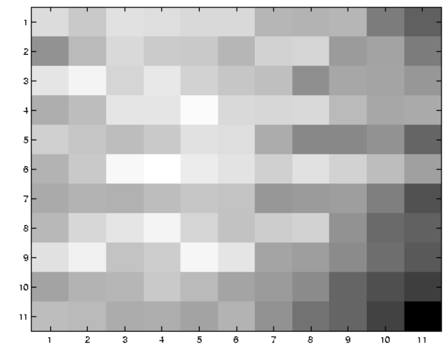


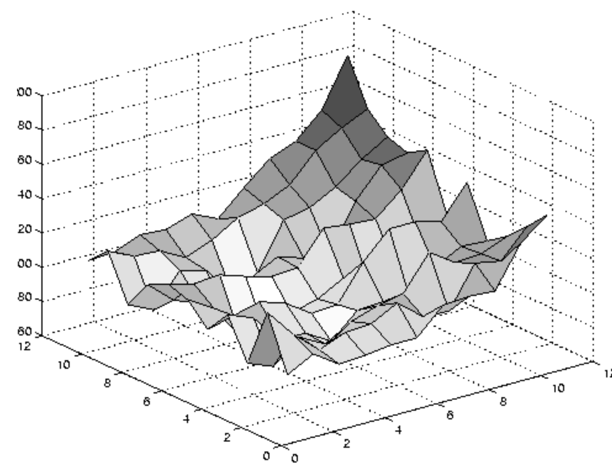
– large gradients, all the same direction

– large l_1, small l_2

Slide adapted from Kostas Daniilidis

Low Texture Region



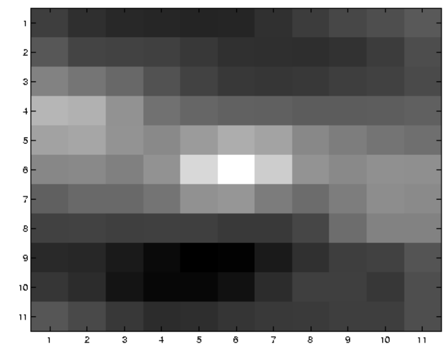


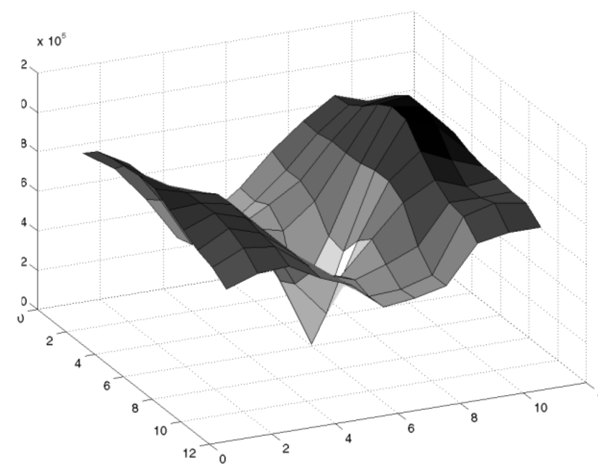
– gradients have small magnitude

– small l_1, small l_2

Slide adapted from Kostas Daniilidis

High Texture Region





– gradients are different, large magnitudes

– large l_1, large l_2

Slide adapted from Kostas Daniilidis

Step 4: Iterative Refinement

- Iterative refinement
  - Estimate velocity at pixels of interests using one iteration of Lucas-Kanade algorithm
  - Transform pixels using the estimated flow field
  - Refine estimate by repeating the process

Step 4: Iterative Refinement

$$\int_W g(g^T d) w \, dx = \int_W [I(x) - J(x)] g \, w \, dx$$

$$A = \sum_{i,j} \begin{bmatrix} g_x(i,j) g_x(i,j) & g_y(i,j) g_x(i,j) \\ g_x(i,j) g_y(i,j) & g_y(i,j) g_y(i,j) \end{bmatrix}$$

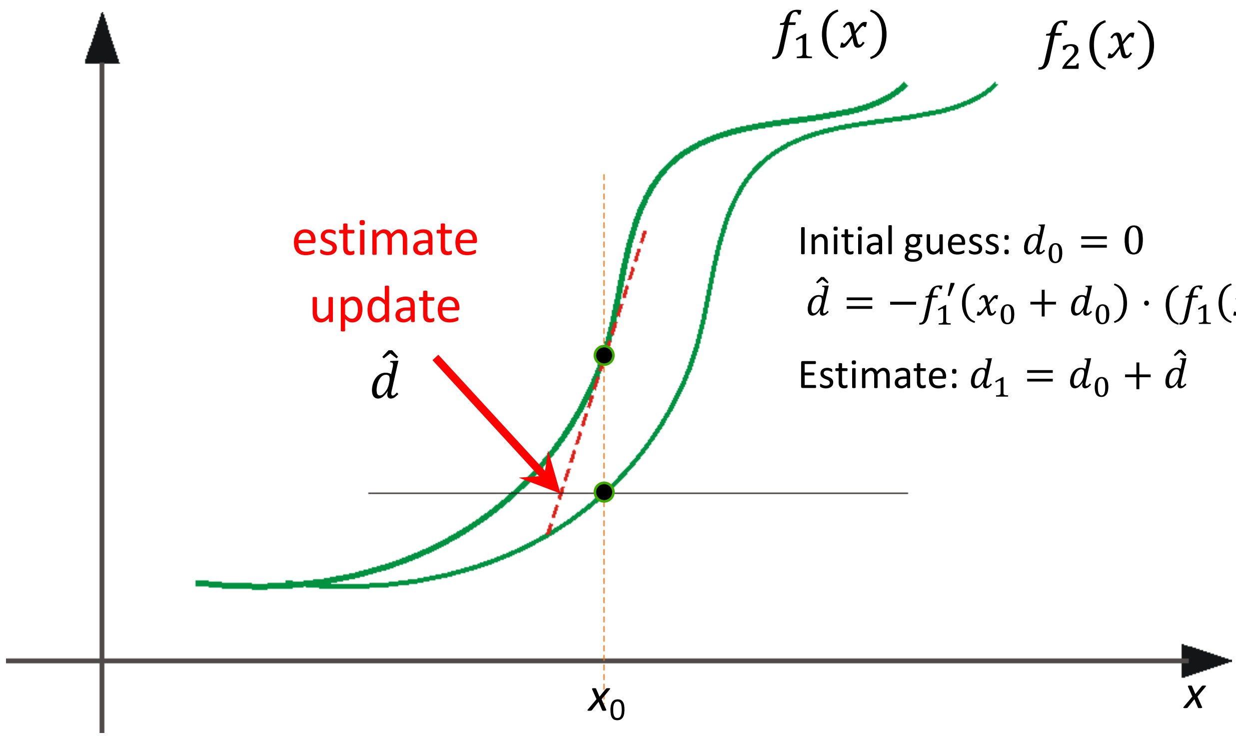
$$b = \sum_{i,j} \begin{bmatrix} g_x(i,j)[I(i,j) - J(i,j)] \\ g_y(i,j)[I(i,j) - J(i,j)] \end{bmatrix}$$

$$d = A^{-1} b$$

- Iterate:
  - Update $J_{i+1}(x) \rightarrow J_i(x+d)$
  - Recompute d between $J_{i+1}(x)$ and $I(x)$

Step 4: Iterative Refinement

Compute d to minimize $\| f_1(x+d) - f_2(x) \|^2$



Initial guess: $d_0 = 0$

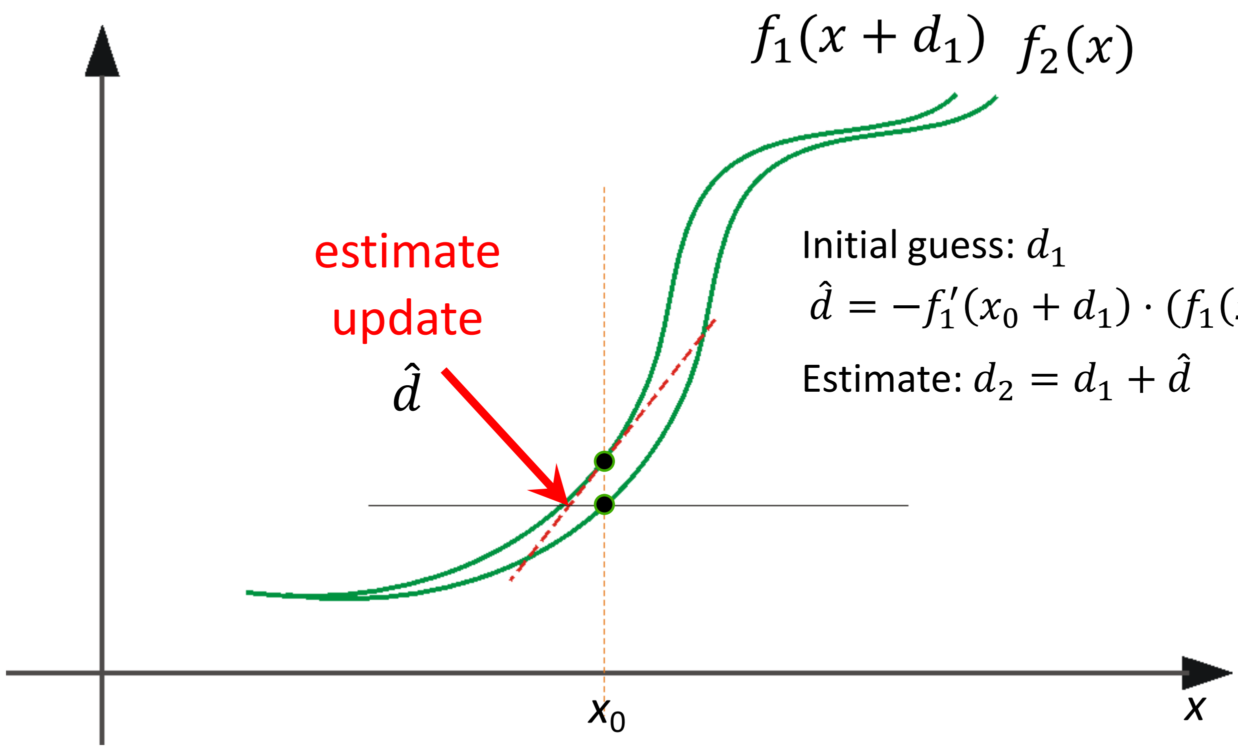
$\hat{d} = -f_1'(x_0 + d_0) \cdot (f_1(x_0 + d_0) - f_2(x_0))$

Estimate: $d_1 = d_0 + \hat{d}$

Slide adapted from Richard Szeliski

Step 4: Iterative Refinement

Compute d to minimize $\| f_1(x+d) - f_2(x) \|^2$



Initial guess: d_1

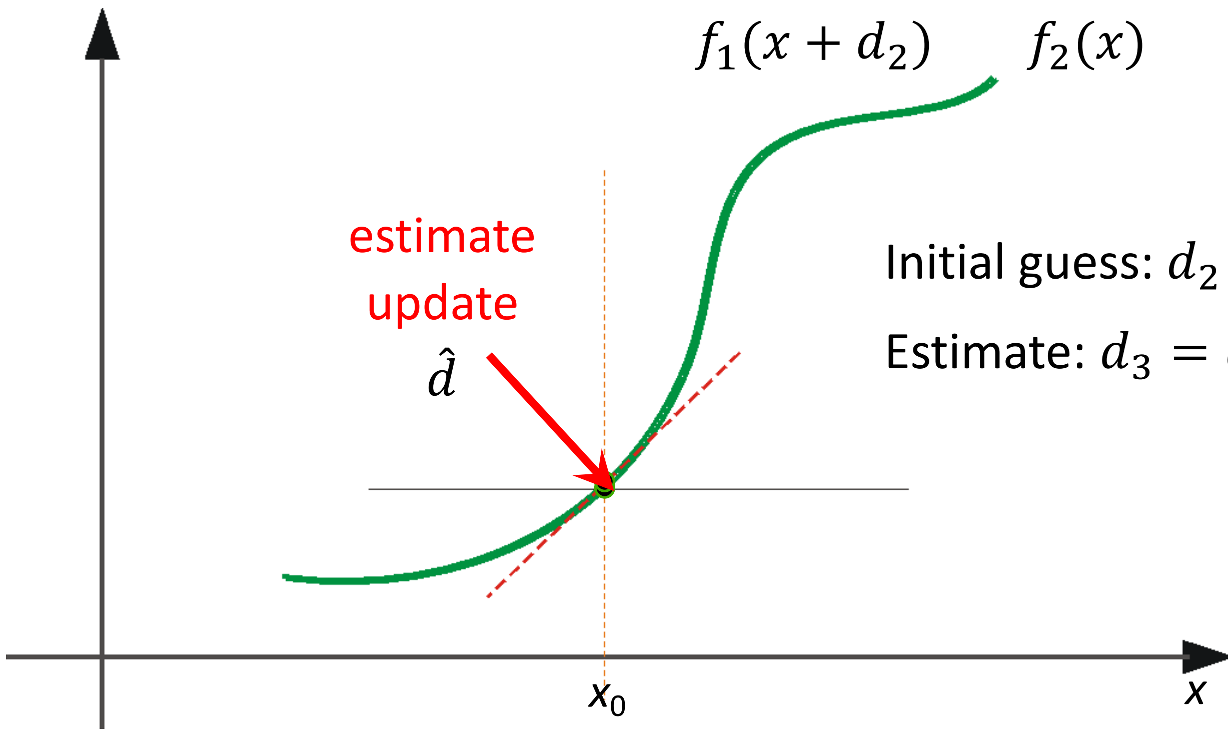
$\hat{d} = -f_1'(x_0 + d_1) \cdot (f_1(x_0 + d_1) - f_2(x_0))$

Estimate: $d_2 = d_1 + \hat{d}$

Slide adapted from Richard Szeliski

Step 4: Iterative Refinement

Compute d to minimize $\|f_1(x+d) - f_2(x)\|^2$



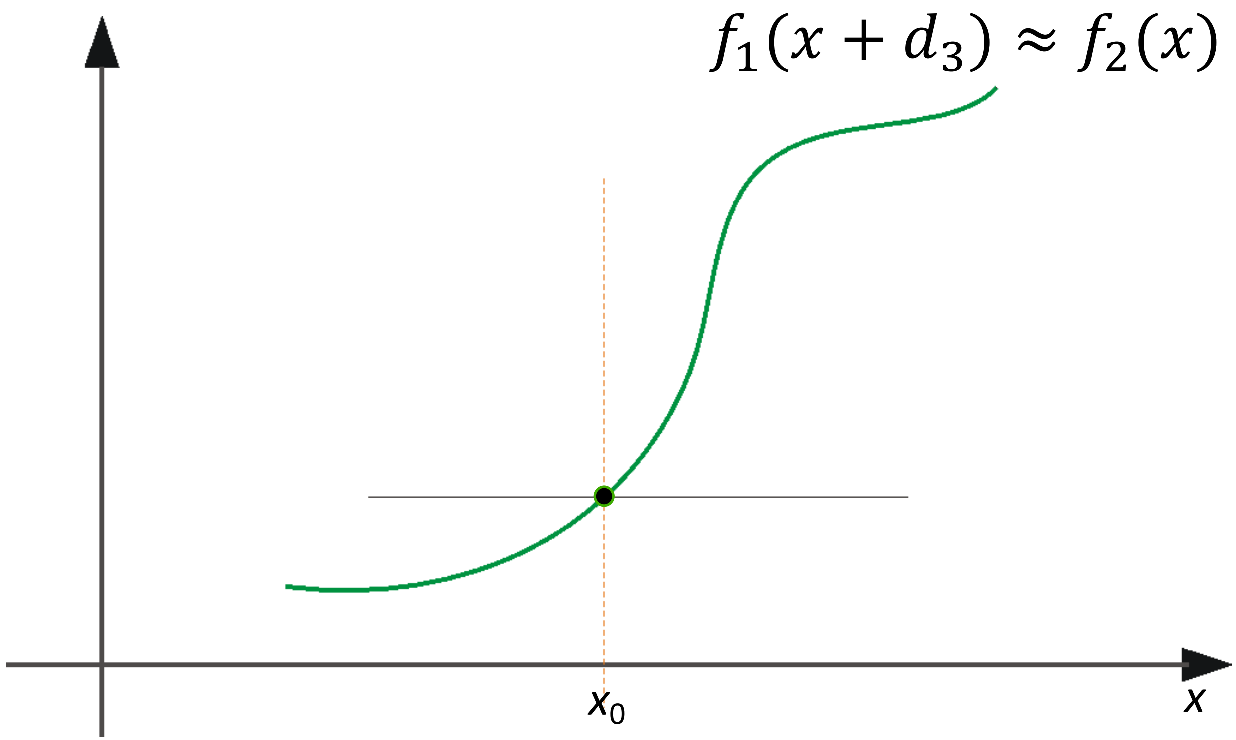
Initial guess: d_2

Estimate: $d_3 = d_2 + \hat{d}$

Slide adapted from Richard Szeliski

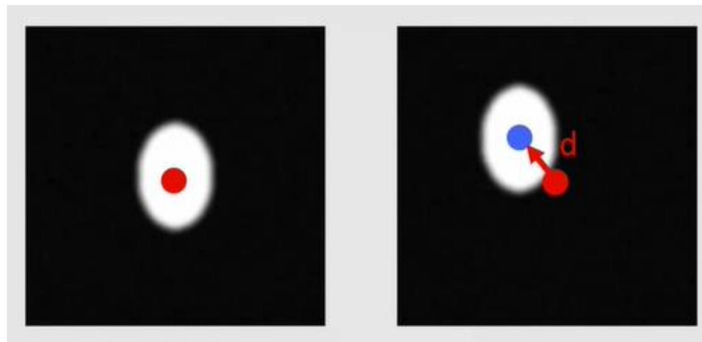
Step 4: Iterative Refinement

Compute d to minimize $\|f_1(x+d) - f_2(x)\|^2$

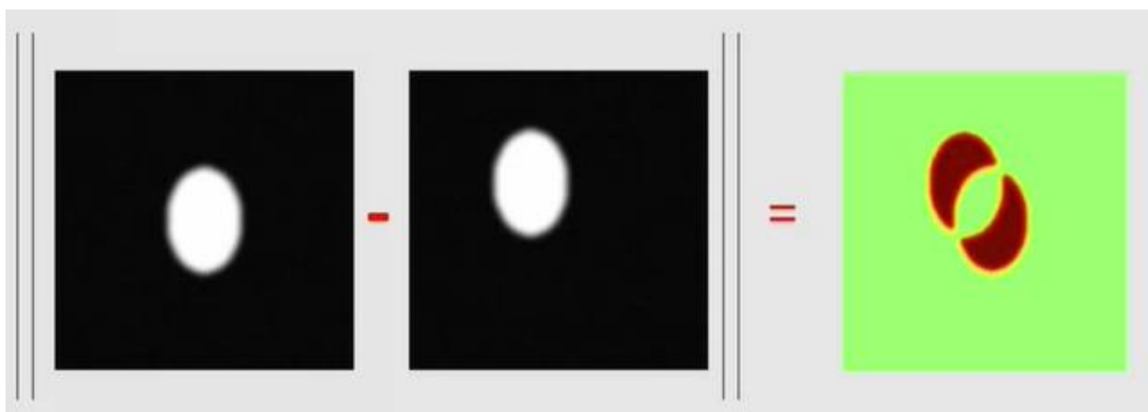


Slide adapted from Richard Szeliski

Case Study



$I(x)$ $J(x+d)$



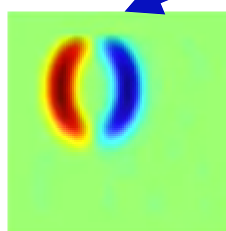
Sum of Squared Difference (SSD) error

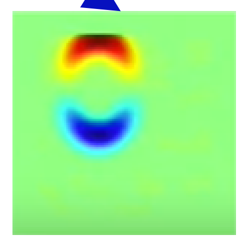
Slide adapted from Jianbo Shi

Case Study

- Gradient

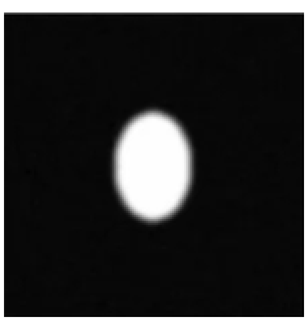
$$g = \left(\frac{\partial J}{\partial x}, \frac{\partial J}{\partial y}\right)^T$$





