Fixed Effects and Difference-in-Differences

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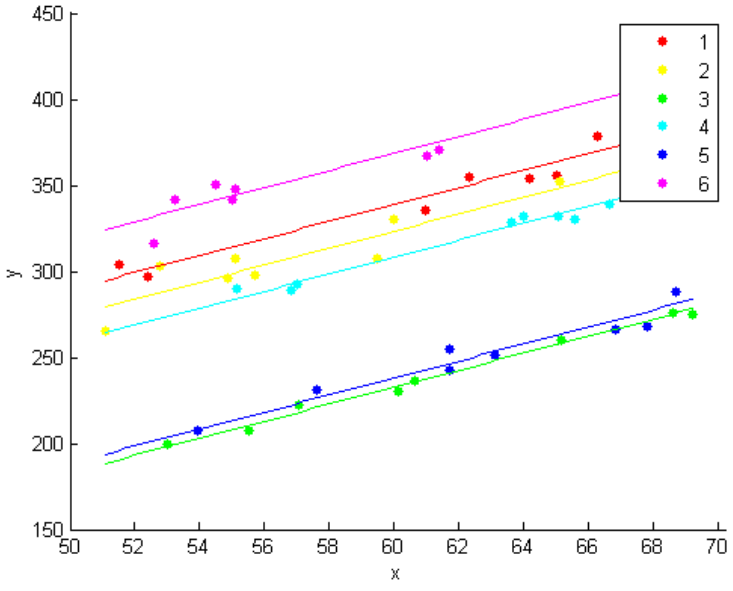
Outline

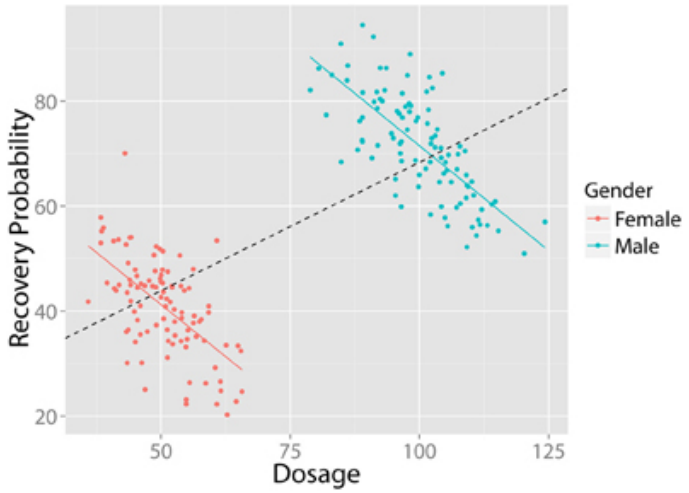
Fixed Effect Regression



- Each group has its own intercepts
- But slope is the same
- Also called varying intercept (same slope) model
  - contrasting varying intercept and varying slope model

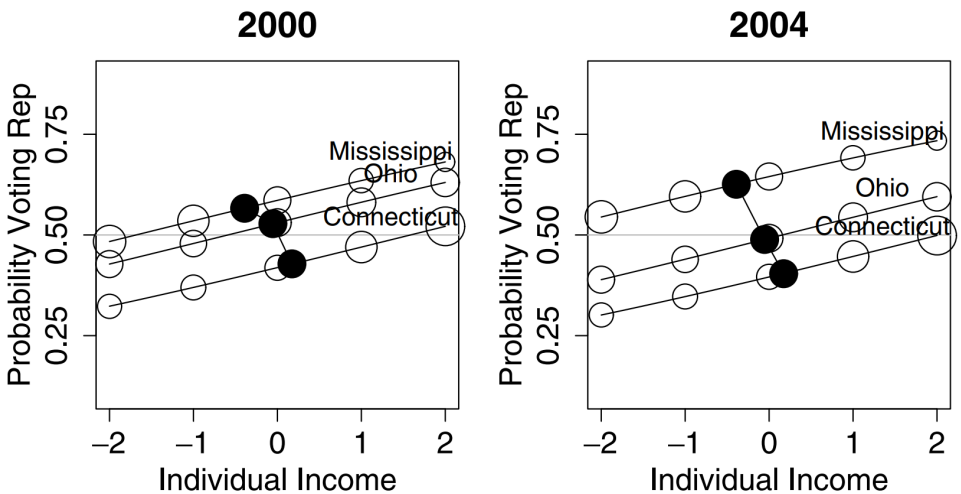
Fixed effect and Simpson's paradox

- Simpson's paradox: correlation between X and Y is reversed at individual and at macro-level
  - Remember the kidney stone example in the 1st lecture?
- Kiev it et al., 2013, "Simpson's paradox in psychological science"
- The mean of male/female suggests that dosage and recovery prob is *positively* correlated
- Within each group, at individual level, dosage and recovery prob is *negatively* correlated



