Causal Inference in Experiments and Observational Studies

Han Zhang

Outline

Readings

Today's topics are drawn from:

- Joshua D. Angrist and Jorn-Steffen Pischke. *Mostly Harmless Econometrics: An Empiricists Companion* . Princeton University Press, 2009. (Chapters 2 - 3)

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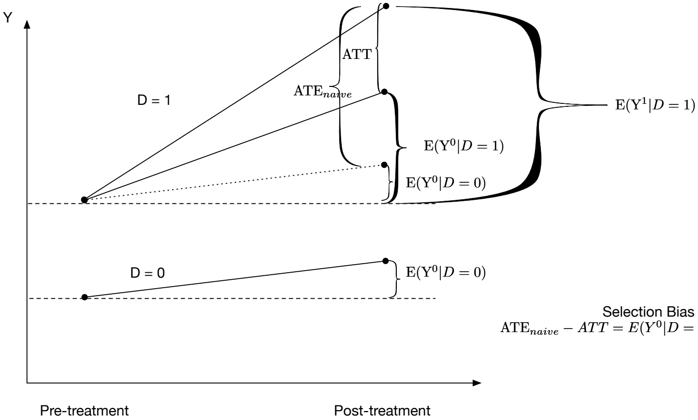
- Proofs:
  - Imbens, Guido W., and Donald B. Rubin. *Causal inference in statistics, social, and biomedical sciences*. Cambridge University Press, 2015 (Chapter 6 - 7).

Observational Studies



Selection Bias =
$\text{ATE}_{naive} - ATT = E(Y^0|D=1) - E(Y^0|D=0)$

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  - $ATT = ATE$

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- Rule of thumb: the gold-standard is always randomized controlled experiment

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- The trend is leaning toward design-based causal inference

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    - Problem: we do not know all factors that determine leadership changes
        - A non-exhaustive lists include economic growth itself, leadership personality, geospatial conditions, etc.

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  - POLITY Score (democracy scores)

Estimating causal effects in natural experiment

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- We can also use regression estimator: $\hat{\rho}$ estimates ATE

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- There is no substitute for a good research design (here, exogenous shocks)

Natural experiments with covariates

- Because natural experiments are not fully controlled by researchers, covariates can have additional help (other than checking pre-treatment balance)

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- In randomized controlled experiments, treatment and control groups were chosen with clear standard so there is no such problem

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- Note that randomized experiments automatically satisfy this assumption

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- That is, we are selecting on unobservables
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- or we have omitted variable bias
- or, there is selection bias due to unobservables

Regression estimator

- If ignorability assumption is true, and you assume the effect of treatment is constant on Y, we can use regression to estimate causal effects:

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  - More X increases the possibility that you do not missed anything important confounders

Regression as Imputation

- If ignorability is true, the regression estimator of ATE is implicitly making counterfactual imputation using linear regression.

Unit	Y_i^0	Y_i^1	D_i	$X_{[1]i}$	$X_{[2]i}$
1	?	2	1	1	7
2	5	?	0	8	2
3	?	3	1	9	3
4	?	10	1	3	1
5	?	2	1	5	2
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4	$\hat{\beta}_0 + \hat{\beta}_1 \cdot 0 + \hat{\beta}_2 \cdot 3 + \hat{\beta}_3 \cdot 1$	10	1	3	1
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- Run a regression as $Y = \beta_0 + \beta_1 D_i + \beta_2 X_{[1]i} + \beta_3 X_{[2]i}$, and impute counterfactual outcome using the linear regression:

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Ignorability: example

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- Jens Hainmueller and Dominik Hangartner, *Who Gets a Swiss Passport? A Natural Experiment in Immigrant Discrimination*, American Political Science Review **107** (2013), no. 01, 159–187

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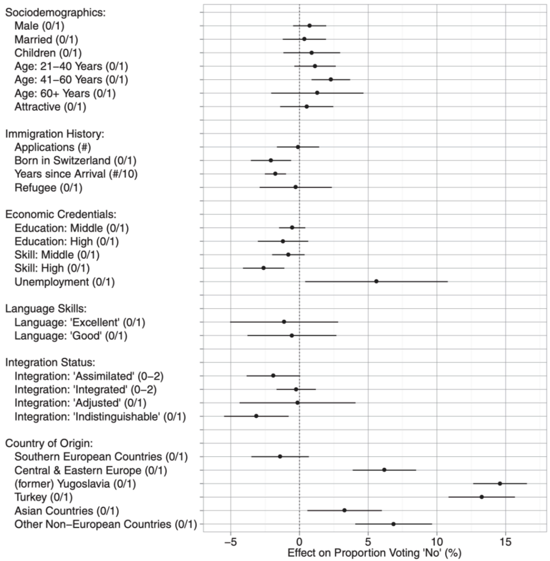
FIGURE 2. Effect of Applicant Characteristics on Opposition to Naturalization Requests



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- That is, they have private information other than that listed in the leaflets

