SOSC 5340 Tutorial Four

Instrumental Variable, and Regression Discontinuity

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Set working directory to the current directory

Remark: Need to save current R file before using getActiveDocumentContext

R Packages

R packages for IV strategy:

- ivreg: https://cran.r-project.org/web/packages/ivreg/index.html
- *lfe*: https://cran.r-project.org/web/packages/lfe/index.html
 - linear models with multiple group fixed effects
 - deals with many levels of "fixed effect"
 - allows for multi-way clustering s.e.
 - can implement IV estimation
- Read the reference manual and vignettes.

R packages for RD strategy:

- $\bullet \ \textit{rdrobust} \colon \texttt{https://cran.r-project.org/web/packages/rdrobust/index.html} \\$
- rddensity: https://cran.r-project.org/web/packages/rddensity/index.html
- Read the reference manual and vignettes.
- Other packages for RD: https://rdpackages.github.io/

Instrumental Variable

Attaching package: 'zoo'

library packages

We will use ivreg, lfe, and plm package to fit instrumental-variable regression by two-stage least squares (2SLS).

Let's use the demand for cigarettes as an example, you can find it in @StockWatson2007 (Chapter 12).

```
## Loading required package: car
## Loading required package: carData
## Loading required package: lmtest
## Loading required package: zoo
##
```

```
## The following objects are masked from 'package:base':
##
       as.Date, as.Date.numeric
##
## Loading required package: sandwich
## Loading required package: survival
library(ivreg)
## Registered S3 methods overwritten by 'ivreg':
##
    method
                         from
##
     anova.ivreg
                         AF.R.
##
    hatvalues.ivreg
                         AER
##
    model.matrix.ivreg AER
##
    predict.ivreg
                         AER
##
                         AER
    print.ivreg
##
    print.summary.ivreg AER
##
     summary.ivreg
                         AER
##
                         AER
     terms.ivreg
##
                         AER
     update.ivreg
##
     vcov.ivreg
                         AER
##
## Attaching package: 'ivreg'
## The following objects are masked from 'package:AER':
##
##
       ivreg, ivreg.fit
# data processing and transformation
data("CigarettesSW", package = "AER")
?CigarettesSW ## type '?CigarettesSW' to see the introduction of the dataset
## packs: Number of packs per capita
## price: Average price during fiscal year, including sales tax;
## cpi: Consumer price index;
## income: State personal income (total, nominal);
## population: State population;
## tax: Average state, federal and average local excise taxes for fiscal year;
## taxs: Average excise taxes for fiscal year, including sales tax.
CigarettesSW$rprice <- with(CigarettesSW, price/cpi) # real average price</pre>
CigarettesSW$rincome <- with(CigarettesSW, income/population/cpi) # real personal income
CigarettesSW$rtax <- with(CigarettesSW, tax/cpi) # real local excise taxes
CigarettesSW$rtdiff <- with(CigarettesSW, (taxs - tax)/cpi) # diff in real local taxes and real taxes
# Estimation
## OLS estimator
ols <- lm(log(packs) ~ log(rprice) + log(rincome),</pre>
          data = CigarettesSW, subset = year == 1995)
ols_se <- coeftest(ols, vcov = vcovHC, type = "HC1")</pre>
## Equation 12.15
ivreg_12.15 <- ivreg(log(packs) ~ log(rprice) + log(rincome) | ## 2nd stage</pre>
                       log(rincome) + rtdiff, ## 1st stage: rtdiff as IV of rprice
                     data = CigarettesSW,
```

```
subset = year == 1995)
ivreg_12.15_se <- coeftest(ivreg_12.15, vcov = vcovHC, type = "HC1")
## Equation 12.16
ivreg_12.16 <- ivreg(log(packs) ~ log(rprice) + log(rincome) |</pre>
                     log(rincome) + rtdiff + rtax, ## 1st stage: rtdiff + rtax as IV of rprice
                   data = CigarettesSW,
                   subset = year == 1995)
ivreg_12.16_se <- coeftest(ivreg_12.16, vcov = vcovHC, type = "HC1")</pre>
## Show the results
library(texreg)
## Version: 1.37.5
## Date:
           2020-06-17
## Author:
          Philip Leifeld (University of Essex)
## Consider submitting praise using the praise or praise_interactive functions.
## Please cite the JSS article in your publications -- see citation("texreg").
screenreg(list(ols, ivreg_12.15, ivreg_12.16),
         custom.model.names = c('OLS',"IV_rtdiff", "IV_rtdiff+rtax"),
         custom.coef.names = c("Constant", "log price", "log income per capita"),
         override.se = list(ols_se[,2], ivreg_12.15_se[,2], ivreg_12.16_se[,2]),
         override.pvalues = list(ols_se[,4], ivreg_12.15_se[,4], ivreg_12.16_se[,4]),
         \#stars = c(0.1, 0.05, 0.01),
         digits = 4)
##
  ______
                                    IV_rtdiff
##
                        OLS
                                                IV rtdiff+rtax
                       10.3420 *** 9.4307 *** 9.8950 ***
## Constant
##
                        (0.9665)
                                    (1.2594)
                                                (0.9592)
## log price
                        -1.4065 *** -1.1434 **
                                                -1.2774 ***
##
                        (0.2609)
                                    (0.3723)
                                                (0.2496)
## log income per capita 0.3439
                                    0.2145
                                                0.2804
                                                (0.2539)
                        (0.2604)
                                    (0.3117)
## --
## R^2
                         0.4327
                                    0.4189
                                                0.4294
## Adj. R^2
                        0.4075
                                    0.3931
                                                 0.4041
## Num. obs.
                        48
## *** p < 0.001; ** p < 0.01; * p < 0.05
# Note: statistical significance level!
```