Another example: blue state vs red state

- Gelman, 2008, "Red State, Blue State, Rich State, Poor State"
- Within each state, rich people have a high prob to vote for Rep (individual level)
- But richer states (e.g., NY, CA) have a higher prob to vote for Dem (macro level)



Group Structure

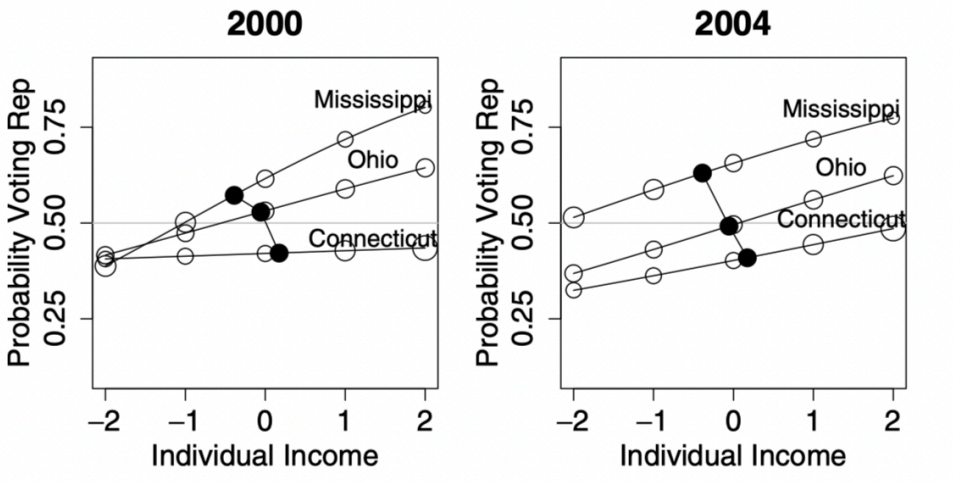
- the correct relationship is what you obtained at the micro level
  - The behavioral theory is people take medicines/votes; not groups do sth
- If we mistakenly model the data at aggregate level, we may obtain a wrong relation
- Two ways of running regression within group structure:

Fixed effect

- Fixed effect:
  - Also called varying-intercept model
  - Or least square dummy variable model (LSDV)
  - Add group as additional regressor
- i indexes individuals and j indexes groups
- simple OLS: $Y_{ij} = \alpha + \rho D_{ij} + \epsilon_{ij}$
  - or $Y_i = \alpha + \rho D_i + \epsilon_i$
  - because you don't care about groups so we do not need to index them
- fixed effect: $Y_{ij} = \alpha_j + \rho D_{ij} + \epsilon_{ij}$

Random effect

- Random effect:
  - Also called varying-intercept, varying-slope model
  - Equation: $Y_{ij} = \alpha_j + \rho_j D_{ij} + \epsilon_{ij}$
  - It's hard to fit; if you are interested, read Gelman and Hill, *Data Analysis using Regression and Multilevel Hierarchical Models*, 2007.



Random effect vs. Fixed effect

- Random effect vs. fixed effect
  - If you are more from a data modeling perspective (e.g., you care about final prediction performance), then random effect makes more sense
  - If you are more from a causal inference perspective, then fixed effect is more widely used

Counterfactual view of fixed effect

- Selection on observables is often a very strong assumption; how do we make it more realistic?
- Intuition: cluster-randomized experiments, with conditional random assignment
  - Within a certain group (e.g., school), treatment assignment is randomized
- Similarly, we may find natural grouping in real worlds that let you make selection on observable assumptions, within groups

Twin studies

- Example: we are interested in whether college degree leads to higher wages
  - Unobserved factors we cannot control: many family and genetic backgrounds
- Twin studies: find twins that grow up together, but have different schooling length
  - We can assume that twins are very similar in their unobserved family and genetic backgrounds;
  - In other words, within each group (here, twin pair), units share unobserved but fixed characteristics
  - If we compare within twin pairs, we can rule out unobserved but fixed characteristics
  - Then the difference in outcome should be mainly driven by different in schooling length, not other unobserved confounders
- Question: can you think of some cases when this assumption is violated?

