SOSC 5340: Linear regression

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Outline

Recommended readings

- Today's topic deals with prediction and OLS regression, one of the simplest prediction model.
- It is often useful to see how other authors present the same content
- More traditional treatment of the topic
  - Wooldridge, *Introductory Econometrics: A Modern Approach*, 2015. Chapters 2 - 8.
- More modern treatment of the topic
  - Aronow and Miller, *Foundations of Agnostic Statistics*, 2019. Chapters 2 - 4.

Expectation of final papers

- Not doing causal inference?
  - I encourage to think about how to do causal inference
  - It makes your study stronger
  - If you are not sure, come to my office hour and let us discuss it
- Collaboration in final paper?
  - Acceptable; no more than 2 people

Parametric Assumptions

- The theorem, "Conditional Expectation as the Best Predictor", is true in general.
- Nonparametric estimator:
  - Estimate $E(Y|X)$ (conditional mean) from the sample directly
  - Pros: no additional assumption
    - More advanced methods in causal inference often rely on this approach
  - Cons: not easy to estimate (e.g., if you have two X, or X is continuous)
    - With an important exception: experiments and causal inference, where X are mostly binary
- Parametric methods (e.g., regressions):
  - explicitly assume functional form of $E(Y|X = x) = g(X)$
  - E.g., linear regression: $g(X)$ is linear

How do we design the approximating function?

- $E(Y|X)$ is the best predictor of Y given X
- The error of the best predictor $\epsilon = Y - E(Y|X)$ satisfy:
    - $E(\epsilon) = 0$:
    - $E(\epsilon|X)$: the error is mean dependent of X.
    - These two equations are true
- If we assume $E(Y|X = x) = g(X)$, we should also assume that the error $\epsilon = Y - g(X)$ satisfy:
    - $E(\epsilon) = 0$
    - $E(\epsilon|X) = 0$
    - It is important to note that now $E(\epsilon) = 0$, and $E(\epsilon|X) = 0$ are assumptions!

OLS Assumptions

- Linear regression or Ordinary Least Square regression (OLS)
- Assumption 1: the expected error is 0

$$E(\epsilon) = 0$$

- Assumption 2: mean independent between X and the error

$$E(\epsilon | X) = 0$$

OLS Assumptions (cont'd)

- Assumption 3: linear model

$$Y = g(X) = \beta_0 + \beta_1 X + \epsilon$$

- β_0 and β_1 are called parameters
- $Y = \beta_0 + \beta_1 X + \epsilon$ is called (parametric) statistical model

Parameters are unknown constants

- X, Y are random variables; their values are generally unknown
- We can sample $X_1, \cdots, X_n$ from population X;
  - $X_1, \cdots, X_n$ are also random variables
  - Their values are known
- Parameters β are constants; their values are unknown
- [Advanced knowledge]: Bayesian statistics view β as random variables instead of constants

Population Regression Function

- Given the assumptions, it is easy to see that

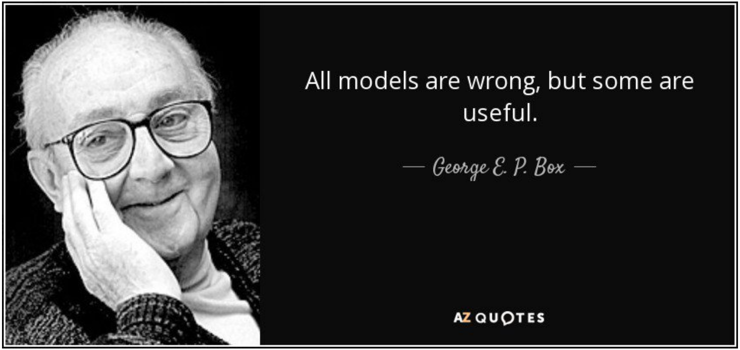
$$
\begin{aligned}
E(Y|X) &= E(\beta_0 + \beta_1 X + \epsilon|X) && (1)\\
&= \beta_0 + \beta_1 X + E(\epsilon|X) && (2)\\
&= \beta_0 + \beta_1 X && (3)
\end{aligned}
$$

- That is, our assumptions lead us to a certain functional form of $E(Y|X) = \beta_0 + \beta_1 X$
- We call $E(Y|X) = \beta_0 + \beta_1 X$ the population regression function
- Note that this is only true when the assumptions are met!

