Chapter 9 Structural Empirical Analysis of Vertical Contracting

Theory

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Outline

Introduction

- One key feature of many industries is a vertical supply chain characterized by an oligopolistic market structure at each level of the chain.
- A model that allows for such margins of adjustment will often be necessary for an accurate prediction of the effects of a policy.
- From theoretical to empirical work,
    - Must work with underlying theoretical models that feature both contracting and competition, yet are tractable and estimable.
    - Requires more detailed data and rich institutional knowledge.

Basics

- Setting
  - A upstream seller U and a downstream buyer D agree on a contract $\mathcal{C}$ from some feasible set.
  - $\mathcal{C} = \{y, t\}$, y includes other provisions and t is a lump-sum transfer.
  - Firms' payoffs: $\Pi_U(\mathcal{C}) \equiv \pi_U(y) + t$; $\Pi_D(\mathcal{C}) \equiv \pi_D(y) - t$
- The maximization problem

$$\begin{aligned} \max_{(y,t)\in\mathcal{Y}\times\mathbb{R}} \quad & \pi_U(y) + t \\ \text{s.t.} \quad & \pi_D(y) - t \geq \overline{\Pi}_D \end{aligned} \tag{1}$$

Basics - example: successive monopoly setting

- A monopolist manufacturer sells a product to a monopolist retailer.
  - $p^m(c) \equiv \arg\max(p - c)D(p)$
  - Vertically integrated: $p^m(c_M + c_R)$ maximizes the bilateral surplus.
  - Price unilaterally: $p^m(w + c_R)$ maximizes the retailer's profit.
  - $p^m(w + c_R) > p^m(c_M + c_R)$: double marginalization problem.

Basics - example: negotiation by Nash bargaining

- The parties will agree to a contract $\mathcal{C} = \{y, t\}$ that solves

$$\max_{C \in \mathcal{C}^+} \left[\Pi_D(y,t) - \overline{\Pi}_D\right]^b \cdot \left[\Pi_U(y,t) - \overline{\Pi}_U\right]^{1-b} \tag{2}$$

- Take derivatives with respect to y and t, and we have

$$\frac{\partial \pi_D(y)}{\partial y_k} + \frac{\partial \pi_U(y)}{\partial y_k} = 0 \quad \text{for } k = 1, \ldots, K \tag{3}$$

  which is the first-order condition for maximizing the bilateral surplus

$$\pi_D(y) + \pi_U(y)$$

Basics - example: negotiation by Nash bargaining (continued)

- Consider the contract $\mathcal{C}$ only includes the wholesale price w.
- We will have

$$(w - c_M)\overline{D}'(w) + \overline{D}(w) = (\frac{b}{1-b})(\frac{w - c_M}{p^m(w + c_R) - (w + c_R)})\overline{D}(w) \qquad (4)$$

where $\overline{D}(w) \equiv D(p^m(w+c_R))$ is the retail demand conditioning on the wholesale price w.

  - $b \to 0$: successive monopoly setting.
  - $b \to 1$: implies $w \to c_M$, maximizing bilateral surplus.

Multilateral settings with externalities

- Previous case: one-to-one vertical contracting.
- Things can be complicated.
  - One-to-many contracting: non-cooperative bargaining
  - Many-to-many contracting: Nash-in-Nash bargaining
- Non-cooperative bargaining: offer & bidding games

The offer game - introduction

- Contracting parties make take-it-or-leave-it offers. Lump-sum transfers are feasible.
- Whether the offer can be known by the other party.
  - Public offers
  - Private offers

The offer game - introduction (continued)

- The principal can sign a bilateral contract $\mathcal{C}_j = (q_j, t_j)$ with agent $j = 1, ..., J$.
- The principal's payoff is $\pi_P(\mathbf{q}) + \sum_j t_j$ and agent j's payoff is $\pi_j(\mathbf{q}) - t_j$.
- Condition W: the joint payoff depends only on the aggregate trade $Q \equiv \sum_j q_j$. When it holds, we also assume that all efficient trade profiles $\mathbf{q}^* \in \mathcal{Q}^*$ have the same aggregate trade Q^*.

$$\mathcal{Q}^* \equiv \arg\max_{\mathbf{q} \in \mathbb{R}^J} \pi_P(\mathbf{q}) + \sum_j \pi_j(\mathbf{q})$$

The offer game - public offers

- Focus on the equilibria where all agents accept the contract.
- Agent j will accept the contract if and only if

$$\pi_j(\mathbf{q}) - t_j \geq \pi_j(0, \mathbf{q}_{-j})$$

- Given this, the principal will offer $\hat{\mathbf{q}}$ solving

$$\max_{\mathbf{q}\in\mathcal{R}^J}\{\pi_P(\mathbf{q}) + \sum_{j=1} \pi_j(\mathbf{q})\} - \sum_{j=1} \pi_j(0, \mathbf{q}_{-j}) \tag{5}$$

- The inefficiency comes from the externalities on non-traders. If there are externalities, the principal has the incentive to distort to lower the reservation payoffs.