Diagnostics: Weak IV and Over-identification test

A good instrumental variable is highly correlated with one or more of the explanatory variables while remaining uncorrelated with the errors.

If an endogenous regressor is only weakly related to the instrumental variables, then its coefficient will be estimated imprecisely. We hope for a large test statistic and small p-value in the diagnostic test for **weak** instruments (Weak instruments: \mathbf{F} -stat > 10).

Applied to 2SLS regression, the **Wu–Hausman test** is a test of **endogenity**. If all of the regressors are exogenous (a small test statistics and large p-value), then both the OLS and 2SLS estimators are consistent, and the OLS estimator is more efficient. But if one or more regressors are endogenous (a large test statistic and small p-value), then 2SLS estimator may be better than the OLS estimator.

The **Sargan test** is a test of **overidentification**. When there are more instrumental variables than coefficients to estimate, it's possible that the instrumental variables provide conflicting information about the values of the coefficients. A large test statistic and small p-value for the Sargan test suggest, therefore, that the model is misspecified. The Sargan test is **inapplicable** to a just-identified regression equation, with an equal number of instrumental variables and coefficients.

```
summary(ivreg_12.15, vcov = vcovHC, diagnostics = TRUE)
##
##
  Call:
   ivreg(formula = log(packs) ~ log(rprice) + log(rincome) | log(rincome) +
##
##
       rtdiff, data = CigarettesSW, subset = year == 1995)
##
##
  Residuals:
##
         Min
                    1Q
                           Median
                                         3Q
                                                  Max
   -0.611000 -0.086072
                        0.009423
                                  0.106912
                                             0.393159
##
##
  Coefficients:
##
                Estimate Std. Error t value Pr(>|t|)
                  9.4307
                              1.3538
                                       6.966 1.15e-08 ***
## (Intercept)
## log(rprice)
                 -1.1434
                              0.4032
                                      -2.836
                                             0.00682 **
                              0.3319
                                       0.646 0.52137
## log(rincome)
                  0.2145
##
## Diagnostic tests:
##
                    df1 df2 statistic
                                       p-value
## Weak instruments
                      1
                         45
                                 40.03 1.02e-07 ***
## Wu-Hausman
                         44
                                  0.96
                                          0.332
                      1
## Sargan
                      0
                         NA
                                    NA
                                             NA
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1896 on 45 degrees of freedom
## Multiple R-Squared: 0.4189, Adjusted R-squared: 0.3931
## Wald test: 7.262 on 2 and 45 DF, p-value: 0.001848
summary(ivreg_12.16, vcov = vcovHC, diagnostics = TRUE)
##
## Call:
  ivreg(formula = log(packs) ~ log(rprice) + log(rincome) | log(rincome) +