Counterfactual framework

- Use i to denote individuals, and j for twin pairs, X_{ij} for observed individual-level variables, and Y_{ij} is outcome
- $D_{ij} = 1$ if i received college education and 0 if not
- Twin studies assume that:
  - Unobserved factors α_j are constant for each group
  - Selection bias is 0 within each group, and conditional on X
    - In math, $E(Y_{ij}^0 | D_{ij} = 1, \alpha_j, X_{ij}) = E(Y_{ij}^0 | D_{ij} = 0, \alpha_j, X_{ij})$
    - That is, if A and B are twins, and A is treated and B is not. We expect that A's counterfactual outcome can be predicted by B's observed outcome
- With the above assumptions, we can estimate ATT using non-parametric difference-in-means estimator
  - Calculate difference in outcomes for each twin pair
  - Take the mean of these differences

Fixed Effects Regression

- If we want to derive a regression estimator, we need additional assumptions.
- Linear and additive counterfactual outcome for untreated units

$$E(Y_{ij}^0|j, X_{ij}) = \alpha_j + X_{ij}\beta$$

- Constant individual-level treatment effect ρ

$$E(Y_{ij}^1|A_j, X_{ij}, D_{ij}) = E(Y_{ij}^0|A_j, X_{ij}) + \rho = \alpha_j + X_{ij}\beta + \rho$$

- Constant treatment effect assumptions is shared by regression estimator of experiments
- Linear and additive counterfactual is unique here; we do not need it to derive regression estimator for randomized experiments

Fixed effects Regressions

- The above assumptions mean that:

$$E(Y_{ij}|X_{ij}, D_{ij}) = \alpha_j + D_{ij}\rho + X_{ij}\beta \quad (1)$$

- Or alternatively, $Y_{ij} = \alpha_j + D_{ij}\rho + X_{ij}\beta + \varepsilon_{ij}$, $E(\varepsilon_{ij}) = 0$, $E(\varepsilon_{ij}|D_{ij}) = 0$ and $E(\varepsilon_{ij}|X_{ij}) = 0$
- This is a regression with group-specific fixed effects (α_j);
- From a pure prediction perspective, fixed-effect regressions means there is separate intercept α_j for each group j
- From causal inference perspective, α_j were motivated as unobserved factors at the group-level

Fixed effect regression

- With all assumptions we have so far, regression coefficient ρ identifies causal effect ATT

$$\begin{aligned}\rho &= E(Y_{ij}^1|A_j, X_{ij}) - E(Y_{ij}^0|A_j, X_{ij}) \\ &= E(Y_{ij}^1|A_j, X_{ij}, D_{ij} = 1) - E(Y_{ij}^0|A_j, X_{ij}, D_{ij} = 1) \qquad (2) \\ &= ATT\end{aligned}$$

- A recap of assumptions we needed:
  - Assumptions about treatment assignment process:
    - Unobserved factors are at the group level;
    - Selection on observables, within groups
  - Assumptions about parametric model (non-parametric estimator does not require these; regression estimator need these)
    - Linear and additive counterfactual outcome
    - Constant treatment effect

Group structure in panel data

- Panel data often exhibit natural group structure
  - Repeated measure for a single unit
- Dell, Jones, and Olken (2011): Temperature Shocks and Economic Growth: Evidence from the Last Half Century, AEJ.
- Key question: whether hot temperature leads to poor economy

  There are countries where the excess of heat enervates the body, and renders men so slothful and dispirited that nothing but the fear of chastisement can oblige them to perform any laborious duty. [Montesquieu, 1750]

- Problem: many unobserved factors can explain poor economy
- Solution: assume that unobserved factors are at the country level but time-invariant
- Then, within each country:
  - Whether hotter years result in lower GDP

Fixed effects as a dummy variable model

$$g_{ct} = \alpha_c + \beta TEMP_{ct} + \varepsilon_{ct} \tag{3}$$

- We have learned that $\beta = ATT$, if all assumptions hold

$$g_{ct} = \alpha_c + \beta TEMP_{ct} + \varepsilon_{ct}$$

- α_c are unobserved unit-level factors.
- How do we estimate the above model in software?
- We can simply treat α_c as parameters to estimate for each unit (i.e., least square dummy variable model)

	g_{ct}	$TEMP_{ct}$	α_{USA}	α_{INDO}	a_{NIGER}
USA $_{2010}$	3	12	1	0	0
USA $_{2011}$	3.2	14	1	0	0
Indonesia2010	1	22	0	1	0
Indonesia$_{2011}$	1.3	23	0	1	0
Niger $_{2010}$	0.1	28	0	0	1
Niger $_{2011}$	0.1	27	0	0	1

(4)

Fixed effects as deviation from means

- We have learned that $\beta = ATT$, if all assumptions hold

$$g_{ct} = \alpha_c + \beta TEMP_{ct} + \varepsilon_{ct}$$

- α_c are unobserved unit-level factors.
- In fact, α_c do not matter, if the goal is to estimate β
- Take the mean within each country:

$$\overline{g_c} = \overline{\alpha_c} + \beta \overline{TEMP_c} + \bar{\varepsilon}_c = \alpha_c + \beta \overline{TEMP_c} + \bar{\varepsilon}_c \quad (5)$$