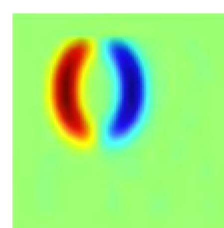
- Taylor Expansion

$$I(x) = J(x+d) = J(x) + g^T d$$

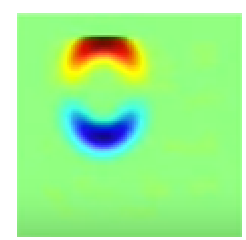


$=$

$+ \; d_x$



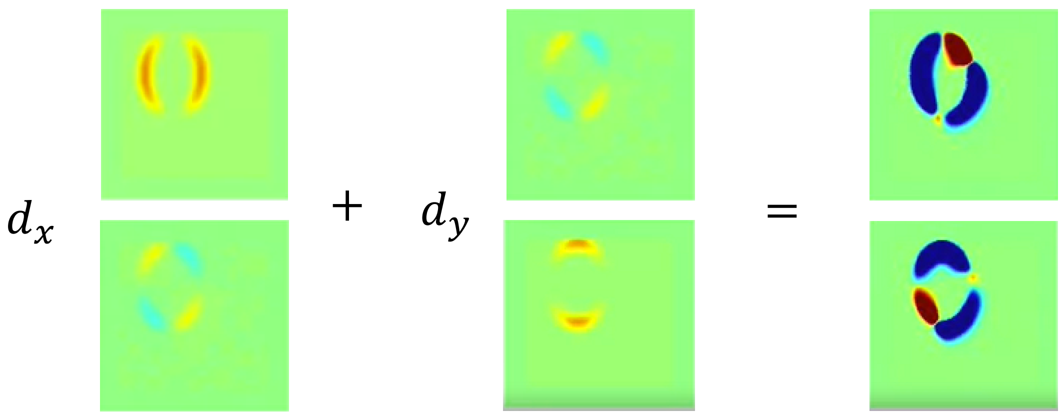
$+ \; d_y$



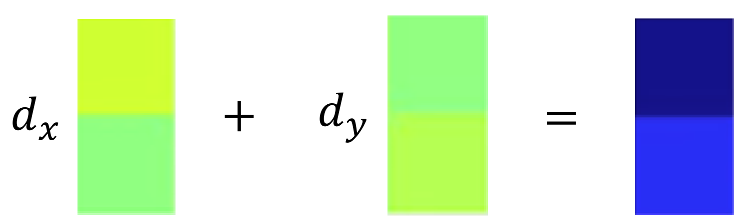
Slide adapted from Jianbo Shi

Case Study

$$\int_W g(g^T d)w\,dx = \int_W [I(x) - J(x)]g\,w\,dx$$

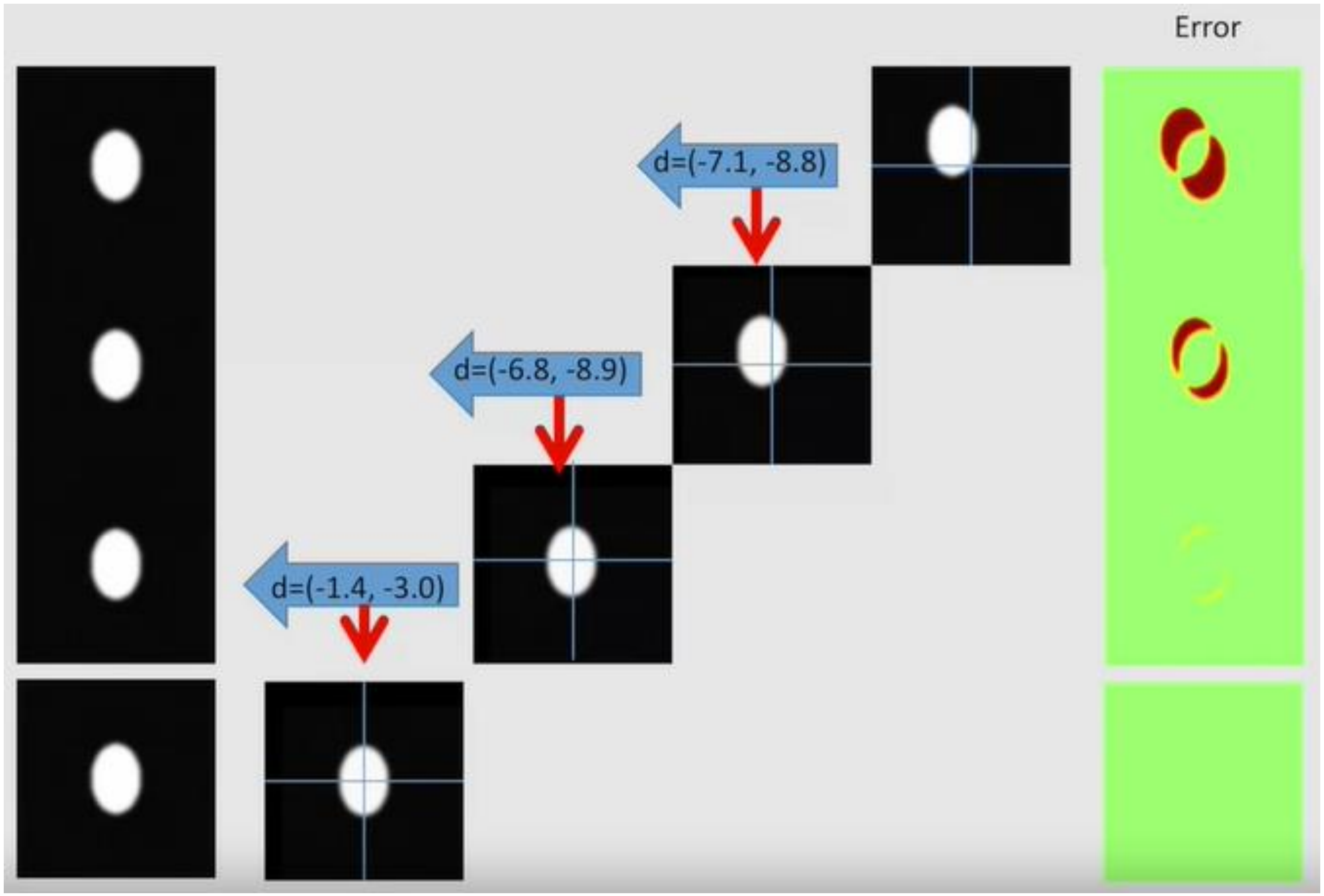


- Over determined, sum up and solve



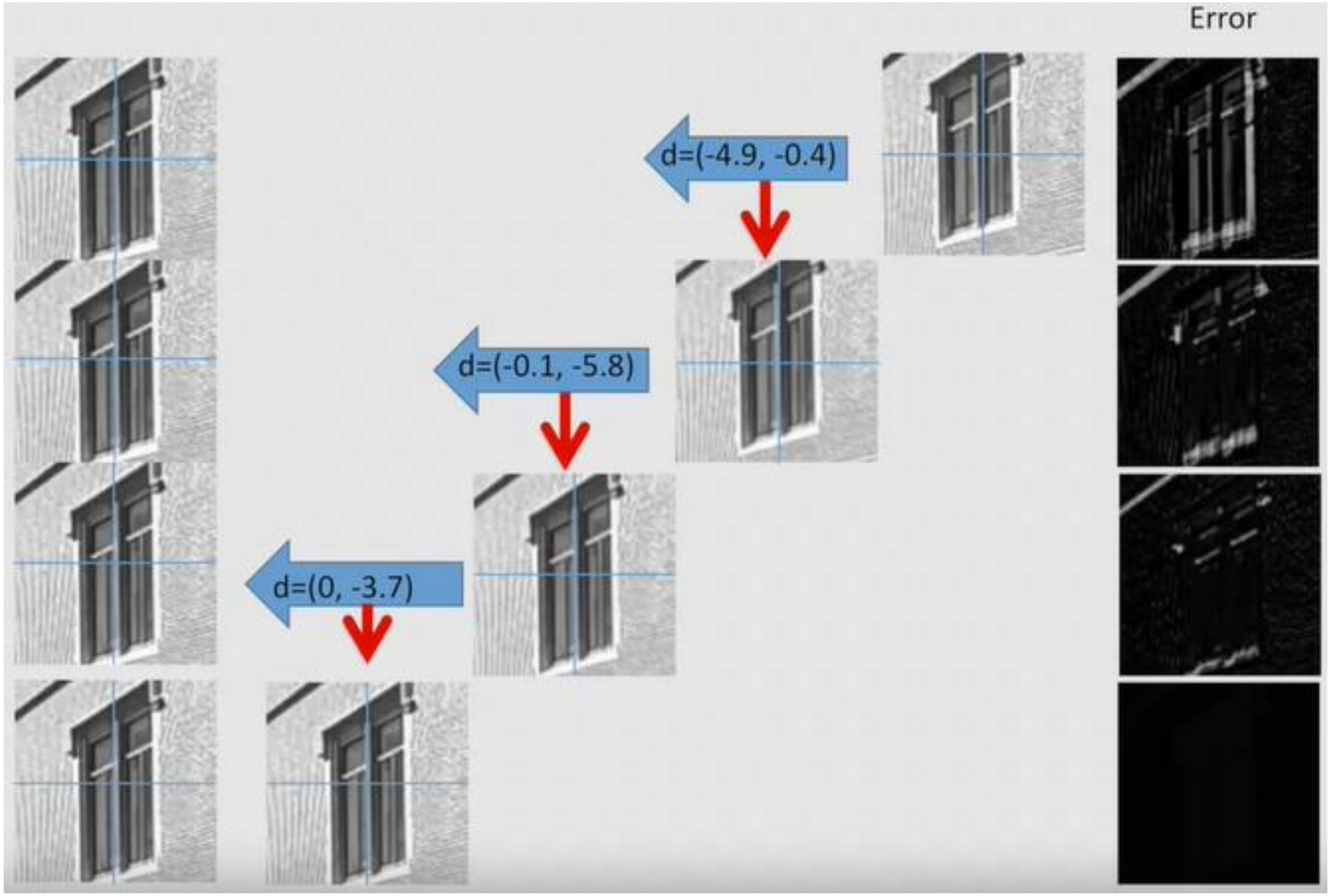
Slide adapted from Jianbo Shi

Case Study



Slide adapted from Jianbo Shi

Another Example



Slide adapted from Jianbo Shi

Motion Models

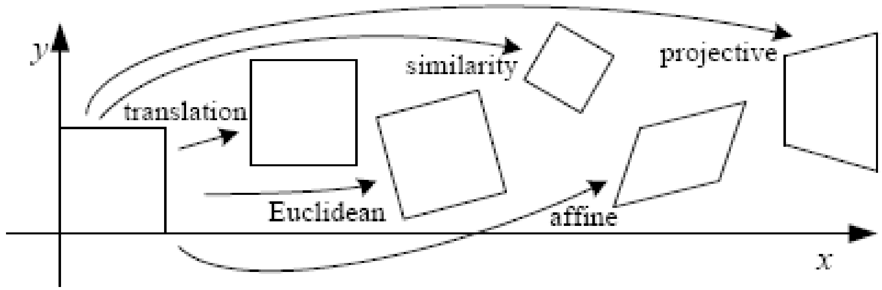
- 2D Models:
  - Affine
  - Quadratic
  - Planar projective transform (Homography)

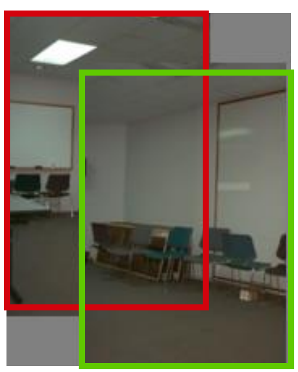
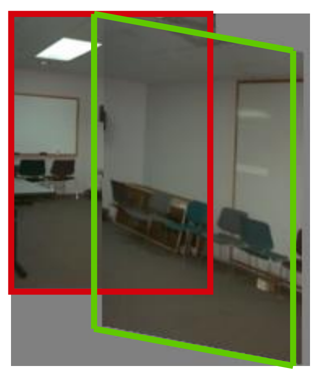
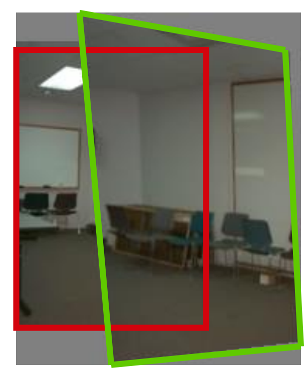
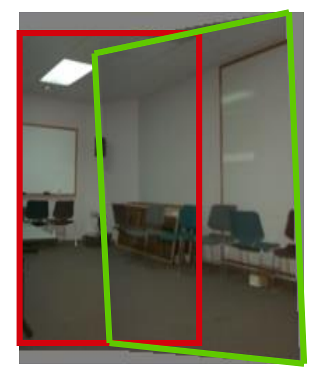
- 3D Models:
  - Instantaneous camera motion models
  - Homography+epipole
  - Plane+Parallax

Model Selection Problem

Slide adapted from Richard Szeliski

Motion Models

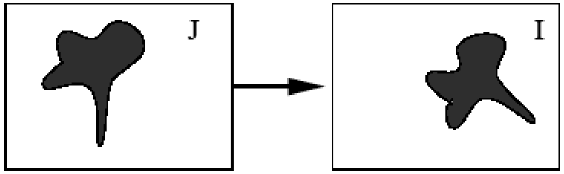


Translation	Affine	Perspective	3D rotation
			
2 unknowns	6 unknowns: $x' = Ax + d$	8 unknowns	3 unknowns

Slide adapted from Richard Szeliski

Compute Affine Motion

- Intensity constancy constraint: $J(Ax + d) = I(x)$



- Define Sum of Square Difference, SSD, error as:

$$\in = \int_W [J(Ax + d) - I(x)]^2 w(x) dx \quad (1)$$

Let$A = I + D$, min. $\in$ with respect to $D \in R^{2\times 2}$, and $d \in R^{2\times 1}$

- Three steps for solving this problem:
  - Set $\frac{\partial_\in}{\partial_D}, \frac{\partial_\in}{\partial_d}$ to 0;
  - Taylor expression on J(Ax+d) respect to x;
  - Solve for A(D) and d

Slide adapted from Kostas Daniilidis

Compute Affine Motion

$$\in = \int_W [J(Ax+d) - I(x)]^2 w(x) dx$$

- Differentiate $\in$ with respect to D and d,

$$\frac{1}{2}\frac{\partial_\in}{\partial_D} = \int_w [J(Ax+d) - I(x)]\; g\; x^T\; w\; dx = 0 \qquad (2)$$

$$\frac{1}{2}\frac{\partial_\in}{\partial_d} = \int_w [J(Ax+d) - I(x)]\; g\; w\; dx = 0 \qquad (3)$$

where $g = (\frac{\partial J}{\partial x}, \frac{\partial J}{\partial y})^T$.

- Assume small motion, $Ax + d = x + (Dx + d) = x + u$,
  - Taylor expression of $J(Ax+d)$ is: $J(Ax+d) = J(x) + g^T u$

Slide adapted from Kostas Daniilidis

Minimize $\epsilon = \int_W [J(Ax+d) - I(x)]^2 w(x) dx$

- From previous slide, we have:

$$\int_w [J(Ax+d) - I(x)]\, g\, x^T\, w\, dx = 0$$
$$\int_w [J(Ax+d) - I(x)]\, g\, w\, dx = 0$$
$$J(Ax+d) = J(x) + g^T u$$

where $g = (\frac{\partial J}{\partial x}, \frac{\partial J}{\partial y})^T$.

- Combining them, we have:

$$\int_W g\, x^T\, (g^T u) w\ \ dx = \int_W [I(x) - J(x)] g\, x^T\, w dx \qquad (5)$$
$$\int_W g\, (g^T u) w\ \ dx = \int_W [I(x) - J(x)] g\, w\, dx \qquad (6)$$

- Can rewrite (5) and (6) as a linear system of 6 equations and unknowns.
- Jianbo Shi and Carlo Tomasi. **Good Features to Track**. *1993.* Appendix

Slide adapted from Kostas Daniilidis

Minimize $\epsilon = \int_W [J(Ax+d) - I(x)]^2 w(x) dx$

- $Tz = a$:

$$T = \int_W \begin{bmatrix} g_x^2 x^2 & g_x g_y xy & g_x^2 xy & g_x g_y x^2 & g_x^2 x & g_x g_y x \\ g_x g_y xy & g_y^2 y^2 & g_x g_y y^2 & g_y^2 xy & g_x g_y y & g_y^2 y \\ g_x^2 xy & g_x g_y y^2 & g_x^2 y^2 & g_x g_y xy & g_y^2 y & g_x g_y y \\ g_x g_y x^2 & g_y^2 xy & g_x g_y xy & g_y^2 x^2 & g_x g_y x & g_y^2 x \\ g_x^2 x & g_x g_y x & g_x^2 y & g_x g_y x & g_x^2 & g_x g_y \\ g_x g_y x & g_y^2 x & g_x g_y y & g_y^2 x & g_x g_y & g_y^2 \end{bmatrix} w\, dx \quad (7)$$

and

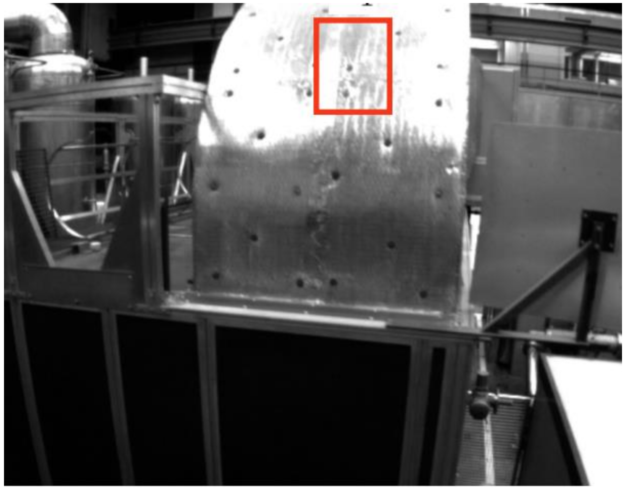
$$z = [D(1,1), D(2,2), D(1,2), D(2,1), d(1), d(2)]^T \quad (8)$$

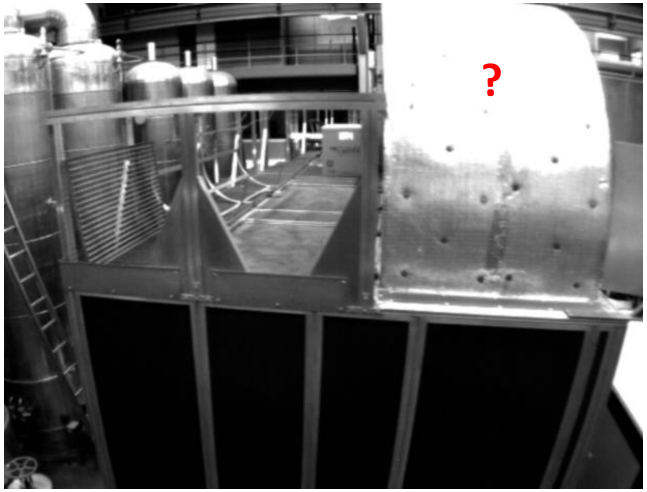
$$a = \int_W (I(x) - J(x)) \begin{bmatrix} g_x x \\ g_y y \\ g_x y \\ g_y x \\ g_x \\ g_y \end{bmatrix} dx \quad (9)$$