##
       rtdiff + rtax, data = CigarettesSW, subset = year == 1995)
##
##
## Residuals:
##
                      1Q
                             Median
                                                        Max
                                             30
   -0.6006931 -0.0862222 -0.0009999
                                     0.1164699
                                                 0.3734227
##
  Coefficients:
##
##
                Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                  9.8950
                              1.0313
                                       9.595 1.87e-12 ***
## log(rprice)
                                      -4.750 2.10e-05 ***
                 -1.2774
                              0.2689
## log(rincome)
                  0.2804
                              0.2640
                                                0.294
                                       1.062
```

```
##
## Diagnostic tests:
                   df1 df2 statistic p-value
                    2 44
                            167.988 <2e-16 ***
## Weak instruments
## Wu-Hausman
                     1 44
                              3.052 0.0876 .
                     1 NA
                              0.333 0.5641
## Sargan
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 0.1879 on 45 degrees of freedom
## Multiple R-Squared: 0.4294, Adjusted R-squared: 0.4041
## Wald test: 13.88 on 2 and 45 DF, p-value: 2.019e-05
```

Panel IV

```
Now, let's use lfe and plm package to fit a panel IV regression.
library(lfe)
## Loading required package: Matrix
##
## Attaching package: 'lfe'
## The following object is masked from 'package:lmtest':
##
##
       waldtest
library(plm)
##
## Attaching package: 'plm'
## The following object is masked from 'package:lfe':
##
##
       sargan
felm_12.16 <- felm(log(packs) ~ log(rincome) |</pre>
                      0 | # Fixed Effects
                      (log(rprice) ~ rtdiff + rtax) | # Instruments
                      0,
                    data = CigarettesSW,
                    subset = year == 1995)
## Stock and Waston 2007: Table 12.1 Column 1
felm_panel <- felm(log(packs) ~ log(rincome) |</pre>
                      year | # Fixed Effects
                      (log(rprice) ~ rtdiff + rtax) | # Instruments
                      0,
                    data = CigarettesSW)
## try `plm`, firstly transform our original data to panel data
CigarettesSW_panel <- pdata.frame(CigarettesSW, index = c("state", "year"), drop.index = TRUE)</pre>
## Stock and Waston 2007: Table 12.1 Column 1
plm_within <- plm(log(packs) ~ log(rprice) + log(rincome) |</pre>
                    log(rincome) + rtdiff + rtax, ## IV
```

##					
##		ivreg 1995	felm 1995		
## ##	(Intercept)		9.8950 ***		
##		(0.9592)	(1.0586)		
## ##	log(rprice)	-1.2774 *** (0.2496)			-1.2675 *** (0.1604)
## ##	log(rincome)		0.2804 (0.2386)		
##	`log(rprice)(fit)`		-1.2774 *** (0.2632)	-1.1996 ***	(,
	R^2	0.4294			0.8994
##	Adj. R^2	0.4041			0.7922
##	Num. obs.	48	48	96	96
##	R^2 (full model)		0.4294	0.5495	
	R^2 (proj model)		0.4294	0.3837	
##	Adj. R^2 (full model)		0.4041	0.5348	
##	Adj. R^2 (proj model)		0.4041	0.3636	
	Num. groups: year	=========	=========	2	:=======
	*** p < 0.001; ** p <				

Remark: How to argue the exogeneity of instruments?

