

# SOSC 4300/5500: Networks

Han Zhang

# Outline

Networks

Small-world network: empirical approach

Small-world network: theoretical approach

Summary of what we have learned so far

## Capturing inter-connections

- Network analysis is a general theoretical approach to study connections

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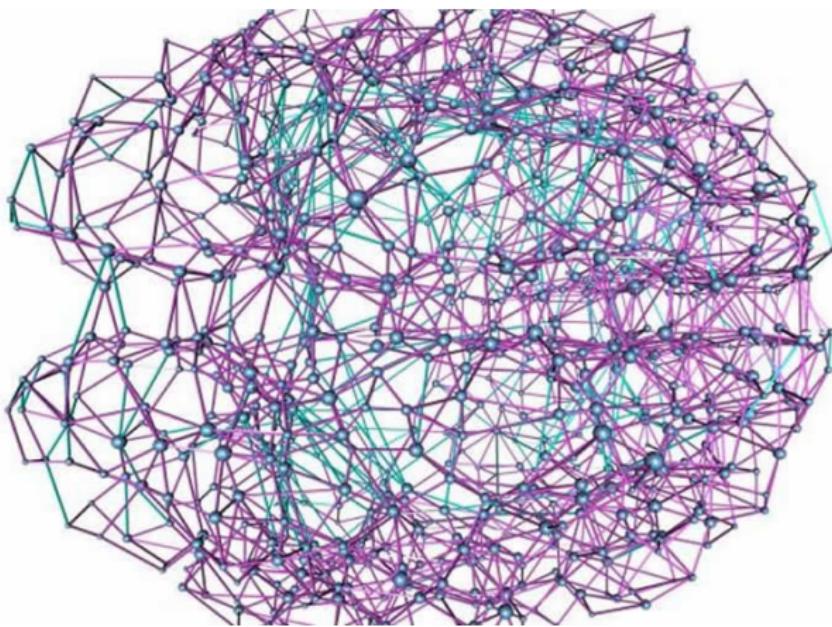
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- David Lazer, Alex Pentland, Lada Adamic, Sinan Aral, Albert-László Barabási, Devon Brewer, Nicholas Christakis, Noshir Contractor, James Fowler, Myron Gutmann, Tony Jebara, Gary King, Michael Macy, Deb Roy, and Marshall Van Alstyne, *Computational Social Science*, *Science* **323** (2009), no. 5915, 721–723

## Facebook network



## Brain neural networks



Global Aviation Network



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  - $G = \langle V, E \rangle$

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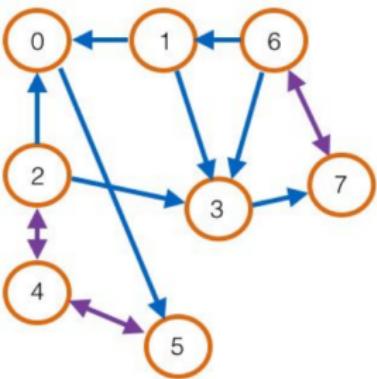
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  - Especially when the network is sparse
  - Usually social networks are sparse

## Edge list

### Graph Representation: Edge List



From	To
0	5
1	3
1	0
2	0
2	3
2	4
3	7
4	2
4	5
5	4
6	1
6	3
6	7
7	6

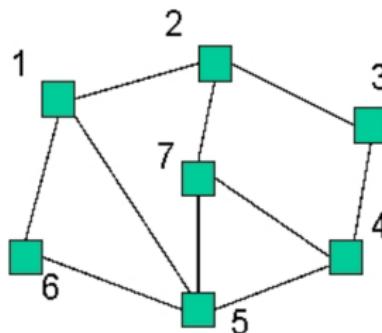
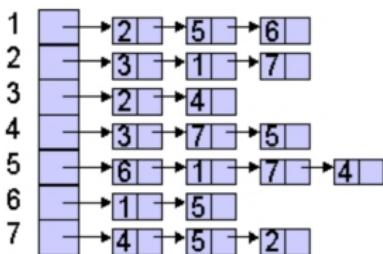
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### Edge List and Adjacency Lists

- List of edges

1	5	1	2	2	3	5	7	5	5
2	1	6	7	3	4	6	4	7	4

- Adjacency lists



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- Degree characterizes the importance of a node in a network

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- Slightly more complicated version:
  - Ask whether two friends know each other

## Ego-centered network: example

- GSS (General Social Survey)

*From time to time, most people discuss important matters with other people. Looking back over the last six months - who are the people with whom you discussed matters important to you? Just tell me their first names or initials. If LESS THAN 5 NAMES MENTIONED, PROBE, Anyone else? ONLY RECORD FIRST 5 NAMES.*

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- Why don't they directly ask "who is your friend?"

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- It's just an interviewer effect: the question in 2004 survey appeared at the end so people were tired and did not want to answer

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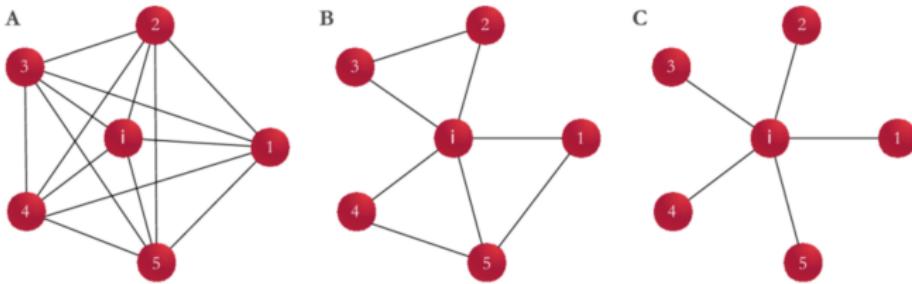
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- The numerator, number of possible edges for a node with  $d$  edges is  $\frac{d*(d-1)}{2}$

