An Industrial Organization Perspective on Productivity

Total Factor Productivity

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Outline

Productivity Concepts

- Definition: Productivity measures input-output efficiency ($\Omega = \frac{Q}{F(K,L,M)}$) A higher value of Ω implies that the producer will obtain more output from a given set of inputs.
- Three perspectives:
  - Production function shifter (Hicks-neutral)
  - Output-input ratio (TFP($\frac{Q}{F(K,L,M)}$) vs. single-factor productivity($\frac{Q}{(L)}$))
  - Cost curve shifter(TFP$\uparrow$, Overall cost curve $\downarrow$)
- Importance: Micro-level productivity differences affect market equilibrium and welfare

Estimating and Addressing Productivity Challenges

Estimating Total Factor Productivity (TFP)

$$\Omega_{it} = \frac{Q_{it}}{F(K_{it}, L_{it}, M_{it})} \tag{1}$$

Log-linearized Production Function

$$q_{it} = \underbrace{\beta_V x_{it}^V}_{\text{Variable}} + \underbrace{\beta_F x_{it}^F}_{\text{Fixed}} + \omega_{it} + \epsilon_{it} \tag{2}$$

Key Challenges

- Endogeneity bias (input choices correlated with productivity)
- Selection bias (exit of low-productivity firms)

Operating Environment and Data Challenges

Table: 1 Market Structure and Unit of Analysis

	Producer Level (1)	Product Level (2)
Perfect Competition (A)	A.1 (Traditional)	A.2 (Transformation)
Imperfect Competition (B)	B.1 (Control function)	B.2 (Quality-adjusted)
Homogeneous Products	B.1.1	B.2.1
Differentiated Products	B.1.2	B.2.2

- **Homogeneous Products (A/B.1.1/B.2.1)**:
- **Differentiated Products (B.1.2/B.2.2)**:
  - Requires explicit quality adjustment.
- **Multi-product Producers (Product Level)**:
  - Input allocation problem.

Core Problem: Missing Measurement Dimension

- Original classification (Table 1) does not distinguish:
  - Physical quantity data (directly observed input/output quantities)
  - Monetary value data (revenues/expenditures reflecting monetary values)

Research Premises

- Implicit assumption: Applied research predominantly uses **deflated monetary variables**
- Exception: Labor inputs typically observed directly as employment counts
- Key implications for scenarios:
  - A.1 (Perfect competition - Producer level)
  - B.1.1 (Homogeneous goods - Producer level)

Valid simplifications

- Price deflation can reconcile revenue vs quantity differences
- Homogeneous goods assumption within industries

Methodological Constraints

Differentiated products (B.1.2/B.2.2) require

- Quality heterogeneity from product attributes
- Price signals reflecting quality differences (Klette-Griliches paradox)
- Input allocation challenges in multi-product firms

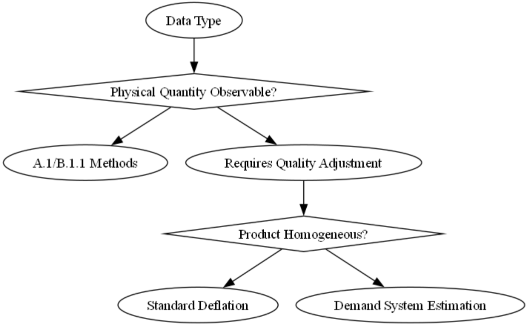
Key breakthroughs (De Loecker et al., 2016)

- Quality-adjusted measurement methods
- Transformation function approach
- Persisting limitation: Cross-product input allocation

Empirical Implementation Guide

Method selection decision tree

1. Data type identification
2. Physical quantity observability test
3. Product homogeneity assessment
4. Demand system estimation (if needed)



Factor Shares Approach(A.1)

Theoretical Basis

Under cost minimization:

$$\beta_X^H = \frac{1}{N_t}\sum_i \frac{P_{it}^{X^H} X_{it}^H}{TC_{it}}$$

At the firm level:

$$\beta_X^H = \frac{P_{it}^{X^H} X_{it}^H}{TC_{it}}$$

Applicability Conditions

- Perfect competition (A.1): Input's cost share equals its revenue share $\frac{P^X X}{PQ}$.
- Constant returns to scale.
- No adjustment costs.

