Applications of dynamic game model

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Outline

Hospital industry: Gowrisankaran and Town (1997)

Cement industry: Ryan (2012)

Hard disk drive industry: Igami & Uetake (2020)

Summary of estimation methodology of dynamic game model

Reference

Hospital industry: Gowrisankaran and Town (1997)

- Two types of hospitals (players): for-profit (FP) and non-profit (NP).
- Three types of patients in a market: with Medicare (MD,Θ_{MD}), private insurance (PI, Θ_{PI}), and no insurance (UI, Θ_{UI}).
- Hospitals makes static decisions (price) and dynamic decisions (investment, exit and entry)

Patient's decision

- In a market, a fixed population X is ill. So each type of patients has total number: $\Theta_{MD}M$, $\Theta_{PI}M$, $\Theta_{UI}M$.
- Hospital j is free to set p_j^{NM}, the base price that it charges for non-medicare patients.
- Poor patients of all medicare types don't have to necessarily pay the full price. The price that each patient must pay is:

$$p_{ij}^{MD} = \max\left\{\min\left(d, Y_i^{MD} - Y_{MIN}\right), 0\right\}$$
$$p_{ij}^{PI} = \max\left\{\min\left(cp_j^{NM}, Y_i^{PI} - Y_{MIN}\right), 0\right\}$$
$$p_{ij}^{UI} = \max\left\{\min\left(p_i^{NM}, Y_i^{UI} - Y_{MIN}\right), 0\right\}$$

Patient's decision

- Hence, the utility for patient i of type T choosing hospital j is

$$U_{ij}^{T} = k_j + \gamma_1 \ln(Y_i^{T} - \gamma_2 p_{ij}^{T}) + \epsilon_{ij}^{T}$$

- The utility for patient i of type T buying outside good is

$$U_{i0}^{T} = \gamma_1 ln(Y_i^{T} - \gamma_2 p_{i0}^{T}) + \epsilon_{i0}^{T}$$

- We can obtain the probability (share) of patient i choosing hospital j:

$$s_{ij}^{T}(p_{ij}^{T}) = \frac{\exp\left\{ k_j + \gamma \ln(\frac{Y_i^{T} - \gamma_2 p_{ij}^{T}}{Y_i^{T} - \gamma_2 p_{i0}^{T}}) \right\}}{1 + \sum_{k=1}^{J} [k_k + \exp(\frac{Y_i^{T} - \gamma_2 p_{ij}^{T}}{Y_i^{T} - \gamma_2 p_{i0}^{T}})]}$$

Hospital's static production decision

- Each hospital j has identical fixed costs F and differing marginal costs (on each patient) $mc_j = \bar{mc} + bk_j$
- Hence, the gross variable profit of j can be constructed as $\pi_j^{GV}(p^{NM})$
- Hospitals' static decision is to choose p_j^{NM} to maximize the static profits.

Hospital's dynamic decision

- State variable vector $s_t = \{k_{it}\}_{i=1,2..n}$
- Timing: in each period, players take actions in this order:
  - Existing hospitals decides whether to exit. Each firm simultaneously receives an i.i.d. scrap value draw from a uniform distribution $U(\phi - \sigma_\phi, \phi + \sigma_\phi)$, and immediately decides whether to accept its scrap value draw and exit forever or stay alive in the current period.
  - Then, the remaining incumbents simultaneously invest. Each hospital j invests amount x_{jt} at a cost of c_I^{FP} or c_I^{FP} .
  - Next, potential entrants decide whether to enter. There are two potential entrants in each period, one NP and one FP. Each of them receive a sunk cost from distribution $U(S - \sigma_S, S + \sigma_S)$. If it enters, it would have an initial quality in the next period.
  - Finally, a static game starts.

Transition between states

- Hospital's investment converts to quality in a certain way:

$$k_j = g(w_j)$$

$$w_{j,t+1} = w_{jt} + v_{jt} - \bar{v}_t$$

$$v_j = \begin{cases} 1, \text{with prob. } ax_{jt}/(1 + ax_{jt}) \\ 0, \text{with prob. } 1/(1 + ax_{jt}) \end{cases}$$

$$\bar{v} = \begin{cases} 1, \text{with prob. } \delta \\ 0, \text{with prob. } 1 - \delta \end{cases}$$

Estimation method

Nested fixed-point algorithm (NFXP)'s central idea:

- Given a structural parameter vector, solve the Bellman equation, get the value function.
- Based on the solution, compute the conditional choice probabilities (CCPs) of actions, examine how well these CCPs match observations.
- Find a parameter vector that causes the predictions to most closely match the data.