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  - And people may not want to give their friends list, or they simply do not remember it
- Using surveys to measure whole network has only been successful in small and closed communities, such as schools or small villages

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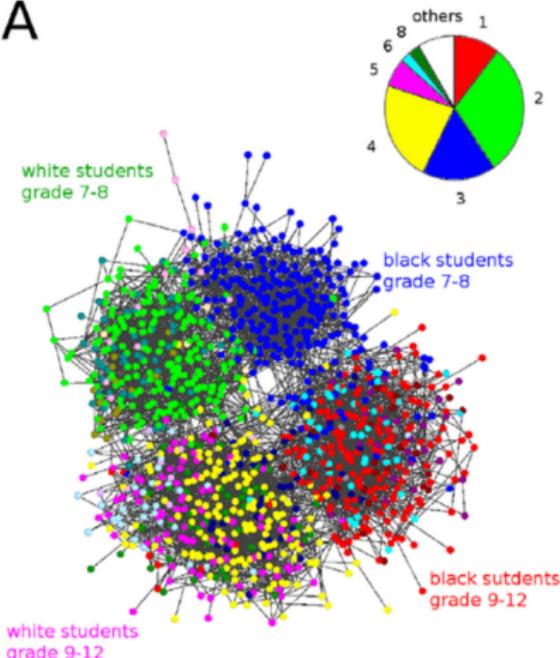
## Community structure with whole network

- With whole network, we can explore **community structure**
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- This provides information on social groups, not defined by their attributes but by their connections

## Add Health Friendship

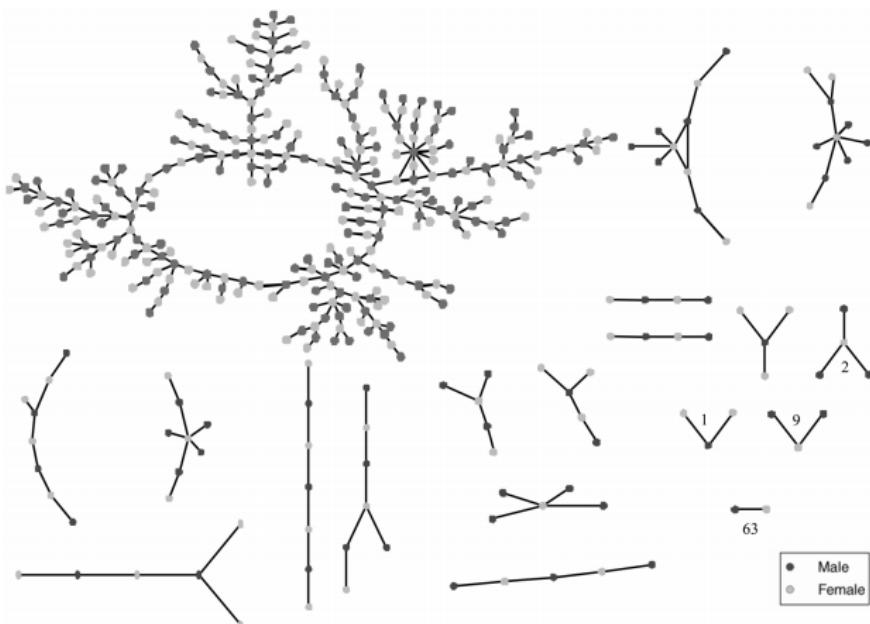
- Add Health is a famous survey dataset that sampled 80 US schools and construct whole network within each school

A



## Add Health Dating Network

- Peter S. Bearman, James Moody, and Katherine Stovel,  
*Chains of Affection: The Structure of Adolescent Romantic and Sexual Networks*, American Journal of Sociology **110** (2004), no. 1, 44–91



## Measuring whole networks in the age of big data

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  - But remember these are **re-purposed** data. Do Facebook capture all my friendships? Certainly not

## Small-world experiment

- People found out how to study a particular aspect of the whole network structure—the distance between two nodes—without actually measuring the whole network

*I read somewhere that everybody on this planet is separated by only six other people. Six degrees of separation between us and everyone else on this planet. The President of the United States, a gondolier in Venice, just fill in the name... It's not just the big names. It's everyone. A native in the rain forest. An Eskimo. I am bound to everyone on this planet by a trail of six people. It's a profound thought.*

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- [https://www.imdb.com/title/tt0108149/?ref\\_=tt\\_ch](https://www.imdb.com/title/tt0108149/?ref_=tt_ch)

# History of Small-World Experiment

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## An Experimental Study of the Small World Problem\*

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Harvard University

AND

STANLEY MILGRAM

The City University of New York

*Arbitrarily selected individuals ( $N=296$ ) in Nebraska and Boston are asked to generate acquaintance chains to a target person in Massachusetts, employing "the small world method" (Milgram, 1967). Sixty-four chains reach the target person. Within this group the mean number of intermediaries between starters and targets is 5.2. Boston starting chains reach the target person with fewer intermediaries than those starting in Nebraska; subpopulations in the Nebraska group do not differ among themselves. The funneling of chains through sociometric "stars" is noted, with 48 per cent of the chains passing through three persons before reaching the target. Applications of the method to studies of large scale social structure are discussed.*

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# Questionnaire used



We need your help in an unusual scientific study carried out at Harvard University. We are studying the nature of social contact in American society. Could you, as an active American, contact another American citizen regardless of his walk of life? If the name of an American citizen were picked out of a hat, could you get to know that person using only your network of friends and acquaintances? Just how open is our "open society"? To answer these questions, which are very important to our research, we ask for your help.