Key Challenges

It requires accurate measurement of input cost shares.

Production Function Estimation(A.1/A.2)

Core Specification

$$q_{it} = \underbrace{\beta_V x_{it}^V}_{\text{Variable}} + \underbrace{\beta_F x_{it}^F}_{\text{Fixed}} + \omega_{it} + \epsilon_{it}$$

Input Classification

Variable Inputs (x^V)

- Labor (L_{it})
- Materials (M_{it})

Fixed Inputs (x^F)

- Capital (K_{it})

Key Parameters

- **Output Elasticities**:

$$\beta_V = \frac{\partial \ln Q}{\partial \ln x^V}, \quad \beta_F = \frac{\partial \ln Q}{\partial \ln x^F}$$

- **TFP Decomposition**:

$$\omega_{it} = \ln \left(\frac{Q_{it}}{F(\cdot)} \right)$$

- **Error Term**:

$$\epsilon_{it} \sim \mathcal{N}(0, \sigma_\epsilon^2)$$

Production Function Estimation(A.1/A.2)

Key Challenges

- Endogeneity bias (input choices correlated with productivity)
- Selection bias (exit of low-productivity firms)

Approaches to Address Challenges:

- **Instrument Variables**
  - Input price (usually not available to the econometrician)
  - Demand-side shifters (affect the validity of the approach)
- **Control Function Approach (Perfect Competition)**
  - Olley-Pakes (1996): Investment policy function inversion
  - Levinsohn-Petrin (2003): Intermediate inputs as proxy
  - Ackerberg-Caves-Frazer (2015): Unified framework
- **Dynamic Panel Approach (Arellano-Bond 1991)**
  - AR(1) process: $\omega_{it} = \rho\omega_{it-1} + \xi_{it}$
  - Capital coefficients are often low or even negative when estimated.

Production Function Estimation: Control Function Approach

- **Key Equations**:

$$\begin{aligned} \text{Production function} \quad & q_{it} = \beta_V x_{it}^V + \beta_F x_{it}^F + \omega_{it} + \epsilon_{it} \\ \text{Input demand} \quad & d_{it} = d_t(\omega_{it}, x_{it}^V, x_{it}^F) \\ \text{Inverted Input demand} \quad & \omega_{it} = d_t^{-1}(d_{it}, x_{it}^V, x_{it}^F) \\ \text{Productivity dynamics} \quad & \omega_{it} = g(\omega_{it-1}) + \xi_{it} \\ \text{Capital accumulation equation} \quad & K_{it} = (1-\delta)K_{it-1} + I_{it-1} \end{aligned}$$

- **Key Assumptions**:
  - Monotonicity condition: d_{it} strictly monotonic in ω_{it}. Unobserved productivity shock can be controlled and obviate the simultaneity problem.
  - Productivity follows a first-order Markov process $\omega_{it} = g(\omega_{it-1}) + \xi_{it}$, which is a core time assumption and deals with selection bias.

Production Function Estimation: Control Function Approach

Estimation Procedure: two-steps

- **First Step: Output Prediction**
  - Replace unobserved productivity ω_{it} with inverted input (investment or intermediate input) demand equation:
  $$\omega_{it} = d_t^{-1}(d_{it}, x_{it}^V, x_{it}^F)$$
  - Construct predicted output equation:
  $$q_{it} = \phi_t(d_{it}, x_{it}^V, x_{it}^F) + \epsilon_{it}$$
  where $\phi_t(\cdot)$ captures predicted output:
  $$\phi_t(\cdot) = \beta_V x_{it}^V + \beta_F x_{it}^F + \omega_{it}$$
  - Key purposes:
    - Remove measurement error ϵ_{it}
    - Capture market-specific price variation through $\phi_t(\cdot)$

