

Regression Discontinuity

Han Zhang

Outline

Sharp RD

Bandwidth selection

Extensions

Reviews

Recommended Readings

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- Cattaneo, Matias D., Nicolás Idrobo, and Rocío Titiunik. *A Practical Introduction to Regression Discontinuity Designs: Foundations*. Cambridge University Press, 2019.

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- <https://rdpackages.github.io/>

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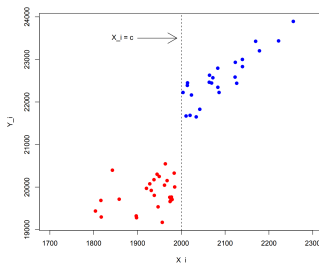
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- Example 2: voting share $\geq 50\%$ \rightarrow win the election

Sharp Regression Discontinuity

- Three core elements

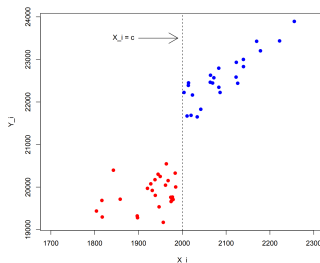
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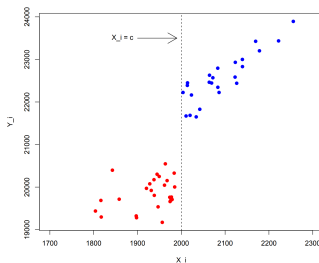
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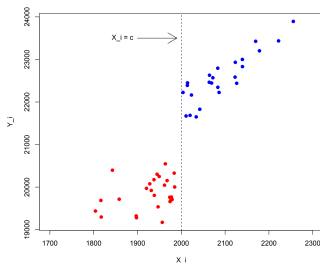
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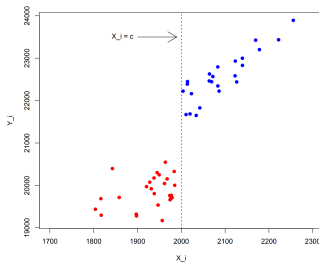
Sharp Regression Discontinuity

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- **Running variable**, or **scores** X
- **Cutoff** or threshold c
- Treatment assignment D , which is fully determined by X based on cutoff c

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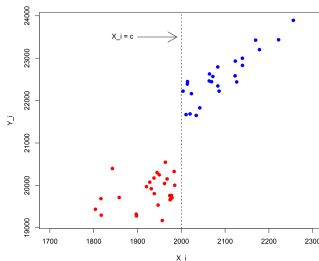


RD Intuition



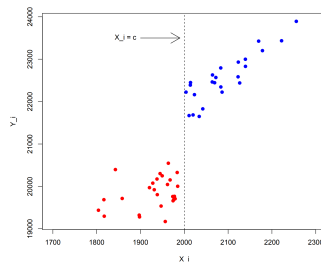
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- But we can compare Y between points on and just below the cutoff
 - $X = c$ vs. $X = c - \epsilon$,

Sharp RD treatment effect

- If ϵ becomes infinitely small, we obtain the causal effect for $X = c$

$$\tau_{\text{SRD}} \equiv \mathbb{E} [Y_i^1 - Y_i^0 | X_i = c] = \lim_{x \downarrow c} \mathbb{E} [Y_i | X_i = x] - \lim_{x \uparrow c} \mathbb{E} [Y_i | X_i = x] \quad (2)$$

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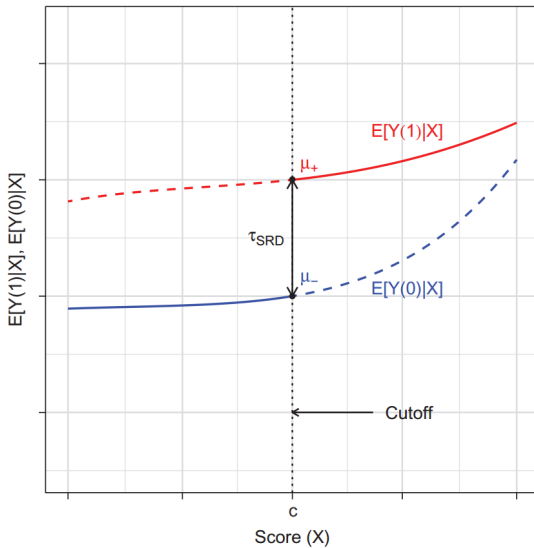
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- In contrast to ATT or ATE, Sharp RD identifies a **local** effect
- The key assumption is that $\mathbb{E} [Y_i^1 | X_i = c]$ and $\mathbb{E} [Y_i^0 | X_i = c]$ are **continuous**