You will notice that this letter has come to you from a friend. He has aided this study by sending this folder on to you. He hopes that you will aid the study by forwarding this folder to someone else. The name of the person who sent you this folder is listed on the Roster at the bottom of this sheet.

In the box to the right you will find the name and address of an American citizen who has agreed to serve as the "target person" in this study. The idea of the study is to transmit this folder to the target person using only a chain of friends and acquaintances.

#### TARGET PERSON

Dana G. Winsor  
32 Harold Street  
Sharon, Massachusetts

Occupation: Investment  
Broker at McDonnell & Co.  
211 Congress Street  
Boston, Mass.

Married: to the former  
Suzel Pike of Randolph, Mass.  
Attended: Northeastern Univ.  
from 1955-56; served in  
U.S. Air Force

#### HOW TO TAKE PART IN THIS STUDY

**1**

ADD YOUR NAME TO THE  
ROSTER AT THE BOTTOM OF  
THIS SHEET, so that the next  
person who receives this letter  
will know who it came from.

**3**

IF YOU KNOW THE TARGET  
PERSON ON A PERSONAL  
BASIS, MAIL THIS FOLDER  
DIRECTLY TO HIM (HER). Do  
this only if you have previously met the target  
person and know each other on a first name  
basis.

**2**

DETACH ONE POSTCARD.  
FILL IT OUT AND RETURN IT  
TO HARVARD UNIVERSITY.  
No stamp is needed. The post-  
card is very important. It allows us to keep  
track of the progress of the folder as it moves  
toward the target person.

**4**

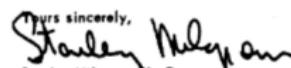
IF YOU DO NOT KNOW THE  
TARGET PERSON ON A PER-  
SONAL BASIS, DO NOT TRY  
TO CONTACT HIM DIRECTLY.  
INSTEAD, MAIL THIS FOLDER (POST CARDS  
AND ALL) TO A PERSONAL ACQUA-  
INTANCE WHO IS MORE LIKELY THAN YOU  
TO KNOW THE TARGET PERSON. You may  
send the folder on to a friend, relative, or  
acquaintance, but it must be someone you know  
on a first name basis.

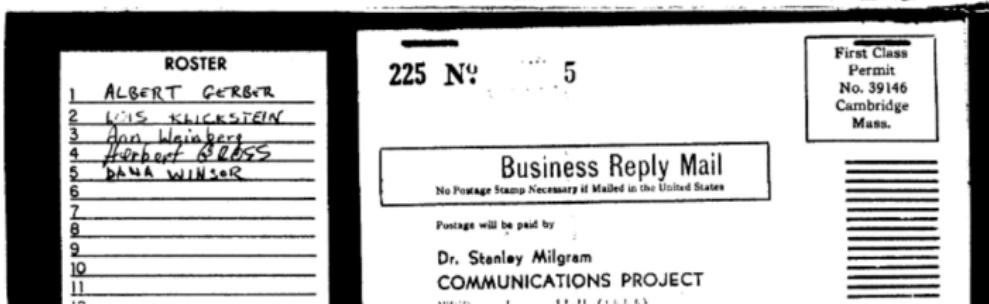
## Questionnaire used

Remember, the aim is to move this folder toward the target person using only a chain of friends and acquaintances. On first thought you may feel you do not know anyone who is acquainted with the target person. This is natural, but at least you can start it moving in the right direction! Who among your acquaintances might conceivably move in the same social circles as the target person? The real challenge is to identify among your friends and acquaintances a person who can advance the folder toward the target person. It may take several steps beyond your friend to get to the target person, but what counts most is to start the folder on its way! The person who receives this folder will then repeat the process until the folder is received by the target person. May we ask you to begin!

Every person who participates in this study and returns the post card to us will receive a certificate of appreciation from the Communications Project. All participants are entitled to a report describing the results of the study.

Please transmit this folder within 24 hours. Your help is greatly appreciated.

Yours sincerely,  
  
 Stanley Milgram, Ph. D.  
 Director, Communications Project



- [https://en.wikipedia.org/wiki/Small-world\\_experiment#/media/File:Experiment\\_Small\\_World\\_\(possible\\_option\).gif](https://en.wikipedia.org/wiki/Small-world_experiment#/media/File:Experiment_Small_World_(possible_option).gif)

## Result 1

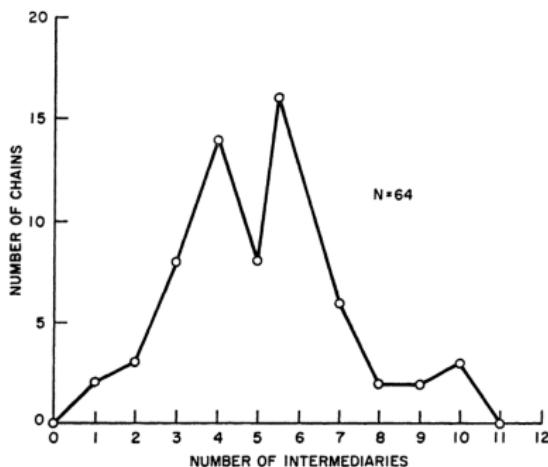
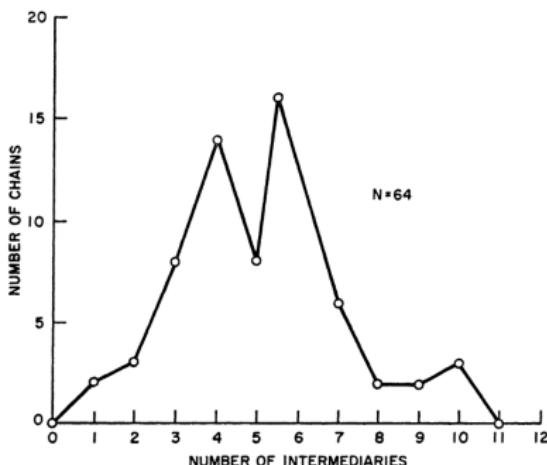


FIGURE 1  
Lengths of Completed Chains

- The mean number of intermediaries: 5.2

## Result 1



*Lengths of Completed Chains*

- The mean number of intermediaries: 5.2
- In other words, **six degrees of separation** to reach a random person in the US

## Result 2

- 29% of chains reached targets.
- [In class discussion]: What are the implications?



