# Chapter 9 Structural Empirical Analysis of Vertical Contracting Theory

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### Outline

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#### Introduction

- One key feature of many industries is a vertical supply chain characterized by an oligopolistic market structure at each level of the chain.
- A model that allows for such margins of adjustment will often be necessary for an accurate prediction of the effects of a policy.
- From theoretical to empirical work,
  - Must work with underlying theoretical models that feature both contracting and competition, yet are tractable and estimable.
  - Requires more detailed data and rich institutional knowledge.

#### Basics

- Setting
  - A upstream seller U and a downstream buyer D agree on a contract  $\mathcal C$  from some feasible set.
  - $C = \{y, t\}$ , y includes other provisions and t is a lump-sum transfer.
  - Firms' payoffs:  $\Pi_U(\mathcal{C}) \equiv \pi_U(y) + t$ ;  $\Pi_D(\mathcal{C}) \equiv \pi_D(y) t$
- The maximization problem

$$\begin{array}{ll} \max\limits_{(y,t)\in\mathcal{Y}\times\mathbb{R}} & \pi_U(y)+t\\ \text{s.t.} & \pi_D(y)-t\geq\overline{\Pi}_D \end{array} \tag{1}$$

# Basics - example: successive monopoly setting

- A monopolist manufacturer sells a product to a monopolist retailer.
  - $p^m(c) \equiv \arg\max(p-c)D(p)$
  - Vertically integrated:  $p^m(c_M + c_R)$  maximizes the bilateral surplus.
  - Price unilaterally:  $p^m(w+c_R)$  maximizes the retailer's profit.
  - $p^m(w + c_R) > p^m(c_M + c_R)$ : double marginalization problem.

## Basics - example: negotiation by Nash bargaining

ullet The parties will agree to a contract  $\mathcal{C}=\{y,t\}$  that solves

$$\max_{C \in \mathcal{C}_{+}^{+}} \left[ \Pi_{D}(y, t) - \overline{\Pi}_{D} \right]^{b} \cdot \left[ \Pi_{U}(y, t) - \overline{\Pi}_{U} \right]^{1-b} \tag{2}$$

• Take derivatives with respect to y and t, and we have

$$\frac{\partial \pi_D(y)}{\partial y_k} + \frac{\partial \pi_U(y)}{\partial y_k} = 0 \quad \text{for } k = 1, \dots, K$$
 (3)

which is the first-order condition for maximizing the bilateral surplus

$$\pi_D(y) + \pi_U(y)$$

# Basics - example: negotiation by Nash bargaining (continued)

- Consider the contract C only includes the wholesale price w.
- We will have

$$(w - c_M)\overline{D}'(w) + \overline{D}(w) = (\frac{b}{1-b})(\frac{w - c_M}{p^m(w + c_R) - (w + c_R)})\overline{D}(w)$$
(4)

where  $\overline{D}(w) \equiv D(p^m(w+c_R))$  is the retail demand conditioning on the wholesale price w.

- $b \rightarrow 0$ : successive monopoly setting.
- $b \to 1$ : implies  $w \to c_M$ , maximizing bilateral surplus.

## Multilateral settings with externalities

- Previous case: one-to-one vertical contracting.
- Things can be complicated.
  - One-to-many contracting: non-cooperative bargaining
  - Many-to-many contracting: Nash-in-Nash bargaining
- Non-cooperative bargaining: offer & bidding games

## The offer game - introduction

- Contracting parties make take-it-or-leave-it offers. Lump-sum transfers are feasible.
- Whether the offer can be known by the other party.
  - Public offers
  - Private offers

# The offer game - introduction (continued)

- The principal can sign a bilateral contract  $C_j = (q_j, t_j)$  with agent j = 1, ..., J.
- The principal's payoff is  $\pi_P(\mathbf{q}) + \sum_i t_i$  and agent j's payoff is  $\pi_j(\mathbf{q}) t_j$ .
- Condition W: the joint payoff depends only on the aggregate trade  $Q \equiv \sum_j q_j$ . When it holds, we also assume that all efficient trade profiles  $\mathbf{q}^* \in \mathcal{Q}^*$  have the same aggregate trade  $Q^*$ .