Slide adapted from Kostas Daniilidis

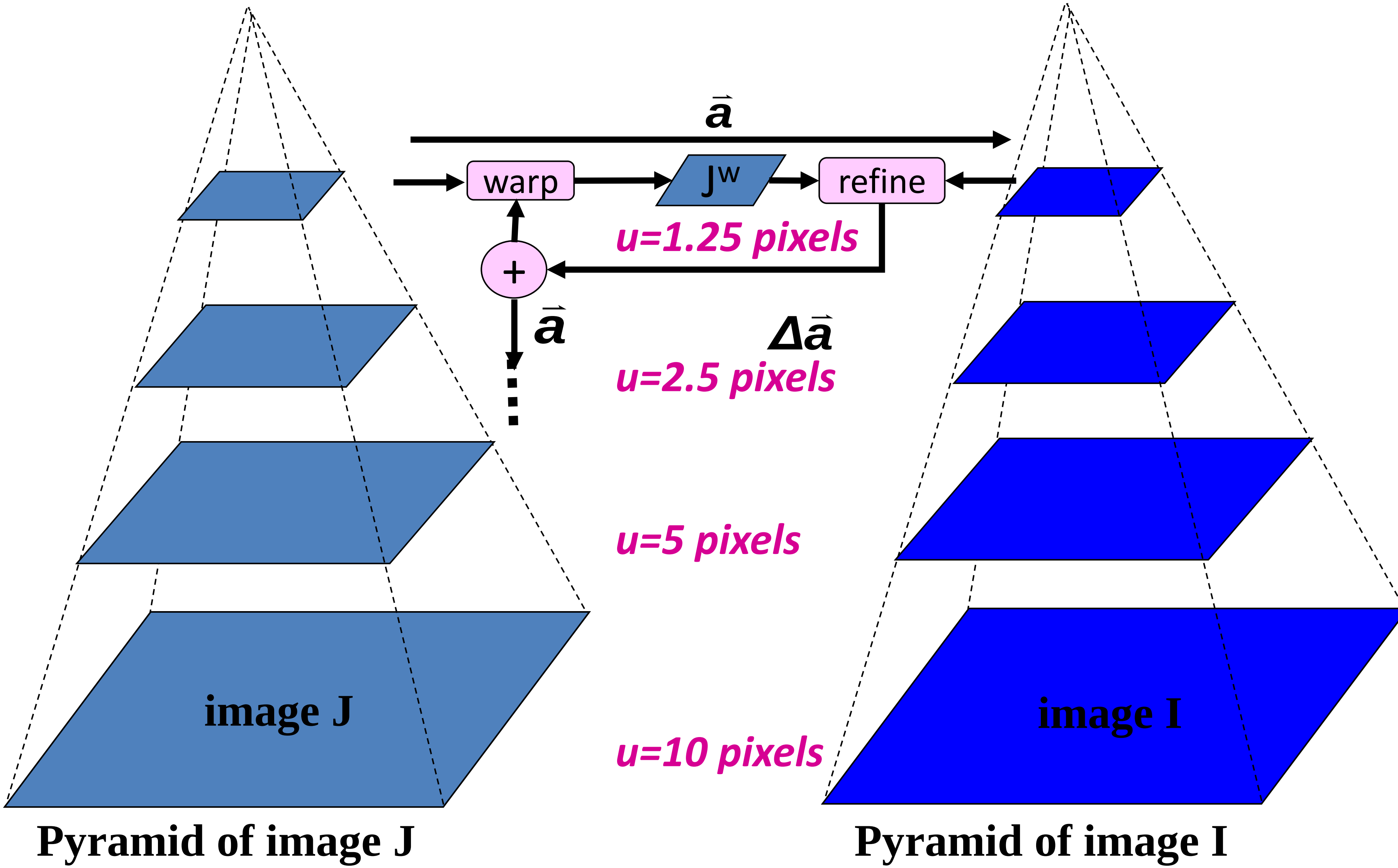
Limits of the KLT Tracker

- Fails when intensity structure in window is poor
- Fails when the displacement is large (typical operating range is motion of 1 pixel
  - Linearization of brightness is suitable only for small displacement
- Brightness is not strictly constant in images
  - Actually less problematic than it appears, since we can filter images to make them look similar



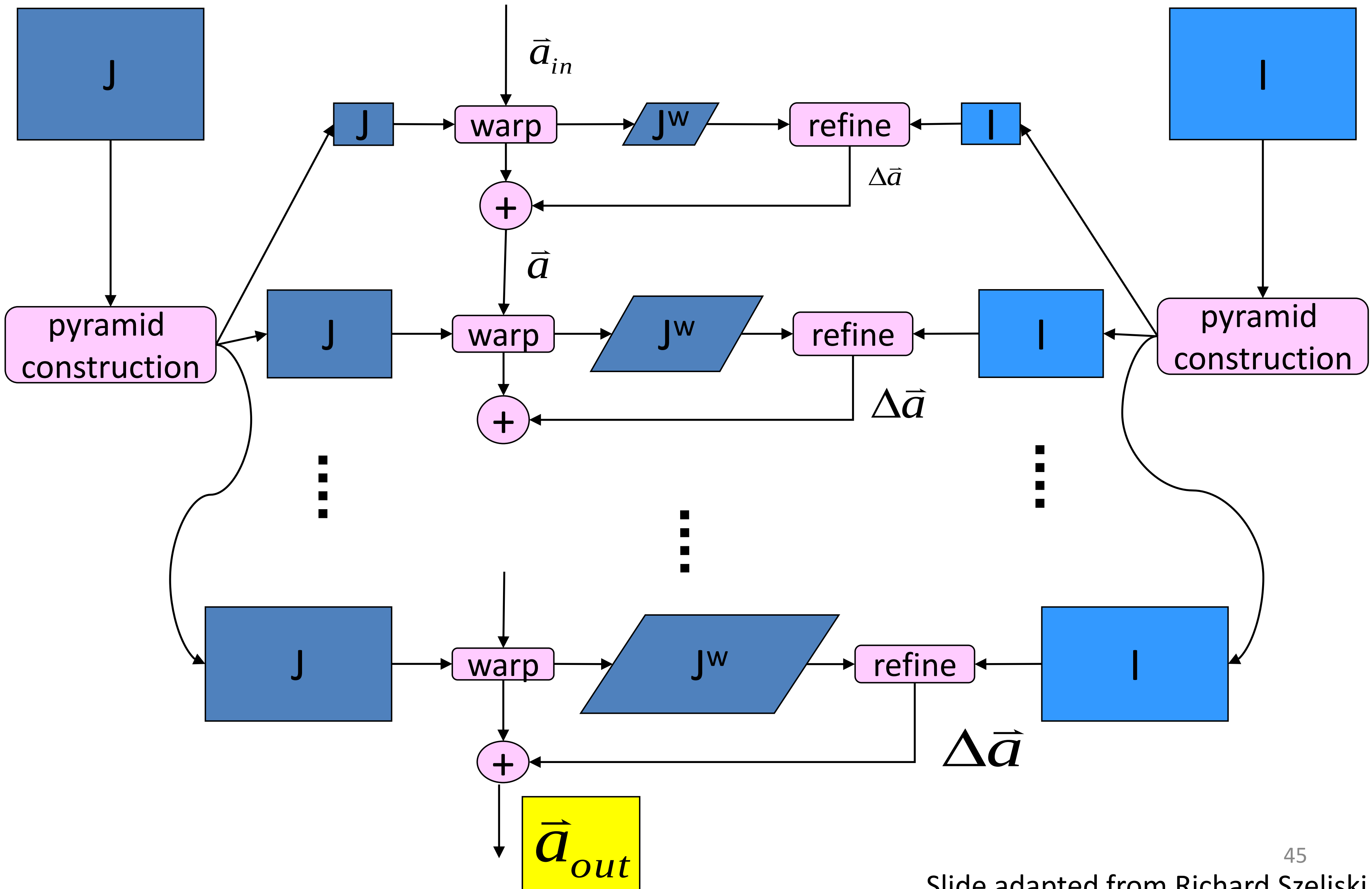


Coarse-to-Fine Estimation



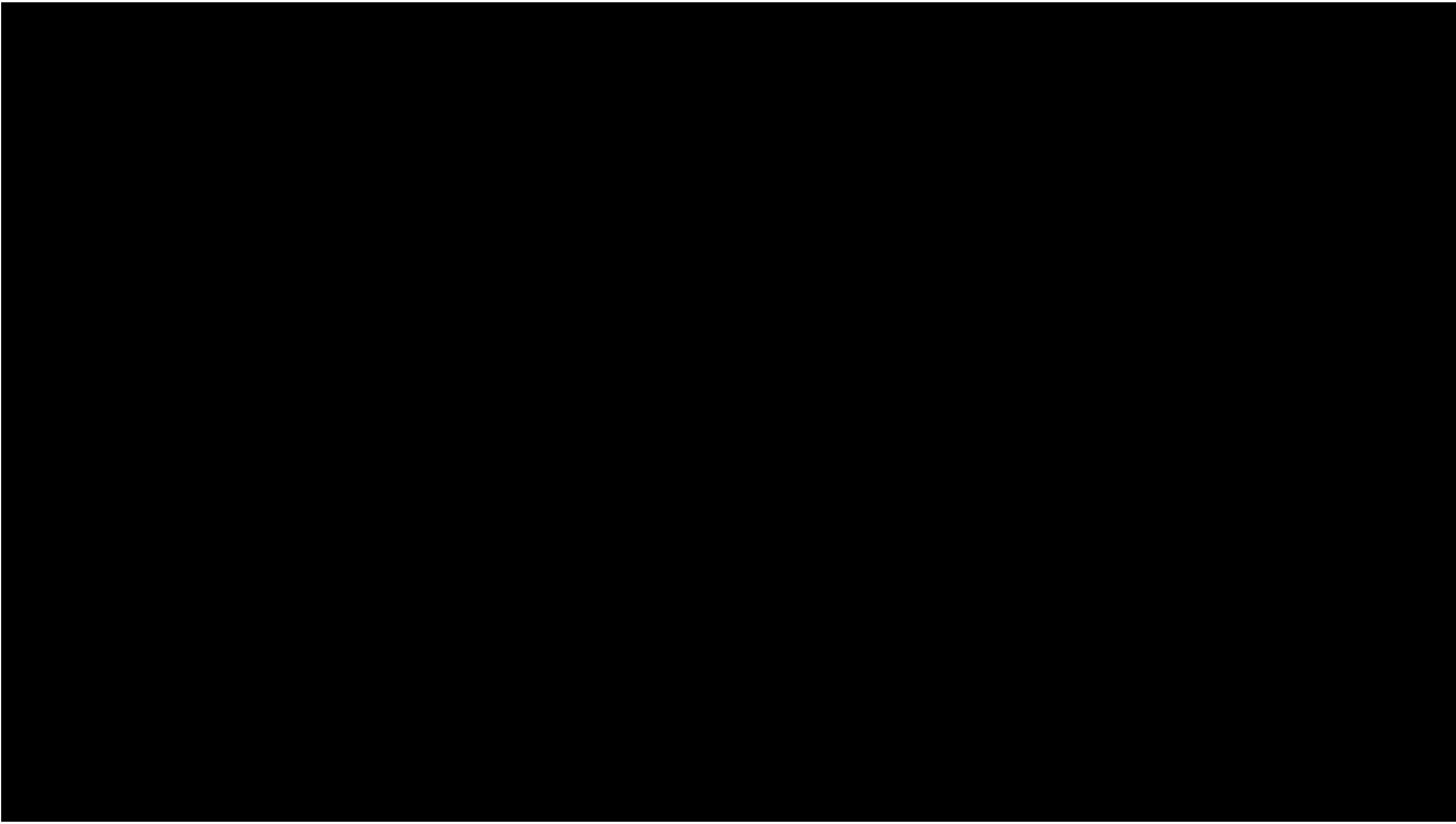
Slide adapted from Richard Szeliski

Coarse-to-Fine Estimation

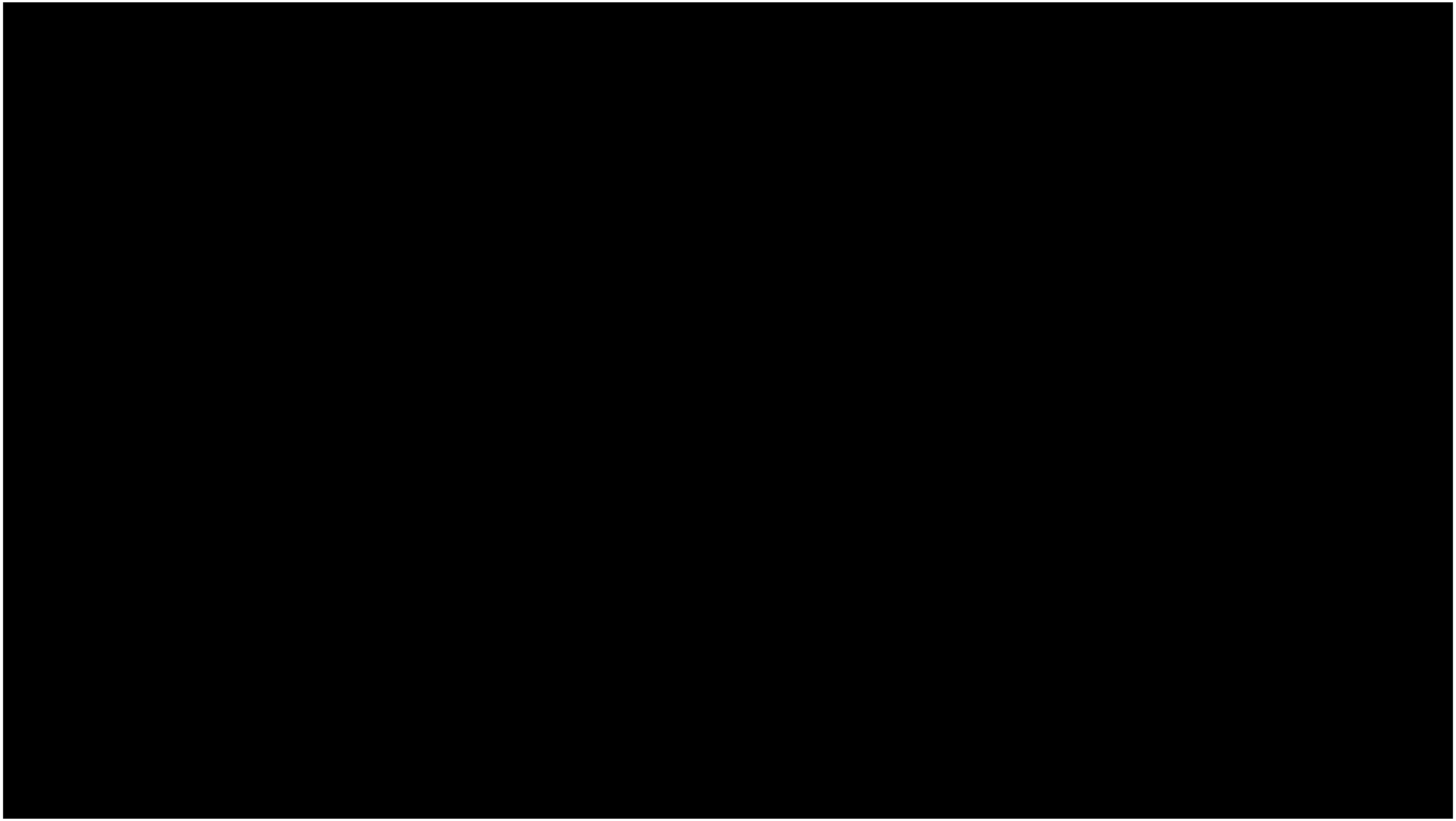


Slide adapted from Richard Szeliski

Optical Flow-based Velocity Estimator



Optical Flow-based Visual Odometry



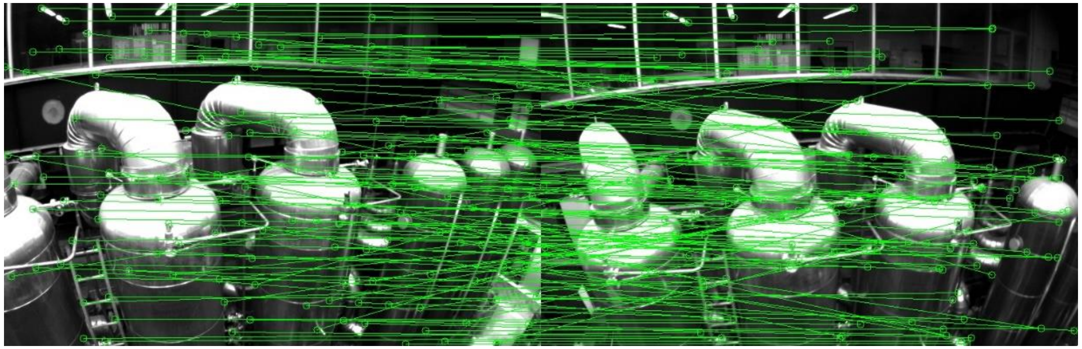
Tracking v.s. Matching

- Feature Matching in VO/SLAM :

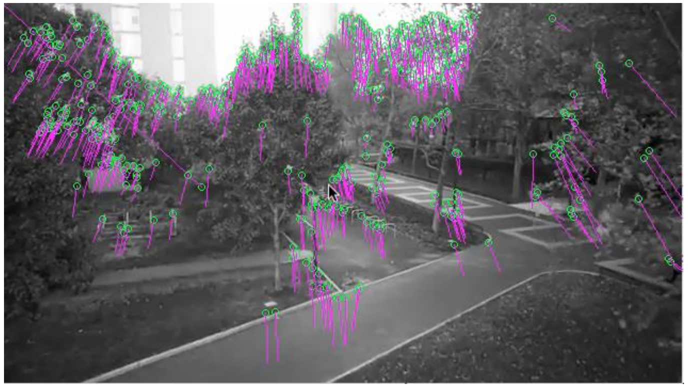
OKVIS in 2016, ORB-SLAM3 in 2020

- Feature Tracking in VO/SLAM:

VINS-Fusion in 2018, Kimera in 2020



Feature matching (Lecture 6)



Optical flow

Ends of Optical Flow

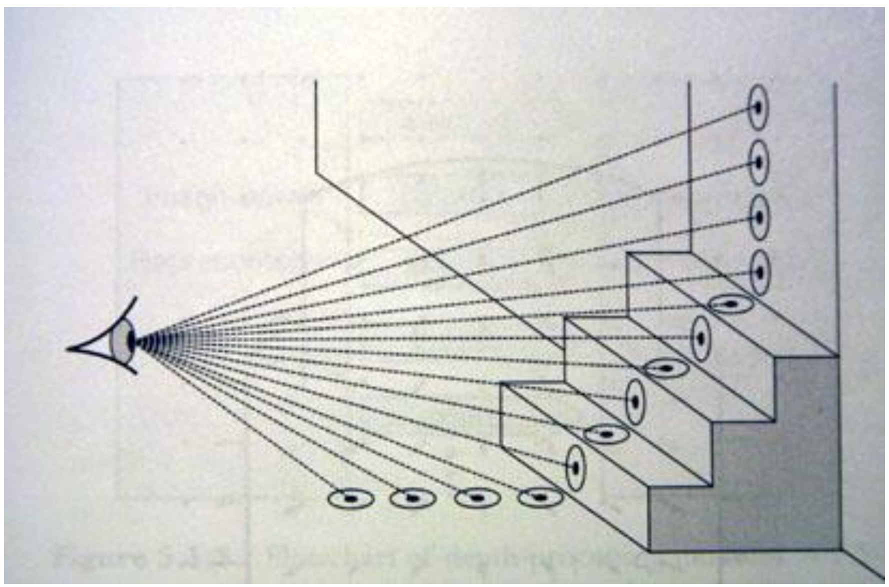
- Student: "What are the three most important problems in computer vision?"
- Takeo Kanade: "Correspondence, correspondence, correspondence!"