## Result 2

- What does this tells you?

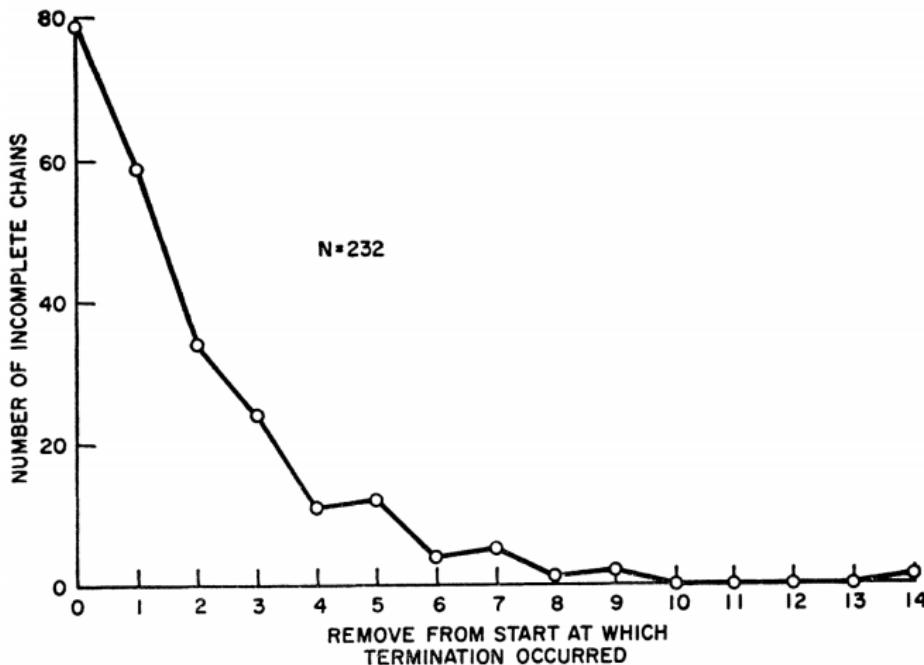


FIGURE 2

*Lengths of Incomplete Chains*

## Result 3

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- Another 10 made contact through a single business associate

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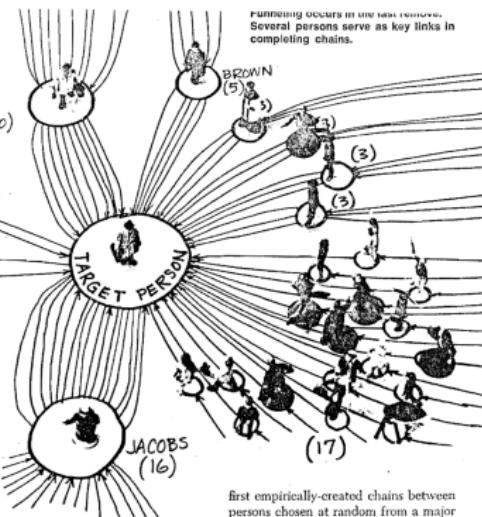
- The 64 letters which reached the target were sent by a total of 26 people.
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- These three superconnectors together accounted for 48% of the total completions

# Result 3

arch extends to an enormous number of persons.  
 If we state there are only 6 acquaintances, this means between the position of man and the target person, one measure of the "radius of acquaintance" of JONES (10) is 3. In different frames of reference, two persons are close, they are linked. Almost all United States is connected by a chain of 5 degrees from me to another person.  
 If a mathematical definition does not make sense, it agrees with that of Nelson. In this case, when we speak of radius, we are talking about psychological distance between the target points, which seems small only because we regard "five" as a small unit. We should think of it as being not five persons but five "circles of acquaintances" apart. Let it in its proper perspective.



# Modern replications of Milgram's Small World Experiment

- Peter Sheridan Dodds, Roby Muhamad, and Duncan J. Watts, *An Experimental Study of Search in Global Social Networks*, Science **301** (2003), no. 5634, 827–829

## An Experimental Study of Search in Global Social Networks

Peter Sheridan Dodds,<sup>1</sup> Roby Muhamad,<sup>2</sup> Duncan J. Watts<sup>1,2\*</sup>

We report on a global social-search experiment in which more than 60,000 e-mail users attempted to reach one of 18 target persons in 13 countries by forwarding messages to acquaintances. We find that successful social search is conducted primarily through intermediate to weak strength ties, does not require highly connected "hubs" to succeed, and, in contrast to unsuccessful social search, disproportionately relies on professional relationships. By accounting for the attrition of message chains, we estimate that social searches can reach their targets in a median of five to seven steps, depending on the separation of source and target, although small variations in chain lengths and participation rates generate large differences in target reachability. We conclude that although global social networks are, in principle, searchable, actual success depends sensitively on individual incentives.