- Subtracting to get deviation from means:

$$g_{ct} - \overline{g_c} = \beta \left(TEMP_{ct} - \overline{TEMP_c} \right) + \varepsilon'_{ct} \quad (6)$$

- In other words, the estimation of causal effect β can be purely done within each group
- this representation is called within estimator

Fixed effects as deviation from means

	$g_{ct} - \overline{g_c}$	$TEMP_{ct} - \overline{TEMP_c}$
USA $_{2010}$	-0.1	-1
USA $_{2011}$	0.1	1
Indonesia$_{2010}$	-0.15	-0.5
Indonesia$_{2011}$	0.15	0.5
Niger $_{2010}$	0	0.5
Niger $_{2011}$	0	-0.5

(7)

- Is the overall relationship between temperature and growth positive or negative?

Fixed effects as difference in time

$$g_{ct} = \alpha_c + \beta TEMP_{ct} + \varepsilon_{ct}$$

- In panel data, from a causal inference perspective, an alternative way to rule out unit-level unobserved factors is to take difference by time
- Use Δ to represent difference between the time t and time $t-1$
  - e.g., $\Delta g_{ct} = g_{ct} - g_{c,t-1}$
- Apply Δ on both sides:

$$\Delta g_{ct} = \beta \Delta TEMP_{ct} + \Delta \varepsilon_{ct} \quad (8)$$

- Again, taking the first difference within groups cancels out the group-level unobserved factors
- In panel data, this is often known as taking the first difference

Fixed effects as different in time

- Taking the first difference with respect to time:

	Δg_{ct}	$\Delta TEMP_{ct}$
USA	0.2	2
Indonesia	0.3	1
Niger	0	-1

(9)

- This is known as first difference estimator
- Is the overall relationship between temperature and growth positive or negative?

Different setup of fixed effect

- Different setups of fixed effect typically result in the same point estimate of coefficients
  - This statement may not be true in real studies, when you have missing data
  - For instance, for a country across 5 years, the outcome is 1,1,2,?,3 (with one missing data)
    - first-difference estimator will probably remove (2,?) and (?,3)
    - within estimator only remove 1 unit
- Standard errors are typically different
  - E.g., within-estimator effectively keeps all observations in a group
  - But first-difference estimator reduces the number of observation by 1

Standard errors in fixed effects

- Standard errors should be clustered at group level
- For one-way fixed effect, you naturally have one group
- For two-way fixed effect, group and time level
- Caution: if the number of groups or number of time periods are too small
  - even though we know theoretically it's a correct thing to cluster the standard errors at group and time level
  - numerically, it's often not feasible to obtain a stable estimates
  - typically, if the number of groups is larger than 30, you should be fine
  - Implication: if you have a short panel (like 10 years), even if you know that you should cluster the standard errors also at year level, in practice the results may subject to this small-group-number bias

Fixed effect

- Think of the twin studies, if a twin pair have the same length of education (very likely), they basically do not contribute any useful information
- This means that fixed effect only uses units that have variations within each group
  - put it differently, it discards many data from your datasets
- This also explains why you often see very high R^2 from fixed effect fits, because you do not seek to explain the unobserved (but not varying) confounder; you just removed them so that you do not need to explain them

Violation of one-way fixed effect assumption in panel data

- We have seen cases with one grouping (one-way fixed effect). One key assumption is
  - unobserved confoundings is fixed within group
  - E.g., in Dell et al. example, group is country
    - all unobserved factors are fixed for each country
- What if the assumption is not correct? Say, unobserved factors can also vary by time?
- E.g., each year there may be some common global trends (e.g., financial crisis in 2008) that impact all countries

Two-way fixed effects

- In such case, we have two natural groups: country and time
- If we further assume that unobserved factors are either fixed by country (but vary by time) or fixed by time (but vary by country)
- Two-way fixed effect regression

$$Y_{it} = \alpha_i + \beta_t + D_{it}\rho + X_{it}\beta + \varepsilon_{it} \tag{10}$$

- α_i: country fixed effect
- β_t: time fixed effect
- $\rho = ATT$ if:
  - Unobserved confounders are either fixed by country or fixed by time
  - all other assumptions of one-way fixed effects (linear and additive counterfactual outcome, constant treatment effect) are also needed.

