

# Chapter 9 Structural Empirical Analysis of Vertical Contracting Theory

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# Outline

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# Introduction

- One key feature of many industries is a vertical supply chain characterized by an oligopolistic market structure at each level of the chain.
- A model that allows for such margins of adjustment will often be necessary for an accurate prediction of the effects of a policy.
- From theoretical to empirical work,
  - Must work with underlying theoretical models that feature both contracting and competition, yet are tractable and estimable.
  - Requires more detailed data and rich institutional knowledge.

# Basics

- Setting
  - An upstream seller  $U$  and a downstream buyer  $D$  agree on a contract  $\mathcal{C}$  from some feasible set.
  - $\mathcal{C} = \{y, t\}$ ,  $y$  includes other provisions and  $t$  is a lump-sum transfer.
  - Firms' payoffs:  $\Pi_U(\mathcal{C}) \equiv \pi_U(y) + t$ ;  $\Pi_D(\mathcal{C}) \equiv \pi_D(y) - t$
- The maximization problem

$$\begin{aligned} & \max_{(y,t) \in \mathcal{Y} \times \mathbb{R}} \pi_U(y) + t \\ & \text{s.t.} \quad \pi_D(y) - t \geq \bar{\Pi}_D \end{aligned} \tag{1}$$

## Basics - example: successive monopoly setting

- A monopolist manufacturer sells a product to a monopolist retailer.
  - $p^m(c) \equiv \arg \max(p - c)D(p)$
  - Vertically integrated:  $p^m(c_M + c_R)$  maximizes the bilateral surplus.
  - Price unilaterally:  $p^m(w + c_R)$  maximizes the retailer's profit.
  - $p^m(w + c_R) > p^m(c_M + c_R)$ : double marginalization problem.

## Basics - example: negotiation by Nash bargaining

- The parties will agree to a contract  $\mathcal{C} = \{y, t\}$  that solves

$$\max_{\mathcal{C} \in \mathcal{C}^+} [\Pi_D(y, t) - \bar{\Pi}_D]^b \cdot [\Pi_U(y, t) - \bar{\Pi}_U]^{1-b} \quad (2)$$

- Take derivatives with respect to  $y$  and  $t$ , and we have

$$\frac{\partial \pi_D(y)}{\partial y_k} + \frac{\partial \pi_U(y)}{\partial y_k} = 0 \quad \text{for } k = 1, \dots, K \quad (3)$$

which is the first-order condition for maximizing the bilateral surplus

$$\pi_D(y) + \pi_U(y)$$

## Basics - example: negotiation by Nash bargaining (continued)

- Consider the contract  $\mathcal{C}$  only includes the wholesale price  $w$ .
- We will have

$$(w - c_M)\bar{D}'(w) + \bar{D}(w) = \left(\frac{b}{1-b}\right)\left(\frac{w - c_M}{p^m(w + c_R) - (w + c_R)}\right)\bar{D}(w) \quad (4)$$

where  $\bar{D}(w) \equiv D(p^m(w + c_R))$  is the retail demand conditioning on the wholesale price  $w$ .

- $b \rightarrow 0$ : successive monopoly setting.
- $b \rightarrow 1$ : implies  $w \rightarrow c_M$ , maximizing bilateral surplus.

# Multilateral settings with externalities

- Previous case: one-to-one vertical contracting.
- Things can be complicated.
  - One-to-many contracting: non-cooperative bargaining
  - Many-to-many contracting: Nash-in-Nash bargaining
- Non-cooperative bargaining: offer & bidding games



# The offer game - introduction

- Contracting parties make take-it-or-leave-it offers. Lump-sum transfers are feasible.
- Whether the offer can be known by the other party.
  - Public offers
  - Private offers

## The offer game - introduction (continued)

- The principal can sign a bilateral contract  $\mathcal{C}_j = (q_j, t_j)$  with agent  $j = 1, \dots, J$ .
- The principal's payoff is  $\pi_P(\mathbf{q}) + \sum_j t_j$  and agent  $j$ 's payoff is  $\pi_j(\mathbf{q}) - t_j$ .
- Condition  $W$ : the joint payoff depends only on the aggregate trade  $Q \equiv \sum_j q_j$ . When it holds, we also assume that all efficient trade profiles  $\mathbf{q}^* \in \mathcal{Q}^*$  have the same aggregate trade  $Q^*$ .