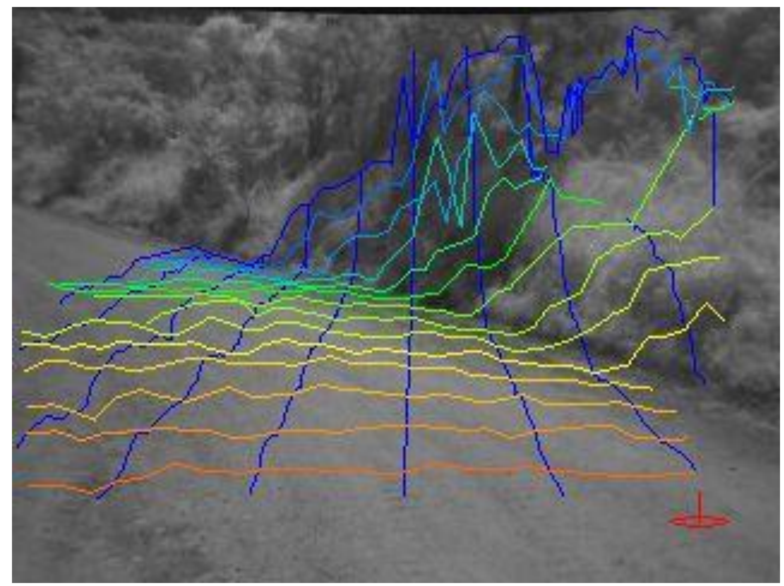
Takeo Kanade

Stereo Vision

3D Shape perception

- Depth: the distance of the surface from the observer
- Surface orientation: the slant and tilt of the surface with respect to observers' sight

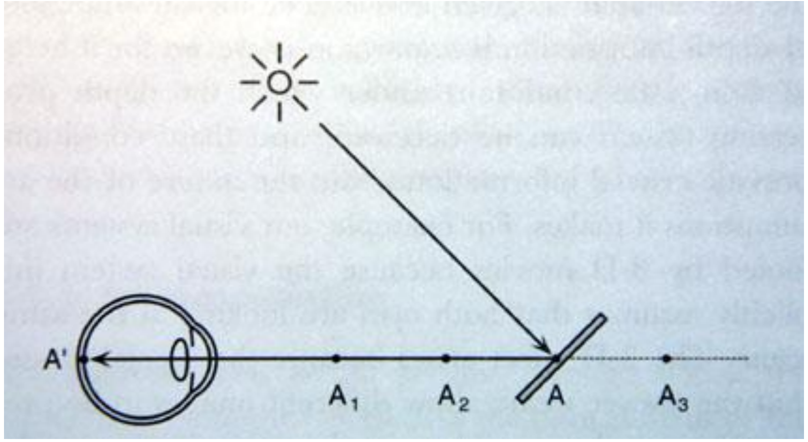


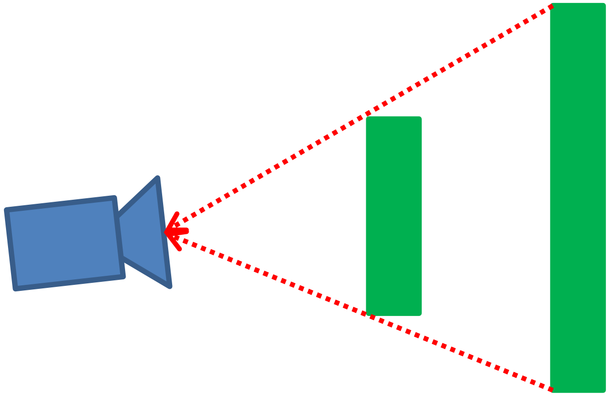


Slide adapted from Kostas Daniilidis

Depth ambiguity

Inverse problem: multiple solution exists



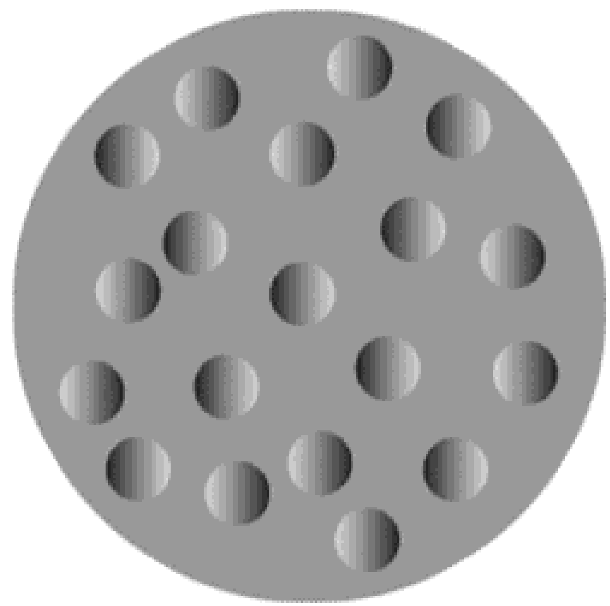


Slide adapted from Kostas Daniilidis

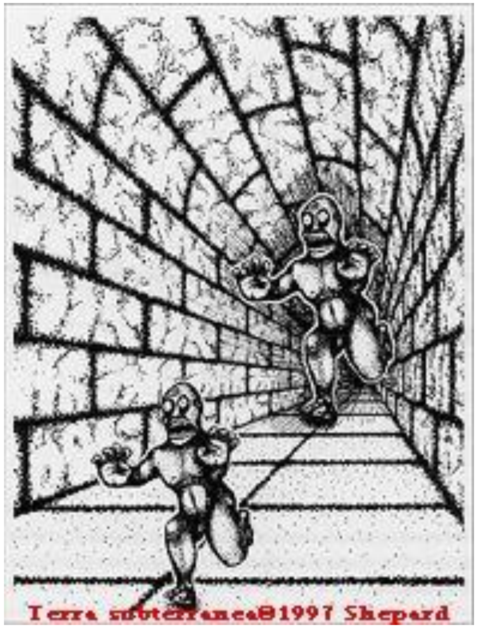
Pictorial cues for 3D shape

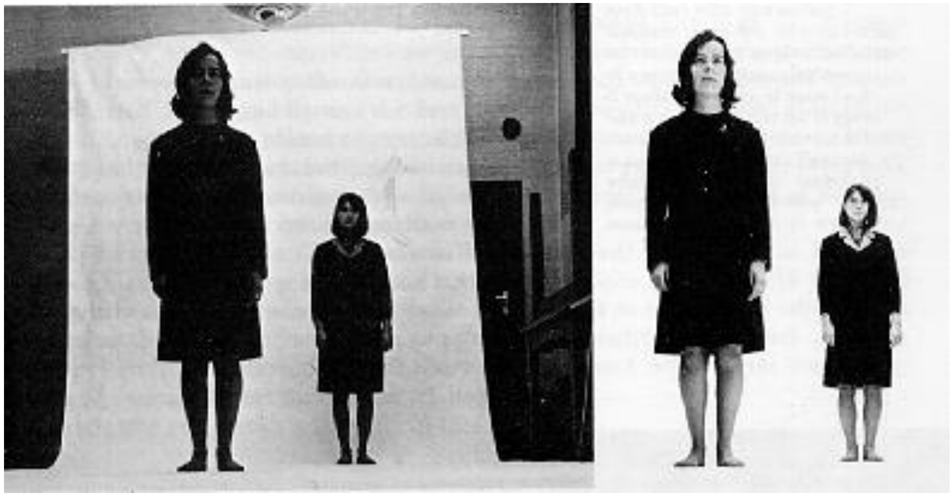
- Perspective projection gives us the relative position to horizon, therefore we can deduce its physical size

- Shading also reveal shape using illumination model



Slide adapted from Kostas Daniilidis





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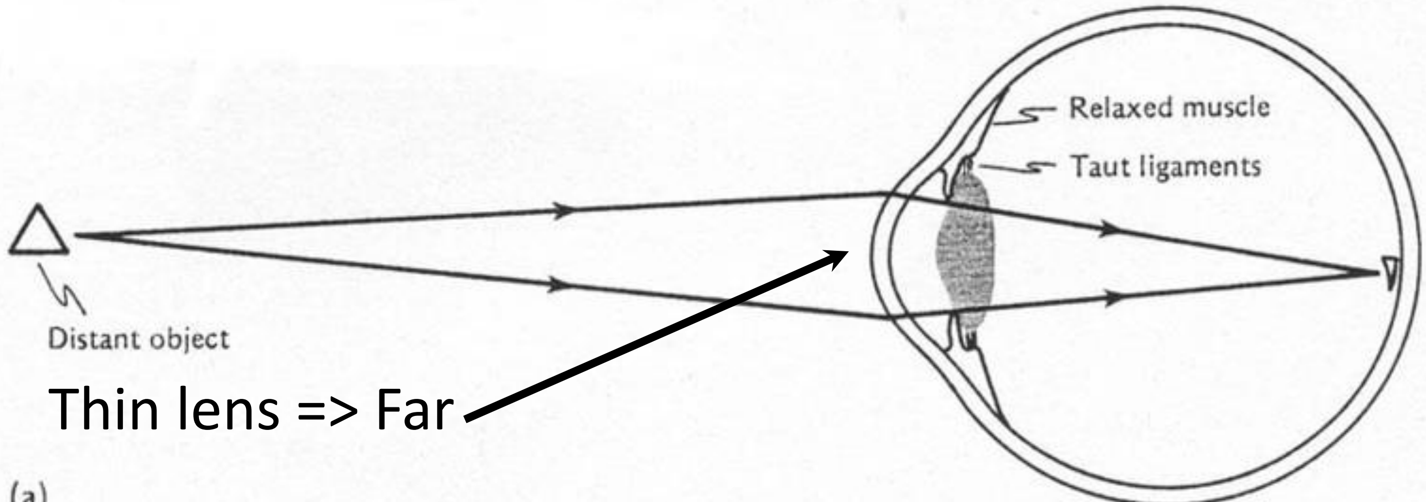
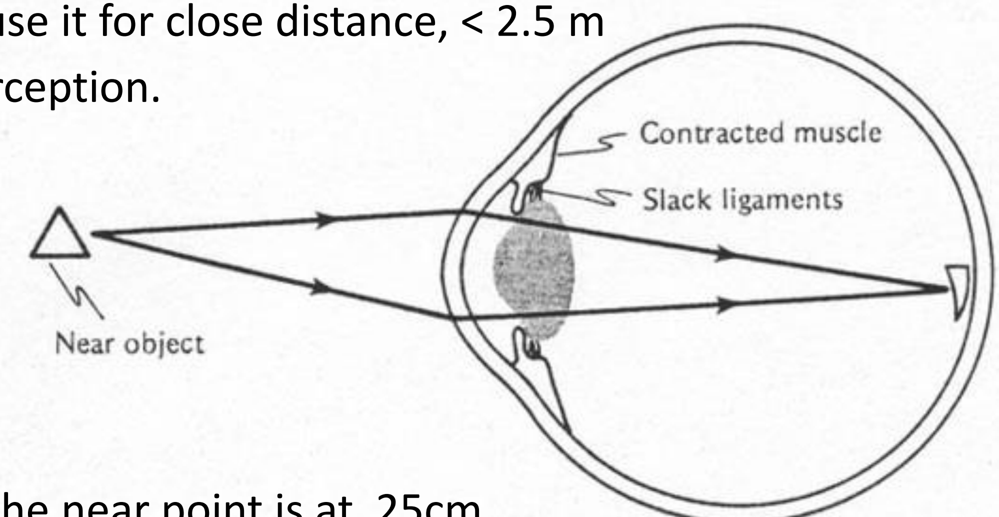
Shape from Focus, Accommodation



(a)

Human performance: use it for close distance, < 2.5 m

Use it more for size perception.



(b) For a normal eye, the near point is at 25cm

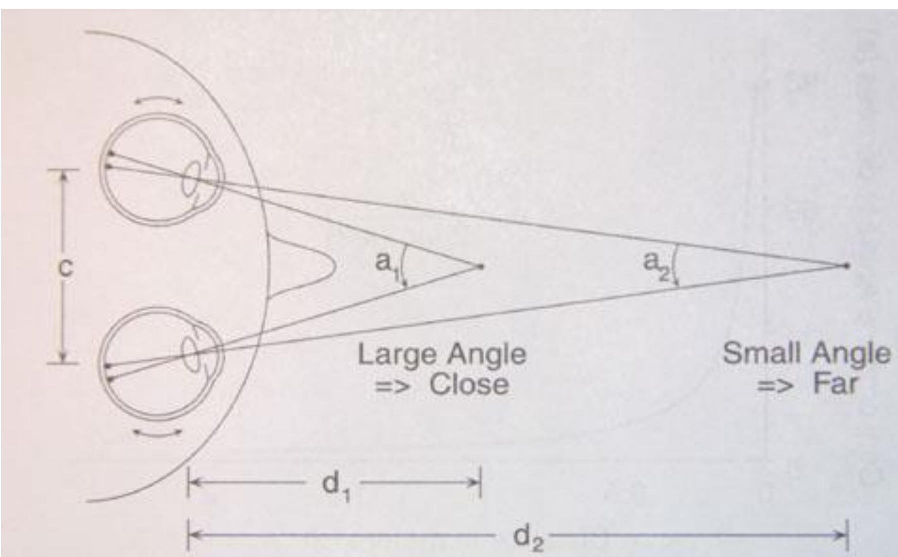
Slide adapted from Kostas Daniilidis

- Because of its restricted range (6-8 feet), accommodation is rarely a crucial source of depth in humans.
- In the chameleon, it is of paramount importance, for it controls this organism's ability to feed itself. A chameleon catches its prey by slicking its sticky tongue out just the right distance to catch an insect.
- When chameleons were outfitted with prisms and spectacles that manipulated the accommodation and convergence of their eyes, the distance they flicked their tongues was changed.

Slide adapted from Kostas Daniilidis

Convergence

- The eyes fixate a given point in external space when both of them, are aimed directly at the point so that light coming from it falls on the centers of both foveae simultaneously.
- The crucial fact about the convergence that provides information about fixation depth is that the angle formed by the two lines of sight varies systematically with distance between the observer and the fixated point.

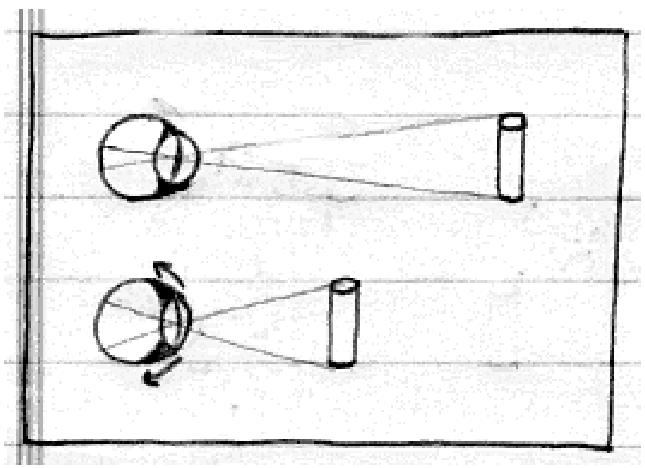


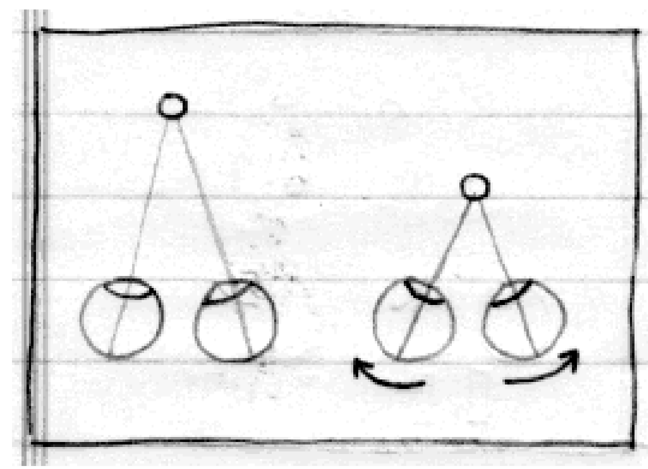
$$d = \frac{c}{2tan(\frac{a}{2})}$$

Slide adapted from Kostas Daniilidis

Accommodation and Convergence

- Accommodation and convergence normally change in lock steps. For human, they are important sources of depth information at close distance.