Two-way fixed effect

- We have already taken the first difference by time

	Δg_{ct}	$\Delta TEMP_{ct}$
USA	0.2	2
Indonesia	0.3	1
Niger	0	-1

(11)

- We can take the second difference by country, which removes time-fixed unobserved factors

	Δg_{ct}	$\Delta TEMP_{ct}$
Indonesia - USA	0.1	-1

(12)

- negative relationship between temperature and growth

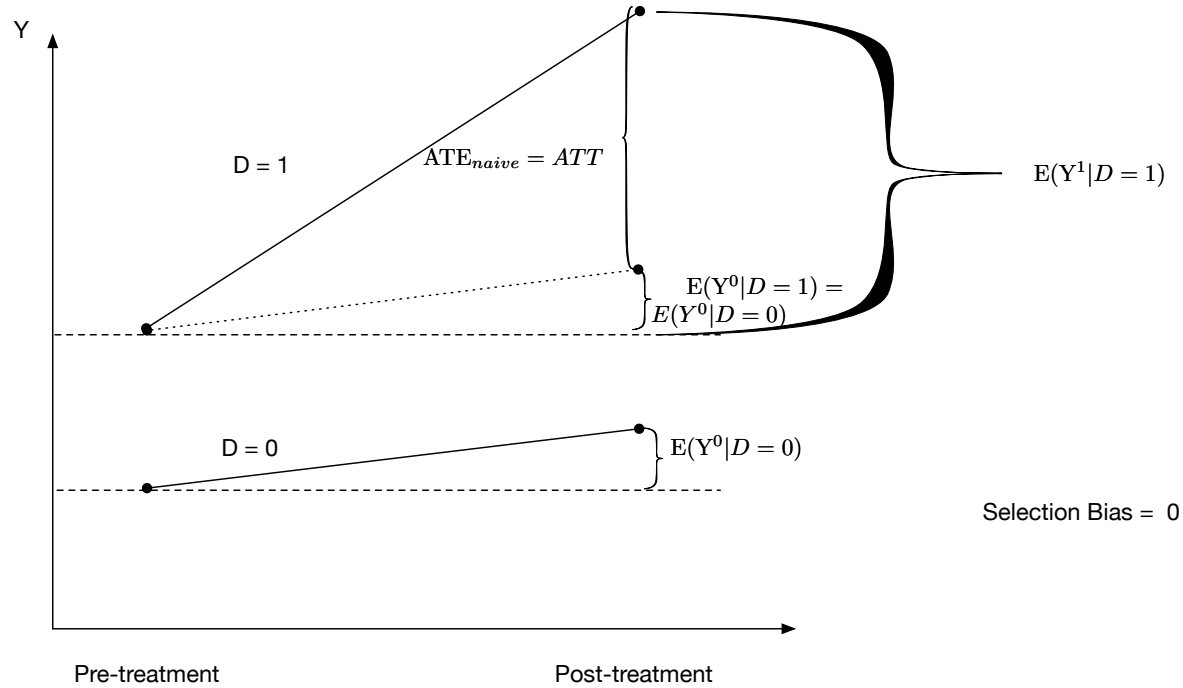
Difference-in-differences

- With two units and two periods, two-way fixed effects become difference-in-differences (DiD) estimator
- Card and Kruger, 1994, "Minimum wages and employment", AER
- Whether raising mimimum wage lead to decrease in employment?
- Treatment:
  - $D = 1$: New Jersey raised the state minimum wage from \$4.25 to \$5.05 on April 1, 1992.
  - $D = 0$: Nearby Pennsylvania, although very similar to New Jersey, did not.
- Outcome Y: average change in the number of Full-time Equivalent (FTE) jobs in fast food restaurants

DiD and experimental ideal

- DiD approximates experimental ideal explicitly:
  - One treatment and one control group
  - Treatment and control groups are very similar; they would change in the same speed had the treatment group not being treated
  - Outcome is measured as change of Y after/before the treatment
- Fixed effect implicitly approaches experiments
- It's much easier to make your study cleaner in DiD's framework
  - like when DiD assumptions is correct, and when DiD assumption is not valid
- But in fixed-effect framework, you are just adding "fixed effects" in a regression, which makes it harder to understand whether the assumptions are correct or not

DiD Assumptions: Parallel Trends



- In DiD setting, selection bias $= 0$ ($E(Y^0|D=1) = E(Y^0|D=0)$) assumption can be visualized
  - $Y^0 = Y^0_{i,t=1} - Y^0_{i,t=0}$ is the change of i after/before treatment
- It's often called parallel trends assumption: if there were no treatment, treated units ($D=1$)'s counterfactual outcome Y^0 would be the same as that of the control units

DiD as differences-in-means

- MHE, Table 5.2.1

time/state	PA (control)	NJ (treated)	NJ - PA
Before (t = 0)	23.33	20.44	-2.89
After (t = 1)	21.17	21.03	-0.14
After - Before	-2.16	0.59	2.76

- First calculate difference within state by time, which cancels out state fixed effect
- Next calculate difference of the differences, which cancels out time fixed effect

DiD as two-way fixed effect

$$Y_{st} = \alpha_s + \beta_t + \rho D_{st} + \epsilon_{st}, D_{st} = 1 \text{ when } s = \text{treated } \& t = 1 \quad (13)$$