Sharp RD



Example

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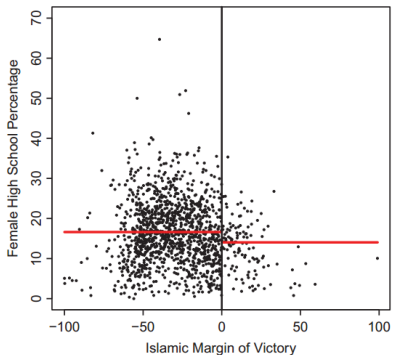
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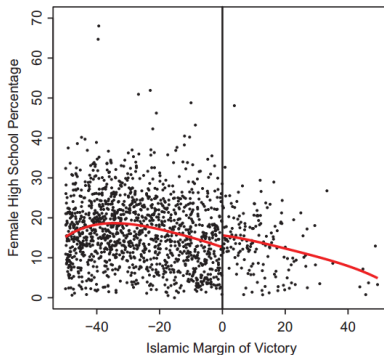
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- X : vote share margin; vote percentage of an Islamic mayor candidates - vote percentage of a secular candidate, in 1994 Turkish local elections
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- Y : share of local women aged 15 to 20 in 2000 who had competed high school by 2000

RD plot: global vs local

- Globally: negative impact



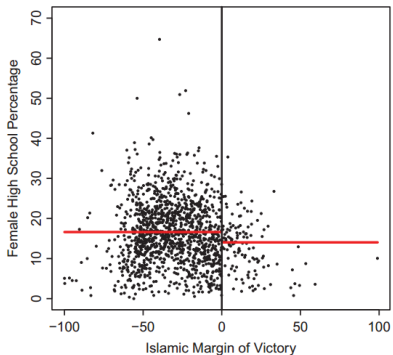
(a) Raw Comparison of Means



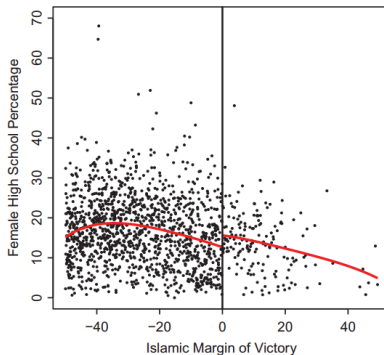
(b) Local Comparison of Means

RD plot: global vs local

- Globally: negative impact
- Locally: positive impact



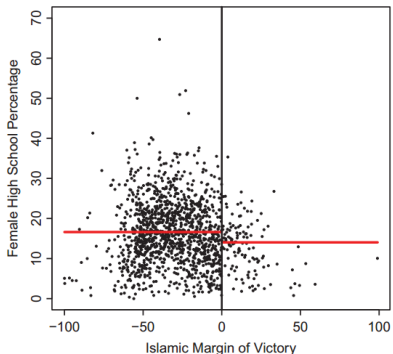
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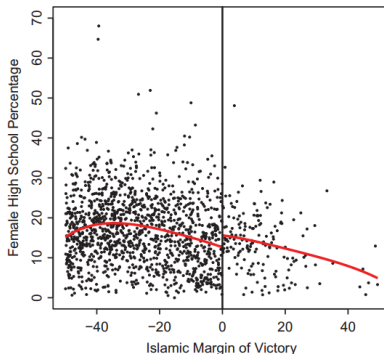
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RD plot: global vs local

- Globally: negative impact
- Locally: positive impact
- Since RD is about local effect, data far away from the cutoff are not useful

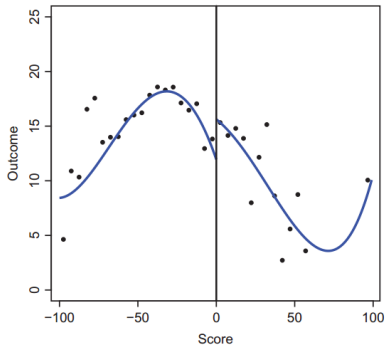


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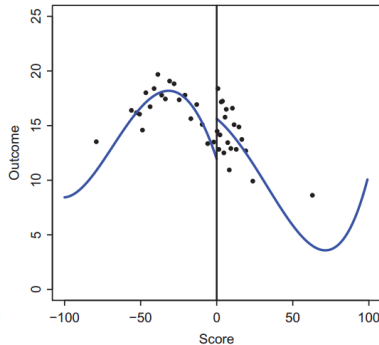