## Result 1

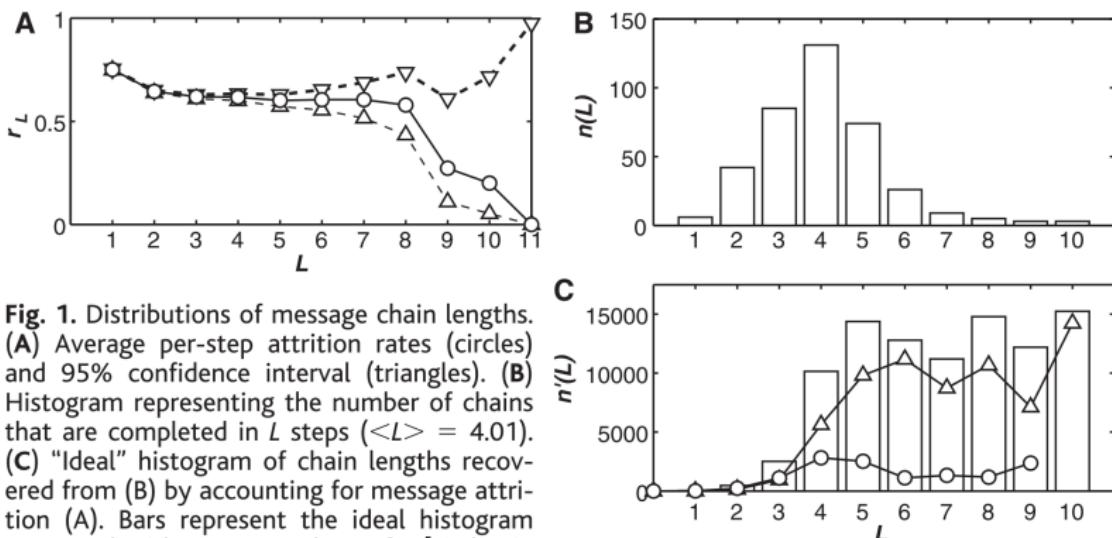
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- “Although the average participation rate (about 37%) was high relative to those reported in most e-mailbased surveys (26), the compounding effects of attrition over multiple links resulted in exponential attenuation of chains as a function of their length and therefore an extremely low chain completion rate (384 of 24,163 chains reached their targets).”

## Result 2

- The mean length is 4.05.

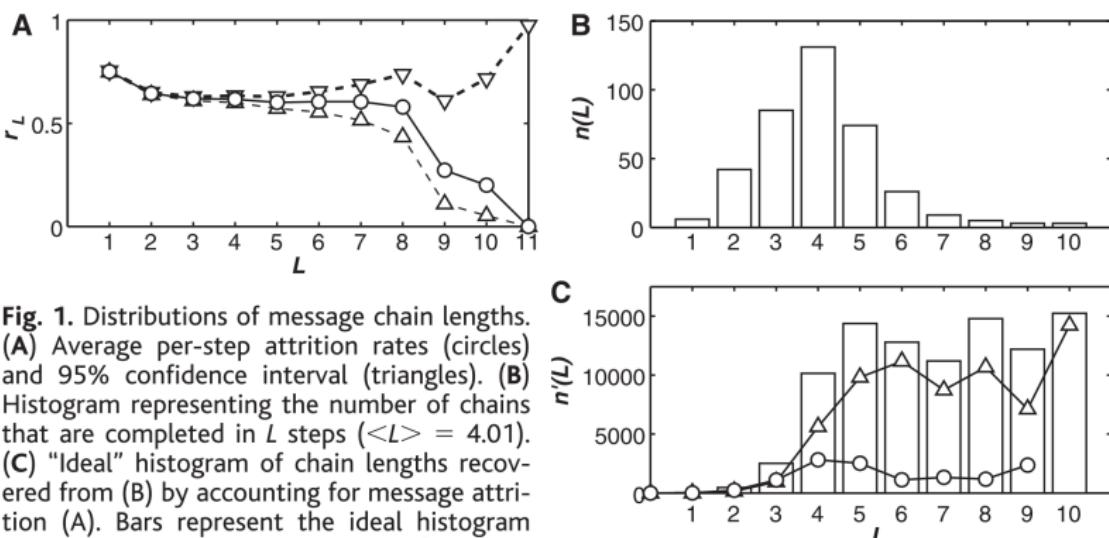


**Fig. 1.** Distributions of message chain lengths. **(A)** Average per-step attrition rates (circles) and 95% confidence interval (triangles). **(B)** Histogram representing the number of chains that are completed in  $L$  steps ( $\langle L \rangle = 4.01$ ). **(C)** “Ideal” histogram of chain lengths recovered from (B) by accounting for message attrition (A). Bars represent the ideal histogram recovered with average values of  $r$  [circles in (A)] for the histogram in (B); lines represent a decomposition of the complete data into chains that start in the same country as the target (circles) and those that start in a different country (triangles).



## Result 2

- The **mean** length is 4.05.
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**Fig. 1.** Distributions of message chain lengths. (A) Average per-step attrition rates (circles) and 95% confidence interval (triangles). (B) Histogram representing the number of chains that are completed in  $L$  steps ( $\langle L \rangle = 4.01$ ). (C) “Ideal” histogram of chain lengths recovered from (B) by accounting for message attrition (A). Bars represent the ideal histogram recovered with average values of  $r$  [circles in (A)] for the histogram in (B); lines represent a decomposition of the complete data into chains that start in the same country as the target (circles) and those that start in a different country (triangles).



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**Table 2.** Reason for choosing next recipient. All quantities are percentages. Location, recipient is geographically closer; Travel, recipient has traveled to target's region; Family, recipient's family originates from target's region; Work, recipient has occupation similar to target; Education, recipient has similar educational background to target; Friends, recipient has many friends; Cooperative, recipient is considered likely to continue the chain; Other, includes recipient as the target.