- First calculate difference within state by time, which cancels out state-fixed effect α_s
  - $Y_{NJ,1} - Y_{NJ,0} = \beta_1 - \beta_0 + \rho(1 - 0) = -2.16$
  - $Y_{PA,1} - Y_{PA,0} = \beta_1 - \beta_0 + \rho(0 - 0) = 0.59$
- Next calculate difference of the differences, which cancels time-fixed effect β_t
  - $(Y_{NJ,1} - Y_{NJ,0}) - (Y_{PA,1} - Y_{PA,0}) = \rho$

Difference-in-differences as a linear regression with interaction

- We can further simplify DiD as an OLS with an interaction term
  - For fixing fixed effect, you need special packages (in R, `plm` or `fixest`)
  - If you could fit DiD by OLS, it's easier coding wise
- Define d_s as treatment unit dummy (here, $d_s = 1$ if the state is NJ)
- Define d_t as time dummy($d_t = 1$ if after treatment)
- $D_{st} = d_s \cdot d_t$ ($D_{st} = 1$ when both s is treated and t is 1)

DiD as a linear regression with interaction

The two models are equivalent:

$$\begin{aligned} Y_{st} &= \alpha_s + \beta_t + \rho D_{st} + \epsilon_{st}, \text{ fixed effect} \\ Y_{st} &= \gamma + \lambda d_s + \tau d_t + \delta(d_s \cdot d_t) + \epsilon_{st}, \text{ interaction OLS} \end{aligned} \tag{14}$$

- Because the previous DiD table can be exactly expressed as a linear regression with interactive model

time/state	PA (control)	NJ (treated)	NJ - PA
Before (t = 0)	γ	$\gamma + \lambda$	λ
After (t = 1)	$\gamma + \tau$	$\gamma + \lambda + \tau + \delta$	$\lambda + \delta$
After - Before	τ	$\tau + \lambda$	δ

- And estimate effect $\rho = \delta = ATT$

Additional Controls

- Non-parametric model needs fewer assumptions
- But with the regression setup (either fixed-effect or interaction model), it is easier to add additional controls
- E.g., we may have some individual-level controls X_{ist} (characteristics of employers);
- We can easily include these X_{ist}, and use micro-level data (instead of aggregate state-year level)

$$
\begin{aligned}
Y_{ist} &= \alpha_s + \beta_t + \rho D_{st} + \mu X_{ist} + \epsilon_{ist}, \text{ fixed effect} \\
&= \text{or, equivalently} \\
Y_{ist} &= \gamma + \lambda d_s + \tau d_t + \delta (d_s \cdot d_t) + \mu X_{ist} + \epsilon_{ist}, \text{ interaction OLS}
\end{aligned} \tag{15}
$$

Standard errors in fixed effects and DiD

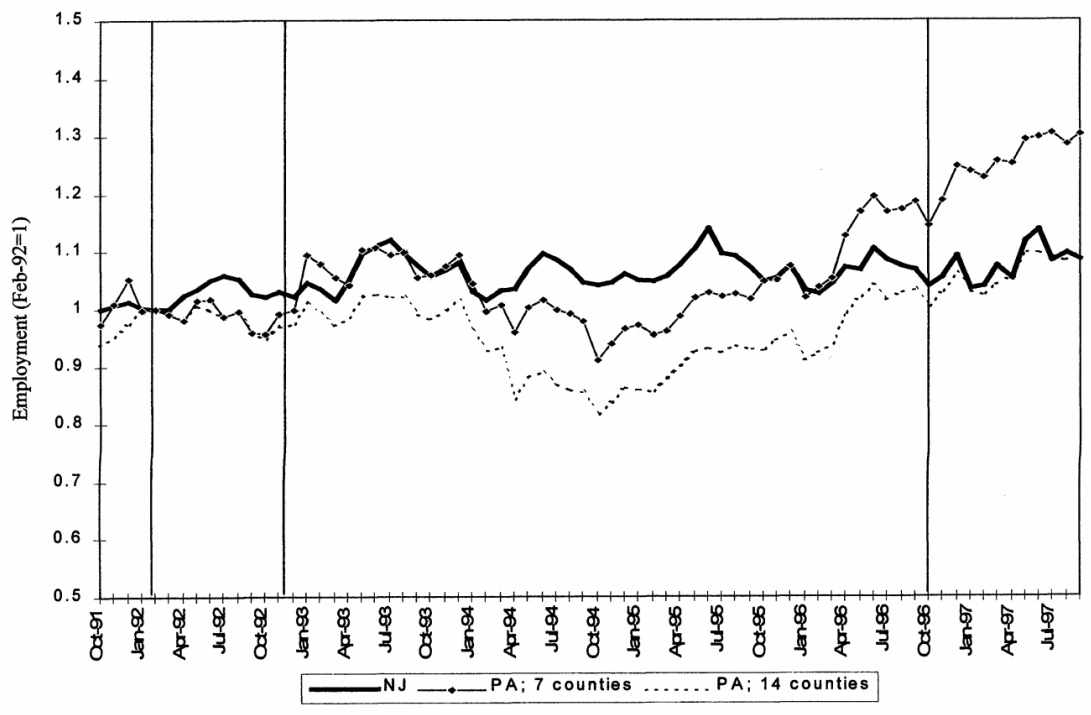
- Bertrand, Duflo and Mullainathan, 2004, "How much should we trust differences-in-differences estimates", QJE
  - If you have added additional control variables within each group
  - If you had use two-way fixed effects setup
    - group structure are clearly specified
    - you should cluster standard errors
  - But, if you had used an OLS with interaction model
    - It is not immediately evident that you need to cluster standard errors
  - if standard errors are not clustered at group level
  - DiD estimates from a linear regression with interactions will seriously underestimate standard error of coefficients