If you suspect that IV impacts on Y through A other than the endogenous variable, then regress A on the IV: will be fine if the result is not significant!

Regression Discontinuity

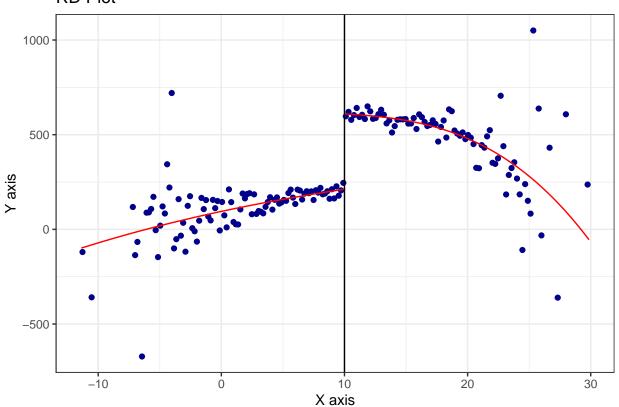
We will use rdrobust and rddensity to deal with RD related analysis.

```
## library packages and load data
library(rdrobust)

## simulated the data
set.seed(3333)
```

```
s = 10 + 5*qnorm(runif(10000)) # running variable
x = s - 10 \# covariate
w = ifelse(s>10, 1, 0) # treatment: threshold = 10
y1 = 600 + 6.5*x - 2*x^2 + 0.001*x^3 + 300*qnorm(runif(10000)) # treated outcome
y0 = 200 + 6.5*x - 0.20*x^2 + 0.01*x^3 + 300*qnorm(runif(10000)) # control outcome
y = y0 + w*(y1-y0) # Rubin Causal Model
rd_data <- data.frame(s, x, w, y1, y0, y)
head(rd_data)
                                       уO
                     x w
                               у1
## 1 13.144164 3.1441638 1 726.3628 476.6828 726.3628
## 2 9.211231 -0.7887692 0 607.6854 616.9262 616.9262
## 3 7.099556 -2.9004436 0 423.9116 455.4765 455.4765
## 4 14.099379 4.0993793 1 450.1038 346.2623 450.1038
## 6 5.619238 -4.3807620 0 245.9212 -15.3554 -15.3554
summary(rd_data)
                                                            у1
##
                         x
                    Min. :-21.367491
                                             :0.0000
                                                            :-605.7
## Min. :-11.367
                                       Min.
                                                      Min.
                   1st Qu.: -3.370934
                                      1st Qu.:0.0000 1st Qu.: 345.1
## 1st Qu.: 6.629
## Median : 9.991 Median : -0.009284
                                       Median: 0.0000 Median: 555.0
## Mean
        : 9.979 Mean : -0.020945
                                       Mean :0.4991 Mean : 550.9
## 3rd Qu.: 13.353 3rd Qu.: 3.352615
                                       3rd Qu.:1.0000 3rd Qu.: 760.1
                    Max. : 19.857085
## Max. : 29.857
                                       Max. :1.0000 Max.
                                                            :1587.6
##
         y0
                          У
## Min.
        :-900.243
                   Min. :-900.2
## 1st Qu.: -8.241 1st Qu.: 118.9
## Median : 194.033 Median : 371.0
## Mean : 196.327 Mean : 370.8
## 3rd Qu.: 401.281 3rd Qu.: 623.3
         :1303.145 Max. :1537.8
## Step 1: Visualizing outcome discontinuity
attach(rd_data)
## The following objects are masked _by_ .GlobalEnv:
##
##
      s, w, x, y, y0, y1
rdplot(y=y, x=s, c=10, p=3)
```