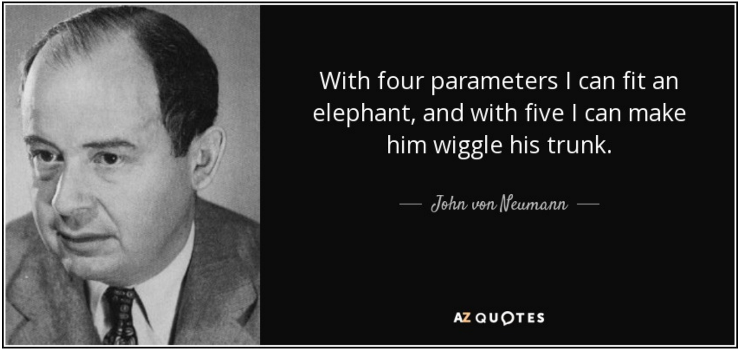
Compare parametric and non-parametric methods

- X is father's height and can take 240 values (every centimeter from 0 to 240); Y is son's height
- Nonparametric: you need a table of 240 cells to represent $E(Y|X = x)$
- Parametric: OLS regression characterizes the relationship between Y and X with 2 parameters: β_0 and β_1
- Parametric model is simpler but need assumptions of the linear relationship

All models are wrong; some are useful

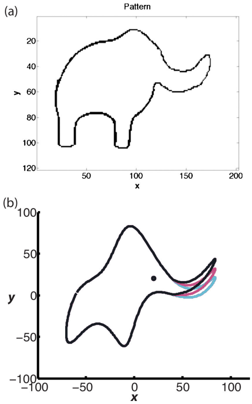


Parametric models can be powerful



Examples

Jürgen Mayer, Khaled Khairy, and Jonathon Howard, "Drawing an elephant with four complex parameters" *American Journal of Physics*, 78, 648 (2010);



Univariate OLS estimation: set up

- Now we go to the problem of estimating population-level conditional expectation $E(Y|X)$ with samples
- Our samples are i.i.d. random samples from X and Y: $(X_1, Y_1), \cdots, (X_n, Y_n)$.
- With a assumed statistical model $Y = \beta_0 + \beta_1 X + \epsilon$
- And two assumptions about the error
  1. $E(\epsilon) = 0$
  2. $E(\epsilon|X) = 0$, which implies $E(\epsilon X) = 0$

Univariate OLS estimation (cont'd)

- $E(\epsilon) = 0 \implies E(Y - \beta_0 - \beta_1 X) = 0$
- $E(X\epsilon) = 0 \implies E\big[X(Y - \beta_0 - \beta_1 X)\big] = 0$
- Now we plug-in sample analog

Population	Sample
$E(Y - \beta_0 - \beta_1 X) = 0$	$\sum_{i=1}^{n} \frac{1}{n}(Y_i - \hat{\beta}_0 - \hat{\beta}_1 X_i) = 0$
$E\big[X(Y - \beta_0 - \beta_1 X)\big] = 0$	$\sum_{i=1}^{n} \frac{1}{n} X_i(Y_i - \hat{\beta}_0 - \hat{\beta}_1 X_i) = 0$

Univariate OLS estimation (solution)

- Solving these two equations give:

$$\widehat{\beta}_0 = \bar{Y} - \widehat{\beta}_1 \bar{X}, \quad \text{and} \quad \widehat{\beta}_1 = \frac{\sum_{i=1}^{N} \left(Y_i - \bar{Y}\right)\left(X_i - \bar{X}\right)}{\sum_{i=1}^{N} \left(X_i - \bar{X}\right)^2} \tag{4}$$

Univariate OLS estimation: Least Square Perspective

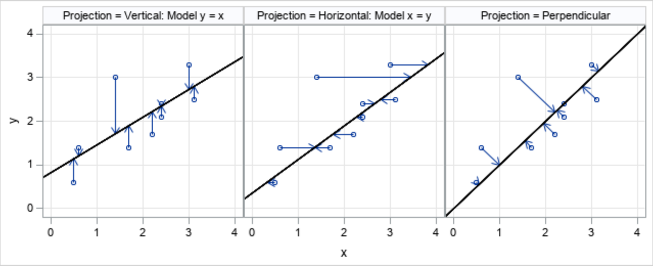
- We have an alternative view of the OLS estimation
- The best predictor of Y, $E(Y|X)$, minimizes Mean Squared Error (MSE), $E[(Y - E(Y|X))^2]$
- Now with samples
  - We will try to minimize the sample MSE (also called empirical MSE)

$$\text{MSE}_{sample} = \frac{1}{n}\sum_{i=1}^{n}(Y_i - \beta_0 - \beta_1 X_i)^2 \tag{5}$$