Production Function Estimation: Control Function Approach

- **Second Stage: Productivity Dynamics**
  - Assume Markov productivity process:

$$\omega_{it} = g(\omega_{it-1}) + \xi_{it}$$

  - Compute productivity for parameter guess β:

$$\omega_{it} = \hat{\phi}_{it} - \beta_V x_{it}^V - \beta_F x_{it}^F$$

  - Form moment conditions:

$$E\left[\xi_{it}(\beta)\begin{pmatrix} x_{it-s}^V \\ x_{it}^F \end{pmatrix}\right] = 0$$

  - Critical timing assumptions:
    - Variable inputs (x^V) react contemporaneously to productivity shock (ξ_{it})
    - Fixed inputs (x^F) follow dynamic accumulation:

$$K_{it} = (1-\delta)K_{it-1} + I_{it-1}$$

Production Function Estimation: Control Function Approach

Key Implementation Features

- **Input Differentiation**:
  - Variable inputs (x^V): Intra-period adjustment (e.g., labor)
  - Fixed inputs (x^F): Inter-temporal accumulation (e.g., capital)
- **Identification Strategy**:
  - Leverage differential timing of input adjustments
  - Use lagged flexible inputs as instruments
  - Exploit exogenous variation in fixed input accumulation
- **Computational Procedure**:
  1. Nonparametric estimation of $\phi_t(\cdot)$ (typically polynomial approximation)
  2. Nonlinear GMM estimation using moment conditions
  3. Bootstrap standard errors to account for generated regressors

Production Function Estimation: Imperfect Competition (B)

Core Challenges

- **Output-Input Comparability**:
  - Differentiated products create non-comparable physical units(B.2)
  - Revenue/expenditure data introduce price heterogeneity (Price Deflation Approaches)
  - Example: Luxury vs economy cars cannot be directly compared
- **Price Heterogeneity Bias**:
  - Unobserved price variations contaminate productivity measures (Demand System Integration)
  - High prices may reflect quality/market power rather than efficiency (Quality-Adjusted)
  - TFPR (Revenue-based TFP) vs TFPQ (Quantity-based TFP) distinction (Demand System Integration)

Price Deflation Approaches

Converts observed revenue to output data using additional price data

- **Industry-level Deflation**:

$$Q_{it} = \frac{R_{it}}{P_t} \quad \text{(Standard practice but insufficient)}$$

  - Removes common price trends from revenue data
  - Fails to address cross-producer price differences

- **Producer-level Deflation**:

$$Q_{it} = \frac{R_{it}}{P_{it}} \quad \text{(Requires product-specific price data)}$$

  - Enables quality-adjusted quantity comparison
  - Implementation challenges in multi-product firms

Demand System Integration

- **Motivation**:
  - Traditional approaches ignore price measurement error, leading to:
    - Downward bias in returns-to-scale estimates
    - Confounded demand shocks and productivity effects
- **Nested CES demand system (De Loecker, 2011)**:

$$Q_{it} = Q_t \left(\frac{P_{it}}{P_t} \right)^{\psi} \exp(\nu_{it})$$

  - ψ: Demand elasticity
  - ν_{it}: Idiosyncratic demand shock
- **Revenue production function with price correction:**:

$$r_{it} - p_t = \alpha_V x_{it}^V + \frac{1}{|\psi|} q_t + \tilde{\omega}_{it} + \tilde{\epsilon}_{it}$$

- Joint identification of: Production elasticity ($\alpha_V = (\frac{\psi+1}{\psi})\beta_V$)

Quality-Adjusted Production Functions (Product Differentiation)