$$\mathcal{Q}^* \equiv rg \max_{\mathbf{q} \in \mathbb{R}^J} \pi_P(\mathbf{q}) + \sum_j \pi_j(\mathbf{q})$$

## The offer game - public offers

- Focus on the equilibria where all agents accept the contract.
- Agent j will accept the contract if and only if

$$\pi_j(\mathbf{q}) - t_j \geq \pi_j(0, \mathbf{q}_{-j})$$

• Given this, the principal will offer  $\hat{\mathbf{q}}$  solving

$$\max_{\mathbf{q}\in\mathcal{R}^J} \{\pi_P(\mathbf{q}) + \sum_{j=1} \pi_j(\mathbf{q})\} - \sum_{j=1} \pi_j(0, \mathbf{q}_{-j})$$
 (5)

• The inefficiency comes from the externalities on non-traders. If there are externalities, the principal has the incentive to distort to lower the reservation payoffs.

## Proposition 1

In the public-offer game with lump-sum transfers and absent externalities on non-traders, the equilibrium trade profile  $\hat{\mathbf{q}}$  is efficient, i.e.,  $\hat{\mathbf{q}} \in \mathcal{Q}^*$ .

#### Proposition 2

Assume Condition W holds and suppose that the aggregate trade in an equilibrium trade profile of the public-offer game is  $\hat{Q}$ . Then with positive (or negative) externalities on non-traders,  $Q \leq (or \geq) Q^*$ .

#### Proof of Proposition 2

- Suppose that externalities on non-traders are positive.
- The minimized value of reservation utility becomes

$$R(Q) \equiv \min_{oldsymbol{q} \in \mathbb{R}^J} \sum_j \pi_j(0, oldsymbol{q}_{-j})$$
 s.t.  $\sum_i q_j = Q$ 

Note that  $R(\cdot)$  is a non-decreasing function.

#### Proof of Proposition 2 (cont'd)

• The principal's problem becomes

$$\max_{Q} \ \Pi(Q) - R(Q)$$

• Suppose that  $\hat{Q} > Q^*$ . By definition of  $Q^*$  and the fact that  $R(\cdot)$  is a non-decreasing function, we have  $\Pi(\hat{Q}) - R(\hat{Q}) < \Pi(Q^*) - R(Q^*)$ , which contradicts to  $\hat{Q}$  solving equation (5). So we must have  $\hat{Q} \leq Q^*$ .

- Negative externalities will lead to large trade, while positive externalities will lead to small trade.
  - Exclusive contract: hinder potential entrants and reduce potential benefits to nontraders.
  - M&A contract: reduce competition and increase the benefits to other competing firms.

## The offer game - private offers

- The offer can only be observed by the agent.
- Assume agents hold passive beliefs: they believe other agents received their equilibrium offers even when they receive an unexpected offer.
- The equlibrium trade profile  $\hat{\mathbf{q}} = \{\hat{q_1},...,\hat{q_J}\}$

$$\hat{\mathbf{q}} \in \arg\max_{q \in \mathbb{R}^J} \pi_P(\mathbf{q}) + \sum_j [\pi_j(q_j, \hat{\mathbf{q}}_{-j}) - \pi_j(0, \hat{\mathbf{q}}_{-j})]$$

$$\arg\max_{q \in \mathbb{R}^J} \pi_P(\mathbf{q}) + \sum_j \pi_j(q_j, \hat{\mathbf{q}}_{-j})$$
(6)

The inefficiency comes from the externality on efficient traders.