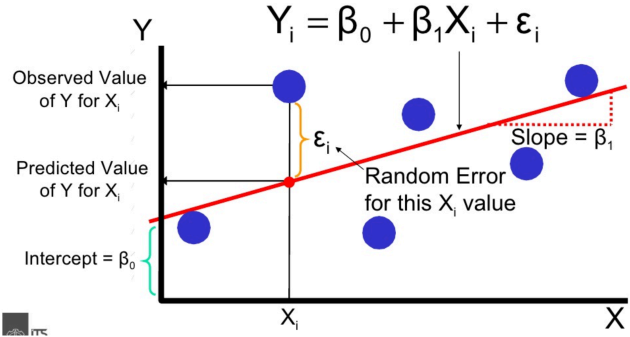
- Note that the sample MSE is almost surely to be larger than the MSE of the best predictor; but this is the best we can do given the assumptions (linear model)
- The problem now is to find $\hat{\beta}_0, \hat{\beta}_1$ that minimize MSE_{sample}.
  - Use standard calculus; let the derivative of MSE with respect to β to be 0.
- The solutions will be the same

Univariate OLS estimation: geometric perspective

- We have yet another view of the OLS estimation
- Linear regression project observation points vertically onto the "fitted line"
- The left and middle one are linear regressions
- The right one is a special case of a famous machine learning algorithm, "Support Vector Machine" (SVM)



OLS geometry (cont'd)



Multivariate OLS Assumptions (cont'd)

- Now we shift from using a single variable X to predict Y, to use k variables $X_1, \cdots, X_k$ to predict Y
- Extend the assumptions of univariate OLS to multivariate OLS case:
- Assumption 1: the expected error is 0

$$E(\epsilon) = 0$$

- Assumption 2: mean independent between X and the error

$$E(\epsilon | X_1, \cdots, X_k) = 0$$

- This assumption implies that the error and any covariate is uncorrelated, that is, $E(\epsilon X_i)$ for any i

Multivariate OLS Assumptions (cont'd)

- Assumption 3: linear model

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots \beta_k X_k + \epsilon$$

- Specific to OLS regression
  - β_0 is called intercept
  - And the rest are also called slope
  - Together, they are called regression coefficients

Multivariate OLS Estimation

- Now going from population to samples
- We sample $(Y, X_1, \cdots, X_k)$ for n times
- The sampled data look like

$$\begin{bmatrix} X_{11} & X_{12} & \dots & X_{1k} \\ X_{21} & X_{22} & \dots & X_{2k} \\ \dots & \dots & \dots & \dots \\ \dots & & & \\ X_{n1} & X_{n2} & \dots & X_{nk} \end{bmatrix} \tag{6}$$

- and $(Y_1, Y_2, \cdots, Y_n)$

Multivariate OLS Estimation (cont'd)

- Again, we plug-in the sample analog in place of poplation equations

Population	Sample
$E(\epsilon) = 0$	$\sum_{i=1}^{n} \frac{1}{n}(Y_i - \hat{\beta}_0 - \hat{\beta}_1 X_{11} - \cdots - \hat{\beta}_k X_{1k}) = 0$
$E(X_1\epsilon) = 0$	$\sum_{i=1}^{n} \frac{1}{n} X_{i1}(Y_i - \hat{\beta}_0 - \hat{\beta}_1 X_{i1} - \cdots - \hat{\beta}_k X_{ik}) = 0$
$\cdots$	$\cdots$
$E(X_k\epsilon) = 0$	$\sum_{i=1}^{k} \frac{1}{n} X_{in}(Y_i - \hat{\beta}_0 - \hat{\beta}_1 X_{i1} - \cdots - \hat{\beta}_k X_{ik}) = 0$

- Now we have $k+1$ parameters, from β_0 to β_k
- And we have $k+1$ equations
- Solving these equations will give the solution to OLS regression

Multivariate OLS Estimation (matrix notations)