- **Assumption**:
  - products where higher-quality version (whose producers can sell for higher prices) require higher-quality inputs (whose producers must pay more to obtain)
- **Vertical Differentiation Model**:

$$Q(\nu) = F(X^V(\nu))\Omega$$

  - ν: Product quality (inferred from prices)
  - Price-quality mapping: $\nu = v(p_{it})$
- **Empirical Implementation**:

$$q_{it} = \beta_V x_{it}^V + \omega_{it} + v(p_{it}) + \epsilon_{it}$$

- Identification via: uses lagged prices as instruments for quality shocks and price quality mapping $\nu = v(p_{it})$

Method Comparison

Table: 2 Method Comparison

Approach	Data Requirements	Key Limitation
Industry Deflation	Aggregate price indices	Ignores firm-level price variation
Demand System	Product-level quantities	Requires demand shifters
Quality Adjustment	Firm-specific prices	Assumes vertical differentiation

Estimation Methods: Multi-product Production (A.2/B.2)

Basic Framework: Given a multi-product producer, we start with the product-level production function:

$$Q_j = \left(X_j^V\right)^{\beta_V} \left(X_j^F\right)^{\beta_F} \Omega$$

where:

- Q_j: Output of product j
- X_j^V, X_j^F: Variable and fixed inputs allocated to product j
- β_V, β_F: Output elasticities of inputs

Under two critical assumptions:

1. **Common Productivity**: Productivity ω_{it} is identical across all products within a firm
2. **Neutral Input Allocation**: Inputs allocated proportionally to products via shares a_j:

$$X_j^H = a_j X^H \quad \forall H \in \{V, F\}$$

Aggregation Procedure

1. **Data Inputs**:
   - Product-level outputs (Q_j): Typically revenue data
   - Firm-level inputs (X^V, X^F): No product breakdown
2. **Allocation Rules**:

$$a_j = \begin{cases} \frac{1}{J} & \text{(Equal share, number of products)} \\ \frac{Q_j}{Q} & \text{(Output share, Revenue/Physical output)} \end{cases}$$

3. **Aggregation Formula** (CRS):

$$Q = \sum_j Q_j = \sum_j \alpha_j^{\beta_V + \beta_F} (X^V)^{\beta_V} (X^F)^{\beta_F} \Omega \quad \text{if } \beta_V + \beta_F = 1$$

Non-CRS Case: Requires additional terms (e.g., log number of products $\ln J$) and addresses endogeneity of product count J

Estimate Transformation Function (A.2)

Study Reference: Dhyne et al. (2014)

- Focus: Production in Belgian bakeries producing bread and pastries.
- Assumption: Both products (bread and pastries) are homogeneous.

$$q_1 = \gamma_1 q_2 + \beta_V^1 x^V + \beta_F^1 x^F + \omega_1$$
$$q_2 = \gamma_2 q_1 + \beta_V^2 x^V + \beta_F^2 x^F + \omega_2$$

Parameter	Meaning
γ_j	Inter-product interaction effect: $\gamma_j < 0$: Resource competition (e.g., $\gamma_1 < 0$ implies 1 output decreases with 2 production) $\gamma_j > 0$: Synergy effect (e.g., $\gamma_2 > 0$ suggests 2 efficiency gains from 1 production)
β_V^j	Variable input elasticity for product j
β_F^j	Fixed input elasticity for product j
ω_j	Product-specific productivity shock

Fundamental Issues

- **Multiple Unobservables**:
  - J productivity terms: $\omega_{i1t}, \omega_{i2t}, \ldots, \omega_{iJt}$
  - Traditional single-product models only handle ω_{it}
  - Total parameters must be estimated: J (products) $+ 2J$ (input elasticities) $= 3J$ *(Example: 6 parameters when $J = 2$)*
- **Endogeneity Problems**:
  - Simultaneity: $q_{-j,t}$ as endogenous regressor
  - High-dimensional instrumentation: Requires $(J - 1)$ valid instruments

Key Complexity

Model complexity grows exponentially with product count J

Control Function Extension

Data Description:

- Quantities produced for each product are reported.
- Total input use is reported, distinguishing between:
  - Variable factors of production.
  - Fixed factors of production.