Testing Parallel Trends Assumption

- Strictly speaking, parallel trends assumption (or more general, unobserved factors are either fixed at unit or time-level) are not testable
  - Because it assumes something about $E(Y^0|D=1)$ which can never be observed
- In simple setting such as DiD, and when we have more than one period before the treatment
- We can perform check on the pre-trends
  - If the two groups truly would have exhibited parallel trends in the absence of treatment
  - Then their trends in Y would be parallel prior to treatment
- Note that parallel pre-trends does not mean parallel trends
  - To have the parallel trends assumption we need help from theory
- But violation of parallel pre-trends suggests that parallel trends assumption do not hold

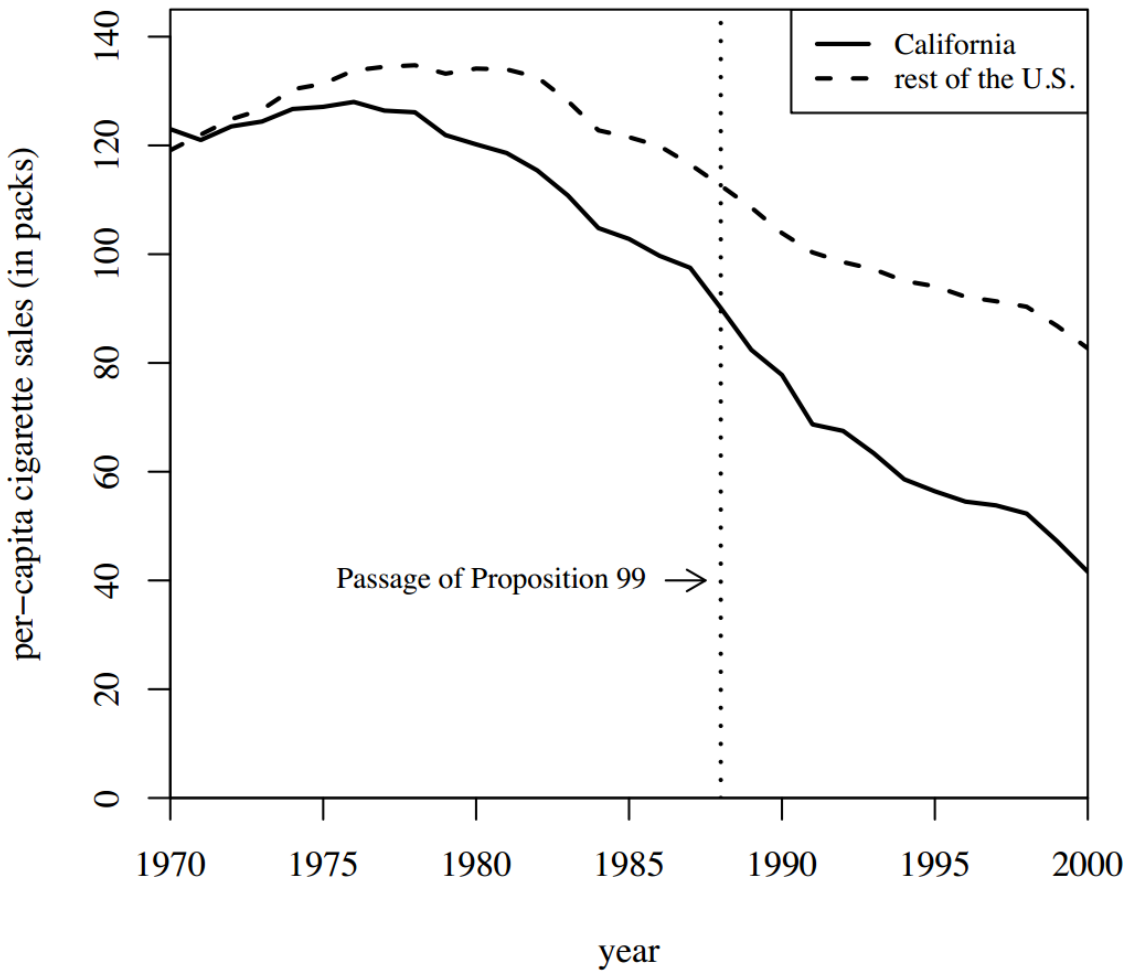
Graphical Test of parallel pre-trends

- A bad one: MHE, Card and Krueger, 1994, AER (Figure 5.2.2 in MHE)



Graphical test of parallel pre-trends

- A good one: Abadie, Diamond and Hainmuller, 2010, JANA.
- Effect of Proposition 99 (a tobacco control program in Cali.)

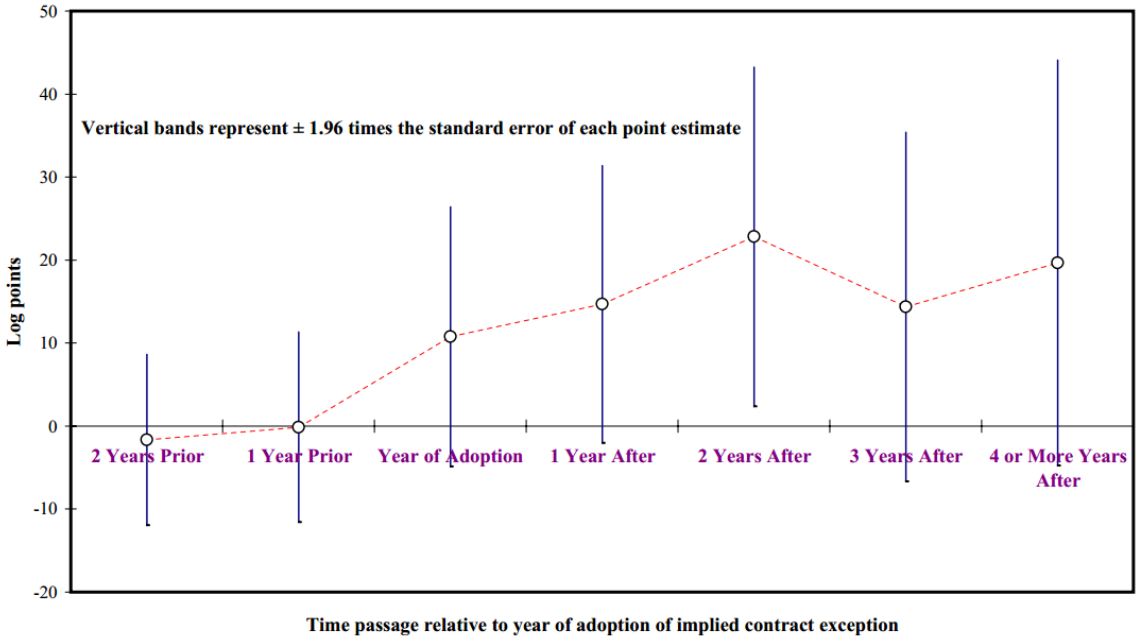


Event study

- In addition to eyeballing, you can perform event study to test the parallel pre-trends
- Regular DiD:
  - $d_t = 1$ if after treatment and 0 otherwise
  - $Y_{st} = \gamma + \lambda d_s + \tau d_t + \delta(d_s \cdot d_t) + \epsilon_{st}$
- Event study model: from binary time dummy to time indexes before/after treatment
  - $d_t = \cdots -3, -2, 0, 1, 2, 3 \cdots$ (-1 used for reference group)
  - Now, from one δ to many δ_t, one for each time index
  - You should expect δ_t before the treatment to be not statistically significant
    - Thus treatment/control are similar before treatment
  - And δ_t after the treatment to be statistically significant

Event study

- One of the early study that adopted event study model is
- Autor, David H. "Outsourcing at will: The contribution of unjust dismissal doctrine to the growth of employment outsourcing." *Journal of Labor Economics* 21, no. 1 (2003): 1-42.



Fixed Effects and DiD

- Two-way Fixed Effects and DiD share the same set of assumptions (if you use regression estimator)
- DiD is
  - simpler (two periods, two units)
  - approximates experimental ideal explicitly; force you to think through the language of randomized controlled experiments
  - More easily to visualize and thus to recognize wrong assumptions
- Two-way fixed effects can incorporate more years and units and thus produce more precise and robust results
  - But you may just easily assume that the unobserved factors are fixed at unit or time level, without thinking it clearly as you would do for a DiD