(b) Local Comparison of Means

RD plot: aggregated individual data



(a) 40 Evenly-Spaced Bins



(b) 40 Quantile-Spaced Bins

Figure 7 RD Plots (Meyersson Data)

Estimating Sharp RD Treatment Effect

- Theory: calculate the vertical distance between those on the boundary and those just below the boundary

$$Y = \beta_0 + \beta_1 X + \beta_2 X^2 + \cdots + \beta_p X^p$$

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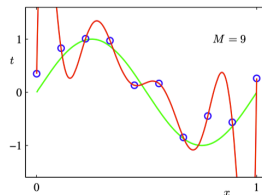
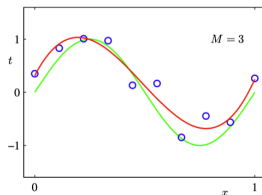
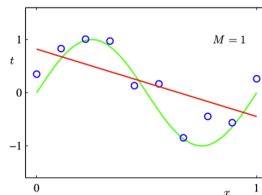
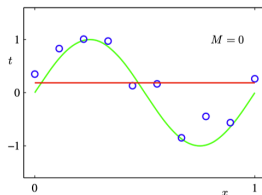
Estimating Sharp RD Treatment Effect

- Theory: calculate the vertical distance between those on the boundary and those just below the boundary
- Reality: if X is continuous, there are no (or sometimes in practice very few) observations around c
- Solution: use data in a small region $[c - h, c + h]$ and a prediction function to predict $E(Y^1|X = c)$ and $E(Y^0|X = c)$

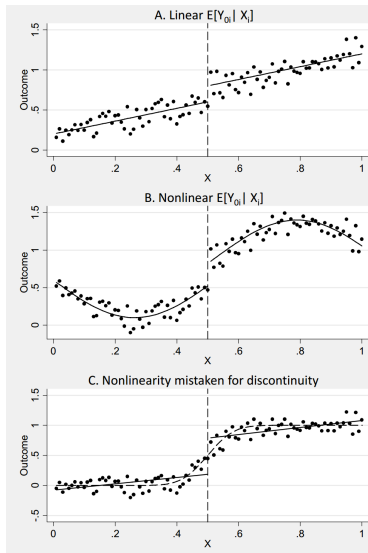
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Polynomial example

- here M is p : order of polynomial

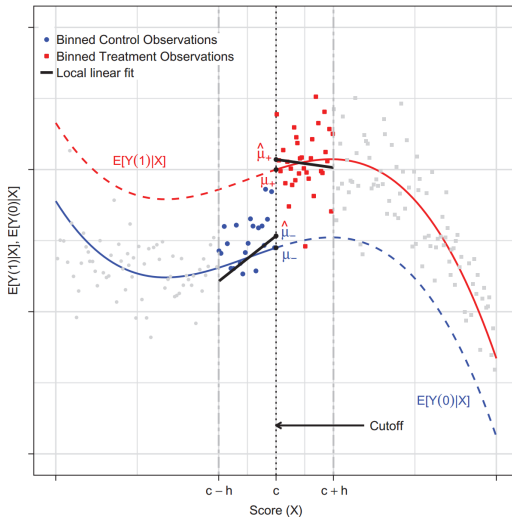


Extrapolation function matters



Using Local Polynomial Regression to predict

- here $p = 1$: linear prediction



Sharp RD procedures

1. Choose a polynomial **order** p and a kernel function $K(\cdot)$.
 - Kernel essentially add weights to points according to their distances to c
2. Choose a **bandwidth** h
3. For $X_i \in [c - h, c)$, Fit a **weighted** linear regression of Y on $X_i - c, (X_i - c)^2, \dots, (X_i - c)^p$, use weights based on $K(\frac{X_i - c}{h})$.
Estimate of $E(Y^0|X = c) = \hat{\mu}_-$

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4. The same thing for $X_i \in [c, c + h]$; obtain estimate of $E(Y^1|X = c) = \hat{\mu}_+$
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Different kernels