Cement industry: Ryan (2012)

Some features of cement market:

- Cement is a fine mineral dust with binding properties, the key ingredient of concrete. Its production generates significant emissions, making it a frequent target of environmental regulations.
- As a concentrated industry, sunk costs of entry and costly investment are important determinants of market structure. 1990 Clean Air Act (CAA) amendment affects the cost structure.
- Cement is a largely homogeneous commodity.
- Cement easily absorbs water from air, making storage expensive. Short shipping distance makes cement markets quasi-independent geographically.

Model's setup

- Model's key setup:
  - firm's static decision: quantity of output
  - firm's dynamic decision: exit or not, enter or not, invest/divest or not
  - state: firms' capacity in the market. Firms' actions affect the state in the next period.
- In each period, the sequence of events are:
  - incumbent firms decide whether to exit the industry
  - potential entrants and incumbents who decided not to exit simultaneously make entry and investment decisions.
  - incumbent firms compete over quantities in the market.

Firms' static decision

- Each market is full described by the state vector s_t. s_{it} is firm i's capacity in t.
- In each regional market m, firms face a constant elasticity of demand curve

$$\ln Q_m(\alpha) = \alpha_{0m} + \alpha_1 \ln P_m,$$

- the cost of production is:

$$C_i(q_i; \delta) = \delta_0 + \delta_1 q_i + \delta_2 1(q_i > \nu s_i)(q_i - \nu s_i)^2,$$

  where δ_0 is fixed cost, δ_1 is constant marginal cost. If output level q_i reaches a threshold νs_i, it would cause extra costs.
- Then, in each period, solve the static Cournot competition, firm i's profit is determined: $\bar{\pi}_i(s; \alpha, \delta)$

Firms' investment decision

- Recall that firms can adjust their capacity by investment or divestment x_i. The cost of adjustments is:

$$\Gamma(x_i;\gamma) = 1(x_i > 0)(\gamma_{i1} + \gamma_2 x_i + \gamma_2 x_i^2) + 1(x_i < 0)(\gamma_{i4} + \gamma_5 x_i + \gamma_6 x_i^2)$$

if invest, $x_i > 0$, fixed cost γ_1 is drawn each period from the common distribution $F_\gamma \ N(\mu_\gamma^+, \sigma_\gamma^+)$. if divest, $x_i < 0$, fixed cost γ_4 is drawn each period from the common distribution $G_\gamma \ N(\mu_\gamma^-, \sigma_\gamma^-)$.

Firms' entry and exit decision

- Sunk cost of entry and scrap value

$$\Phi_i(a_i; k_i, \phi_i) = \begin{cases} -k_i, & (a_i = enter) \\ \phi_i, & (a_i = exit) \end{cases} \tag{1}$$

  fixed cost of entry k_i is private information, drawn from common distribution F_k. ϕ_i is the payment firms receive when exiting the market (scape value), drawn from common distribution F_ϕ.

- Collecting all the parameters, firm i's per-period payoff function is

$$\pi_i(\mathbf{s}, a; \theta) = \pi_i(\mathbf{s}, a; \alpha, \delta, \gamma_i, k_i, \phi_i) = \bar{\pi}_i(\mathbf{s}; \alpha, \delta) - \Gamma(x_i; \gamma_i) + \Phi_i(a_i; k_i, \phi_i)$$

Transitions between states

- actions a_i (investment, entry and exit) in t take one period to occur in s_{t+1} , such as $s_{i,t+1} = s_{it} + x_{it}$

Equilibrium

- In a Markovian setting, firms only condition on the current state and their private shocks when making decisions.
  Each firm's strategy $a_i = \sigma(s, \epsilon_i)$
- Then, we get value functions of incumbents and potential entrants

$$V_i(\mathbf{s}; \sigma(\mathbf{s}), \theta, \epsilon_i)$$

- MPNE requires each firm's strategy profile is the best response of competitors' strategy profiles.

Estimation method

- Use Bajari, Benkard, and Levin (2007)'s two-step estimation method (BBL)
- Intuition of BBL: the econometrician lets the agents in the model solve the dynamic program, and finds parameters of the underlying model such that their behavior is optimal.
  1^{st} step: estimate the reduced-form policy functions $\sigma(s)$
  2^{nd} step: Based on $\sigma(s)$, recover parameters θ.