- It is too complex to write down all these equations every time
- We rewrite the sample data, using matrix notation
- The first column of **X**, is added artificially (that is, we just assume $X_0 = 1$)

$$\mathbf{X} = \begin{bmatrix} 1 & X_{11} & X_{12} & \dots & X_{1k} \\ 1 & X_{21} & X_{22} & \dots & X_{2k} \\ \dots & \dots & \dots & \dots & \dots \\ 1 & X_{n1} & X_{n2} & \dots & X_{nk} \end{bmatrix}; \quad \beta = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \dots \\ \dots \\ \beta_k \end{bmatrix} \tag{7}$$

- The matrix product:

$$\mathbf{X}\beta = \begin{bmatrix} 1 * \beta_0 + X_{11} * \beta_1 + \dots + X_{1k}\beta_k \\ \dots \\ 1 * \beta_0 + X_{n1} * \beta_1 + \dots + X_{nk}\beta_k \end{bmatrix} \tag{8}$$

Multivariate OLS Estimation (matrix version)

- With the notation $X_0 = 1$, our sample estimation equations can be written as:

$$\begin{bmatrix} 1 & 1 & \cdots & 1 \\ X_{11} & X_{12} & \ldots & X_{1n} \\ X_{21} & X_{22} & \ldots & X_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ X_{k1} & X_{k2} & \ldots & X_{kn} \end{bmatrix} \begin{bmatrix} Y_i - \hat{\beta}_0 - \hat{\beta}_1 X_{i1} - \cdots - \hat{\beta}_k X_{ik} \\ \vdots \\ \vdots \\ Y_i - \hat{\beta}_0 - \hat{\beta}_1 X_{i1} - \cdots - \hat{\beta}_k X_{ik} \end{bmatrix} = 0 \tag{9}$$

- Note that $\mathbf{X}$ is transposed here; we write it as $\mathbf{X}^T$
- And in the matrix term, we write the estimation equations as:

$$\mathbf{X}^T(\mathbf{Y} - \mathbf{X}\hat{\beta}) = 0 \tag{10}$$

Multivariate OLS Estimation (matrix algebra)

- Using matrix notation, we can write the estimates of parameters easily:

$$\begin{aligned} \mathbf{X}^T(\mathbf{Y} - \mathbf{X}\hat{\beta}) &= 0 \\ \mathbf{X}^T\mathbf{Y} &= \mathbf{X}^T\mathbf{X}\hat{\beta} \\ \hat{\beta} &= \left[\mathbf{X}^T\mathbf{X}\right]^{-1}\mathbf{X}^T\mathbf{Y} \end{aligned} \tag{11}$$

Multivariate OLS Estimation (unbiasedness)

- Remember, a good estimator has three properties: unbiasedness, consistent, and asymptotically normal
- Does OLS estimator have these good properties?

$$\begin{aligned}\widehat{\beta} &= \left(\mathbf{X}^\top \mathbf{X}\right)^{-1} \mathbf{X}^\top \mathbf{Y}, \text{(then substitutue } \mathbf{Y} = \mathbf{X}\beta + \epsilon) \\ &= \left(\mathbf{X}^\top \mathbf{X}\right)^{-1} \mathbf{X}^\top \mathbf{X}\beta + \left(\mathbf{X}^\top \mathbf{X}\right)^{-1} \mathbf{X}^\top \epsilon \\ &= \beta + \mathbf{X}\left(\mathbf{X}^\top \mathbf{X}\right)^{-1} \epsilon\end{aligned} \tag{12}$$

- β is a constant (unknown)
- It is easy to see that $E(\hat{\beta}) = \beta$, hence $\hat{\beta}$ is unbiased estimator of β

Multivariate OLS Estimation (consistency)

- $\hat{\beta} - \beta = \mathbf{X}\epsilon \left(\mathbf{X}^{\top}\mathbf{X}\right)^{-1}$
- As n goes to infinity, sample analog of $E(X\epsilon) \rightarrow 0$
- Why?
- Assumption 2
- So $\hat{\beta}$ is consistent

Multivariate OLS in action

```
library(AER)
data(CASchools)
model <- lm(math ~ income + english, data = CASchools)
coef(model)
```

```
(Intercept)      income     english
636.6293146   1.5035886  -0.4005886
```