<i>L</i>	<i>N</i>	Location	Travel	Family	Work	Education	Friends	Cooperative	Other
1	19,718	33	16	11	16	3	9	9	3
2	7,414	40	11	11	19	4	6	7	2
3	2,834	37	8	10	26	6	6	4	3
4	1,014	33	6	7	31	8	5	5	5
5	349	27	3	6	38	12	6	3	5
6	117	21	3	5	42	15	4	5	5
7	37	16	3	3	46	19	8	5	0



## Measuring degree of separation with social media

- [https://research.fb.com/  
three-and-a-half-degrees-of-separation/](https://research.fb.com/three-and-a-half-degrees-of-separation/)

Year	Distance	Number of Facebook users
2008	5.28	5.8 million users
2011	4.74	721 million users
2016	4.57	1.6 billion users

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    - i.e., you should have higher probability to befriend with your friends of friends, than some random person

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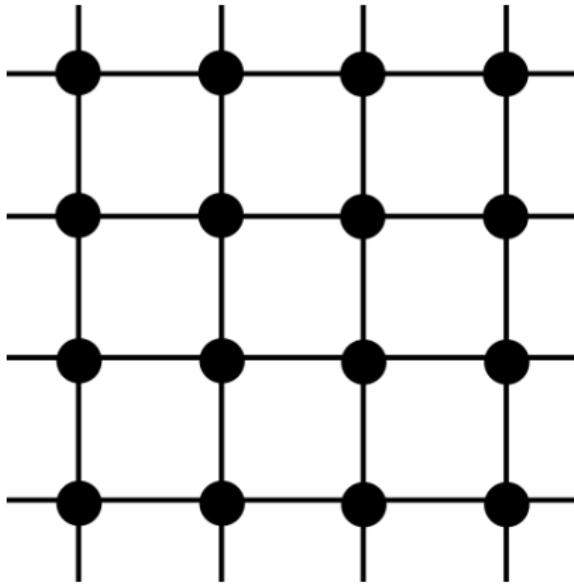
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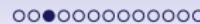
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- Before answering these questions, we first look at two simple network examples that are not small-world networks

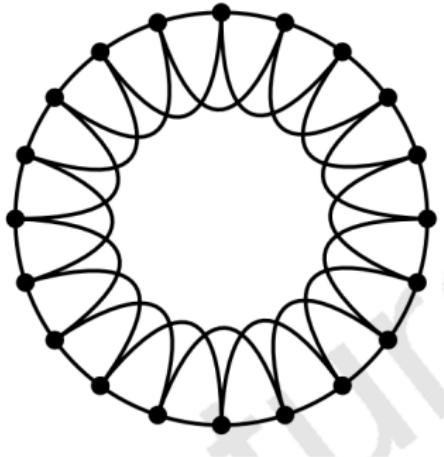
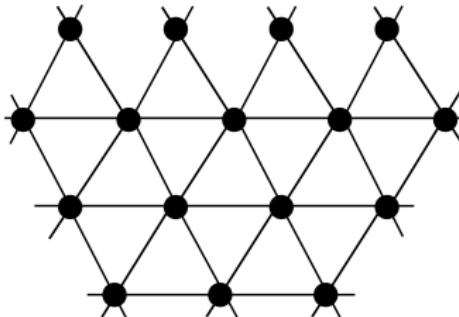
## Simplest network: regular network

- Every node has the exact same number of edges





## Simplest network: regular network



# Regular networks are not small-world networks

- Why?

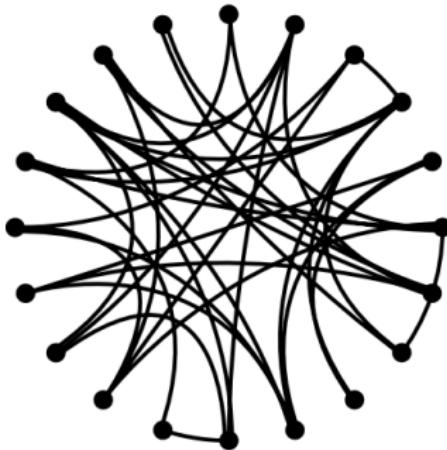
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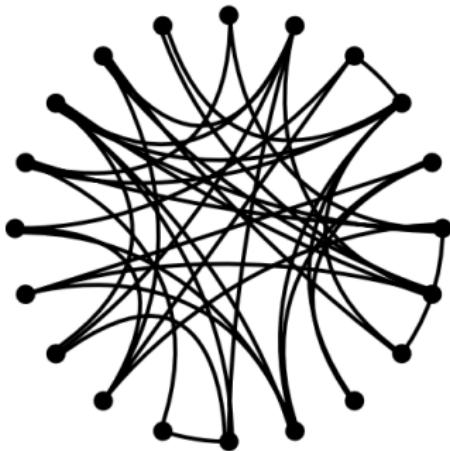
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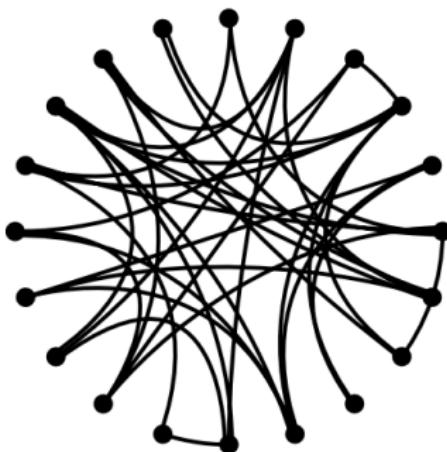
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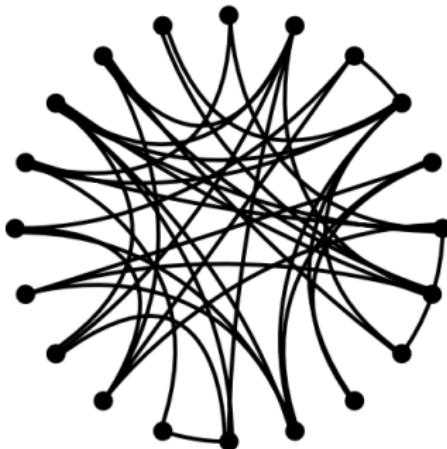
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- $N \cdot p$  edges in expectation