$$Q^* \equiv \arg \max_{\mathbf{q} \in \mathbb{R}^J} \pi_P(\mathbf{q}) + \sum_j \pi_j(\mathbf{q})$$

## The offer game - public offers

- Focus on the equilibria where all agents accept the contract.
- Agent  $j$  will accept the contract if and only if

$$\pi_j(\mathbf{q}) - t_j \geq \pi_j(0, \mathbf{q}_{-j})$$

- Given this, the principal will offer  $\hat{\mathbf{q}}$  solving

$$\max_{\mathbf{q} \in \mathcal{R}^J} \{ \pi_P(\mathbf{q}) + \sum_{j=1} \pi_j(\mathbf{q}) \} - \sum_{j=1} \pi_j(0, \mathbf{q}_{-j}) \quad (5)$$

- The inefficiency comes from the externalities on non-traders. If there are externalities, the principal has the incentive to distort to lower the reservation payoffs.

## The offer game - public offers (continued)

### Proposition 1

*In the public-offer game with lump-sum transfers and absent externalities on non-traders, the equilibrium trade profile  $\hat{\mathbf{q}}$  is efficient, i.e.,  $\hat{\mathbf{q}} \in \mathcal{Q}^*$ .*

### Proposition 2

*Assume Condition W holds and suppose that the aggregate trade in an equilibrium trade profile of the public-offer game is  $\hat{Q}$ . Then with positive (or negative) externalities on non-traders,  $Q \leq$  (or  $\geq$ )  $Q^*$ .*

# The offer game - public offers (continued)

## *Proof of Proposition 2*

- Suppose that externalities on non-traders are positive.
- The minimized value of reservation utility becomes

$$\begin{aligned} R(Q) &\equiv \min_{\mathbf{q} \in \mathbb{R}^J} \sum_j \pi_j(0, \mathbf{q}_{-j}) \\ &\text{s.t. } \sum_j q_j = Q \end{aligned}$$

Note that  $R(\cdot)$  is a non-decreasing function.

## The offer game - public offers (continued)

### *Proof of Proposition 2 (cont'd)*

- The principal's problem becomes

$$\max_Q \Pi(Q) - R(Q)$$

- Suppose that  $\hat{Q} > Q^*$ . By definition of  $Q^*$  and the fact that  $R(\cdot)$  is a non-decreasing function, we have  $\Pi(\hat{Q}) - R(\hat{Q}) < \Pi(Q^*) - R(Q^*)$ , which contradicts to  $\hat{Q}$  solving equation (5). So we must have  $\hat{Q} \leq Q^*$ .  $\square$

## The offer game - public offers (continued)

- Negative externalities will lead to large trade, while positive externalities will lead to small trade.
  - Exclusive contract: hinder potential entrants and reduce potential benefits to non-traders.
  - M&A contract: reduce competition and increase the benefits to other competing firms.

## The offer game - private offers

- The offer can only be observed by the agent.
- Assume agents hold passive beliefs: they believe other agents received their equilibrium offers even when they receive an unexpected offer.
- The equilibrium trade profile  $\hat{\mathbf{q}} = \{\hat{q}_1, \dots, \hat{q}_J\}$

$$\begin{aligned} \hat{\mathbf{q}} \in \arg \max_{\mathbf{q} \in \mathbb{R}^J} \pi_P(\mathbf{q}) + \sum_j [\pi_j(q_j, \hat{\mathbf{q}}_{-j}) - \pi_j(0, \hat{\mathbf{q}}_{-j})] \\ \arg \max_{\mathbf{q} \in \mathbb{R}^J} \pi_P(\mathbf{q}) + \sum_j \pi_j(q_j, \hat{\mathbf{q}}_{-j}) \end{aligned} \tag{6}$$

- The inefficiency comes from the externality on efficient traders.