Multivariate OLS in action

Now we get the matrix estimates

```
X <- model.matrix(model) # this is our X
Y <- CASchools$math
# solve() is to take X^{-1}
# %*% is matrix product
# t() is transpose
beta = solve(t(X) %*% X) %*% t(X) %*% Y
beta
```

```
                  [,1]
(Intercept) 636.6293146
income        1.5035886
english      -0.4005886
```

Variance of OLS estimators

- We have talked about how to estimate our parameters β
- But how confidence are we?
- How can we estimate the variance and confidence intervals?

$$\begin{aligned} V(\widehat{\beta}) &= V\left(\underbrace{\beta}_{\text{Variance of 0}} + \left(\mathbf{X}^\top \mathbf{X}\right)^{-1} \mathbf{X}^\top \epsilon\right) \\ &= \left(\mathbf{X}^\top \mathbf{X}\right)^{-1} \mathbf{X}^\top V(\epsilon) \mathbf{X} \left(\mathbf{X}^\top \mathbf{X}\right)^{-1} \end{aligned} \tag{13}$$

- $V(\epsilon)$ is the variance of the population error. We have to estimate it!

Assumption 4: homoskedastic error

- We have to add assumptions to estimate $V(\epsilon)$
- Assumption 4 (homoskedasticity): for every sample, they have the same variance of the error V(ϵ). We also write $V(\epsilon) = \sigma^2$,
- Under Assumption 4: our estimate of σ^2 is:
  1. $\hat{\epsilon}_i = Y_i - \hat{\beta}_0 - \hat{\beta}_1 X_{i1} - \cdots - \hat{\beta}_k X_{ik}$; $\hat{\epsilon}_i$ is called residuals
  2. $\hat{V}(\epsilon) = \hat{\sigma}^2 = \frac{1}{n-k}(\hat{\epsilon_1}^2 + \cdots + \hat{\epsilon_n^2})$ (hint: $n-k$ makes this quantity unbiased)
- That is, the estimate of the variance of the error is the (weighted) sample mean of residual squares.
- With the estimate $\hat{\sigma}$, we have the classical standard error of β

$$\hat{V}(\widehat{\beta}) = \left(\mathbf{X}^\top \mathbf{X}\right)^{-1} \mathbf{X}^\top \hat{\sigma}^2 \mathbf{X} \left(\mathbf{X}^\top \mathbf{X}\right)^{-1} = \hat{\sigma}^2 \left(\mathbf{X}^\top \mathbf{X}\right)^{-1} \tag{14}$$

Alternative Assumption 4: heteroskedastic error

- Alternative Assumption 4 (heteroskedasticity): the sample's error can be different (but they are uncorrelated with each other)
- Under Alternative Assumption 4,
  1. $\hat{\epsilon}_i = Y_i - \hat{\beta}_0 - \hat{\beta}_1 X_{i1} - \cdots - \hat{\beta}_k X_{ik}$
  2. we do not calculate the sample mean; instead:

$$\hat{V}(\epsilon) = \begin{bmatrix} \hat{\epsilon}_{i1}^2 & 0 & 0 \\ 0 & \hat{\epsilon}_{i2}^2 & 0 \\ 0 & 0 & \hat{\epsilon}_{in}^2 \end{bmatrix} \tag{15}$$

- With the estimate of $V(\epsilon)$, we have heteroskedasticity-robust standard error of β

$$\hat{V}(\widehat{\beta}) = \left(\mathbf{X}^\top \mathbf{X}\right)^{-1} \mathbf{X}^\top \hat{V}(\epsilon) \mathbf{X} \left(\mathbf{X}^\top \mathbf{X}\right)^{-1} \tag{16}$$

Homoskedasticity vs heteroskedasticity

- Homoskedastic errors are almost surely to be wrong in reality
- So why we still care about the classical standard error?
- Some math reasons: classical standard errors (and homoskedasticity) make the inference easier, as will be seen later.
- Some historical reasons: it is harder to calculate the robust standard error in the old days (but now with computers, is it very easy)
- In default statistical packages (R, Stata or most other languages), when you run a simple OLS regression, the default standard error you get is still the classical standard error

Assumption 5: normality

- We have derived the estimator for β and its variance
- Now we want to construct confidence intervals
- Assumption 5: normal error assumptions (with homoskedasticity)

$$\epsilon \sim N(0, \sigma^2)$$

- Note that Assumption 5 directly implies Assumption 1 (error has mean 0) and Assumption 4 (sample error is the same)
- But not the other way around
- So Assumption 5 is a very strong assumption
- Effectively this also makes the estimator asymptotically normal