Random



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- But the clustering coefficient  $\rightarrow 0$  when  $N$  increases

## Comparisons

- The two ideal types, regular networks and random networks, looks very different

	Diameter $L$	Clustering Coefficient $C$
Random	small	small
Regular	large	large
Small-world	small, around $\log(N)$	large

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- And they are all different from small-world networks

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## Small-world Phenomena beyond social networks

- “the small-world phenomenon is not merely a curiosity of social networks, nor an artefact of an idealized model; it is probably generic for many large, sparse networks found in nature”

**Table 1 Empirical examples of small-world networks**

	$L_{\text{actual}}$	$L_{\text{random}}$	$C_{\text{actual}}$	$C_{\text{random}}$
Film actors	3.65	2.99	0.79	0.00027
Power grid	18.7	12.4	0.080	0.005
<i>C. elegans</i>	2.65	2.25	0.28	0.05

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- Local clustering coefficient is easier to measure than diameter

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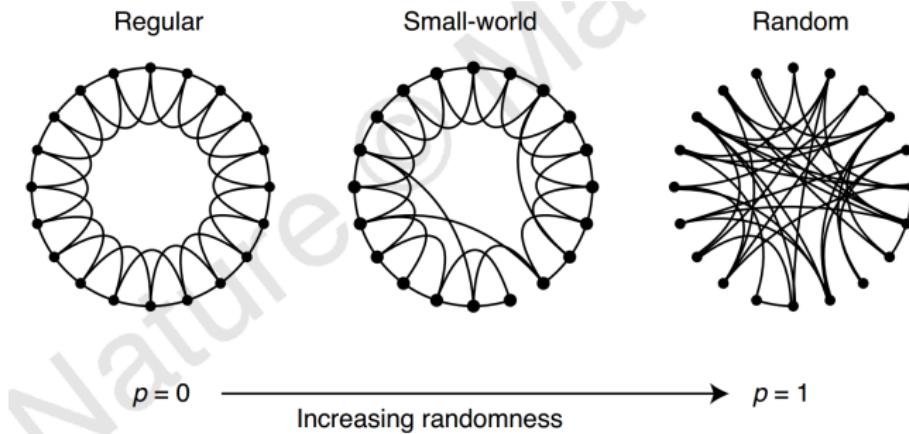
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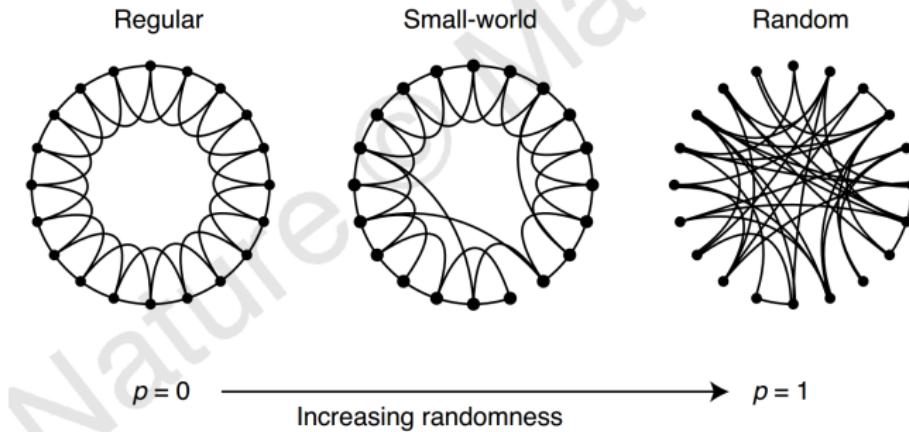
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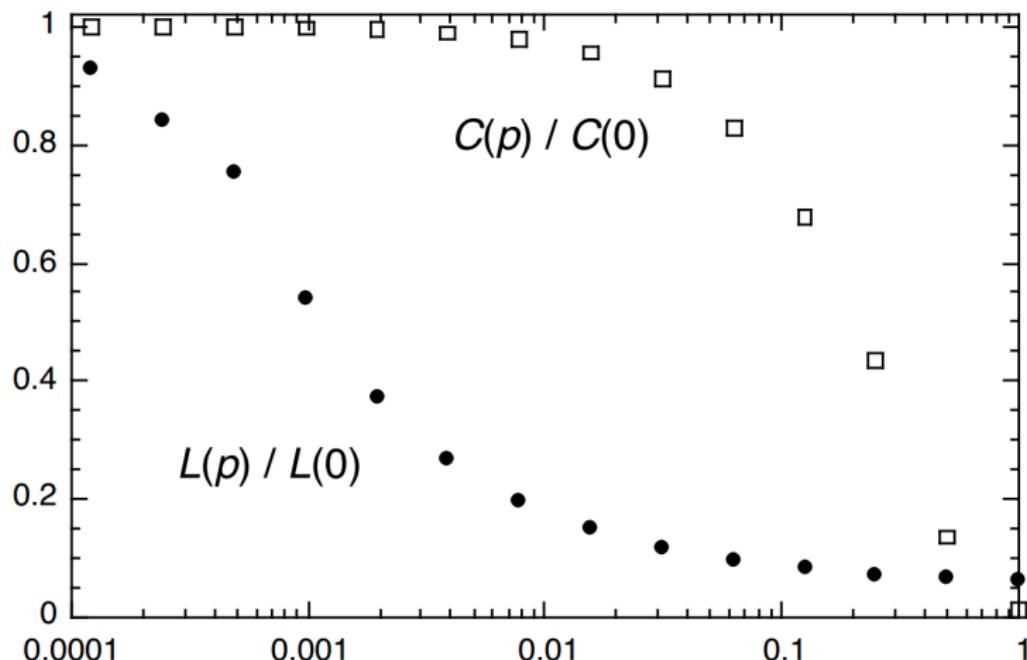
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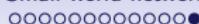
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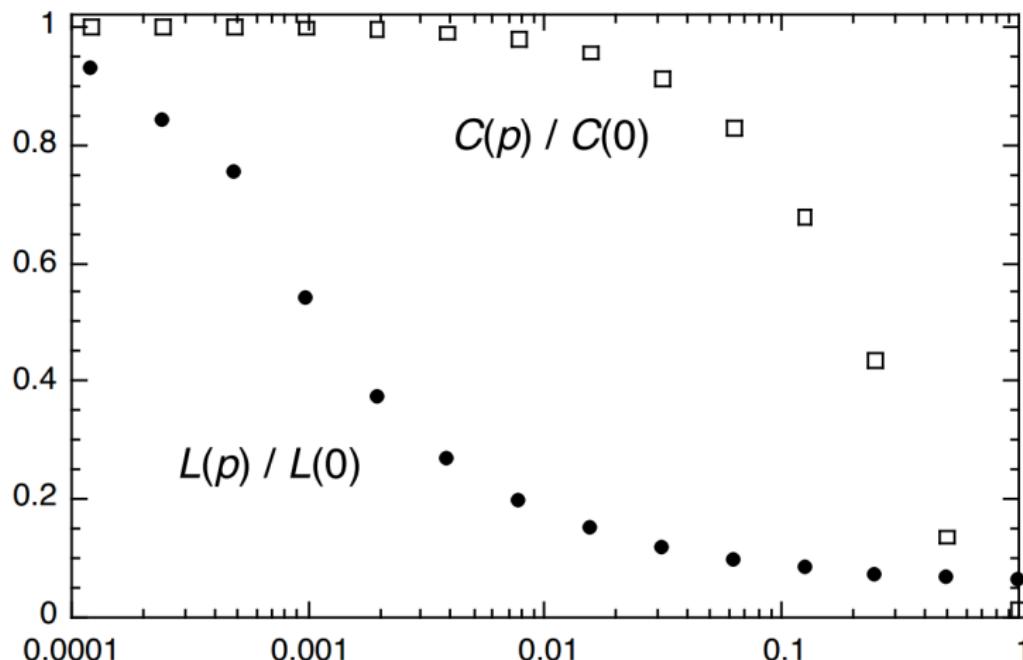
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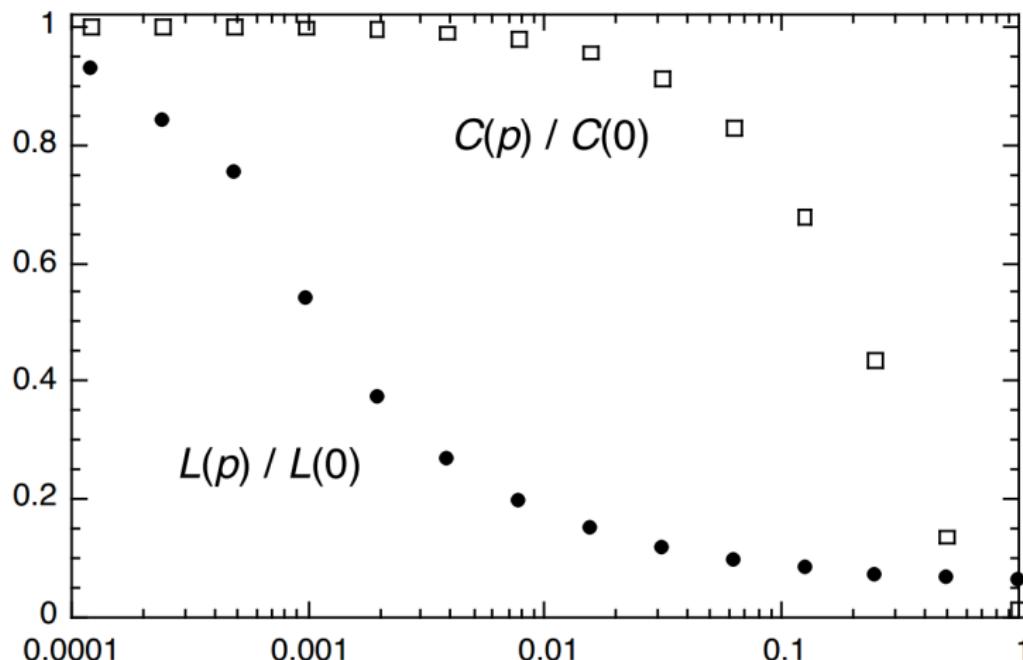
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  - E.g., applying machine learning techniques on text data to generate some variables, and then put these variables into a linear regressions to test some theories
  - Mostly an empirical approach: **old theory + new data**