The offer game - public offers (continued)

Proposition 1

In the public-offer game with lump-sum transfers and absent externalities on non-traders, the equilibrium trade profile $\hat{\mathbf{q}}$ *is efficient, i.e.,* $\hat{\mathbf{q}} \in \mathcal{Q}^*$.

Proposition 2

Assume Condition W *holds and suppose that the aggregate trade in an equilibrium trade profile of the public-offer game is* $\hat{Q}$. *Then with positive (or negative) externalities on non-traders,* $Q \leq$ *(or* $\geq$*)* Q^*.

The offer game - public offers (continued)

Proof of Proposition 2

- Suppose that externalities on non-traders are positive.
- The minimized value of reservation utility becomes

$$R(Q) \equiv \min_{q \in \mathbb{R}^J} \sum_j \pi_j(0, \mathbf{q}_{-j})$$
$$\text{s.t. } \sum_j q_j = Q$$

Note that $R(\cdot)$ is a non-decreasing function.

The offer game - public offers (continued)

Proof of Proposition 2 (cont'd)

- The principal's problem becomes

$$\max_{Q} \quad \Pi(Q) - R(Q)$$

- Suppose that $\hat{Q} > Q^*$. By definition of Q^* and the fact that $R(\cdot)$ is a non-decreasing function, we have $\Pi(\hat{Q}) - R(\hat{Q}) < \Pi(Q^*) - R(Q^*)$, which contradicts to $\hat{Q}$ solving equation (5). So we must have $\hat{Q} \leq Q^*$. $\square$

The offer game - public offers (continued)

- Negative externalities will lead to large trade, while positive externalities will lead to small trade.
  - Exclusive contract: hinder potential entrants and reduce potential benefits to non-traders.
  - M&A contract: reduce competition and increase the benefits to other competing firms.

The offer game - private offers

- The offer can only be observed by the agent.
- Assume agents hold passive beliefs: they believe other agents received their equilibrium offers even when they receive an unexpected offer.
- The equlibrium trade profile $\hat{\mathbf{q}} = \{\hat{q}_1, ..., \hat{q}_J\}$

$$\begin{aligned} \hat{\mathbf{q}} \in \arg\max_{q \in \mathbb{R}^J} \pi_P(\mathbf{q}) + \sum_j [\pi_j(q_j, \hat{\mathbf{q}}_{-j}) - \pi_j(0, \hat{\mathbf{q}}_{-j})] \\ \arg\max_{q \in \mathbb{R}^J} \pi_P(\mathbf{q}) + \sum_j \pi_j(q_j, \hat{\mathbf{q}}_{-j}) \end{aligned} \tag{6}$$

- The inefficiency comes from the externality on efficient traders.

The offer game - private offers (continued)

Proposition 3

In the private-offer game with lump-sum transfers:
(i) If there are no externalities on efficient traders, then any passive beliefs equilibrium trade profile is efficient.
(ii) Assume Condition W holds and and let $\hat{Q}$ be the aggregate trade in a passive beliefs equilibrium. If externalities on efficient traders are positive (or negative), then $\hat{Q} \leq$ (or $\geq$) Q^*.

The offer game - private offers (continued)

Proof of Proposition 3 (i)

- Notice that for any passive beliefs equilibrium trade profile $\hat{q}$, and any efficient trade profile $q^* \in Q^*$, we have

$$\begin{aligned} \pi_P(\hat{\mathbf{q}}) + \sum_j \pi_j(\hat{q}_j, \hat{\mathbf{q}}_{-j}) &\geq \pi_P(\mathbf{q}^*) + \sum_j \pi_j(q_j^*, \hat{\mathbf{q}}_{-j}) \\ &= \pi_P(\mathbf{q}^*) + \sum_j \pi_j(q_j^*, \mathbf{q}^*_{-j}) \end{aligned} \tag{7}$$

Together they imply $\hat{\mathbf{q}}$ is efficient.