Confidence intervals

- With the normality assumption, the α confidence interval of regression coefficients β:

$$\left(\hat{\beta} - z_{\frac{1+\alpha}{2}}\sqrt{\hat{V}(\hat{\beta})}, \hat{\beta} + z_{\frac{1+\alpha}{2}}\sqrt{\hat{V}(\hat{\beta})}\right) \tag{17}$$

- z is the quantile function of a standard normal distribution
  - $\alpha = 0.95$; $z_{0.975} = 1.96$
  - $\alpha = 0.99$; $z_{0.995} = 2.58$

Hypothesis Testing

- Confidence interval can be used to judge the possibility of a particular guess of β
- This is known as Hypothesis Testing
- For instance, we think that X_1 is very predictive of outcome Y, so we hypothesize that its coefficient β_1 is not 0.
    - alternative hypothesis: $\beta_1 \neq 0$. (this is the hypothesis you truly believe)
    - null hypothesis: $\beta_1 = 0$. (this is the "boring" or default hypothesis)

Use confidence interval to perform hypothesis testing (example)

- Our alternative hypothesis is $\beta_1 \neq 0$
- And null hypothesis is $\beta = 0$
- Example: our point estimate of β_1 is $\hat{\beta}_1 = 0.8$, and our estimated 95% confidence interval is $[0.2, 1.4]$
- This indicates 95% of times, our true value of β_1 will be in the range of $[0.2, 1.4]$ over repeated samples
- Therefore, the chance our true β_1 will be 0 (null hypothesis) is small (say 0.05 on average)
- So we reject the null hypothesis and find support for our alternative hypothesis

Use confidence interval to perform hypothesis testing

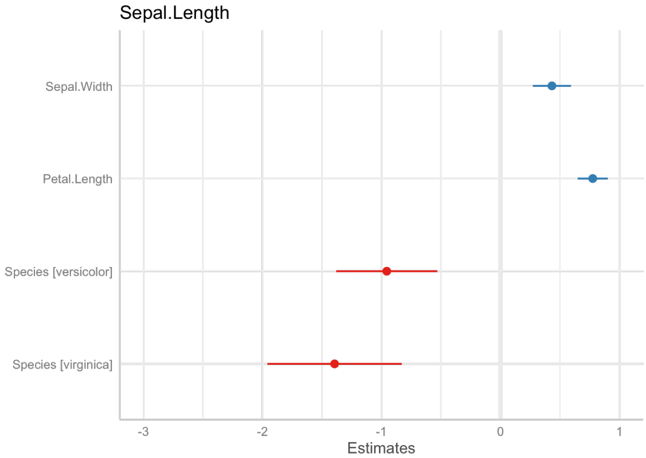
- In general: if we have 95% confidence interval for some quantity θ, $[\theta_{min}, \theta_{max}]$
- And our alternative hypothesis is that $\theta = \theta_{test}$
- Then
  - if θ_{test} does not belong to $[\theta_{min}, \theta_{max}]$, we find support for the null hypothesis
  - if θ_{test} falls into $[\theta_{min}, \theta_{max}]$ or not, we find support for our alternative hypothesis

Illustration

- $\theta_{test} = 0$; we want to test whether regression coefficient differs from 0

```
library (sjPlot); data(iris)
m2 <- lm(Sepal.Length ~ Sepal.Width +
    Petal.Length + Species, data = iris)
plot_model(m2)
```

Illustration



- If 95% confidence interval does not touch 0, we can reject the null and say that coefficients are statistically significantly different from 0

Hypothesis Testing using test statistics

- Alternatively, Hypothesis Testing can be done by calculating test statistics of β
- Null Hypothesis: $\beta = 0$
- Alternative Hypothesis: $\beta \neq 0$
- t -statistics is a commonly used test statistics:

$$t = \frac{\hat{\beta} - \beta}{\sqrt{\hat{V}(\beta)}} \tag{18}$$

- If Null Hypothesis is true, we would expect that t is small; otherwise, t should be large
- With homoskedastic error, t follows a student t distribution
- With heteroskedastic error, t distribution is more complex
- Student t distribution depends on data size as well as model; it's not comparably across different data sets
    - E.g., are 10 large enough? 100 large enough?