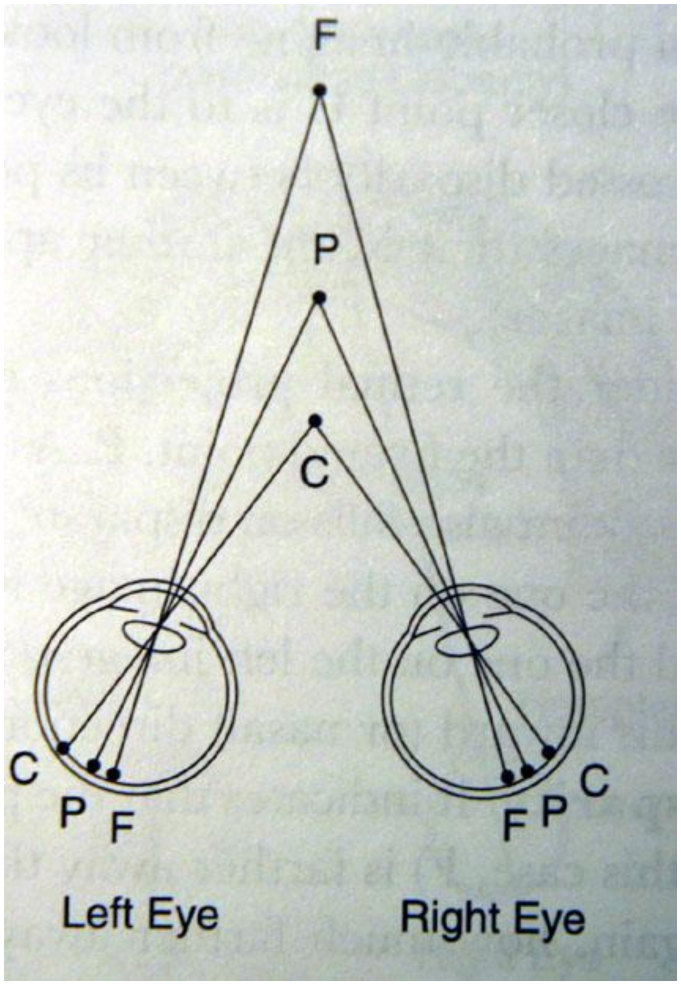


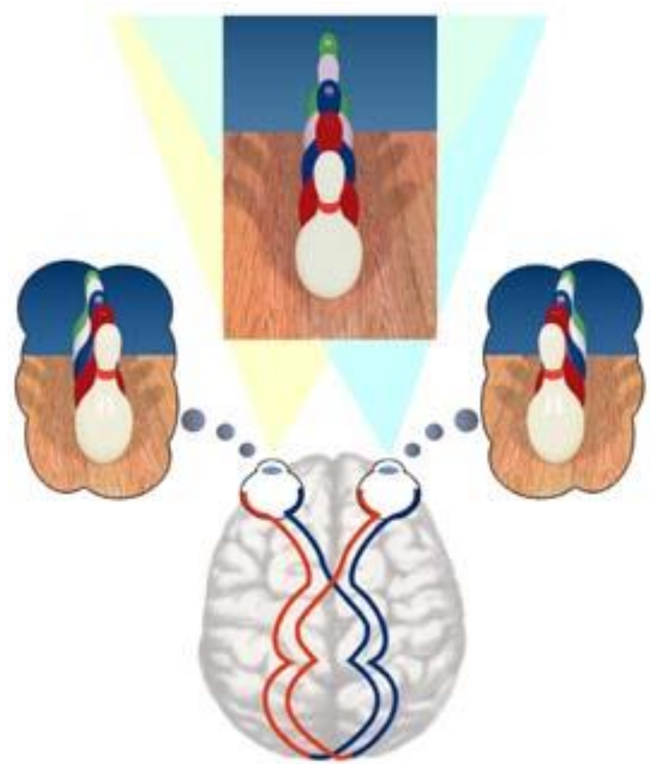


Human performance: up to a few meters

Slide adapted from Kostas Daniilidis

Stereo Vision

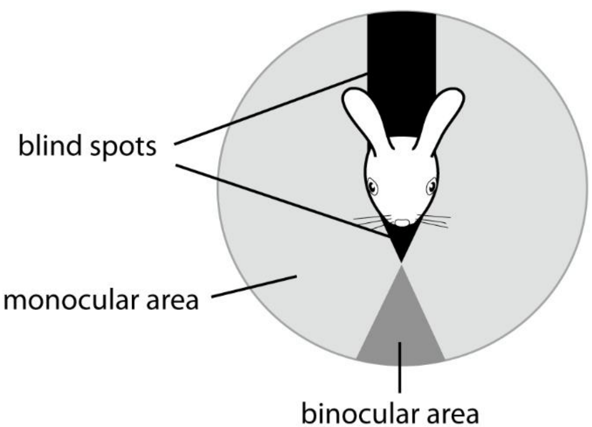


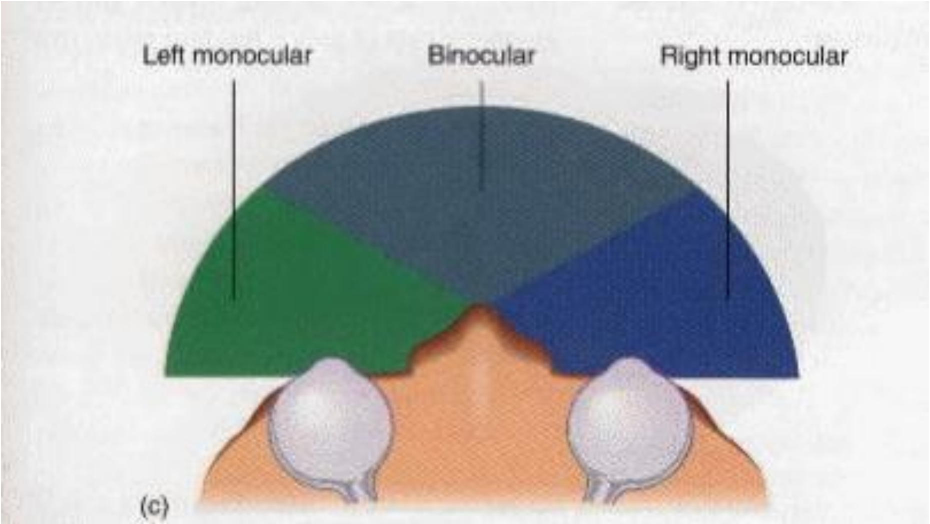


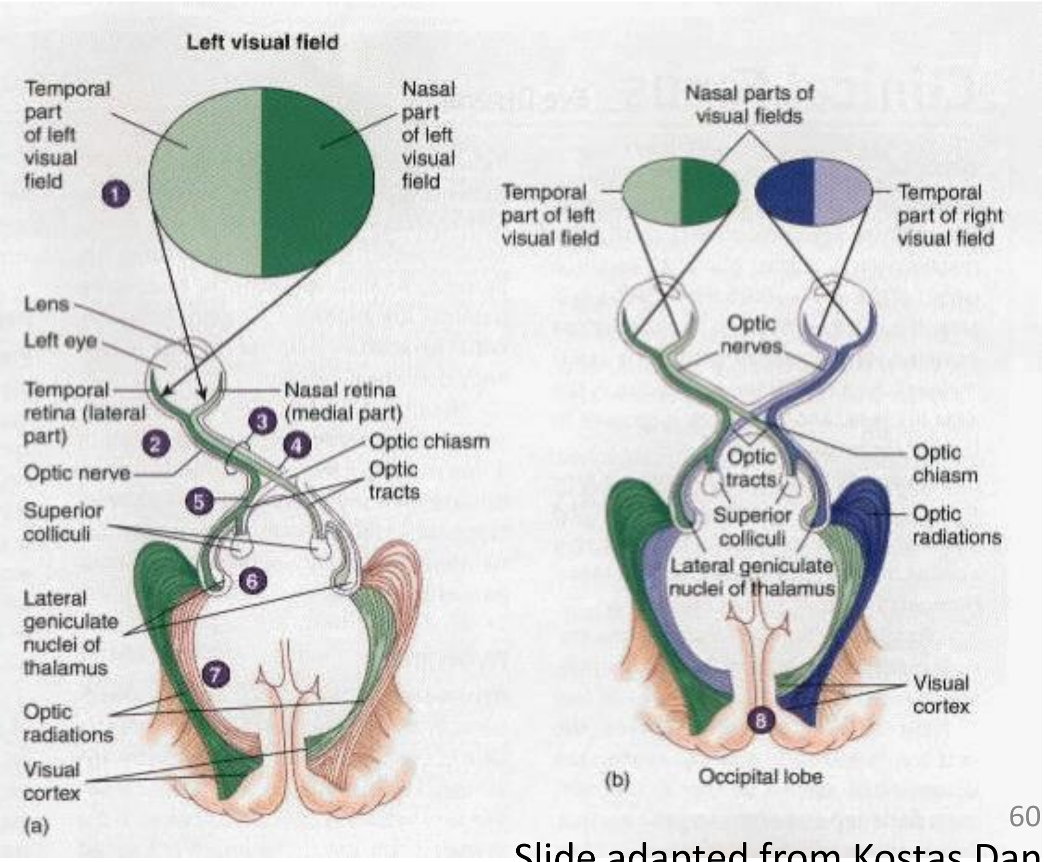
Slide adapted from Kostas Daniilidis

Our visual angle is 104d, and it is facing forward.

What happened to rabbit's vision?

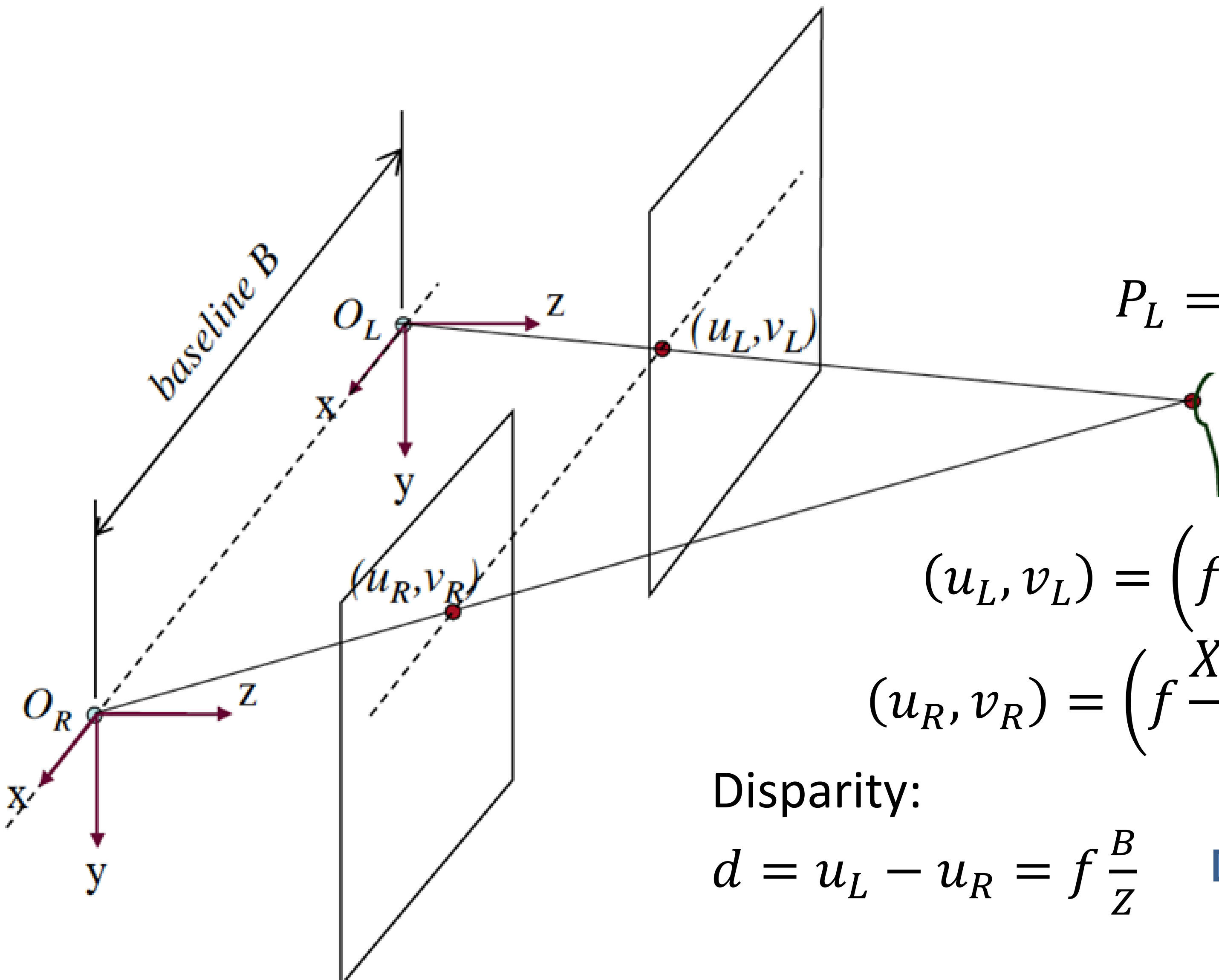






Slide adapted from Kostas Daniilidis

Basic Stereo Derivations



$P_L = (X, Y, Z)$

$$(u_L, v_L) = \left(f\frac{X}{Z}, f\frac{Y}{Z}\right)$$

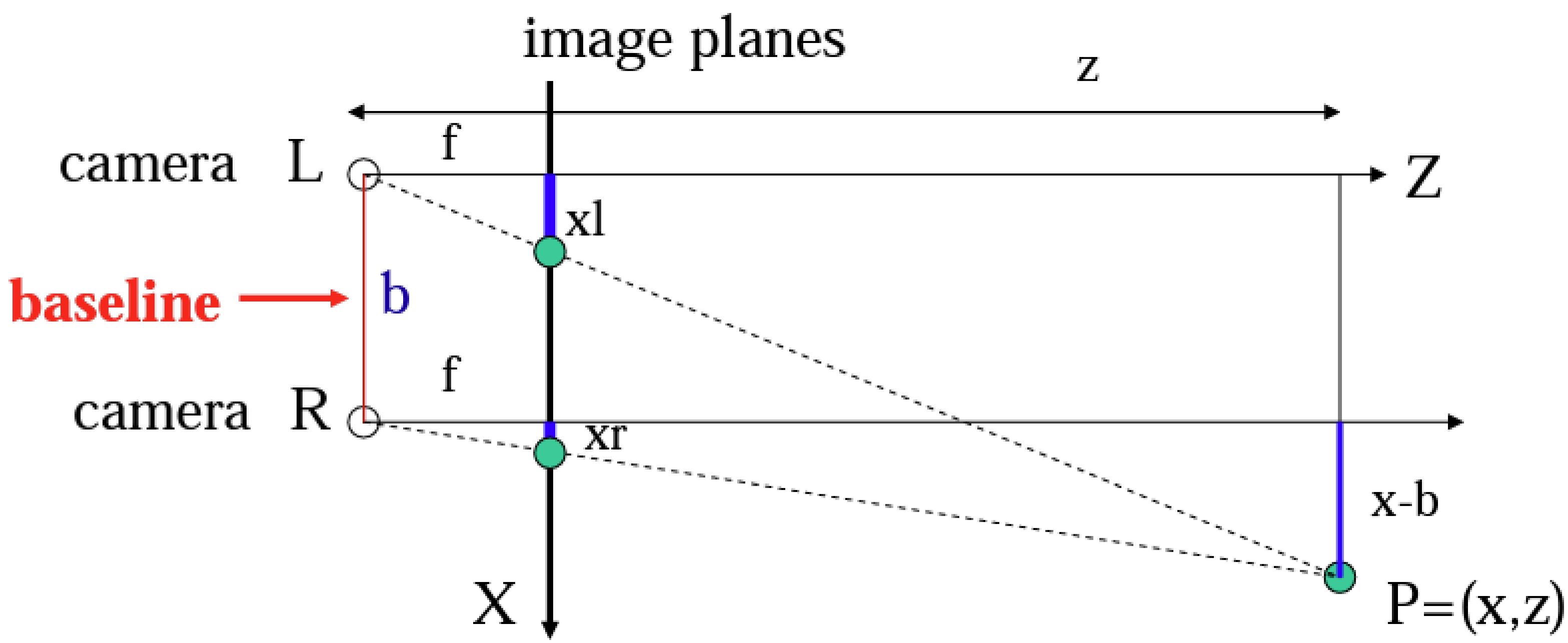
$$(u_R, v_R) = \left(f\frac{X-B}{Z}, f\frac{Y}{Z}\right)$$

Disparity:

$$d = u_L - u_R = f\frac{B}{Z} \Rightarrow \boxed{Z = f\frac{B}{d}}$$

Slide adapted from Kostas Daniilidis

Basic Stereo Derivations (BEV)



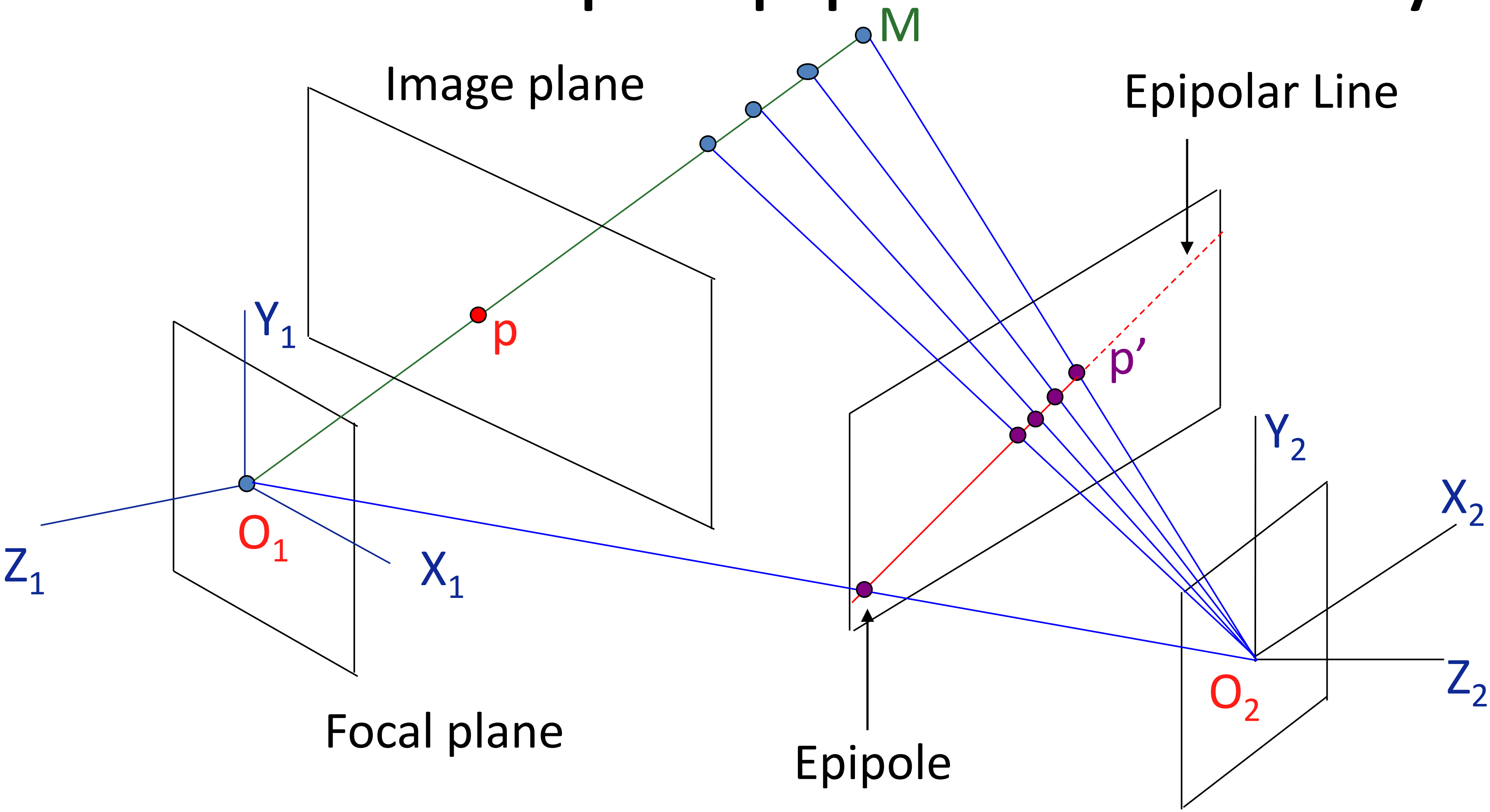
Disparity:

$$d = u_L - u_R = f\,\frac{B}{Z} \Rightarrow \boxed{Z = f\,\frac{B}{d}}$$

Slide adapted from University of Washington

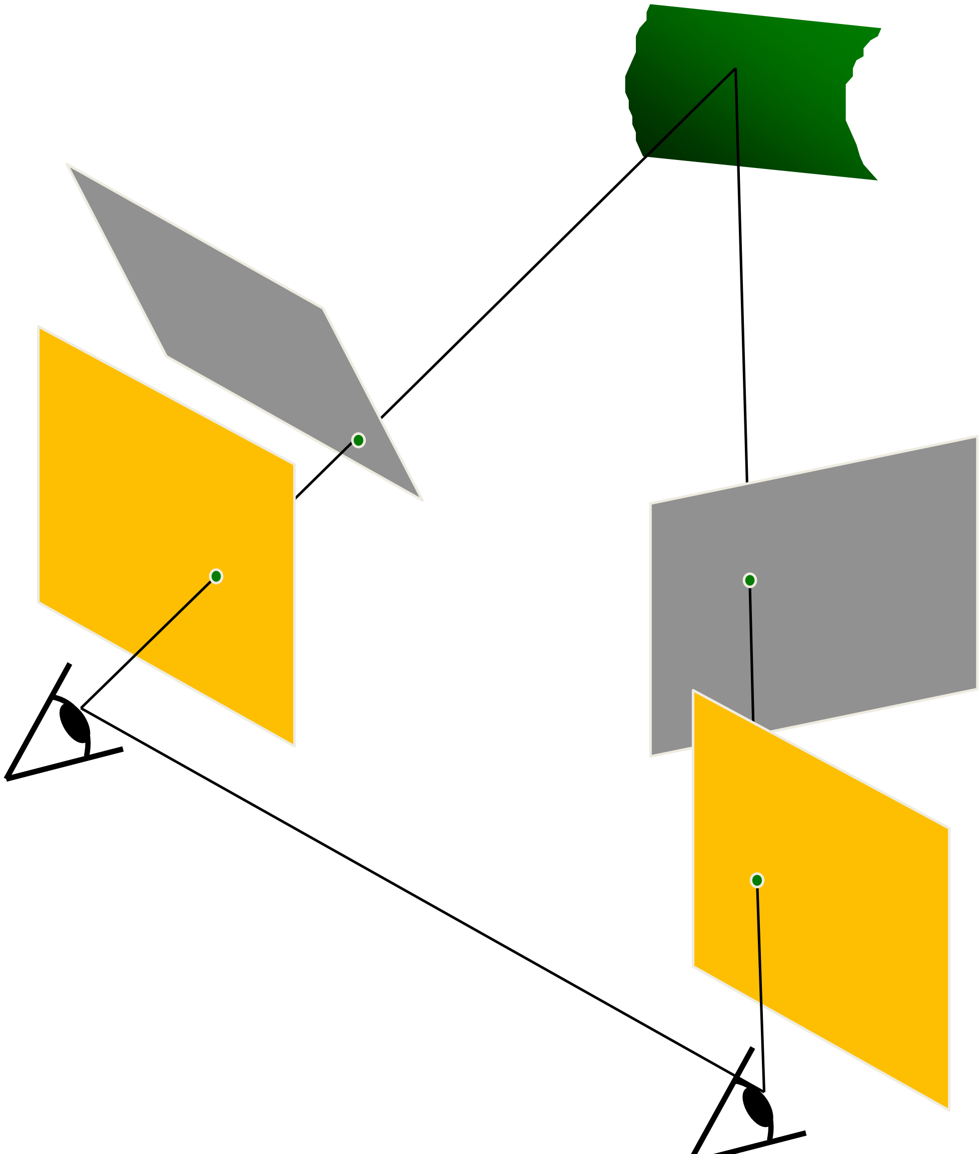
What if the optical axes of the 2 cameras are not parallel to each other?

