# Instrumental Variables

Han Zhang

# Outline

# Recommend Readings

- Joshua D. Angrist and Jorn-Steffen Pischke. *Mostly Harmless Econometrics: An Empiricists Companion* . Princeton University Press, 2009. (Chapter 4)

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  - General principle: approximates experiment ideal

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- IV exploits exogenous variation that drive the treatment but do not otherwise affect the outcome.

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  - Later you will become more clear what this means

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  - US compulsory schooling laws are in terms of age (16), not number of years of schooling completed. You can drop out on your 16th birthday (even if in the middle of the school year).
  - So those born in 4th quarter are compelled to take more educations than those born in 1st quarter

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   - Reasonable? Will parents selectively give birth in certain months because they think children will be more beneficial?

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- But if you pass DWH test (p-value $< 0.05$), it does not mean that you have a valid IV
- You still need to have a story on why IV is as-if randomly assigned

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        - e.g., those who were born in 1st quarter are older so that they will be more mentally developed compared with those born in 4th quarter

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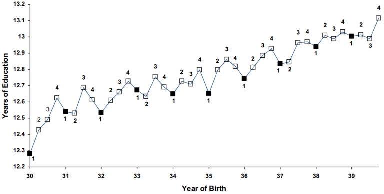
   - Plot $D$ against $Z$; visually inspect correlation (MHE, Fig. 4.1.1)



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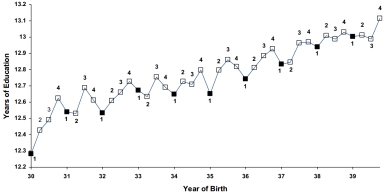

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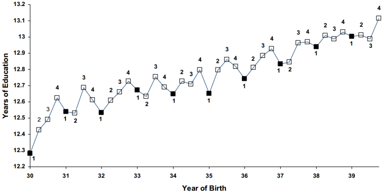


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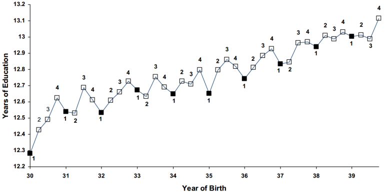


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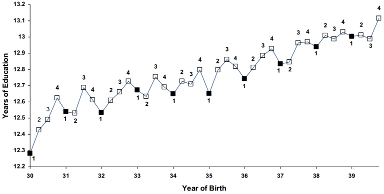


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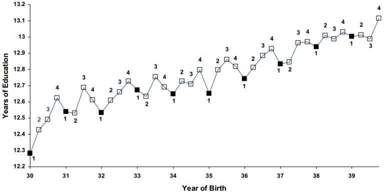


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  - Some recent work suggests that F-statistics $> 100$

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- Hence regressing $Y$ on instrument $Z$ identifies $\rho\beta$

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- Because we are interested in the effect of $D$ on $Y$, but we run two other regression $Y$ on $Z$, and $D$ on $Z$, and then take their ratios

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- Which treatment effect do we use? Not so easy under the indirect least square

# Wald estimator (Wald, 1940)

- In the case of binary instrument, assuming the same linear model:

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\hat{\rho} &= \frac{\hat{E}(Y|Z=1) - \hat{E}(Y|Z=0)}{\hat{E}(D|Z=1) - \hat{E}(D|Z=0)}, \text{(Wald Estimator)} && (2) \\
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- Note: Wald estimator is a more general non-parametric estimator; it equals to Indirect Least Square in binary case only when the linear model assumption is true

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- Software (e.g. R's `ivreg` or Stata's `ivregress`) will correct this for you.

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        - It will be more clear what this assumption entails later

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- In many randomized experiments, you can assign treatment, but cannot force people to take the treatment

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  - Or put it differently, the effect of treatment assignment on outcome only matters through actual treatment

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- Effect of $D$ on $Y$ measures actual treatment effect
- We formally develop the idea using counterfactual framework

# New IV assumptions

1. Exogeneity: $Z$ is as-if randomly assigned
   - Compare with the previous exogenous regressor assumption, which one do you prefer?

2. Exclusion restriction: $Z$ does not has a direct effect on $Y$

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   - Under counterfactual framework, once we fix the value of the actual treatment $D$, $Z$ does not impact $Y$
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# Potential Outcome Framework under Non-Compliance

- Previously we know potential outcome for outcome $Y$

$$Y_i = \begin{cases} Y_i^0 : D_i = 0 \\ Y_i^1 : D_i = 1 \end{cases}$$

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- We only observed $D_i$, not $D_i^0$ and $D_i^1$

# Compliance types in the ideal world

| | $D_i^0 = 0$ | $D_i^0 = 1$ |
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| $D_i^1 = 0$ | never-taker | defier |
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- Defier: those who would always take the treatment if not assigned to, and vice versa

# Compliance type in observed data

- Each time we only observe one of $D_i^0$ and $D_i^1$, never both

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- And it is easier to understand

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- Angrist, Imbens, and Rubin, 1996, JASA

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- In general, $LATE \neq ATT = E(Y^1 - Y^0|D=1))$
- Traditional IV "cheats" by assuming constant treatment effect, thus forcing the effect to be the same for compliers and always-takers

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- $LATE = ATT$ when there is no always-taker

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- Let us have a thought exercise
- Who are compliers?
  - people would be actually utilize the "advantage" that they were born in the 1st quarter to dropout after 16

# ITT vs LATE

- To understand why LATE may not generalize to the whole population, there is an alternative view

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- LATE is almost always larger than ITT

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  - That is, even among the group that were assigned to treatment, due to self-selection into taking the treatment, ATT will be a lot bigger than ITT

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- ITT: if gov want to implement the policy (e.g., providing Vitamin A for everyone), they must take into consideration that some people may not follow the instruction. So ITT provides a more faithful evaluation of the effect on the population affected by the policy
- So be careful how compliers differ from the general population if you care more about ITT, instead of LATE

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  - Also continuous instrument/multiple instruments/adding covariates/HTE with IV

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- And the combination of the two exacerbates each other

# Types of IV

Table 2. Types of IVs

| IV Type | #Papers | Percentage% |
|---|---|---|
| **Theory** | 40 | 62.5 |
| Geography/climate/weather | 10.5 | 16.4 |
| History | 10 | 15.6 |
| Treatment diffusion | 2.5 | 3.9 |
| Others | 17 | 26.6 |
| **Experiment** | 12 | 18.8 |
| **Rules & policy changes** | 5 | 7.8 |
| Change in exposure | 3 | 4.7 |
| Fuzzy RD | 2 | 3.1 |
| **Econometrics** | 7 | 10.9 |
| Interactions/"Bartik" | 5 | 7.8 |
| Lagged treatment | 1 | 1.6 |
| Empirical test | 1 | 1.6 |
| **Total** | 64 | 100.0 |

***Note:*** One paper uses both geography-based instruments and an instrument based on treatment diffusion from neighbors. We count 0.5 for each category.

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- In practice, you should use a more conservation standard error, which also lead to smaller $F$ statistics calculation

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- Note that it's easier to address weak instrument issue, but it's much harder to see if there is violation of exogeneity

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  - note: you also need to use theory to know who are likely to be never-takers