Applications

- $\theta = \{\alpha, \delta, \gamma, k, \phi\}$
- 1^{st} step: recover the policy functions governing entry, exit and investment, along with the production relevant parameters $\Rightarrow$ *get* $\sigma(s)$, α, δ
- 2^{nd} step: take policy functions and restrictions of MPNE to recover the dynamic parameters $\Rightarrow$ *get* $\mu_\gamma^+, \sigma_\gamma^+, \mu_\gamma^-, \sigma_\gamma^-, \mu_\phi, \sigma_\phi, \mu_k, \sigma_k$

Results

- Finding: the CAA increased the mean entry cost by 22 percent, with no significant change on variance. Moreover, the increase in entry costs greatly reduces the chances that marginal firms enter a market, and this has significant effects on product market competition.
- Counterfactual analysis: As a result of entry rates, the overall welfare in the US cement market decreased by at least $810M.

Hard disk drive industry: Igami & Uetake (2020)

- Antitrust towards mergers is the most important area in which IO economists shape the debate on policy.
- Conventional merger analysis takes a proposed merger as given and focuses on its immediate effects on competition. Such a static analysis could be appropriate only if mergers were completely random events. However, **mergers can be endogenous!**

Model's setup

- Time is discrete with a finite horizon, $t = 0, 1, 2..., T$. The final period T is the time at which the demand for HDDs becomes zero (the industry comes to an end).
- Finite number of incumbent firms, $i = 1, 2..., n_t$. Potential entrant $i = 0$
  Each firm has productivity on a discretized grid with unit interval $\omega_{it} \in \{\omega^1, \omega^2, ...\}$, representing its tacit knowledge.
- State variable is ω_t
- A potential entrant exists in every period $a^0 \in A^0 = \{enter, out\}$.
  a^0 has sunk cost $k^{a^0} + \epsilon(a^0_{it})$.
  An incumbent take actions in every period $a \in A =$
  $\{\text{exit}, \text{innovate}, (\text{propose merger to j})_{ji}, (\text{innovate and propose to j})_{ji}, idle\}$.
  a has sunk cost $k^a + \epsilon(a_{it})$.

Transitions between state

- If incumbent i $a_{it} = exit$, then $\omega_{i,t+1} = \omega^{00}$ ("dead")
- If $a_{it} = innovate$, then $\omega_{i,t+1} = \omega_{it} + 1$
- If $a_{it} =$ propose merger to j, and firm j takes the deal, then $\omega_{i,t+1} = \{\omega_{it}, \omega_{jt}\} + \triangle_{i,t+1}$. $\triangle_{i,t+1}$ reflects "synergies".

Timing

- Instead of assuming players simultaneous move in each period, here the authors consider an alternating-move game, in which the time interval is relatively short and only one firm can make a dynamic discrete choice within a period.
- The timeline of the stochastically alternating moves:
  - At the beginning of each period, nature equally chooses one firm to make a move.
  - If it's i's turn to make a move, it observes the current industry state ω_t, draws sunk costs of actions.
  - Based on these information, i makes the discrete choice $a_{it} \in A_{it}$. Corresponding sunk costs immediately occurs.

    If i negotiate a merger agreement with j, a acquisition price p_{ij} would be paid if the deal is made. It's a take-it-or-leave-it offer.
  - All active firms participate static competition, earn period profits and pay costs of operation.

Dynamic optimization and equilibrium

- To reach an incumbent firm i's dynamic optimization, the corresponding Bellman equation is:

$$V_{it}(\omega_t, \epsilon_{it}) = \pi_{it}(\omega_t) - \phi_t(\omega_{it}) + \max\left\{\bar{V}_{it}^{x}, \bar{V}_{it}^{c}, \bar{V}_{it}^{i}, \bar{V}_{ijt}^{m}, \bar{V}_{ijt}^{i\&m}\right\}$$

$\bar{V}_{it}^{a}$ represents conditional values of exiting, idling, innovating, proposing merger to rival j, and both merge and innovate. Such as:

$$\bar{V}_{it}^{c}(\omega_t, \epsilon_{it}^{c}) = \epsilon_{it}^{c} + \beta E\left[V_{i,t+1}(\omega_{t+1}) | \omega_t, a_{it} = idle\right],$$

Estimation method

Conditional choice probability (CCP) based method

- 1^{st} step: estimate reduced-form CCP function by using data actions and states.
- 2^{nd} step: Using these CCP functions, calculate value functions, find optimal structural parameters.

Summary of estimation methodology

- Full solution methods
  - Nested fixed point algorithm (Gowrisankaran and Town, 1997)
  - High precision but computationally expensive
- BBL (Ryan, 2012)
  - Computationally efficient
- CCP (Igami and Uetake, 2020)
  - Computationally efficient

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