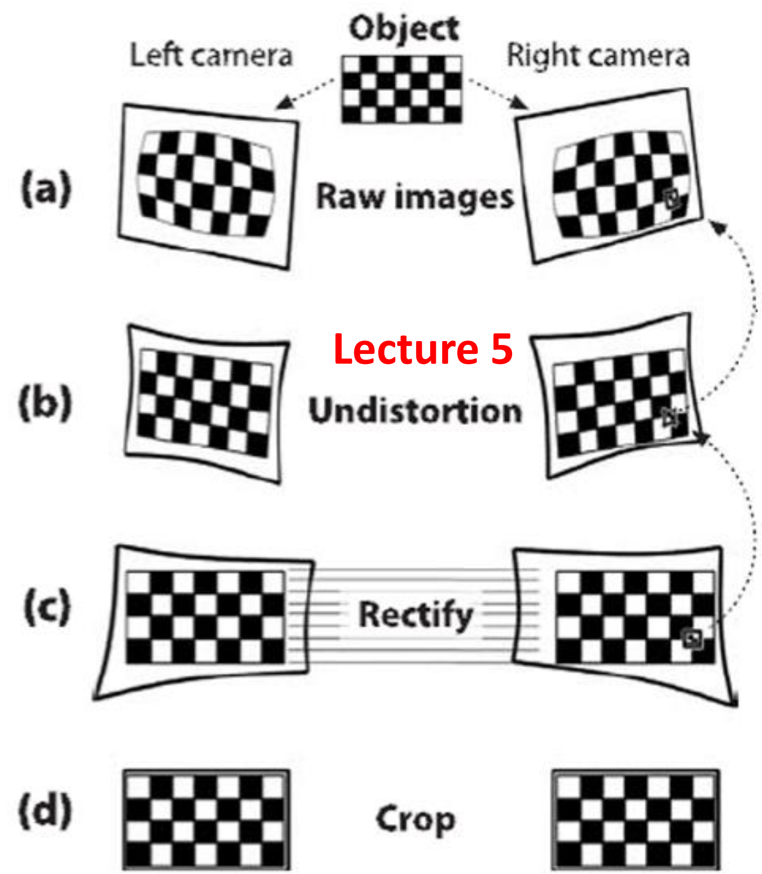
General Setup - Epipolar Geometry



Slide adapted from Kostas Daniilidis

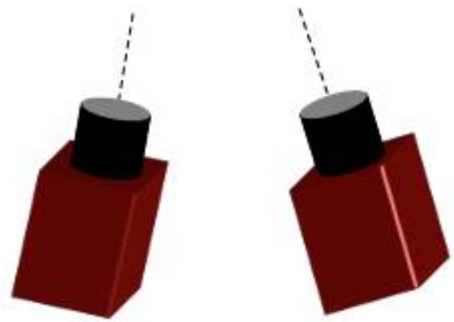
Stereo Rectification

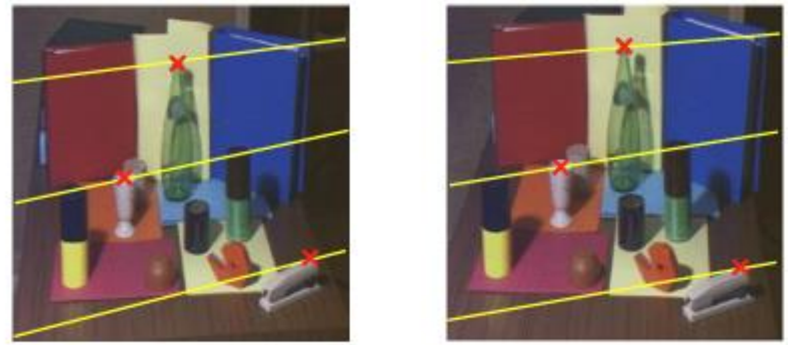




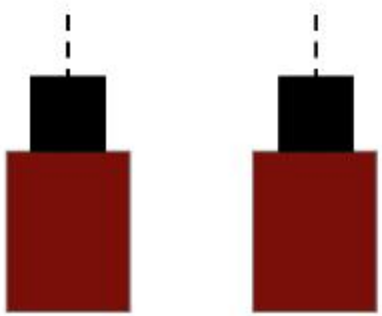
Slide adapted from Kostas Daniilidis and Luca Carlone

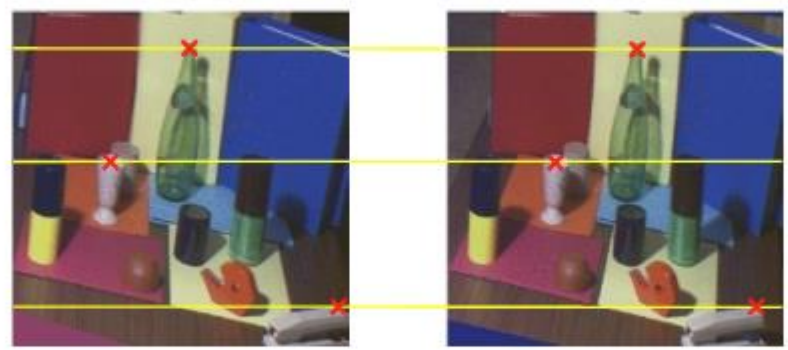
Stereo Rectification





Original stereo pair

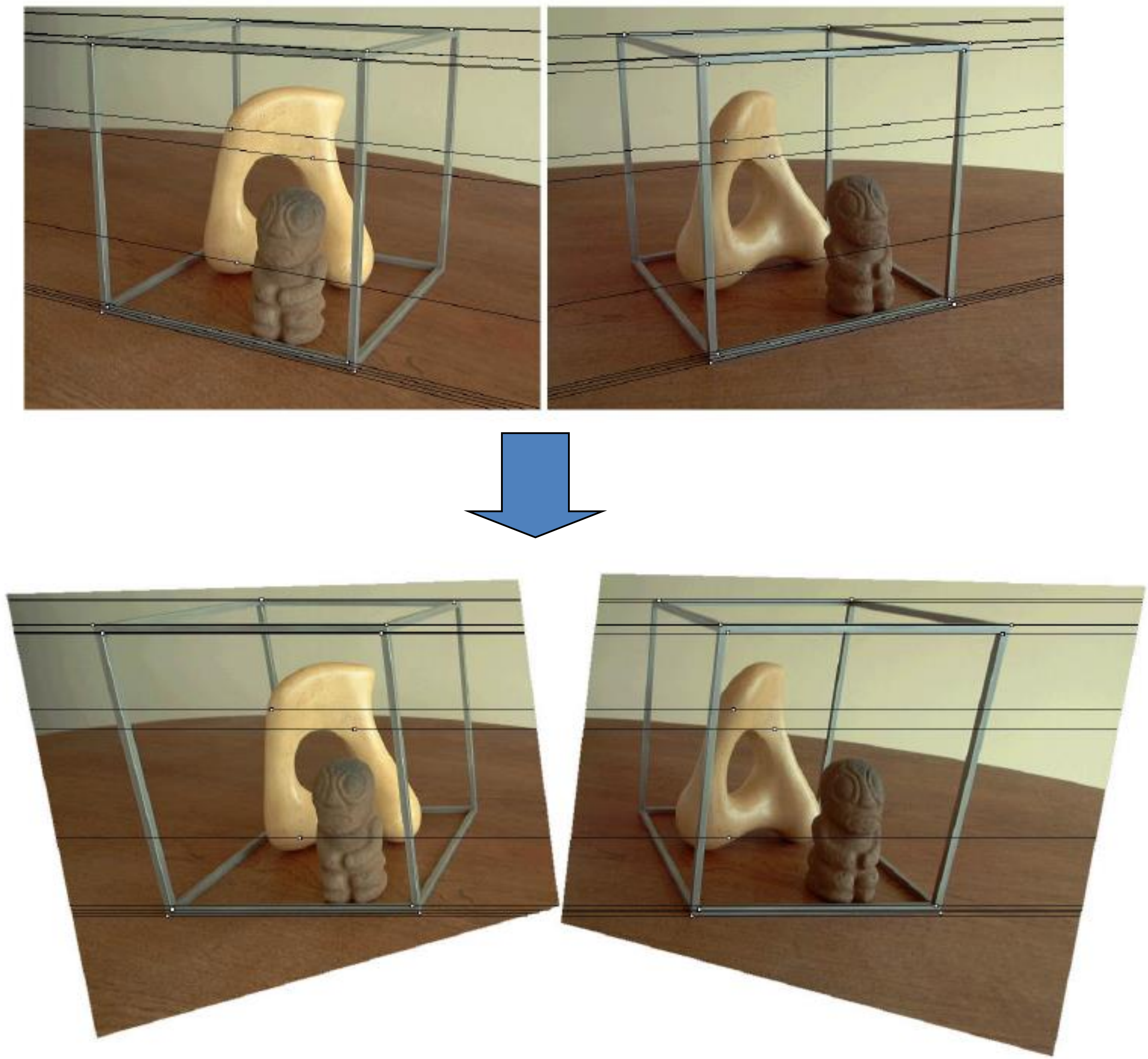




Stereo pair in standard form

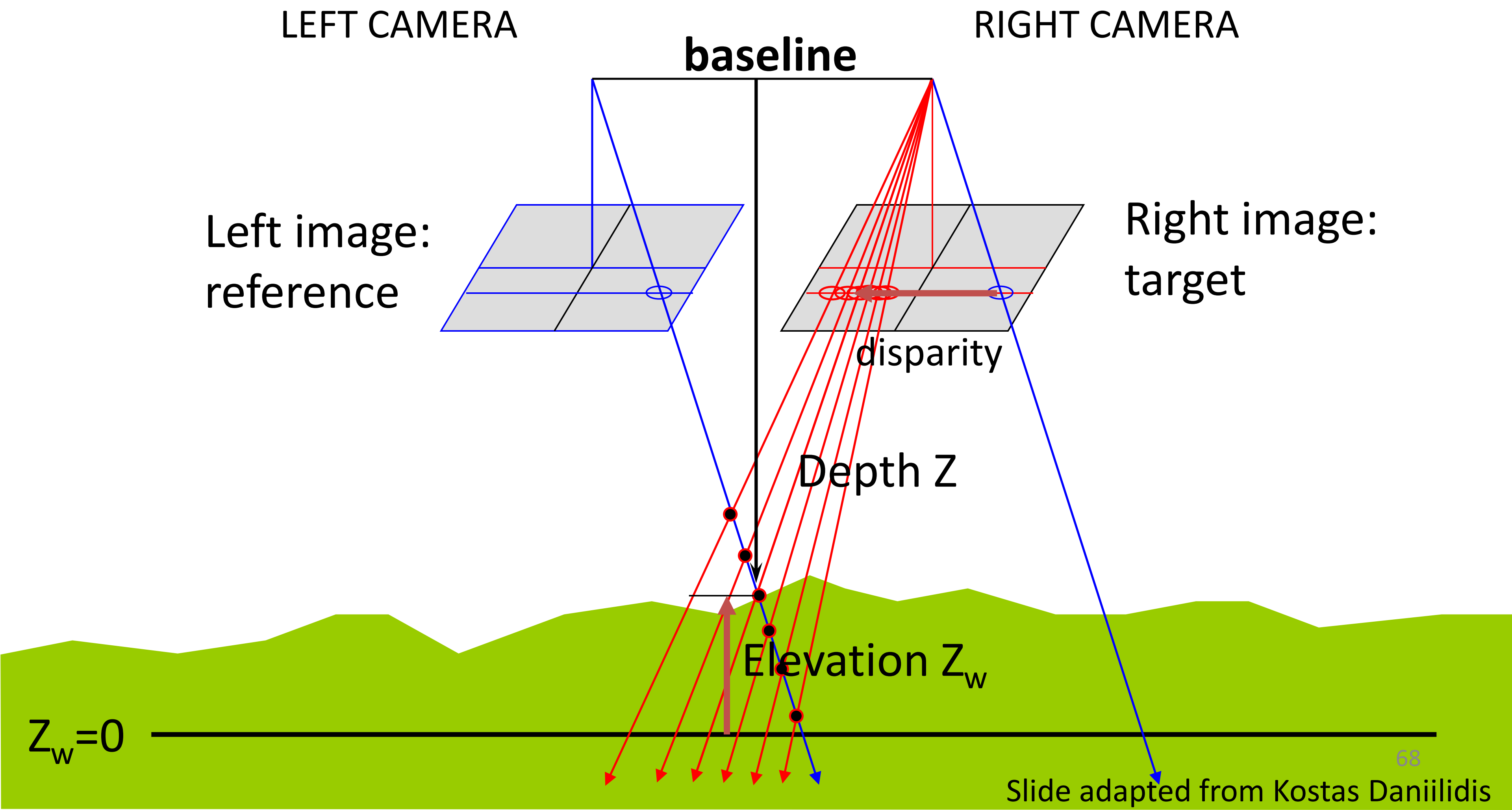
Slide adapted from Stefano Mattoccia

Stereo Rectification

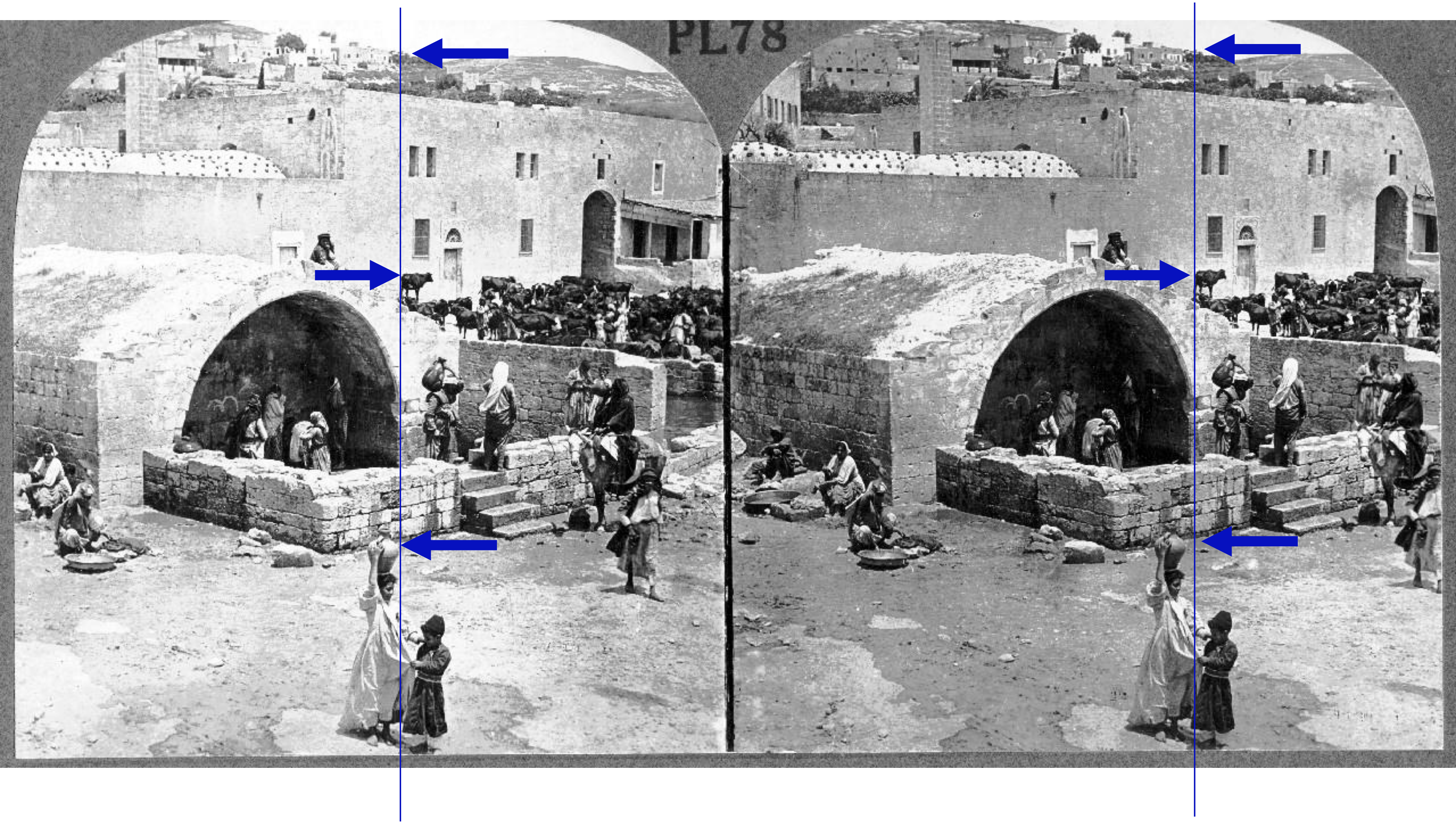


Slide adapted from Kostas Daniilidis

A Simple Stereo System

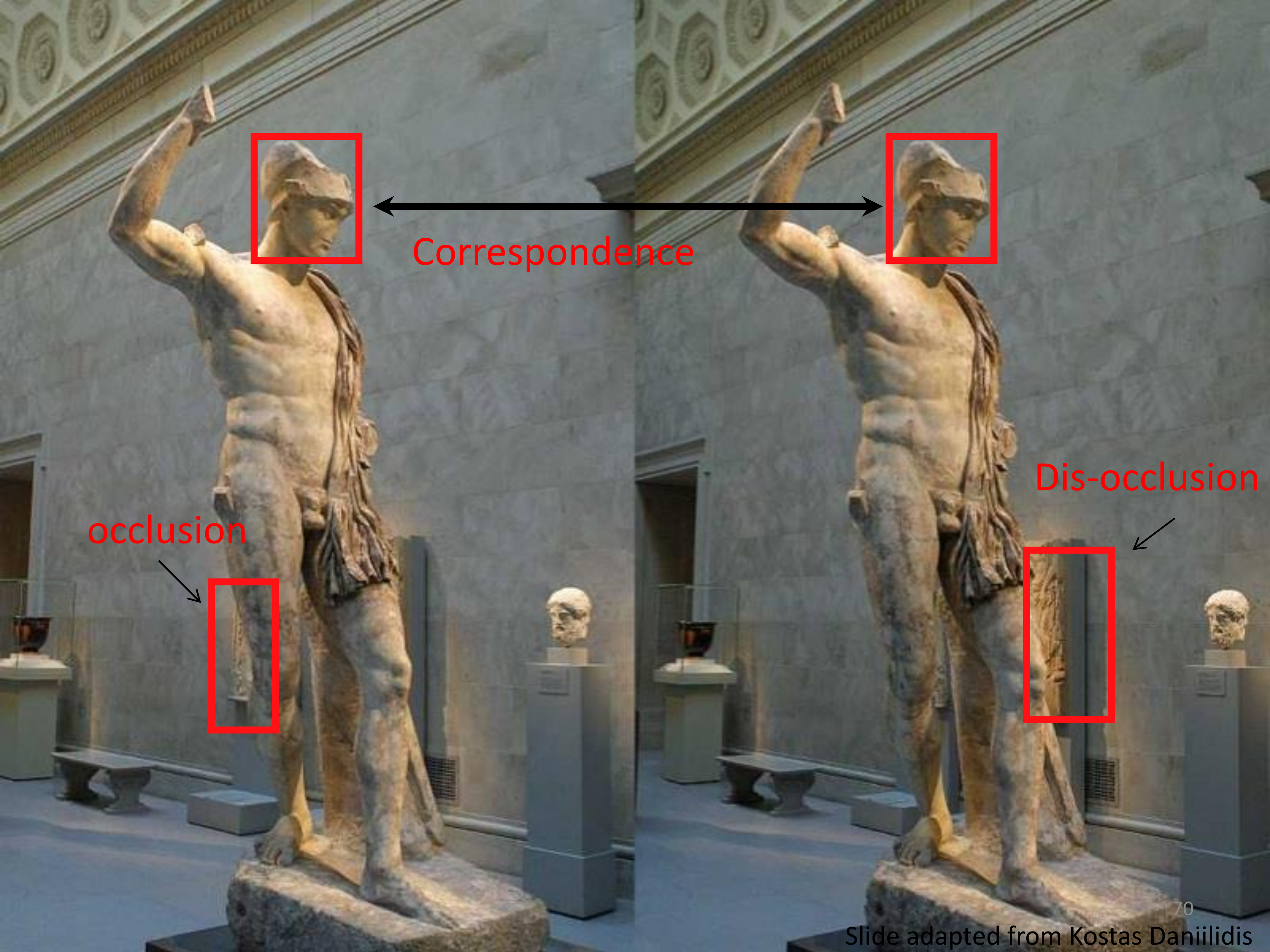


Slide adapted from Kostas Daniilidis



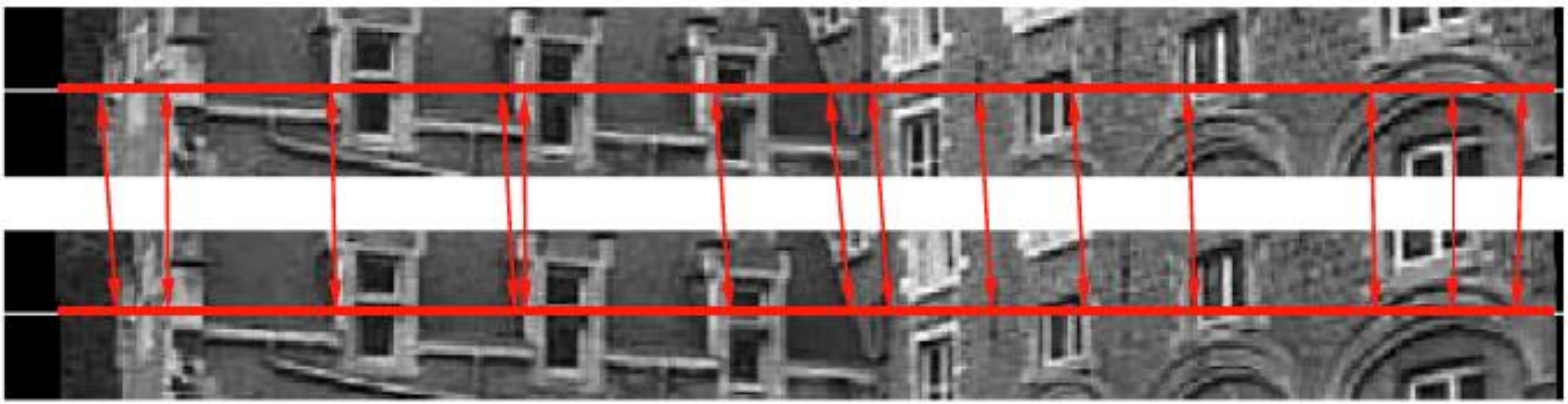
Notice the disparity difference at different distances