The offer game - private offers (continued)

Proof of Proposition 3 (ii)

- Suppose there are negative externalities on efficient traders but $\hat{Q} < Q^*$.
- Under Condition W, there is some efficient trade profile q^* such that $\sum_j q_j^* = Q^*$ and $\hat{q}_j < q_j^*$ for all j.

$$\begin{aligned}\pi_P(\hat{\mathbf{q}}) + \sum_j \pi_j(\hat{q}_j, \hat{\mathbf{q}}_{-j}) &\geq \pi_P(\mathbf{q}^*) + \sum_j \pi_j(q_j^*, \hat{\mathbf{q}}_{-j}) \\ &> \pi_P(\mathbf{q}^*) + \sum_j \pi_j(q_j^*, \hat{\mathbf{q}}_{-j}^*)\end{aligned}$$

which contradicts $\mathbf{q}^*$ being efficient. Hence, we must have $\hat{Q} \geq Q^*$. $\square$

The bidding game

- Multiple principal make offers to the single agent, who then decide whether to accept or reject each offer.
- Only unilateral contract deviations are possible.
- It is possible for deviating contract offer to induce the agent to reject the offer from a rival principal.

The bidding game (continued)

An example

- There are two manufacturers. Each manufacturer j must earn her marginal contribution to the joint monopoly profit given the trade with the other manufacturer.

$$t_j - c_j q_j^* = [P(q_1^* + q_2^*)(q_1^* + q_2^*) - c_1 q_1^* - c_2 q_2^*] - [P(q_j^*, 0) q_j^* - c_j q_j^*] \tag{8}$$

- Have the incentive to provide an exclusive offer.

$$q_k^e = \arg\max_{q_k} P(q_k, 0) q_k - c_k q_k \tag{9}$$

The bidding game (continued)

- Less is known about equilibrium outcomes in settings with contracting externalities for bidding games than for offer games.
- The non-cooperative models discussed above are special.
  - "Triangle" vertical structures.
  - Make take-it-or-leave-it offers.
  - Under the consideration of externalities, possible lump-sum transfers are assumed.

Nash-in-Nash bargaining

- Consider a setting with I sellers and J buyers.
- Each pair ij may agree to a contract $\mathbb{C}_{ij} \in \mathcal{C}_{ij}$. Given a collection of contracts between all pairs i and j, $\mathbb{C} \equiv \{\mathbb{C}_{ij}\}$, downstream firm j's payoff is $\Pi_{Dj}(\mathbb{C})$ and upstream firm i's payoff is $\Pi_{Uj}(\mathbb{C})$.
- Contracts $\hat{\mathbb{C}} \equiv \{\hat{\mathbb{C}}_{ij}\}$ constitute a Nash-in-Nash equilibrium if for all ij such that $\hat{\mathbb{C}}_{ij} \neq \mathbb{C}$,

$$\begin{aligned} \hat{\mathbb{C}}_{ij} \in \arg \max_{\mathbb{C}_{ij} \in \mathcal{C}^{+}_{ij}(\hat{\mathbb{C}}_{-ij})} & [\Pi_{Dj}(\mathbb{C}_{ij}, \hat{\mathbb{C}}_{-ij}) - \Pi_{Dj}(\mathbb{C}_0, \hat{\mathbb{C}}_{-ij})]^{b_{ij}} \\ & \times [\Pi_{Uj}(\mathbb{C}_{ij}, \hat{\mathbb{C}}_{-ij}) - \Pi_{Uj}(\mathbb{C}_0, \hat{\mathbb{C}}_{-ij})]^{1-b_{ij}} \end{aligned} \tag{10}$$

Nash-in-Nash bargaining (continued)

- A collection of contracts is a Nash-in-Nash equilibrium if each pair's contract solves the bilateral Nash bargaining problem *taking the contracts agreed by all other pairs as given.*
- However, Nash-in-Nash equilibria may involve unreasonable payoff predictions.

Nash-in-Nash bargaining (continued)

An example

- There are two manufacturers and one retailer, with equal bargaining power between them. Let Q^m denote the joint monopoly sales level for the vertical structure.
- One possible Nash-in-Nash equilibrium is $q_1 = Q^m$, $q_2 = 0$. Manufacturer 1 and the retailer share the profits equally.
- However, in the bidding/offer game, all of the profits would be earned by the retailer.

Nash-in-Nash bargaining (continued)

Nash-in-Nash with Threat of Replacement (Ho and Lee, 2019)

- The retailer can credibly threaten to replace the manufacturer with a new manufacturer.
- Under the NNTR protocol, the equilibrium becomes $q_1 = Q^m$, but the retailer earns all of the profits.

Nash-in-Nash bargaining (continued)

Contracting dynamics

- Commitment problem: the temptation to contract secretly with one agent at the expense of others.
- Why static models may overstate this temptation?
  - Reputation building.
  - If contracts eventually become observed, contractsmight be structured in ways that curbs opportunism.
  - Contracts may be renegotiated in response to observed changes in rivals' contracts.