P-value

- We use something called p-value; it is calculated from the cumulative distribution of the sampling distribution of t
- For example: if p-value is p , it means that the probability of obtaining a t -statistics that equals to t_0 or higher, when sampling from the same population, is approximately p.
- The smaller the p value, the more evidence that we can reject the Null Hypothesis

Confidence interval, t -statistics, and p-value

- Null Hypothesis: $\beta = 0$
- Alternative Hypothesis: $\beta \neq 0$
- The following statements are equivalent
  - The t -statistics is larger than 1.96 (for linear regression only!)
  - The p-value is small or equal to 0.05
  - Estimated 95% confidence interval does not contain 0
  - And each of the three argument can let us to reject the null hypothesis and find support for the alternative hypothesis

Interpret coefficient

- Marginal effect: how the change in one variable X_i predicts average change in y, holding other variables constant
- Marginal effect is done by taking the partial derivative of Y regarding X_i

$$\frac{\partial Y}{\partial X_i} \tag{19}$$

- In OLS regression, the marginal effect is simple:
- Marginal effect of X_i on Y is its regression coefficient β_i
- Or in other words: one unit change in X_i predicts on average β_i change in Y, given the same values for other variables.

Collinearity

- Estimating multivariate OLS with k variables:
  - $k+1$ equations; $k+1$ unknown parameters
- But if two variables are colinear, we cannot solve the equation (one equation becomes useless)
- Examples of collinearity:
  - $X_2 = 2X_1$
  - $X_2 = 1 - X_1$
- If X_1 and X_2 have very high correlation: multicollinearity
  - Variance estimate of parameters can be very high
  - Good practice: always check correlation before putting variables into regression

Dummy variable

- Note that we make no assumption about how X and Y should be distributed at all
- So X can be categorical, such as gender (man, female, transgender, etc)
- So naturally, gender looks like (man, female, man, transgender, female) (assume that we have five observations)
- Dummy transformation turns the data into (1 means yes and 0 means no); this is also called zero-one representation or indicator representation
- Each row, one and only one cell is 1; the other cells are 0

observation/gender	man	female	transgender
1	1	0	0
2	0	1	0
3	1	0	0
4	0	0	1
5	0	1	0

Dummy variable interpretations

- wage $= \beta_0 + \beta_1$female $+ \beta_2$education $+ \epsilon$
- Female is a dummy variable for females (i.e., $1 =$ females and $0 =$ males), and educ is years of schooling. wage is hourly wage.
- So the reference group is male
- Interpretations:

β_0 *represents the difference in mean hourly wage between females and the reference group, male, given the same amount of education.*

Interactions

- Sometimes, effect of X on Y
- We add interaction terms to capture the idea

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \delta X_1 X_2 \tag{20}$$

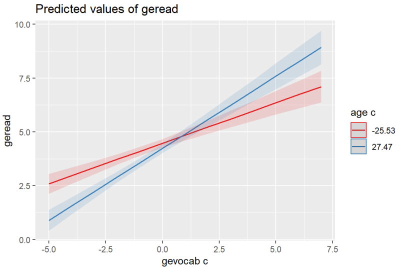
- Effect of X_1 on Y is the marginal effect, which depends on X_2

$$\frac{\partial Y}{\partial X_1} = \beta_1 + \delta X_2 \tag{21}$$

- Now, interpreting β_1 needs some caution
  - it is wrong to say that one unit increase in X_1 predicts β_1 increase in outcome
  - The amount of increase in outcome also depends on the value of X_2
  - It's much easier to plot the marginal effect

Plotting interaction effect

- E.g., Suppose that we want to test whether more vocabulary predicts higher reading scores differently for age groups reading $= \beta_0 + \beta_1\text{age} + \beta_2\text{vocab} + \delta\text{age} \times \text{vocab} + \epsilon$
- Using R package `sjPlot`; many other options, such as `interactions` or `interplot`



Interactions involving categorical variables

$$\frac{\partial \text{wage}}{\partial \text{education}} = \beta_2 + \delta \text{female}$$

- This means that when female $= 0$, the marginal effect is β_2
- When female $= 1$, the marginal effect is $\beta_2 + \delta$
- The marginal effect of education on wage is different, conditional on gender

Wrap Up

- $E(Y|X)$ is the best predictor of Y with MSE
- What are OLS regression's assumptions?
- How to estimate OLS regression coefficient and perform inference?
- How to interpret OLS coefficients