Slide adapted from Kostas Daniilidis



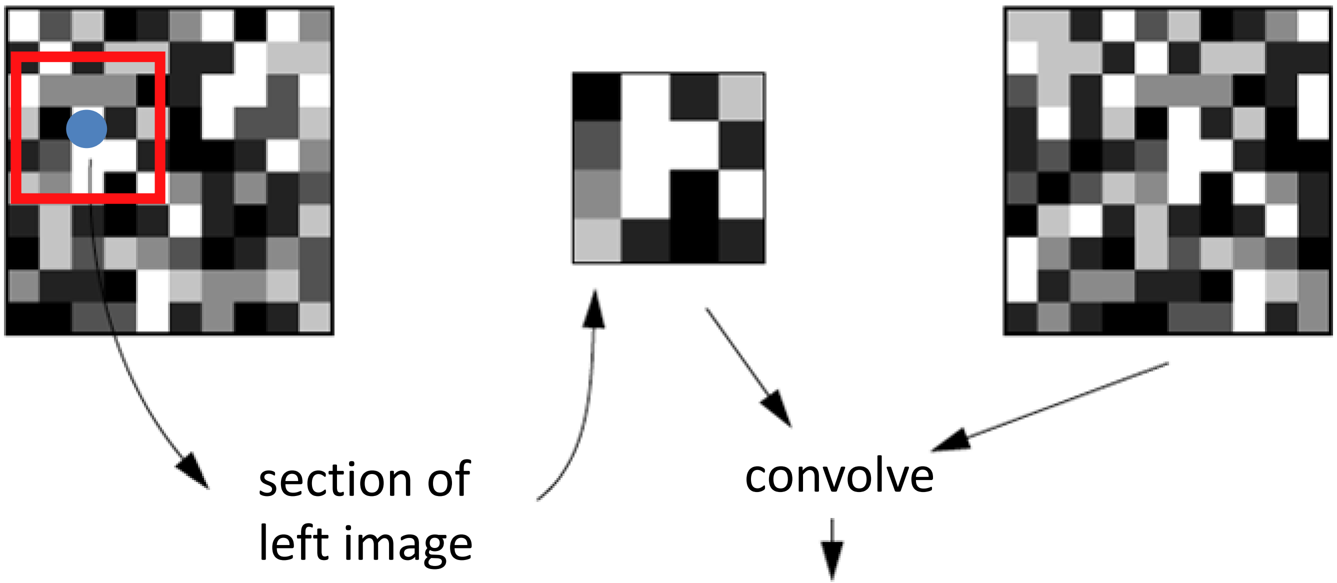
Slide adapted from Kostas Daniilidis

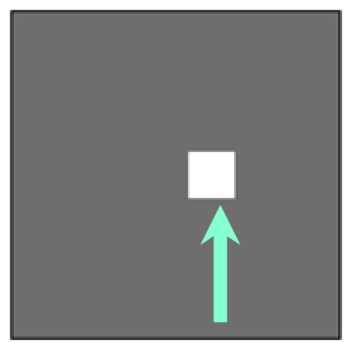
Correspondence



Slide adapted from Kostas Daniilidis

Computing Correspondence





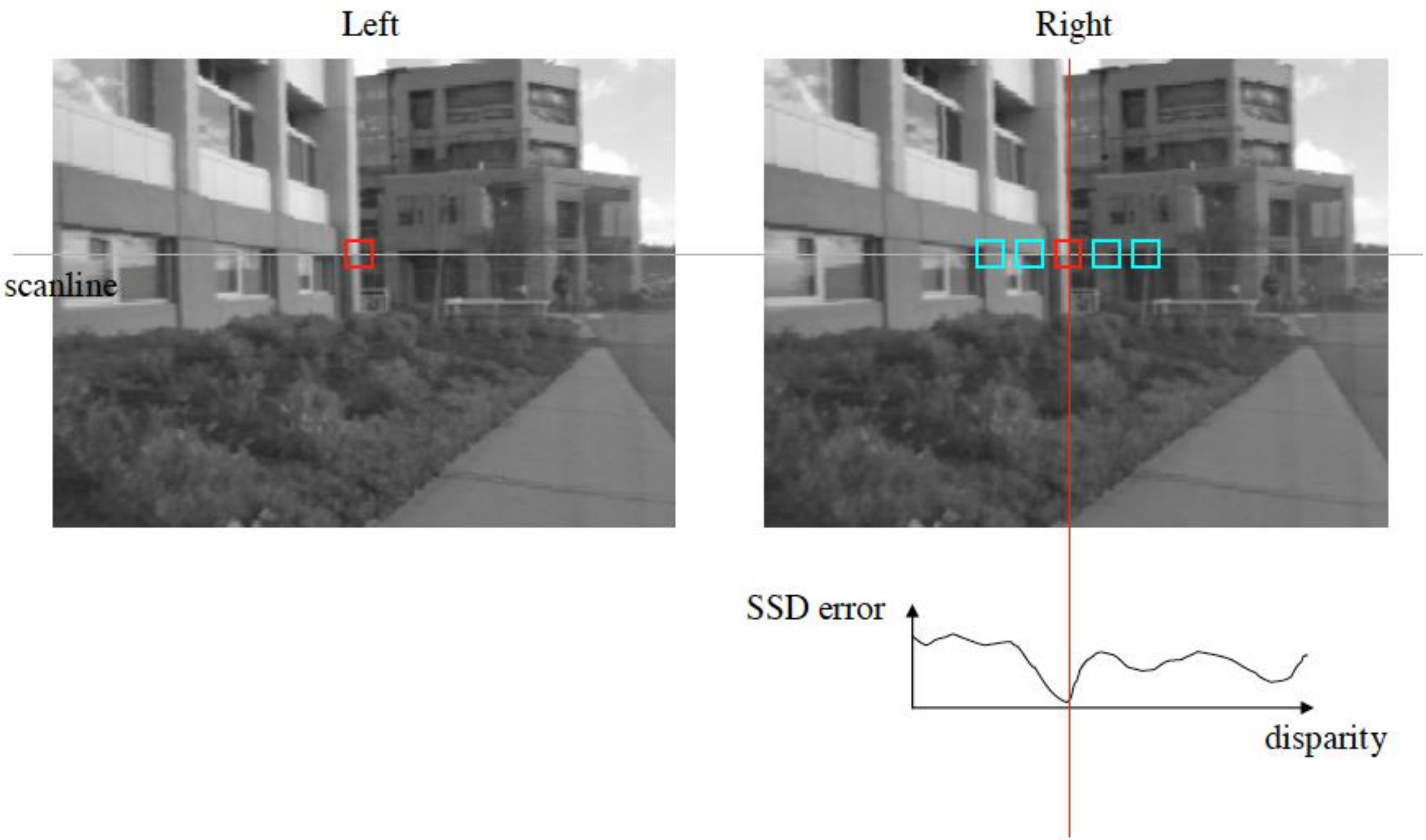
Disparity_x

Take a window (template) around a pixel in the left image, search where that template finds its best match in the right image.

Convolution peak (here schematic) at position of corresponding patch in right image

Slide adapted from Kostas Daniilidis

Correspondence Using Correlation



Slide adapted from Kostas Daniilidis

Choice of similarity function for image patches

Sum of squared differences

$$SSD(f,g) = \sum_{i,j} (f(i,j) - g(i,j))^2$$

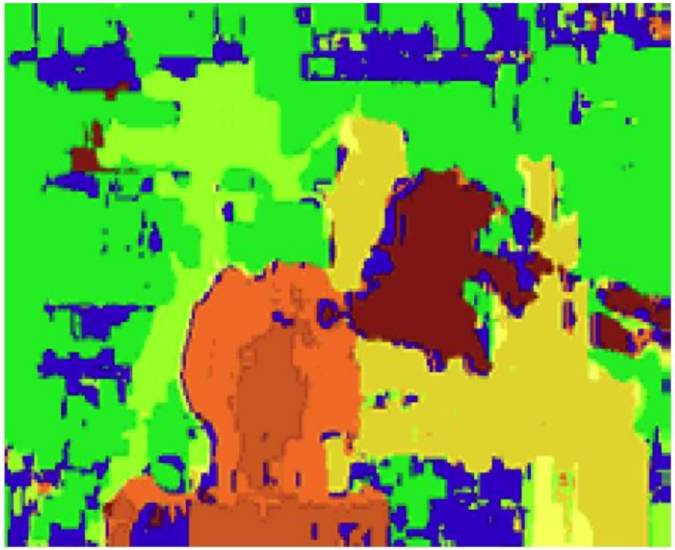
We want similarity function to be resistant to image noise, illumination changes.

Slide adapted from Kostas Daniilidis

Disparity computation using SSD

Scene

Ground truth



Slide adapted from Kostas Daniilidis

Alternative Dissimilarity Measures

- Rank and Census transforms [Zabih ECCV94]
- Rank transform:
  - Define window containing R pixels around each pixel
  - Count the number of pixels with lower intensities than center pixel in the window
  - Replace intensity with rank (0..R-1)
  - Compute SAD on rank-transformed images
- Census transform:
  - Use bit string, defined by neighbors, instead of scalar rank
- Robust against illumination changes

Slide adapted from Kostas Daniilidis

Census Measure

$$\begin{matrix} 127 & 127 & 129 \\ 126 & 128 & 129 \\ 127 & 131 & A \end{matrix} \rightarrow \begin{matrix} 1 & 1 & 0 \\ 1 & & 0 \\ 1 & 0 & a \end{matrix} \rightarrow \{1, 1, 0, 1, 0, 1, 0, a\}$$

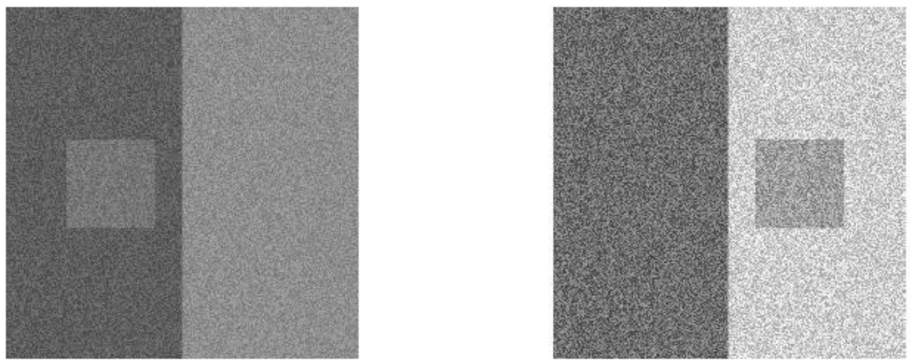


Fig. 2. Right and left random dot stereograms

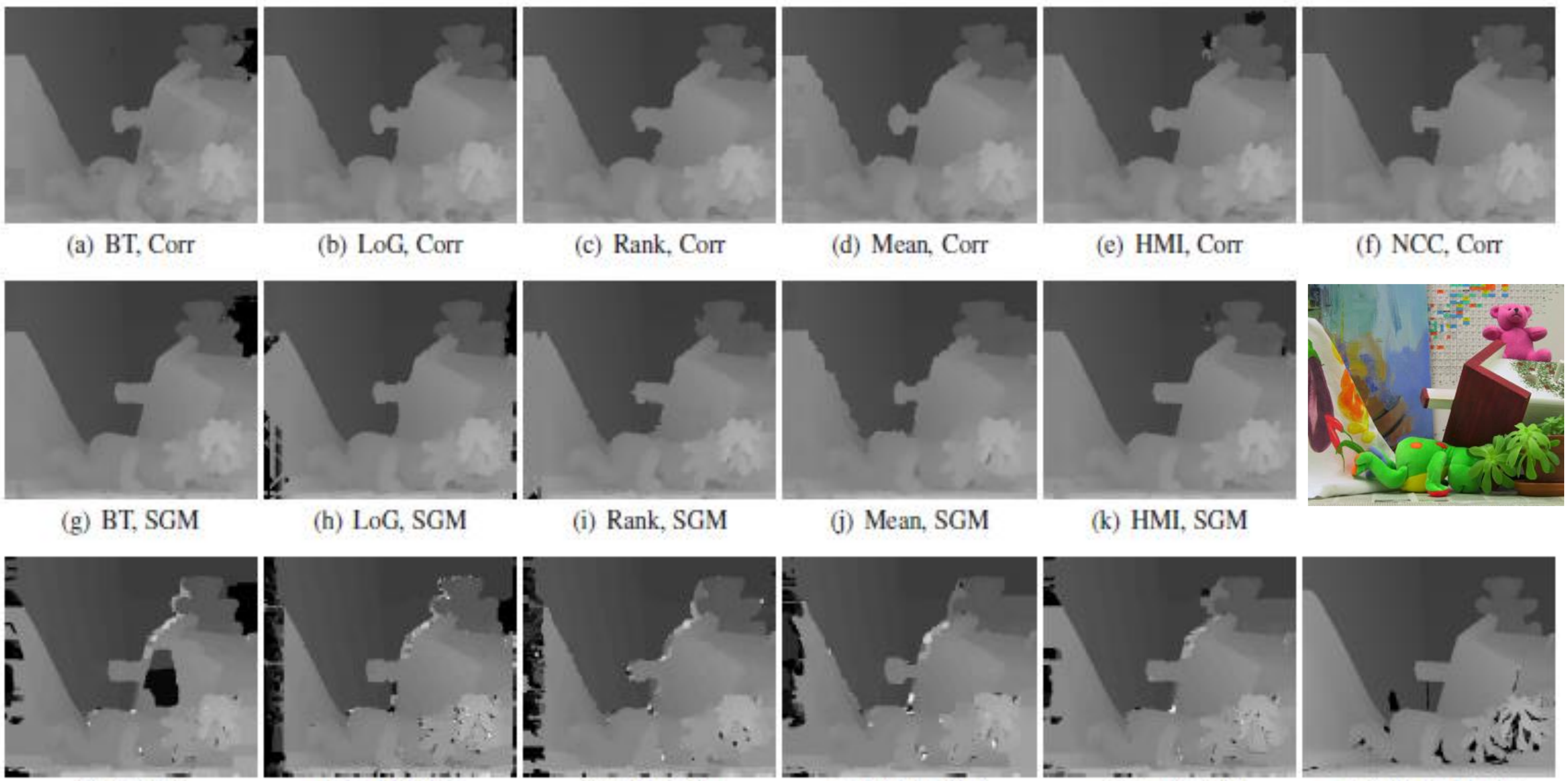
Fig. 3. Disparities from normalized correlation, rank and census transforms

Slide adapted from Kostas Daniilidis

MATCH METRIC	DEFINITION
Normalized Cross-Correlation (NCC)	$\frac{\sum_{u,v}(I_1(u,v)-\bar{I}_1)\cdot(I_2(u+d,v)-\bar{I}_2)}{\sqrt{\sum_{u,v}(I_1(u,v)-\bar{I}_1)^2\cdot\sum_{u,v}(I_2(u+d,v)-\bar{I}_2)^2}}$
Sum of Squared Differences (SSD)	$\sum_{u,v}(I_1(u,v)-I_2(u+d,v))^2$
Normalized SSD	$\sum_{u,v}\left(\frac{(I_1(u,v)-\bar{I}_1)}{\sqrt{\sum_{u,v}(I_1(u,v)-\bar{I}_1)^2}}-\frac{(I_2(u+d,v)-\bar{I}_2)}{\sqrt{\sum_{u,v}(I_2(u+d,v)-\bar{I}_2)^2}}\right)^2$
Sum of Absolute Differences (SAD) Zero Mean SAD	$\sum_{u,v}\lvert I_1(u,v)-I_2(u+d,v)\rvert$ $\sum_{u,v}\lvert(I_1(u,v)-\bar{I}_1)-(I_2(u+d,v)-\bar{I}_2)\rvert$
Rank	$I'_k(u,v)=\sum_{m,n}I_k(m,n)<I_k(u,v)$ $\sum_{u,v}(I'_1(u,v)-I'_2(u+d,v))$
Census	$I'_k(u,v)=BITSTRING_{m,n}(I_k(m,n)<I_k(u,v))$ $\sum_{u,v}HAMMING(I'_1(u,v),I'_2(u+d,v))$

Slide adapted from Kostas Daniilidis

Comparison of different similarity measures



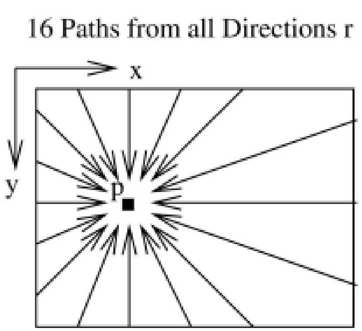
(a) BT, Corr (b) LoG, Corr (c) Rank, Corr (d) Mean, Corr (e) HMI, Corr (f) NCC, Corr

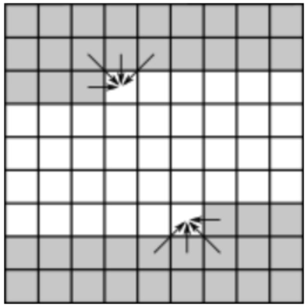
(g) BT, SGM (h) LoG, SGM (i) Rank, SGM (j) Mean, SGM (k) HMI, SGM

(l) BT, GC (m) LoG, GC (n) Rank, GC (o) Mean, GC (p) HMI, GC (q) Ground truth

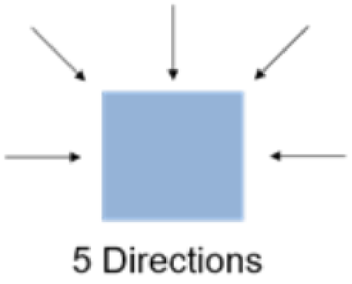
Semi-Global Matching (SGM)

- Disparity estimation approaches
  - Local: only consider neighbouring pixels
  - Global: consider information in the whole image
  - Semi-global: use information from neighboring pixels in multiple directions
- Balances between pixel similarity and disparity continuity over a sum of multiple “scanlines” from multiple directions
- 16 directions give the best quality, fewer directions can be used to achieve faster execution, e.g. 8 or 5 directions.





two-pass SGM with 8 directions



[1] Hirschmuller, H. (2007). Stereo processing by semiglobal matching and mutual information. IEEE Transactions on pattern analysis and machine intelligence, 30(2), 328-341.