Dhyne et al. (2014) Solution:

- Combines OP (investment) and LP (materials) proxies:

$$\begin{pmatrix} \omega_{i1t} \\ \omega_{i2t} \end{pmatrix} = h_t(k_{it}, m_{it}, i_{it})$$

- Requires bijective mapping $h_t(\cdot)$
- Limited to $J = 2$ case (bread & pastries)

This further highlights the additional restrictions required of the control function approach in this scenario.

Product Differentiation (B.2)

- **Product Differentiation Bias**:
  - Traditional deflation methods fail under quality heterogeneity
  - Unobserved input price variations confound elasticity estimates
  - Example: Premium flour inputs correlate with bread quality
- **Identification Complexity**:
  - Requires separating demand shocks from productivity effects
  - Needs product-level price/quality adjustments

Methodological Framework

- **Key Assumptions**:
  1. Product-specific production functions:

$$Q_1 = F_1(X_1)\Omega_1, \; Q_2 = F_2(X_2)\Omega_2$$

  2. Firm-level productivity (ω), no physical synergies

$$q_1 = \beta_{V_1} x_1^V + \beta_{F_1} x_1^F + \omega$$
$$q_2 = \beta_{V_2} x_2^V + \beta_{F_2} x_2^F + \omega$$

Scope Restriction

Excludes equipment sharing efficiencies but allows:

- Shared fixed costs:
- Bulk purchase discounts:

$$C(Q_1, Q_2) \leq C(Q_1) + C(Q_2)$$

Estimation Procedure

1. **Input Allocation**: Define allocation shares (ρ_j) based on expenditure:

$$\exp(\rho_j) = \frac{P_j^H X_j^H}{\sum_j P_j^H X_j^H}, \quad H \in \{V, F\}$$

   This ensures full input allocation:

$$\sum_j \exp(\rho_j) = 1$$

2. **System of Equations**: Solve the following system of equations:

$$\begin{cases} q_1 - \beta_{V_1} x^V - \beta_{F_1} x^F = \rho_1 + \omega \\ q_2 - \beta_{V_1} x^V - \beta_{F_2} x^F = \rho_2 + \omega \end{cases}$$

   Unknowns: ρ_1, ρ_2, ω.

Simplified CRS Illustration

- **Common Technology Assumption**:

$$\beta_V^1 = \beta_V^2, \quad \beta_F^1 = \beta_F^2$$

- **Input Allocation Rule**:

$$\exp(\rho_j) = \frac{Q_j}{\sum_j Q_j}$$

- **Productivity Metric**:

$$\Omega = \frac{\sum_j Q_j}{X}, \quad \ln X = \beta_V x^V + \beta_F x^F$$

Generalized Framework Features

- **Extended Capabilities**:
  - Product-specific technologies ($\beta_V^j \neq \beta_V^k$)
  - Variable returns to scale ($\beta_V^j + \beta_F^j \neq 1$)
  - Quality-adjusted differentiation ($Q_j = \nu_j F_j(X_j)$)
- **Key Requirements**:
  1. Single-product producer samples for baseline estimation
  2. Cost traceability to products
  3. Product-specific (not factor-specific) allocation

Critical Insight

Quality differentiation requires price-quality mapping:

$$\nu_j = v(p_j)$$

Deriving the Cost Function

Starting from the production function and solving a static cost minimization problem (equating the marginal rate of technical substitution to input price ratios), we derive the log-linear cost function:

$$\ln C_{it} = c_0 + \frac{1}{\beta_V + \beta_F} q_{it} + \frac{\beta_V}{\beta_V + \beta_F} p_{it}^V + \frac{\beta_F}{\beta_V + \beta_F} p_{it}^F + \frac{1}{\beta_V + \beta_F} \omega_{it} + \epsilon_{it}^*$$

where:

