Chapter 9 Structural Empirical Analysis of Vertical Contracting

Presented by Siyuan (Bruce) Jin and Yangfan Sun

Hong Kong University of Science and Technology

March 2025

Outline - Theory

Introduction

Basics

Non-cooperative bargaining models

- The offer game
- The bidding game

Nash-in-Nash bargaining

Introduction

-

Basics

- Setting
  - A upstream seller U and a downstream buyer D agree on a contract $\mathcal{C}$ from some feasible set.
  - The contract $\mathcal{C} = \{y, t\}$, y includes other provisions and t is a lump-sum transfer.
  - Firms' payoffs: $\Pi_U(\mathcal{C}) \equiv \pi_U(y) + t$; $\Pi_D(\mathcal{C}) \equiv \pi_D(y) - t$
- The maximization problem

$$\begin{aligned} \max_{(y,t)\in\mathcal{Y}\times\mathbb{R}} \quad & \pi_U(y) + t \\ \text{s.t.} \quad & \pi_D(y) - t \geq \overline{\Pi}_D \end{aligned} \tag{1}$$

Basics - example: successive monopoly setting

- A monopolist manufacturer sells a product to a monopolist retailer.
  - Vertically integrated: $p^m(c) \equiv argmax(p - c_m - c_R)D(p)$, $p^m(c_M + c_R)$ maximizes the bilateral surplus.
  - Price unilaterally: $p^m(w + c_R)$ maximizes the retailer's profit.
  - $p^m(w + c_R) > p^m(c_M + c_R)$: double marginalization problem.

Basics - example: negotiation by Nash bargaining

- The parties will agree to a contract $\mathcal{C} = \{y, t\}$ that solves

$$\max_{C \in \mathcal{C}^+} \left[\pi_D(y) - \overline{\Pi}_D\right]^b \cdot \left[\pi_U(y) - \overline{\Pi}_U\right]^{1-b} \tag{2}$$

- Take derivatives with respect to y and t, and we have

$$\frac{\partial \pi_D(y)}{\partial y_k} + \frac{\partial \pi_U(y)}{\partial y_k} = 0 \quad \text{for } k = 1, \ldots, K \tag{3}$$

Basics - example: negotiation by Nash bargaining (continued)

- Consider the contract $\mathcal{C}$ only includes the wholesale price w.
- We will have

$$(w - c_M)\overline{D}(w) + \overline{D}(w) = \left(\frac{b}{1-b}\right)\left(p^*\left(w + c_R\right) - \left(w - c_R\right)\right)\overline{D}(w) \qquad (4)$$

  where $\overline{D}(w) \equiv D(p^m(w + c_R))$ is the retail demand conditioning on the wholesale price w.

  - $b \to 0$: successive monopoly setting.
  - $b \to 1$: implies $w \to c_M$, maximizing bilateral surplus.

Multilateral settings with externalities

- Previous case: one-to-one vertical contracting.
- Things can be complicated: many-to-many with contracting externalities.
- Two different approaches to analyze more complicated settings.
  - Non-cooperative bargaining
  - Nash-in-Nash bargaining

The offer game - introduction

- Two issues
  - Contracting parties can make take-it-or-leave-it offers or go back and forth.
  - Whether the offer can be known by the other party.
- Begin with non-coorperative approach: only one side of the market has multiple parties, and lump-sum transfers are feasible.
  - Public offers
  - Private offers

The offer game - public offers

- The principal can sign a bilateral contract $\mathcal{C}_j = (q_j, t_j)$ with agent $j = 1, ..., J$.
- Focus on the equilibria where all agents accept the contract.
- Agent j will accept the contract if and only if

$$\pi_j(\mathbf{q}) - t_j \geq \pi_j(0, \mathbf{q}_{-j})$$

- Given this, the principal will offer $\hat{\mathbf{q}}$ solving

$$\max_{\mathbf{q} \in \mathcal{R}^J} \{\pi_P(\mathbf{q}) + \sum_{j=1} \pi_j(\mathbf{q})\} - \sum_{j=1} \pi_j(0, \mathbf{q}_{-j}) \tag{5}$$

where $\sum_{j=1} \pi_j(0, \mathbf{q}_{-j})$ is the reservation payoffs of agents.

The offer game - public offers

- When no externalities on non-traders, the outcome is efficient. However, if there are externalities, the principal has the incentive to distort to lower the reservation payoffs.
- Negative externalities will lead to large trade, while positive externalities will lead to small trade.
    - M&A contract: reduce competition and increase the benefits to other competing firms.
    - Exclusive contract: hinder potential entrants and reduce potential benefits to non-traders.

The offer game - private offers

•

The bidding game

Nash-in-Nash bargaining

Conclusion