Semi-Global Matching (SGM)

- The accumulated cost $S(p,d) = \sum_r L_r(p,d)$
- $L_r(p,d)$ is the cost to reach pixel p with disparity d along direction r, which is expressed recursively as

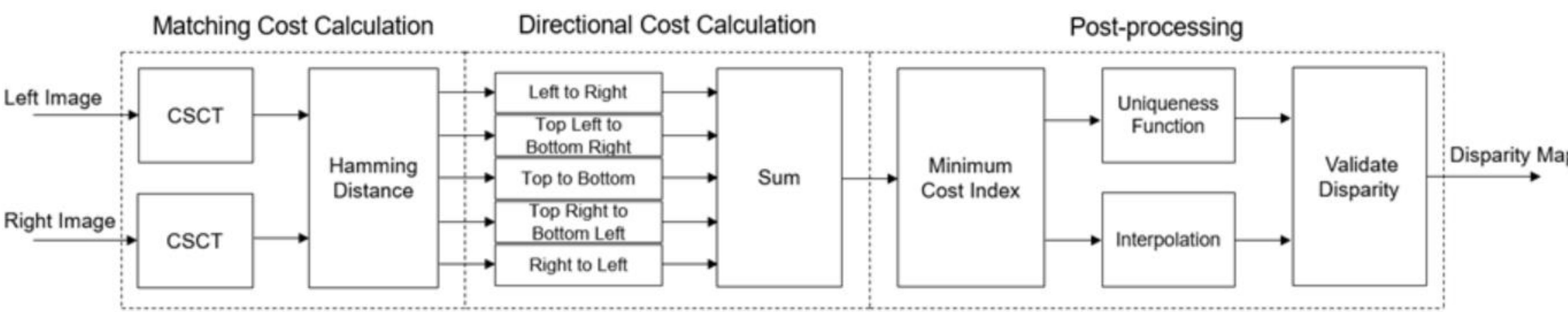
$$L_r(p,d) = D(p,d) + \min\left\{L_r(p-r,d), L_r(p-r,d-1)+P_1, L_r(p-r,d+1)+P_1, \min_i L_r(p-r,i)+P_2\right\} - \min_k L_r(p-r,k)$$

Dissimilarity measure

Regularisation term penalizes jumps in disparity between adjacent pixels. P_1 and P_2 are two constant parameters with $P_1 < P_2$

For numerical stability

- The value of disparity at each pixel is given by $d^*(p) = \mathrm{argmin}_d\, S(p,d)$
- An example of implementation[1]



* CSCT: Center-Symmetric Census Transform

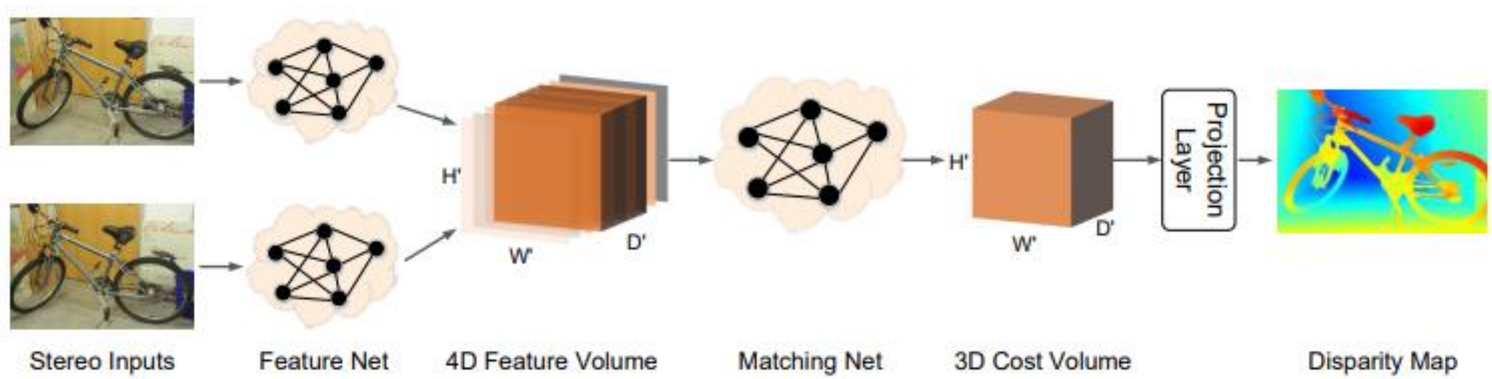
[1] https://www.mathworks.com/help//visionhdl/ug/stereoscopic-disparity.html

Learning-based Stereo Depth Estimation

- Pros: better handling against scene ambiguity, better smoothness
- Cons: same as all learning-based methods: the generalization issue

- Direct regression Methods
  - Fully data-driven, directly get the per-pixel disparity without considering the geometric constraints

- Volumetric methods
  - Build a 4D feature volume, often consists of feature net and matching net

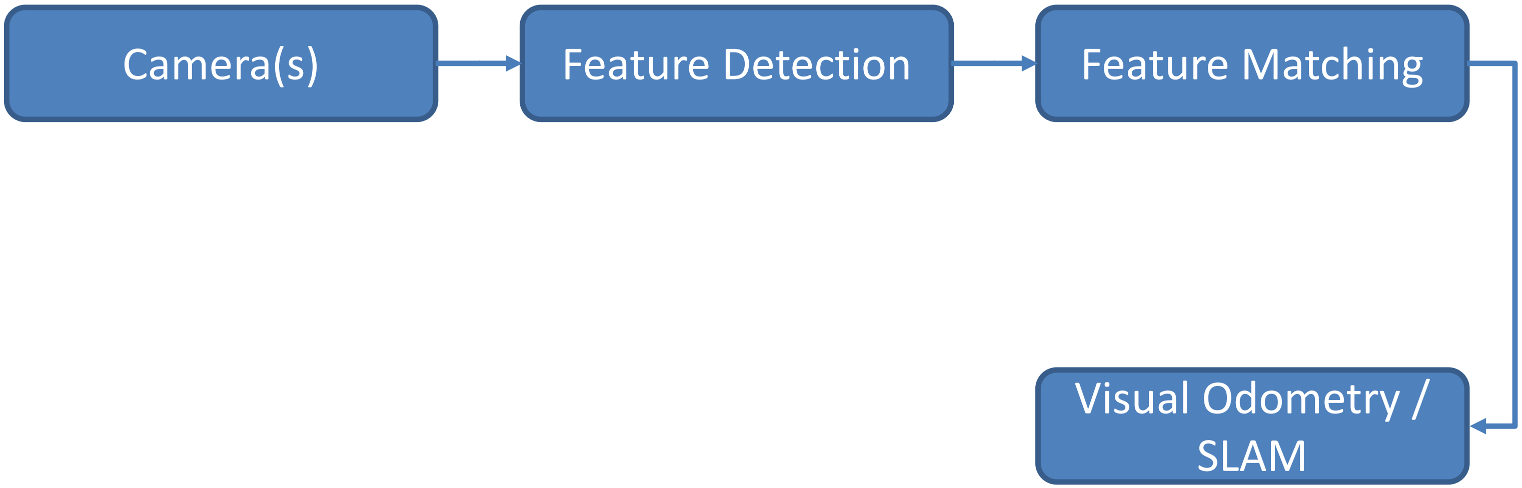


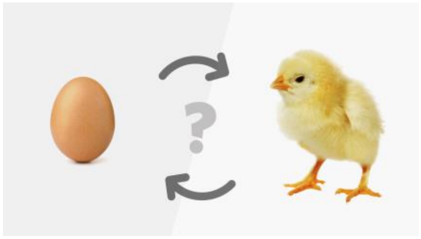
[1] X. Cheng et al., "Hierarchical Neural Architecture Search for Deep Stereo Matching", Advances in Neural Information Processing Systems, vol 33, 2020.

Depth from Stereo Vision

Visual Odometry

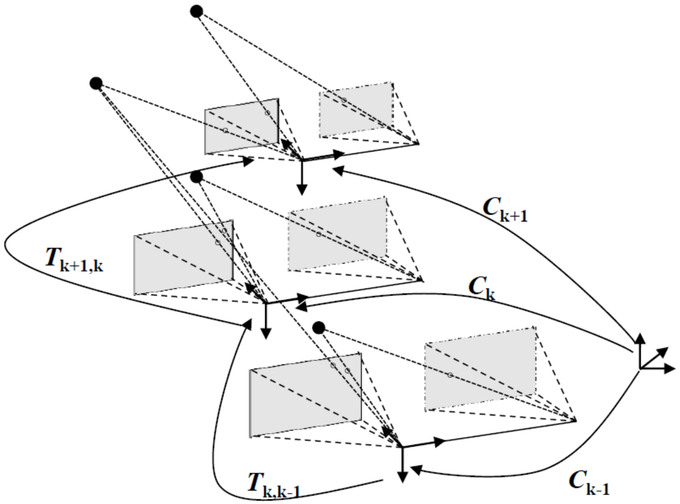
Vision-based Incremental Pose Estimation Pipeline (aka. Visual Odometry)





Visual Odometry

- Visual odometry is the process of real-time estimation of incremental motion of the camera (sensor suite) using only sequential images as input
- Analogy to odometer on cars

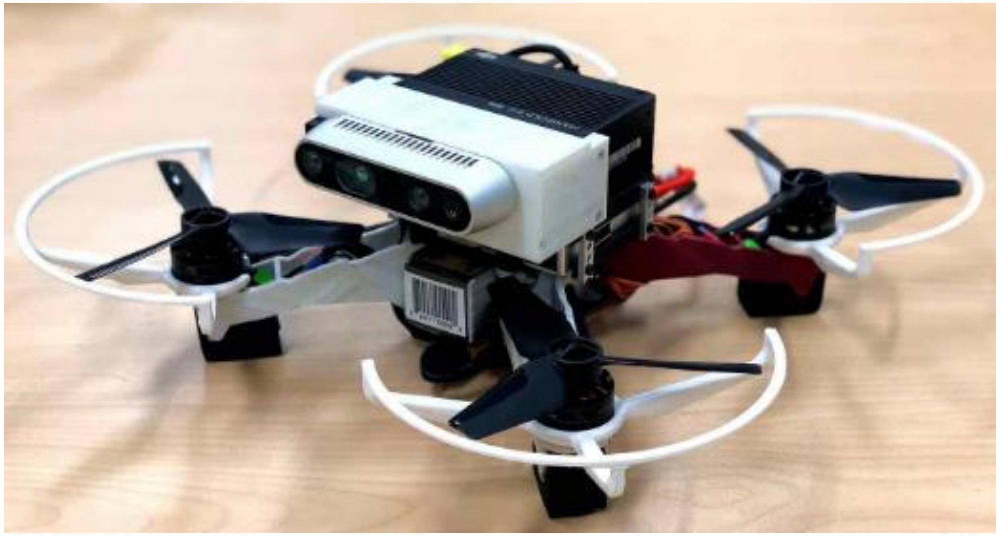


Visual Odometry v.s. Map-based Localization

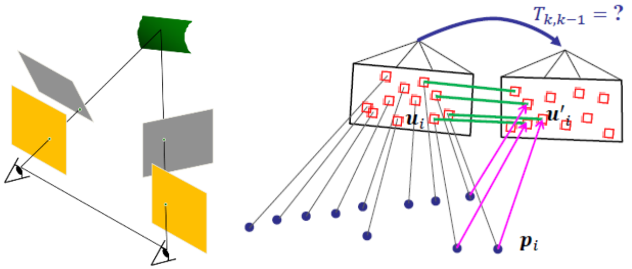
- VO Setup
  - Applicable to different camera configurations (monocular, stereo, etc.)
  - Sufficient illumination and texture
  - Dominance of static scene
  - Unknown environment
  - Sufficient overlapping between consecutive frames
  - Focus on local consistency
- Localization Setup
  - Applicable to different camera configurations (monocular, stereo, etc.)
  - Sufficient illumination and texture
  - Dominance of static scene
  - Known map
  - Sufficient observation of map features
  - Focus on global consistency

Stereo Visual Odometry

- Setup
  - Known stereo intrinsic and extrinsic calibration
  - Rectified stereo image pairs
  - Set starting point of the dataset as the origin
  - Estimate camera movement with respect to the origin



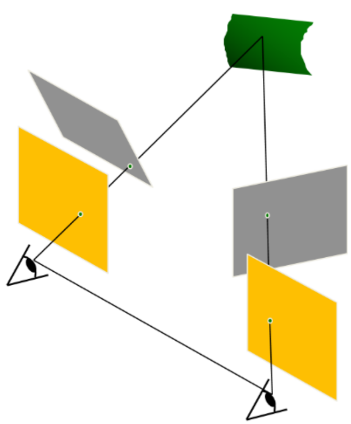
Stereo Visual Odometry

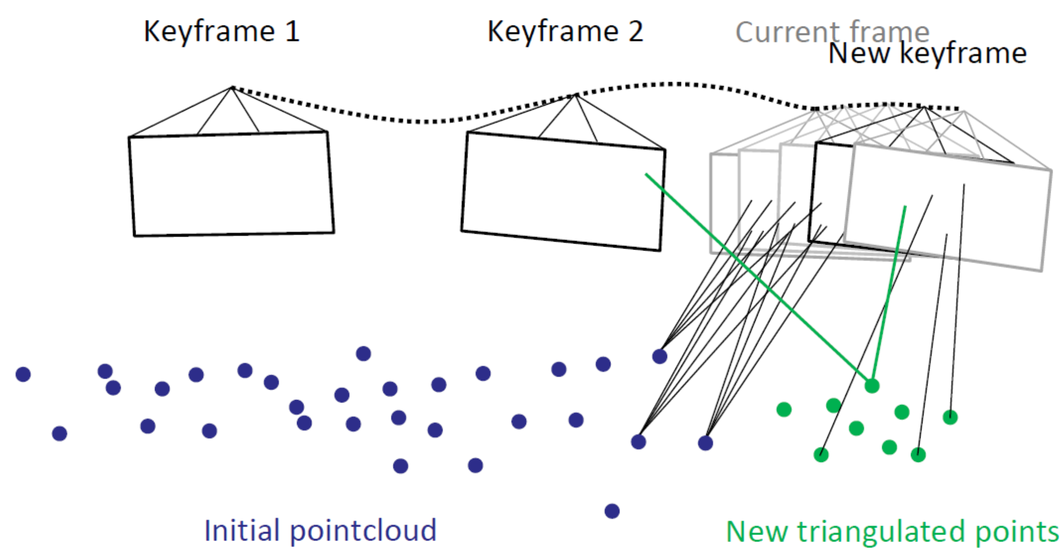


- Frame-to-frame stereo visual odometry
  1. Input Frame1 (two images)
  2. Detect 2D features in Frame1
  3. Compute depth of each 2D feature in Frame1 using feature matching or optical flow between left and right images
  4. Input Frame2 (two images)
  5. Detect 2D features in Frame2 and match with features in Frame 1, or use optical flow-based methods to track features from Frame 1 to Frame 2
  6. Compute the incremental pose displacement between Frame1 and Frame2 using 2D-3D pose estimation
  7. Accumulate incremental pose displacement
  8. Goto Step 1
- Question: Can we reuse feature detection results?
- Question: How to set initial values?
- Question: What are the disadvantages of frame-to-frame setup?

Stereo Visual Odometry

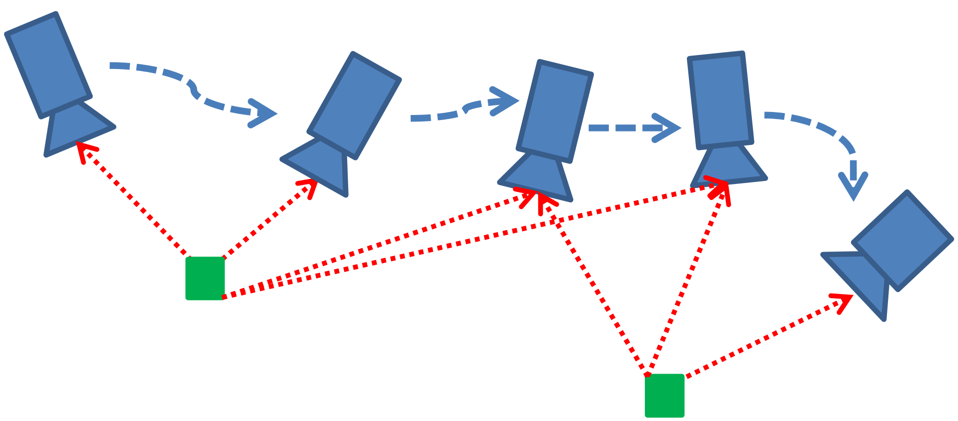
- Keyframe-based stereo visual odometry
  - No pose drift when there is no keyframe change
  - Only initiate new keyframe when:
    - Displacement between the current frame and the latest keyframe is large
    - Number of features between the current frame and the latest keyframe is insufficient
  - Question: Can we do even better?



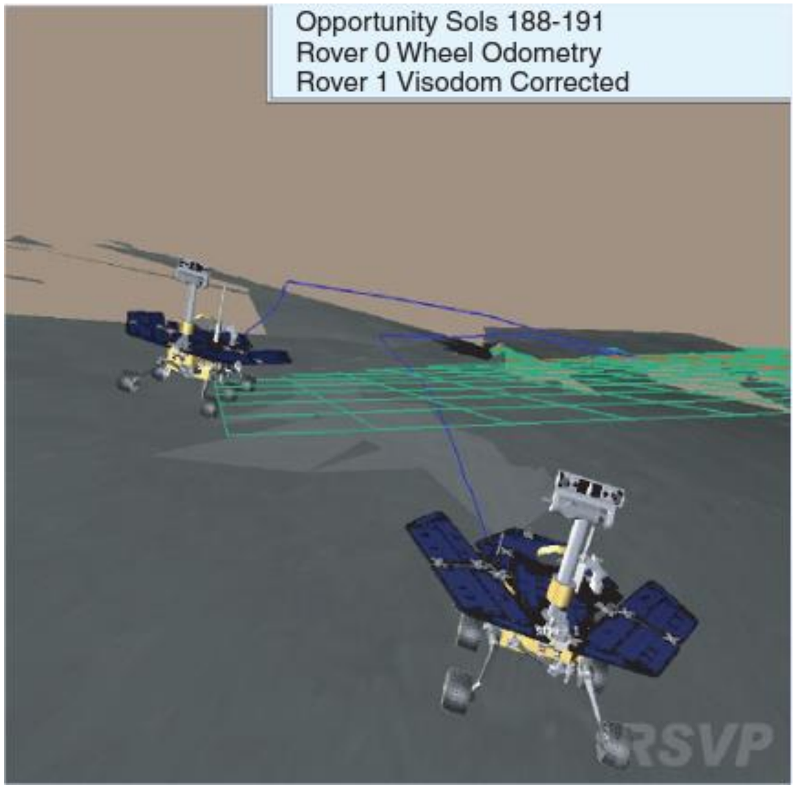


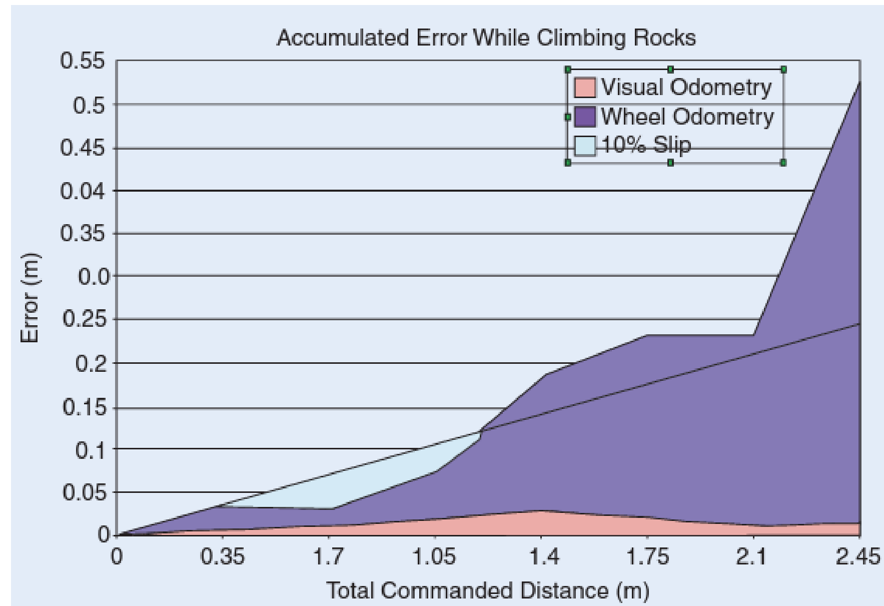
More on Visual Odometry

- Sliding window visual odometry
- Sliding window visual-inertial odometry
- Full visual SLAM
- Full visual-inertial SLAM
- …
- To be covered in Lecture 10



Visual Odometry on Mars





Cheng, Yang, Mark W. Maimone, and Larry Matthies. "Visual odometry on the Mars exploration rovers-a tool to ensure accurate driving and science imaging." *IEEE Robotics & Automation Magazine* 13.2 (2006): 54-62.

Build your own stereo VO

Logistics

- The deadline for Project 1, phase 4 is 03/31
- The deadline for Project 2, phase 1 is 03/31
- Project 2, phase 2 is released (03/28)
  – finish it by 04/07