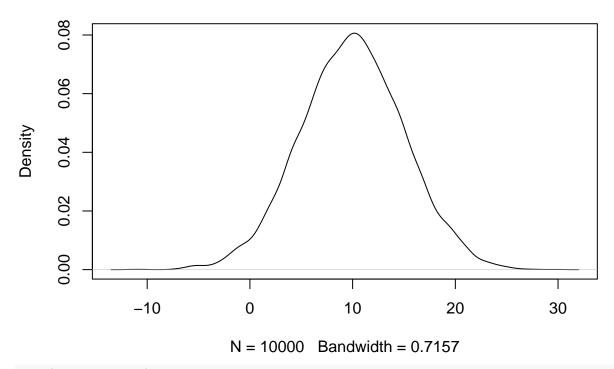
RD Plot



y is the dependent variable, x is the running variable, c is the RD cutoff in x; p specifies the order

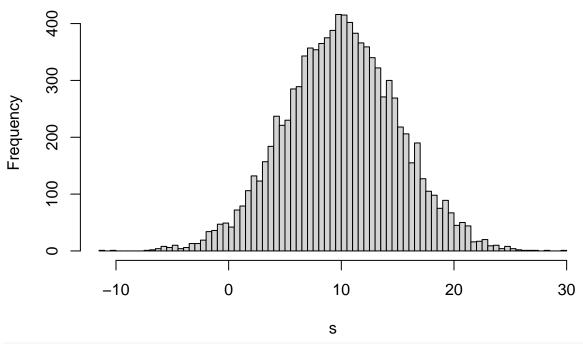
```
## Step 2: Testing balancing at the threshold (covariate balance)
rd_data_balance <- subset(rd_data, s>=9 & s<=11)
t.test(x ~ w, data = rd_data_balance)
##
   Welch Two Sample t-test
##
##
## data: x by w
## t = -68.881, df = 1618.2, p-value < 2.2e-16
\mbox{\tt \#\#} alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
## -1.0204514 -0.9639448
## sample estimates:
## mean in group 0 mean in group 1
        -0.4887857
                         0.5034123
## we want to test whether covariate x is balanced in the range [9,11]
## Step 3: Testing non-manipulation of the running variable (bunching of running variable)
plot(density(s))
```

density.default(x = s)



hist(s, breaks=100)

Histogram of s



library(rddensity)
summary(rddensity(s, c=10))

##

```
## Manipulation testing using local polynomial density estimation.
##
## Number of obs =
                           10000
## Model =
                          unrestricted
## Kernel =
                          triangular
## BW method =
                          estimated
## VCE method =
                          jackknife
##
## c = 10
                          Left of c
                                                Right of c
## Number of obs
                          5009
                                                4991
## Eff. Number of obs
                          2174
                                                2579
## Order est. (p)
                                                2
                          2
## Order bias (q)
                          3
                                                3
                          2.877
## BW est. (h)
                                                3.487
##
## Method
                                                P > |T|
## Robust
                           -0.0809
                                                0.9355
##
##
## P-values of binomial tests (HO: p=0.5).
##
## Window Length / 2
                                       >=c
                                               P>|T|
                                <c
## 0.012
                                         6
                                14
                                               0.1153
## 0.024
                                23
                                         18
                                               0.5327
## 0.037
                                36
                                         25
                                               0.2000
## 0.049
                                44
                                         37
                                               0.5052
## 0.061
                                55
                                         48
                                               0.5546
## 0.073
                                65
                                         56
                                               0.4672
## 0.085
                                74
                                         63
                                               0.3930
## 0.098
                                90
                                         74
                                               0.2414
## 0.110
                                99
                                         83
                                               0.2661
## 0.122
                               111
                                         90
                                               0.1582
```

Now, let's use rdrobust function from rdrobust package to estimate ATE. Type ?rdrobust to see the instruction of the function. Basically, you need to input:

- y: the dependent variable;
- x: the running variable
- c: RD cutoff in x;
- fuzzy: specifies the treatment status variable used to implement fuzzy RD estimation;
- p: specifies the order of the local-polynomial;
- h: specifies the main bandwidth used to construct the RD point estimator. If not specified, bandwidth h is computed by the companion command rdbwselect;
- covs: specifies additional covariates to be used in the polynomial regression.

```
## Step 4: Estimating ATE by sharp RD
summary(rdrobust(y=y, x=s, c=10, p=3, covs=x))
## Call: rdrobust
##
```