## The offer game - private offers (continued)

### Proposition 3

*In the private-offer game with lump-sum transfers:*

*(i) If there are no externalities on efficient traders, then any passive beliefs equilibrium trade profile is efficient.*

*(ii) Assume Condition W holds and let  $\hat{Q}$  be the aggregate trade in a passive beliefs equilibrium. If externalities on efficient traders are positive (or negative), then  $\hat{Q} \leq$  (or  $\geq$ )  $Q^*$ .*

## The offer game - private offers (continued)

### *Proof of Proposition 3 (i)*

- Notice that for any passive beliefs equilibrium trade profile  $\hat{q}$ , and any efficient trade profile  $q^* \in Q^*$ , we have

$$\begin{aligned}\pi_P(\hat{\mathbf{q}}) + \sum_j \pi_j(\hat{q}_j, \hat{\mathbf{q}}_{-j}) &\geq \pi_P(\mathbf{q}^*) + \sum_j \pi_j(q_j^*, \hat{\mathbf{q}}_{-j}) \\ &= \pi_P(\mathbf{q}^*) + \sum_j \pi_j(q_j^*, \mathbf{q}_{-j}^*)\end{aligned}\tag{7}$$

Together they imply  $\hat{\mathbf{q}}$  is efficient.

## The offer game - private offers (continued)

### *Proof of Proposition 3 (ii)*

- Suppose there are negative externalities on efficient traders but  $\hat{Q} < Q^*$ .
- Under Condition  $W$ , there is some efficient trade profile  $q^*$  such that  $\sum_j q_j^* = Q^*$  and  $\hat{q}_j < q_j^*$  for all  $j$ .

$$\begin{aligned}\pi_P(\hat{\mathbf{q}}) + \sum_j \pi_j(\hat{q}_j, \hat{\mathbf{q}}_{-j}) &\geq \pi_P(\mathbf{q}^*) + \sum_j \pi_j(q_j^*, \hat{\mathbf{q}}_{-j}) \\ &> \pi_P(\mathbf{q}^*) + \sum_j \pi_j(q_j^*, \hat{\mathbf{q}}_{-j}^*)\end{aligned}$$

which contradicts  $\mathbf{q}^*$  being efficient. Hence, we must have  $\hat{Q} \geq Q^*$ . □

## The offer game - example: linear Cournot retailer

- One manufacturer and  $J \geq 2$  retailers.
- The inverse demand is  $P(Q) = a - bQ$ . The manufacturer's cost is  $c(Q) = c + dQ$ .
- The joint monopoly aggregate trade is

$$Q^* = \frac{a - c}{2b + d} \quad (8)$$

- The passive beliefs equilibrium aggregate trade is

$$\hat{Q}_J = \frac{a - c}{\frac{J+1}{J}b + d} \quad (9)$$

## The offer game - example: linear Cournot retailer (continued)

- The competitive aggregate trade is

$$Q^c = \frac{a - c}{b + d} \quad (10)$$

- The principal can yield the competitive outcome by offering a contract that gives the retailer the competitive profit.
- Segal and Whinston (2003) shows that the ability to offer such contracts implies the set of equilibrium aggregate trades in the offer game is  $[\underline{Q}_J, Q^c]$  where

$$\underline{Q}_J = \frac{a - c}{(1 + \frac{2b}{dJ})(b + d)}$$

# The offer game - public v.s. private offers

- The source of inefficiency
  - Public offers: externalities on non-traders.
  - Private offers: externalities on efficient traders.
- $t_j = \pi_j(\mathbf{q}) - \pi_j(0, \mathbf{q}_{-j})$ 
  - Public offers: if the principal changes  $q_k$ , this change can be extracted from agent  $j$  by changing  $\pi_j(\mathbf{q})$  or  $\pi_j(0, \mathbf{q}_{-j})$ .
  - Private offers: there is no effect on agent  $j$ 's payoff when not trading, but  $j$  also does not see the change on  $\pi_j(\mathbf{q})$ . The principal will not internalize the effect on  $\pi_j(\mathbf{q})$ .

# The bidding game

- Multiple principal make offers to the single agent, who then decide whether to accept or reject each offer.
- Only unilateral contract deviations are possible.
- It is possible for deviating contract offer to induce the agent to reject the offer from a rival principal.