- in practice, kernel choices are less sensitive

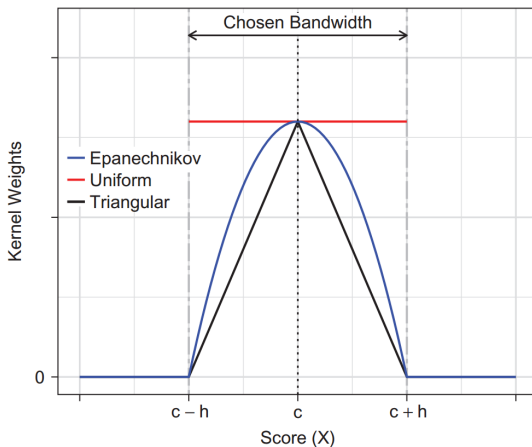
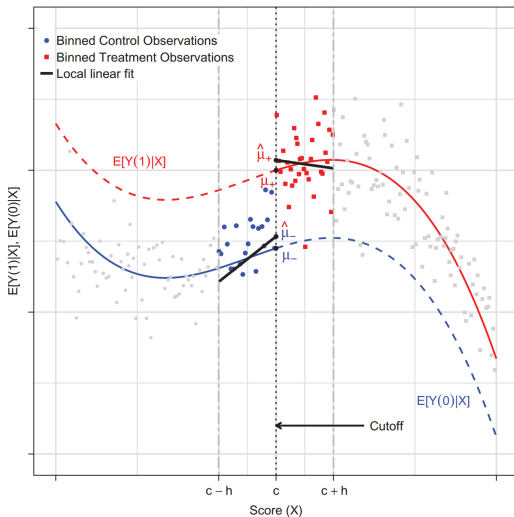


Figure 13 Different Kernel Weights for RD Estimation

Bandwidth matters



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Bandwidth selection

- Key idea: bias-variance trade-off

$$\text{MSE}(\hat{\tau}_{\text{SRD}}) = \text{Bias}^2(\hat{\tau}_{\text{SRD}}) + \text{Variance}(\hat{\tau}_{\text{SRD}}) \quad (4)$$

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- And bias for $X > c$ can be similarly written down
- Total bias is the sum of bias for $X < c$ and bias for $X > c$

Bandwidth selection

- With Taylor Expansion, we can also write $E(Y^0|X = X_i)$ as:

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- Where h is bandwidth and B is asymptotic bias (those cannot be removed by taking limits)

Bandwidth selection

- Using similar idea, variance can be roughly expressed as

$$\text{Variance}(\hat{\tau}_{\text{SRD}}) = \frac{1}{nh} V, \quad (8)$$

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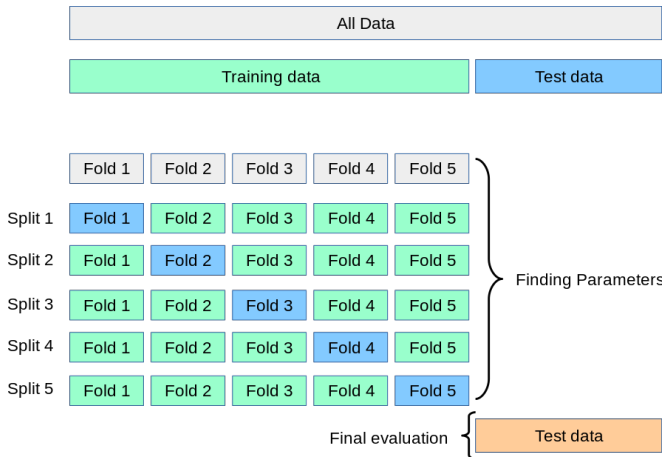
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Cross-validation

- K-fold cross validation (below example shows $K = 5$)



Cross-validation example

- Lee and Lemieux, 2010, *Journal of Economic Literature*

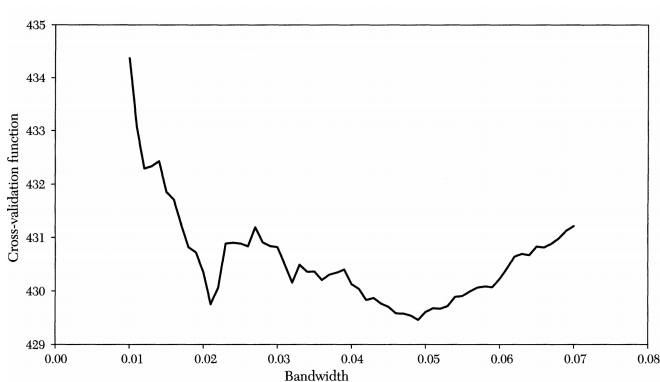


Figure 12. Cross-Validation Function for Choosing the Bandwidth in a RD Graph:
Winning the Next Election

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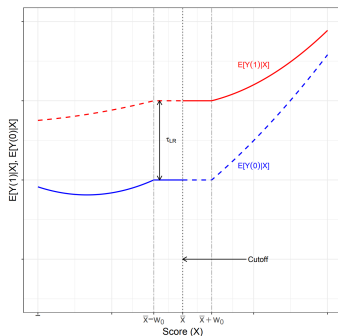
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- The local randomness assumption is **not necessary!** It is more demanding than the continuity assumption
- E.g., in voting example, it's hard to argue that districts in which parties has a narrow win share is due to randomness.