# The offer game - private offers (continued)

## Proposition 3

In the private-offer game with lump-sum transfers:

- (i) If there are no externalities on efficient traders, then any passive beliefs equilibrium trade profile is efficient.
- (ii) Assume Condition W holds and and let  $\hat{Q}$  be the aggregate trade in a passive beliefs equilibrium. If externalities on efficient traders are positive (or negative), then  $\hat{Q} \leq (or \geq) Q^*$ .

# The offer game - private offers (continued)

## Proof of Proposition 3 (i)

• Notice that for any passive beliefs equilibrium trade profile  $\hat{q}$ , and any efficient trade profile  $q^* \in Q^*$ , we have

$$\pi_{P}(\hat{\mathbf{q}}) + \sum_{j} \pi_{j}(\hat{q}_{j}, \hat{\mathbf{q}}_{-j}) \geq \pi_{P}(\mathbf{q}^{*}) + \sum_{j} \pi_{j}(q_{j}^{*}, \hat{\mathbf{q}}_{-j})$$

$$= \pi_{P}(\mathbf{q}^{*}) + \sum_{j} \pi_{j}(q_{j}^{*}, \mathbf{q}_{-j}^{*})$$

$$(7)$$

Together they imply  $\hat{\mathbf{q}}$  is efficient.

# The offer game - private offers (continued)

### Proof of Proposition 3 (ii)

- Suppose there are negative externalities on efficient traders but  $\hat{Q} < Q^*$ .
- Under Condition W, there is some efficient trade profile  $q^*$  such that  $\sum_j q_j^* = Q^*$  and  $\hat{q}_j < q_j^*$  for all j.

$$egin{aligned} \pi_P(\hat{\mathbf{q}}) + \sum_j \pi_j(\hat{q}_j, \hat{\mathbf{q}}_{-j}) &\geq \pi_P(\mathbf{q}^*) + \sum_j \pi_j(q_j^*, \hat{\mathbf{q}}_{-j}) \ &> \pi_P(\mathbf{q}^*) + \sum_j \pi_j(q_j^*, \hat{\mathbf{q}}_{-j}^*) \end{aligned}$$

which contradicts  $\mathbf{q}^*$  being efficient. Hence, we must have  $\hat{Q} \geq Q^*$ .

## The offer game - public v.s. private offers

- The source of inefficiency
  - Public offers: externalities on non-traders.
  - Private offers: externalities on efficient traders.
- $t_j = \pi_j(\mathbf{q}) \pi_j(0, \mathbf{q}_{-j})$ 
  - Public offers: if the principal changes  $q_k$ , this change can be extracted from agent j by changing  $\pi_j(\mathbf{q})$  or  $\pi_j(0, \mathbf{q}_{-j})$ .
  - Private offers: there is no effect on agent j's payoff when not trading, but j also does not see the change on  $\pi_j(\mathbf{q})$ . The principal will not internalize the effect on  $\pi_j(\mathbf{q})$ .

## The offer game - example: linear Cournot retailer

- One manufacturer and J > 2 retailers.
- The inverse demand is P(Q) = a bQ. The manufacturer's cost is c(Q) = c + dQ.
- The joint monopoly aggregate trade is

$$Q^* = \frac{a - c}{2b + d} \tag{8}$$

• The passive beliefs equilibrium aggregate trade is

$$\hat{Q}_J = \frac{a - c}{\frac{J+1}{l}b + d} \tag{9}$$

# The offer game - example: linear Cournot retailer (continued)

• The competitive aggregate trade is

$$Q^c = \frac{a - c}{b + d} \tag{10}$$

- The principal can yield the competitive outcome by offering a contract that gives the retailer the competitive profit.
- Segal and Whinston (2003) shows that the ability to offer such contracts implies the set of equilibrium aggregate trades in the offer game is  $[Q_1, Q^c]$  where

$$\underline{Q}_J = \frac{a - c}{(1 + \frac{2b}{dJ})(b + d)}$$

## The bidding game

- Multiple principal make offers to the single agent, who then decide whether to accept or reject each offer.
- Only unilateral contract deviations are possible.
- It is possible for deviating contract offer to induce the agent to reject the offer from a rival principal.