Number of Obs. 10000
BW type mserd
Kernel Triangular
VCE method NN

##

Number of Obs. 5009 4991

```
## Eff. Number of Obs.
                     3770
                                3780
                       3
                                 3
## Order est. (p)
## Order bias (q)
                          4
## BW est. (h)
                      5.830
                               5.830
## BW bias (b)
                       8.485
                                8.485
## rho (h/b)
                      0.687
                                0.687
## Unique Obs.
                        5009
                                 4991
##
       Method Coef. Std. Err.
                                      P>|z|
                                               [ 95% C.I. ]
  Conventional
              371.567
                       25.762
                                             [321.075 , 422.060]
##
                              14.423
                                      0.000
                                      0.000 [311.289 , 418.988]
        Robust
                              13.290
## Step 5: Checking robustness: varying the bandwidth and the polynomial order
## bandwidth selection
summary(rdbwselect(y, s, c=10, p=3, covs=x))
## Call: rdbwselect
##
                        10000
## Number of Obs.
## BW type
                        mserd
## Kernel
                   Triangular
## VCE method
                        NN
##
## Number of Obs.
                        5009
                                 4991
## Order est. (p)
                        3
                                   3
## Order bias (q)
                          4
## Unique Obs.
                        5009
                                 4991
##
BW est. (h) BW bias (b)
          Left of c Right of c Left of c Right of c
mserd 5.830 5.830 8.485
                                     8.485
## -----
## varying the bandwidth
summary(rdrobust(y, s, c=10, p=3, h=1, covs=x))
## Call: rdrobust
##
## Number of Obs.
                        10000
## BW type
                       Manual
## Kernel
                    Triangular
## VCE method
                         NN
## Number of Obs.
                        5009
                                 4991
## Eff. Number of Obs.
                        804
                                  817
## Order est. (p)
                                   3
## Order bias (q)
                          4
                                   4
## BW est. (h)
                       1.000
                                1.000
## BW bias (b)
                       1.000
                                1.000
## rho (h/b)
                       1.000
                                1.000
## Unique Obs.
                        5009
                                4991
```

```
##
Method Coef. Std. Err. z
                              P>|z|
5.131
##
  Conventional 323.657 63.082
                              0.000
                                  [200.018, 447.295]
   Robust - -
                        4.206
##
                              0.000 [175.818 , 482.728]
## -----
summary(rdrobust(y, s, c=10, p=3, h=5.950, covs=x))
## Call: rdrobust
##
## Number of Obs.
                  10000
## BW type
                  Manual
## Kernel
                Triangular
## VCE method
                  NN
##
## Number of Obs.
                  5009
                          4991
## Eff. Number of Obs.
                   3829
                          3834
## Order est. (p)
                   3
                            3
## Order bias (q)
                   4
## BW est. (h)
                  5.950
                         5.950
## BW bias (b)
                  5.950
                         5.950
## rho (h/b)
                  1.000
                         1.000
                  5009
## Unique Obs.
                         4991
## -----
     Method Coef. Std. Err. z P>|z| [ 95% C.I. ]
[321.987 , 422.025]
  Conventional 372.006
                  25.520 14.577
##
                              0.000
##
  Robust -
                              0.000 [312.053 , 434.334]
                        11.963
summary(rdrobust(y, s, c=10, p=3, h=10, covs=x))
## Call: rdrobust
##
## Number of Obs.
                   10000
## BW type
                  Manual
## Kernel
                Triangular
## VCE method
                   NN
##
## Number of Obs.
                  5009
                         4991
## Eff. Number of Obs.
                  4755
                          4757
                  3
## Order est. (p)
                           3
## Order bias (q)
                    4
## BW est. (h)
                 10.000
                        10.000
                 10.000
## BW bias (b)
                         10.000
## rho (h/b)
                  1.000
                         1.000
                  5009
## Unique Obs.
                          4991