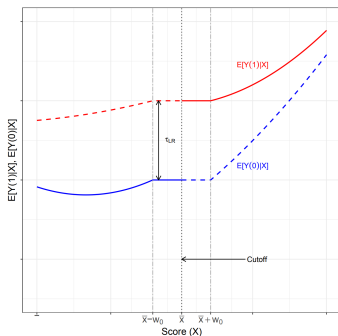
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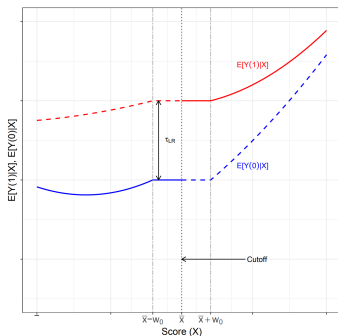
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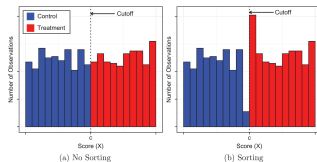
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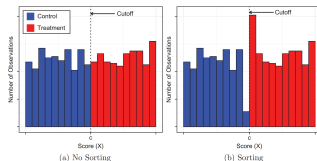
Credibility of RD assumptions

- **Density test** is popularized by McCrary, 2008, *Journal of Econometrics*



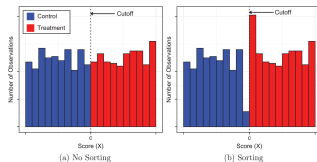
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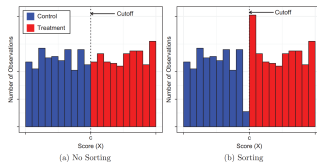
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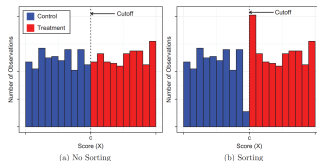
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- Density test: plot X against the number of observations
 - note that RD plot is X against Y



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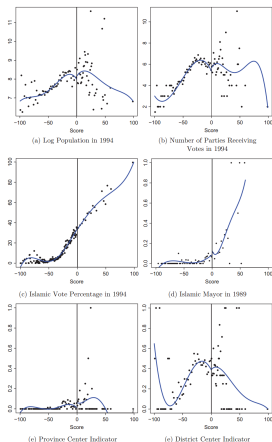


Figure 16 RD Plots for Predetermined Covariates (Meyerson Application)

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- X determines D , but it should not determine other controls

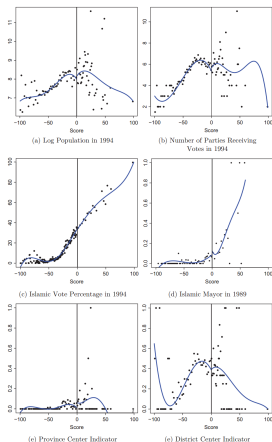


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RD as a linear regression

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$$Y_i = \alpha + \tau_{\text{SRD}} D_i + \mu_{-,1} (X_i - c) + \cdots + \mu_{-,p} (X_i - c)^p + \mu_{+,1} D_i (X_i - c) + \cdots + \mu_{+,p} D_i (X_i - c)^p \quad (11)$$

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- Modern RD packages ('rdrobust' in R and Stata) generally does not use regression under the hood.

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- We have discussed how to estimate RD by comparing means at the cutoff

Analogously to the case of randomized experiments, the generally valid justification for including covariates in RD analysis is the potential for efficiency gains, not the promise to fix implausible identification assumptions.