# The bidding game (continued)

#### An example

 There are two manufacturers. Each manufacturer j must earn her marginal contribution to the joint monopoly profit given the trade with the other manufacturer.

$$t_j - c_j q_j^* = [P(q_1^* + q_2^*)(q_1^* + q_2^*) - c_1 q_1^* - c_2 q_2^*] - [P(q_j^*, 0)q_j^* - c_j q_j^*]$$
 (11)

• Have the incentive to provide an exclusive offer.

$$q_k^e = \arg\max_{q_k} P(q_k, 0) q_k - c_k q_k \tag{12}$$

## The bidding game (continued)

- Less is known about equilibrium outcomes in settings with contracting externalities for bidding games than for offer games.
- The non-cooperative models discussed above are special.
  - "Triangle" vertical structures.
  - Make take-it-or-leave-it offers.
  - Under the consideration of externalities, possible lump-sum transfers are assumed.

## Nash-in-Nash bargaining

- Consider a setting with I sellers and J buyers.
- Each pair ij may agree to a contract  $\mathbb{C}_{ij} \in \mathcal{C}_{ij}$ . Given a collection of contracts between all pairs i and j,  $\mathbb{C} \equiv \{\mathbb{C}_{ij}\}$ , downstream firm j's payoff is  $\Pi_{Dj}(\mathbb{C})$  and upstream firm i's payoff is  $\Pi_{Uj}(\mathbb{C})$ .
- Contracts  $\hat{\mathbb{C}} \equiv \{\hat{\mathbb{C}}_{ij}\}$  constitute a Nash-in-Nash equilibrium if for all ij such that  $\hat{\mathbb{C}}_{ij} \neq \mathbb{C}$ ,

$$\hat{\mathbb{C}}_{ij} \in \arg \max_{\mathbb{C}_{ij} \in \mathcal{C}_{ij}^{+}(\hat{\mathbb{C}}_{-ij})} [\Pi_{Dj}(\mathbb{C}_{ij}, \hat{\mathbb{C}}_{-ij}) - \Pi_{Dj}(\mathbb{C}_{0}, \hat{\mathbb{C}}_{-ij})]^{b_{ij}} 
\times [\Pi_{Uj}(\mathbb{C}_{ij}, \hat{\mathbb{C}}_{-ij}) - \Pi_{Uj}(\mathbb{C}_{0}, \hat{\mathbb{C}}_{-ij})]^{1-b_{ij}}$$
(13)

- A collection of contracts is a Nash-in-Nash equilibrium if each pair's contract solves the bilateral Nash bargaining problem taking the contracts agreed by all other pairs as given.
- However, Nash-in-Nash equilibria may involve unreasonable payoff predictions.

#### An example

- There are two manufacturers and one retailer, with equal bargaining power between them. Let  $Q^m$  denote the joint monopoly sales level for the vertical structure.
- One possible Nash-in-Nash equilibrium is  $q_1 = Q^m$ ,  $q_2 = 0$ . Manufacturer 1 and the retailer share the profits equally.
- However, in the bidding/offer game, all of the profits would be earned by the retailer.

Nash-in-Nash with Threat of Replacement (Ho and Lee, 2019)

- The retailer can credibly threaten to replace the manufacturer with a new manufacturer.
- Under the NNTR protocol, the equilibrium becomes  $q_1 = Q^m$ , but the retailer earns all of the profits.

#### Contracting dynamics

- Commitment problem: the temptation to contract secretly with one agent at the expense of others.
- Why static models may overstate this temptation?
  - · Reputation building.
  - If contracts eventually become observed, contracts might be structured in ways that curbs opportunism.
  - Contracts may be renegotiated in response to observed changes in rivals' contracts.