# The bidding game (continued)

## An example

- There are two manufacturers. Each manufacturer  $j$  must earn her marginal contribution to the joint monopoly profit given the trade with the other manufacturer.

$$t_j - c_j q_j^* = [P(q_1^* + q_2^*)(q_1^* + q_2^*) - c_1 q_1^* - c_2 q_2^*] - [P(q_j^*, 0)q_j^* - c_j q_j^*] \quad (11)$$

- Have the incentive to provide an exclusive offer.

$$q_k^e = \arg \max_{q_k} P(q_k, 0)q_k - c_k q_k \quad (12)$$



# The bidding game (continued)

- Less is known about equilibrium outcomes in settings with contracting externalities for bidding games than for offer games.
- The non-cooperative models discussed above are special.
  - "Triangle" vertical structures.
  - Make take-it-or-leave-it offers.
  - Under the consideration of externalities, possible lump-sum transfers are assumed.

# Nash-in-Nash bargaining

- Consider a setting with  $I$  sellers and  $J$  buyers.
- Each pair  $ij$  may agree to a contract  $\mathbb{C}_{ij} \in \mathcal{C}_{ij}$ . Given a collection of contracts between all pairs  $i$  and  $j$ ,  $\mathbb{C} \equiv \{\mathbb{C}_{ij}\}$ , downstream firm  $j$ 's payoff is  $\Pi_{Dj}(\mathbb{C})$  and upstream firm  $i$ 's payoff is  $\Pi_{Uj}(\mathbb{C})$ .
- Contracts  $\hat{\mathbb{C}} \equiv \{\hat{\mathbb{C}}_{ij}\}$  constitute a Nash-in-Nash equilibrium if for all  $ij$  such that  $\hat{\mathbb{C}}_{ij} \neq \mathbb{C}$ ,

$$\begin{aligned} \hat{\mathbb{C}}_{ij} \in \arg \max_{\mathbb{C}_{ij} \in \mathcal{C}_{ij}^+(\hat{\mathbb{C}}_{-ij})} & [\Pi_{Dj}(\mathbb{C}_{ij}, \hat{\mathbb{C}}_{-ij}) - \Pi_{Dj}(\mathbb{C}_0, \hat{\mathbb{C}}_{-ij})]^{b_{ij}} \\ & \times [\Pi_{Uj}(\mathbb{C}_{ij}, \hat{\mathbb{C}}_{-ij}) - \Pi_{Uj}(\mathbb{C}_0, \hat{\mathbb{C}}_{-ij})]^{1-b_{ij}} \end{aligned} \quad (13)$$

## Nash-in-Nash bargaining (continued)

- A collection of contracts is a Nash-in-Nash equilibrium if each pair's contract solves the bilateral Nash bargaining problem *taking the contracts agreed by all other pairs as given*.
- However, Nash-in-Nash equilibria may involve unreasonable payoff predictions.

## Nash-in-Nash bargaining (continued)

### An example

- There are two manufacturers and one retailer, with equal bargaining power between them. Let  $Q^m$  denote the joint monopoly sales level for the vertical structure.
- One possible Nash-in-Nash equilibrium is  $q_1 = Q^m$ ,  $q_2 = 0$ . Manufacturer 1 and the retailer share the profits equally.
- However, in the bidding/offer game, all of the profits would be earned by the retailer.

## Nash-in-Nash bargaining (continued)

### Nash-in-Nash with Threat of Replacement (Ho and Lee, 2019)

- The retailer can credibly threaten to replace the manufacturer with a new manufacturer.
- Under the NNTR protocol, the equilibrium becomes  $q_1 = Q^m$ , but the retailer earns all of the profits.

# Nash-in-Nash bargaining (continued)

## Contracting dynamics

- Commitment problem: the temptation to contract secretly with one agent at the expense of others.
- Why static models may overstate this temptation?
  - Reputation building.
  - If contracts eventually become observed, contracts might be structured in ways that curbs opportunism.
  - Contracts may be renegotiated in response to observed changes in rivals' contracts.