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- Cattaneo et al., 2019, p. 71:

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 - The only practical difference is that you need to select bandwidth

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- And the causal effect under can again be obtained from Wald estimator:

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- τ_{FRD} again estimates local treatment effect for compliers (those who actually follows the assignment of the running variable)

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- Transform p -values or t -statistics into z -statistics (for larger sample sizes)

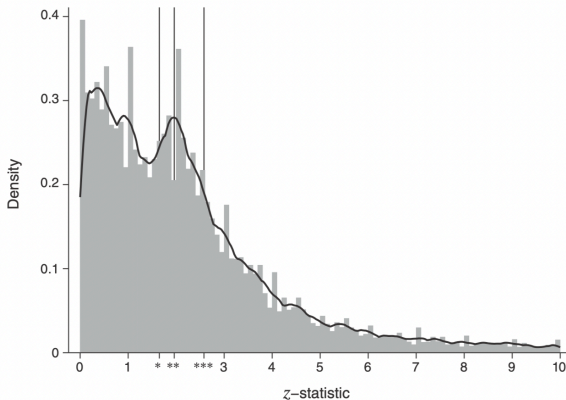
z	p
1.65	< 0.1
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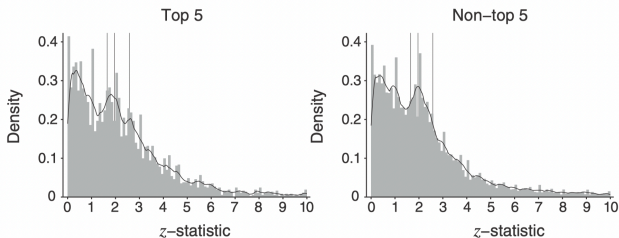
- Transform p -values or t -statistics into z -statistics (for larger sample sizes)
- If there were no p-hacking, we would expect that the probability density distribution around the below cutoffs would be smooth

z	p
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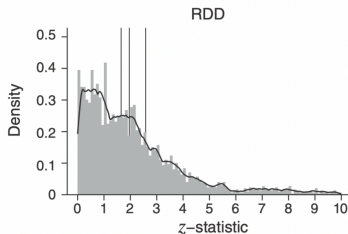
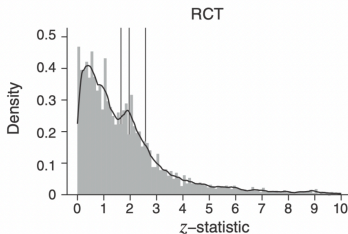
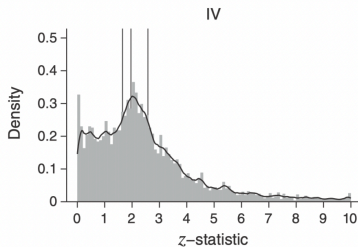
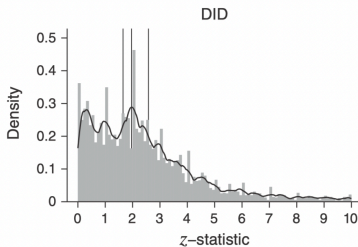
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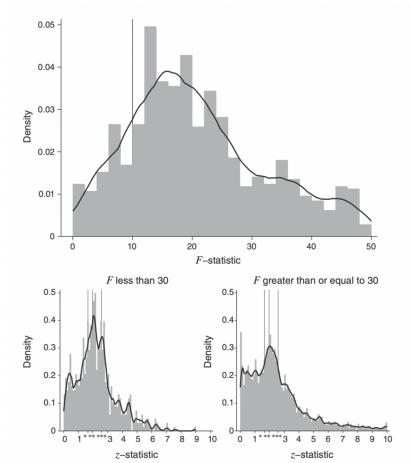
p-hacking by journals



p-hacking by methods

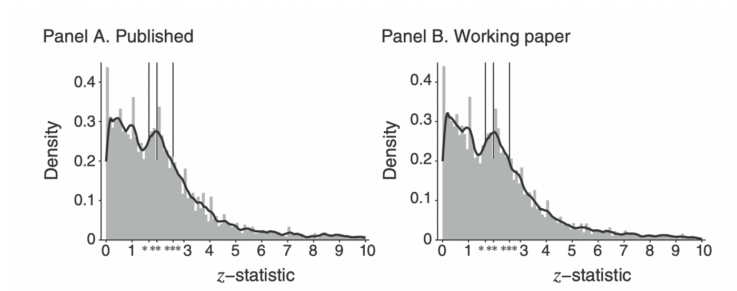


p-hacking and weak instruments



p-hacking and review process

-do people hack because they want their working papers to be published?



Summary

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- Randomized controlled experiments and RD are relatively better