- C_{it}: Total cost of firm i at time t
- q_{it}: Output quantity
- p_{it}^V, p_{it}^F: Prices of variable and fixed inputs
- ω_{it}: Unobserved productivity shocks
- ϵ_{it}^*: Measurement error

Methodological Comparison

- **Data Requirements**:
  - Observability of factor prices (p_{it}^V, p_{it}^F)
  - Accurate measurement of economic costs (C_{it})

Table: 3 Cost Function vs. Production Function Approaches

Cost Function Approach	**Production Function Approach**
Requires cost data and input prices	Requires input quantities and output data
Easily handles scale/scope economies	Requires explicit modeling of multi-product production
Assumes cost minimization	Assumes exogenous productivity
Sensitive to input price measurement	Sensitive to input quantity measurement

Capital Measurement

- **Sources**:
  - Asset heterogeneity (varying equipment lifespans)
  - Depreciation rate misspecification
  - Unadjusted capital price fluctuations
- **Impact**:
  - Biased capital elasticity (β_F)
  - Distorted productivity estimates
- **Solutions**:
  - Instrumental Variables (lagged investment)
  - Nonparametric methods (Kim et al., 2016)

Specification Errors

- **Problem**: Omitting drivers (e.g., R&D A_{it}):

$$\omega_{it} = g(\omega_{it-1}) + \xi_{it} \quad \text{(Missing } A_{it-s}\text{)}$$

- **Solution**: Enhanced dynamic equation

$$\omega_{it} = g(\omega_{it-1}, A_{it-s}) + \xi_{it}$$

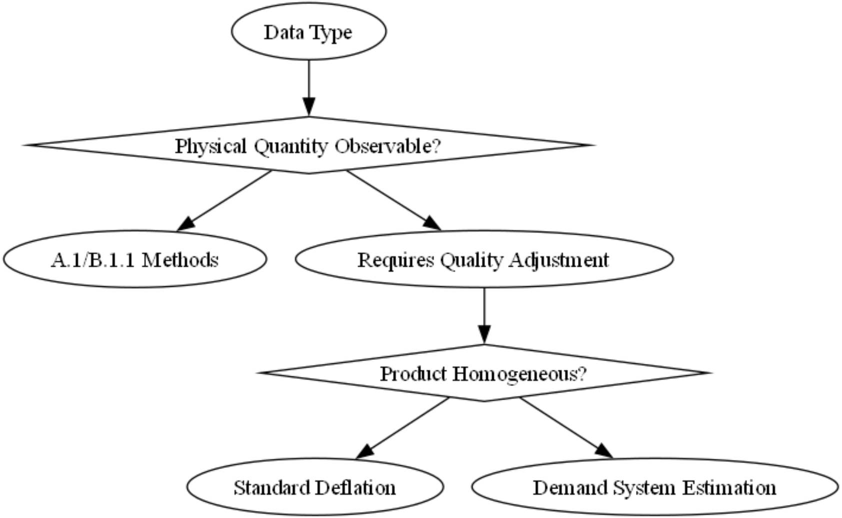
- **Cobb-Douglas Limitation**: Fixed substitution elasticity ($\sigma = 1$)
- **Alternatives**:
  - Translog: Flexible elasticities (De Loecker & Warzynski, 2012)
  - CES: Free substitution parameter
  - Nonparametric: Gandhi et al. (2020)

Empirical Implications & Solutions

Table: 4 Error Types & Remedies

Error Type	**Impact**	**Solution**
Capital Measurement	β_F attenuation	IVs, Nonparametric CF
Output Measurement	ω_{it} volatility	Two-stage CF, GMM
Process Misspecification	R&D effect bias	Enriched dynamics
Technological Heterogeneity	Diffusion misjudgment	Multi-technology models
Functional Form	Scale/elasticity bias	Translog/CES

Decision Tree Visualization



Thank You!