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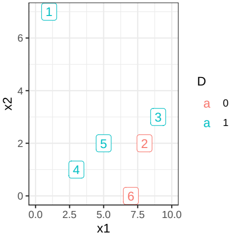
    - Find the j in the control group, whose X_j is the closest X_i
    - Use the Y_j^0 associated with j as the imputed Y_i^0 value for i
- Then we can estimate ATE as the the difference between the mean of Y_i^1 and Y_i^0 for all units

Matching estimator using original data

Unit	Y_i^0	Y_i^1	D_i	$X_{[1]i}$	$X_{[2]i}$
1	?	2	1	1	7
2	5	?	0	8	2
3	?	3	1	9	3
4	?	10	1	3	1
5	?	2	1	5	2
6	0	?	0	7	0

(2)



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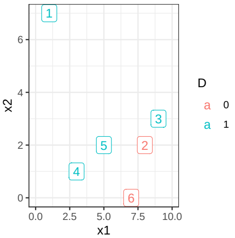

- Unit 3 is treated; it is closest to unit 2; unit 2 is the matched unit of unit 3
- $Y_3^0 \leftarrow Y_2^0 = 5$

Matching estimator using Propensity Score

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  - This is usually called propensity score matching

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1	?	2	1	1	7	0.33
2	5	?	0	8	2	0.14
3	?	3	1	10	3	0.73
4	?	10	1	3	1	0.35
5	?	2	1	5	2	0.78
6	0	?	0	7	0	0.70

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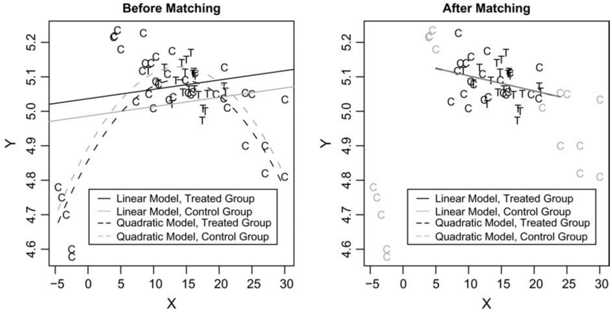

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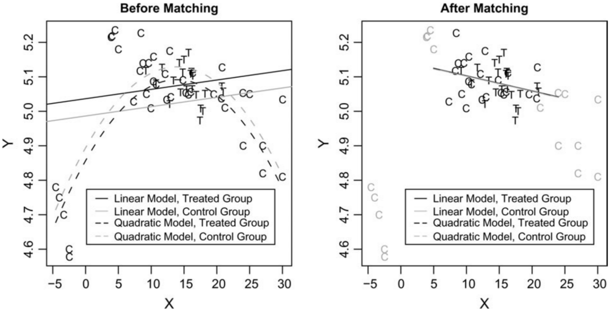

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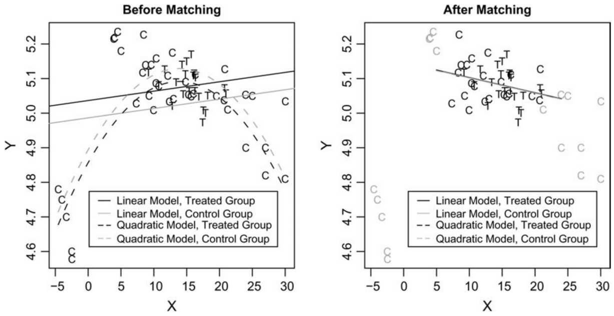

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    - That is, construct a data as (NSW(treated), CPS); CPS replaced NSW control units

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- What the authors did:

TABLE 2.—SAMPLE CHARACTERISTICS AND ESTIMATED IMPACTS FROM THE NSW AND CPS SAMPLES

Control Sample	No. of Observations	Mean Propensity Score[A]	Age	School	Black	Hispanic	No Degree	Married	RE74	RE75	U74	U75	Treatment Effect (Diff. in Means)	Regression Treatment Effect
NSW	185	0.37	25.82	10.35	0.84	0.06	0.71	0.19	2095	1532	0.29	0.40	1794[B]	1672[C]
													(633)	(638)
Full CPS	15992	0.01	33.23	12.03	0.07	0.07	0.30	0.71	14017	13651	0.88	0.89	−8498	1066
		(0.02)[D]	(0.53)	(0.15)	(0.03)	(0.02)	(0.03)	(0.03)	(367)	(248)	(0.03)	(0.04)	(583)[E]	(554)
Without replacement:														
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		(0.03)	(0.79)	(0.23)	(0.04)	(0.03)	(0.05)	(0.04)	(495)	(341)	(0.05)	(0.05)	(733)	(709)

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    - And it will be dangerous to add *occupation* as a control variable, since occupation may be the result of treatment

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- The third approach:
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  - Essentially admit that we cannot control everything; there are some unobserved variables we cannot control for

Econometric tools in working with non-ignorability

- Fixed effect and diff-in-diff

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