##
##
     Method Coef. Std. Err. z
                              P>|z|
                                     [ 95% C.I. ]
## Conventional 397.226
                  20.718 19.173
                              0.000 [356.619 , 437.833]
```

```
- - 14.895 0.000 [324.653 , 423.039]
summary(rdrobust(y, s, c=10, p=3, h=50, covs=x))
## Call: rdrobust
##
## Number of Obs.
                     10000
## BW type
                    Manual
## Kernel
                  Triangular
## VCE method
                      NN
##
## Number of Obs.
                    5009
                             4991
## Eff. Number of Obs.
                    5009
                             4991
## Order est. (p)
                               3
## Order bias (q)
                      4
                               4
## BW est. (h)
                    50.000
                            50.000
## BW bias (b)
                    50.000
                            50.000
## rho (h/b)
                    1.000
                           1.000
## Unique Obs.
                     5009
                             4991
Method
              Coef. Std. Err.
                             Z
                                 P>|z|
                                         [ 95% C.I. ]
Conventional 399.370 16.532
                          24.157
                                  0.000
                                       [366.967 , 431.773]
   Robust -
                   _
                                       [373.133 , 449.265]
##
                          21.172
                                 0.000
## varying the polynomial order
summary(rdrobust(y, s, c=10, p=1, h=5.950, covs=x))
## Call: rdrobust
                     10000
## Number of Obs.
## BW type
                    Manual
## Kernel
                  Triangular
## VCE method
##
## Number of Obs.
                    5009
                             4991
## Eff. Number of Obs.
                    3829
                             3834
## Order est. (p)
                     1
                             1
## Order bias (q)
                      2
                               2
## BW est. (h)
                    5.950
                            5.950
## BW bias (b)
                    5.950
                            5.950
## rho (h/b)
                    1.000
                            1.000
## Unique Obs.
                     5009
                            4991
Coef. Std. Err.
       Method
                                 P>|z|
                                         [ 95% C.I. ]
##
   Conventional
            405.872
                    13.937
                          29.123
                                  0.000
                                       [378.557 , 433.187]
                                       [354.110 , 431.955]
##
       Robust
                          19.792
                                  0.000
summary(rdrobust(y, s, c=10, p=3, h=5.950, covs=x))
```

```
## Call: rdrobust
##
                  10000
## Number of Obs.
## BW type
                  Manual
## Kernel
                Triangular
## VCE method
                   NN
##
## Number of Obs.
                  5009
                         4991
                  3829
## Eff. Number of Obs.
                          3834
                  3
                          3
## Order est. (p)
## Order bias (q)
## BW est. (h)
                 5.950
                         5.950
## BW bias (b)
                 5.950
                         5.950
## rho (h/b)
                  1.000
                         1.000
## Unique Obs.
                  5009
                         4991
##
## Method Coef. Std. Err. z P>|z|
                                     [ 95% C.I. ]
Conventional 372.006 25.520 14.577
                              0.000 [321.987, 422.025]
  Robust - 11.963 0.000 [321.987, 422.025]
##
summary(rdrobust(y, s, c=10, p=5, h=5.950, covs=x))
## Call: rdrobust
## Number of Obs.
                  10000
## BW type
                 Manual
## Kernel
                Triangular
## VCE method
                  NN
##
## Number of Obs.
                  5009
                         4991
## Eff. Number of Obs.
                  3829
                          3834
                  5
## Order est. (p)
                          5
                   6
## Order bias (q)
                           6
## BW est. (h)
                 5.950
                         5.950
## BW bias (b)
                  5.950
                         5.950
## rho (h/b)
                  1.000
                         1.000
## Unique Obs.
                  5009
                         4991
## Method Coef. Std. Err. z P>|z| [ 95% C.I. ]
## -----
## Conventional 372.619 37.154 10.029 0.000 [299.798, 445.439]
                       8.127
      Robust -
                             0.000 [268.006 , 438.369]
```